VIRAM SERIES

ENGINEERING MATHEMATICS-I

As per the Latest Syllabus of Diploma in Engineering Courses Under Jharkhand University of Technology, Ranchi

Syed Ali Hussain

KHANNA PUBLISHERS
Investing in Learning

A Textbook of

Engineering Mathematics-I

As per Syllabus of 1st Semester Diploma in Engineering Course under Jharkhand University of Technology, Ranchi

A Textbook of

Engineering Mathematics-I

As per Syllabus of 1st Semester Diploma in Engineering Course under Jharkhand University of Technology, Ranchi

Syed Ali Hussain

KHANNA PUBLISHERS®
Investing in Learning®
Operational Office :
4575/15, Onkar House, Opp. Happy School,
Ground Floor, Daryaganj, New Delhi 110 002
Phones : 011-45033819 • *Mob.* 09811541460
email : contactus@khannapublishers.in

Published by :

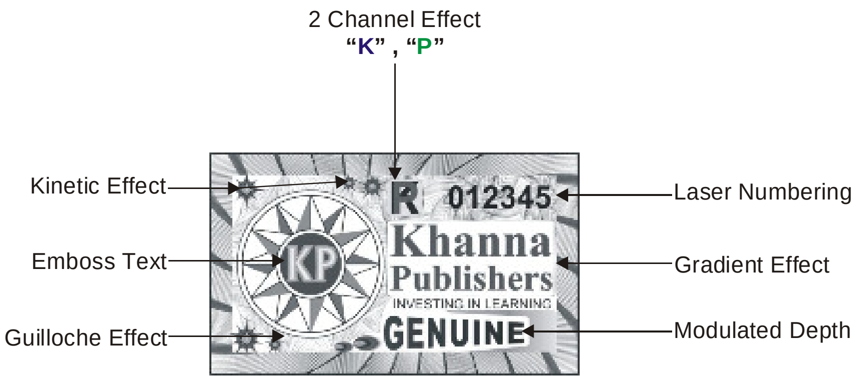
Romesh Chander Khanna & Vineet Khanna
for **KHANNA PUBLISHERS**
2-B, Nath Market, Nai Sarak
Delhi- 110 006 (India)

Website : www.khannapublishers.in

Hologram & Description

To all readers of our books, from yourself if being defrauded by pirates to prevent, please make sure that there is an Hologram on the cover of our books with the below specifications. If you find any book without Hologram and Description, please mail us at **contactus@khannapublishers.in**

Thanking you



ISBN No. 978-81-945380-5-9

***First Edition :* 2020**

Preface

This book presents the subject matter in full conformity with the syllabi prescribed by Jharkhand Technical University, Jharkhand. To keep pace with changing trends in technical education at state level, the whole text has been arranged strictly according to diploma engineering pattern.

The book provides a result oriented training to young students. The text has been studded with simple self-study questions, so as to provide an insight and proper grip over the topic, as one learns it. The article work in each chapter of a unit is coupled with well graded and carefully selected solved numerical problems for easy comprehension for the beginners. These numericals have been focussed on board pattern.

My special thanks to all my colleagues and friends for their creative suggestions.

In spite of all precautions and care taken to produce a clear and accurate book, some errors and misprints might have been left inadvertently. The author will welcome comments and corrections with gratitude.

— Syed Ali Hussain

Syllabus

Chapter No.	*Name of Topics*	*Hours*	*Marks*
	Algebra		
1	**1.1 Prerequisites Revision of** • Arithmetic, Geometric and Harmonic Progressions, • Formula of n^{th} term and sum to n-terms of A.P. and G.P. • Expression of Σn, Σn^2 and Σn^3. • Quadratic equations with real coefficients and relation between their roots and coefficient	01	01
	1.2 Logarithms • Definition of logarithm (Natural and Common logarithm.) • Laws of logarithm • Examples based on 1.2.1 to 1.2.2	03	04
	1.3 Partial Fraction • Definition of Polynomial Fraction Proper and Improper Fractions and definition of Partial fractions. • To resolve proper fraction into partial fraction with denominator containing non repeated linear factors, repeated linear factors and irreducible non repeated quadratic factors. • To resolve improper fraction into partial fraction.	03	06
	1.4 Determinant and Matrices **Determinant** 4 Marks • Definition and expansion of determinants of order 2 and 3. • Cramer's rule to solve simultaneous equations for 2 and 3 unknowns. **Matrices** 12 Marks • Definition of a matrix of order in $m \times n$ and types of matrices with examples. • Algebra of matrices such as equally, addition, subtraction, scalar multiplication and multiplication of two matrices. • Transpose of a matrix. • Minor, Cofactor of an element of a matrix, adjoint of matrix and Inverse of matrix by Adjoint method. • Solution of simultaneous equations containing 2 and 3 unknowns by matrix inversion method. • Idea of Rank of Matrix and their calculation.		
	1.5 Binomial Theorem • Definition of factorial notation, definition of permutation and combinations with formula (without proof). • Derivation of simple identities and solution based on it. • Binomial theorem for positive index. • General term, middle term and independent term and coefficient of x^n • Binomial theorem for negative index (only idea). • Approximate value (only formula).	02	04

2	**Trigonometry**		
	2.1 Revision • Measurement of an angle (degree and radian). Relation between degree and radian. • Trigonometrical ratios of 0°, 30°, 45°, 60°, 90° ± θ, 180° ± θ and 360° ± θ • Fundamental identities.	01	01
	2.2 Trigonometric Ratios of Allied, Compound, Multiple and Submultiple Angles Questions based on numerical computations.	03	06
	2.3 Transformation Formula of Product into Sums or Difference and vice versa, Simple Problems Based on it.	03	06
	2.4 Inverse trigonometric ratios • Definition of inverse trigonometric, ratios, principal values of inverse trigonometric rations.		
	2.5 Properties of Triangle Sine, Cosine, Projection and tangent rules (without proof). Simple problems.	02	04
3	**Coordinate Distances**		
	3.1 Point and Distances • Distance formula, section formula, midpoint, centroid of triangle. • Area of triangle and condition of collinearity	02	04
	3.2 Straight Line • Slope and intercept of straight line. • Equation of straight line in slope point form, slope-intercept form, two-point form, two-intercept form, normal form. General equation of line. • Angle between two straight lines condition of parallel and perpendicular lines. • Intersection of two lines. • Length of perpendicular from a point on the lines and perpendicular distance between parallel lines.	05	10
	3.3 Circle • Equation of circle in standard form, centre–radius formula and diameter formula. • General equation of circle, its centre and radius, simple problem	02	04
4	**Vector Algebra**		
	4.1 Vectors • Definition of vector, position vector, algebra of vectors (equality, addition, subtraction and scalar multiplication) • Dot (Scalar) product with properties. • Vector (Cross) product with properties.	03	03
	4.4 Applications • Workdone and moment of force/s about a point and line	02	04
	Total	**42**	**80**

Contents

Part-I: Algebra

1

CHAPTER

Prerequisites Revision of Arithmetic, Geometric and Harmonic Progressions

1.1 INTRODUCTION

The notions of sets, sequences and series play an important role in mathematics. In this chapter we shall study sequences and series in brief.

Objectives:

After going through this chapter students should be able to:

(*i*) Recognise the difference between sequence and series;

(*ii*) Know an arithmetic progression;

(*iii*) Find the n^{th} term of an arithmetic progression;

(*iv*) Expression of Σn^2 and Σn^3;

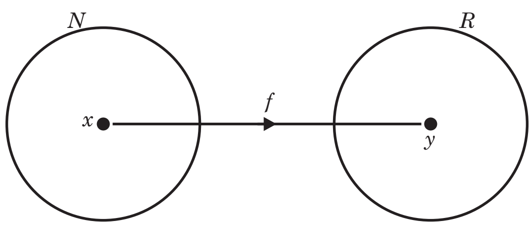
(*v*) Find the sum of a geometries series;

(*vi*) Find the sum of arithmetic series;

(*vii*) Find quadratic equations with real co-efficients:

1.2 SEQUENCE

A sequence is a special class of function whose domain is the set of natural no. N and the range is any sets *i.e.*, a function $f : N \to S$ is called a sequence. If the range of a sequence is any subset to real numbers, then the sequence is called a real sequence. Therefore a real sequence is a function from the set of natural no. N to the set R of real numbers.



A sequence is a rule that assigns to each natural number unique real number *i.e.*, if f is a real sequence, then $f : N \to R$.

The range of the sequence is the set of values $\{f(1), f(2), f(3), \ldots f(n), \ldots\} \subset R$.

For the sake of simplicity and clarity we denote f_n, $\forall n \in N$ and the sequence is written as $< f_n >$ or $\{f_n\}_{n\in N}$

i.e., $$\{f_n\}_{n\in N} = \{f_1, f_2, f_3, ..., f_n, ...\}$$

or $<u_n>$ or $\{u_n\}_{n\in N}$

$$<u_n> = < U_1, U_2, U_3, ... U_n, ...>.$$

Where $f_1, f_2, f_3, ... f_n, ...$ are real nos. are called 1st, second, third, n^{th} element of the sequence.

1.2.1 Different Ways of Describing a Sequence

1. A sequence may be described by listing its few elements.
   e.g. $< 1, 2, 3, 4, ..., n ... >$ is the sequence whose n^{th} element is n.
2. Another way of representing member of the sequence is to specify the rule for its n^{th} element e.g. The sequence $< 1, \frac{1}{2}, \frac{1}{3}, ... >$ can be $< f_n >$, when $f_n = \frac{1}{n}$, $\forall n\in N$.

   Here $f_n = \frac{1}{n}$ gives the rule for forming the n^{th} element of the sequence.

3. A sequence can be described by specifying the first element and stating a rule for determining f_{n+1}, $\forall n \geq 1$ in terms of the elements $f_1, f_2, f_3, ...$
   Ex. The sequence $< f_n >$ for which $f_n = 1, f = 2$, and

   $$f_n = \frac{1}{n}(f_{n-1} + f_{n-2}),\ \forall n \geq 2.$$

   is $< 1, 2, \frac{3}{2}, \frac{7}{4}, ... >$.

1.2.2 Constant Sequence

The sequence $< f_n >$ where $f_n = C \in R\ \forall n \in N$. is called a constant sequence.

In this case $< f_n > = < c, c, c, ... >$

$\therefore$ range of $f = \{c\}$ = Singleton Set = a that is finite set.

Note:

(*a*) It is necessary that all elements of the sequence should be distinct.

(*b*) Range of sequence and the sequence itself are not the same.
Sequence $<f_n>$, where

$$f_n = (-1)^{n-1},\ \forall n\in N \text{ is given by}$$
$$<f_n> = <1, -1, 1, -1,....>$$

Here range of $f = \{1, -1\}$.

(*c*) A sequence is always an infinite set where as range of the sequence need not be infinite.

(*d*) An alternative definition of a sequence as a succession of terms arranged in a definite order and formed according to a definite rule. *i.e.*, the set
$<f_1, f_2, f_3, ...>$ is called a sequence
Where f_1, f_2, f_3 ... are real numbers arranged in definite pattern, and formed according to a definite rule or law.

Thus a special function $f : N \rightarrow S$ is called a sequence whose elements are formed according to some definite rule.

<1, 9, 25, 49, ...> is a sequence and the rule here is that

$$f_n = (2n - 1)^2, \forall n \in N.$$

$<f_n>$ where $f_n = \frac{1}{2^n}, \forall n \in N.$

i.e., $< \frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3} ... >$. is a sequence.

1.3 SERIES

Let $<u_n>$ be a real sequence then expression of the form $U_1 + U_2 + U_3 + ... + U_n$ is called a series and is denoted by $\sum_{n=1}^{n} u_n$ or simply Σu_n. The real nos. U_1, U_2, U_3, ... are various terms of the Series.

Thus, we see that a series is associated with a sequence. Again if a sequence is finite the series associated, with it is called a finite series, otherwise the series is infinite.

(*i*) 2 + 4 + 6 + 8 + ...

(*ii*) 16 + 8 + 4 + 2 + ...

(*iii*) 1/3 + 1/5 + 1/7 + ...

1.4 ARITHMETICAL PROGRESSION

Some important definition and illustrations.

1. **Set:** A set is a well defined collection of distinct and distinguishable objects, called members or elements of the set.

   Ex. (*i*) $V = \{a, e, i, o, u\}$. finite

   (*ii*) Z = {0, ±1, ±2, ±3, ...}. Infinite

   A set is said to be finite if it contains a finite number of elements and infinite, if it contains an infinite number of elements.

2. **Sequence:** A set whose members are formed according to a definite rule is called a sequence. The members are called the term of the sequence.

   Ex. (*i*) Set of odd nos.

   i.e., < 1, 3, 5, 7, ...>

   Here $a_n = 2n - 1 \ \forall n \in N$.

3. **Series:** A series is obtained by adding the terms of the sequence.

   < 2, 4, 6, ...> a sequence.

   2 + 4 + 6 + ... a series.

   A series is finite or infinite according as the number of terms in the series is finite or infinite.

4. **Arithmetical progression:**

   Definition: An arithmetical progression or A.P. is a sequence of nos. in which each term after the first is formed from the preceding one by adding to it a constant quantity. The constant quantity is called the common difference and is usually denoted by the symbol '*d*'.

   If *a, b, c* be in A.P. Then,

   $$b - a = c - b = d.$$

Example of arithmetical progression.

(*i*) 3, 6, 9, 12, ... (*ii*) 1, 3, 5, 7, ...

(*iii*) $a, a + d, a + 2d, \ldots$ etc

1.4.1 General Term of an A.P.

Let a be the first term and d be the common difference. Then by definition,

$$a_2 = a + d = a + (2 - 1) \cdot d$$
$$a_3 = a + 2d = a + (3 - 1) \cdot d$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$a_n = a + (n - 1) \cdot d$$

i.e., $$\boxed{a_n = a + (n - 1) \cdot d}$$

The n^{th} term is called the general term of an A.P. If an A.P has only n terms then the n^{th} term is the last term l and is given by,

$$\boxed{l = a_n = a + (n - 1) \cdot d}$$

1.4.2 Properties of A.P.

1. If every term of an A.P. be increased or decreased by the same quantity, the resulting term will also be in A.P. whose $c.d.$ will be the same as that of the original A.P.
2. If each term of an A.P. be multiplied or divided by the same quantity, the resulting terms will also be in A.P.
3. In an A.P. the sum of any two terms equidistant from the beginning and end is constant.

   $a, a + d, a + 2d, \ldots l.$

   r^{th} term from the beginning

   $$= a + (r - 1) \cdot d$$

   r^{th} term from the end

   $$= l + (r - 1) \cdot (-d)$$

   ∴ There sum

   $$= a + (r - 1) \cdot d + l + (r - 1) \cdot (-d)$$
   $$= a + l = \text{Which is independent of } r.$$
   $$= a \text{ constant}$$

   Hence Sum of any two terms equidistant from the beginning and end is constant.
4. If n^{th} term of a sequence be a linear expression in n, the sequence must be an A.P.

   i.e., $$\boxed{a_n = a \cdot n + b}$$

   $$t_n = a_n + b$$
   $$t_{n-1} = a(n - 1) + b$$
   $$t_n - t_{n-1} = an + b - a(n - 1) - b$$
   $$= a = a \text{ constant.}$$
5. If the sum of a n terms of a sequence be a quadratic expression, in n, the sequence must be an A.P.

   Let $$S_n = an^2 + bn + c$$
   $$S_{n-1} = a(n - 1)^2 + b(n - 1) + c$$

So,
$$\begin{aligned} t_n &= S_n - S_{n-1} \\ &= \{an^2 + bn + c\} - \{a(n-1)^2 + b(n-1) + c\}. \\ &= \{an^2 + bn + c\} - \{a(n^2 - 2n + 1) + bn - b + c\} \\ &= an^2 + bn + c - an^2 + 2an - a - bn + b - c \\ &= 2an - a + b. \\ &= 2an - (a + b) \text{ which is a linear expression in } n. \end{aligned}$$

Hence by property (4) the sequence must be an A.P.

WORKED OUT EXAMPLES

Example 1.1. *Write the first 5 terms of the sequence defined by* $a_n = (-1)^{n-1} \cdot 2^n \ \forall n \in N$.

Solution: Given:

$$a_n = (-1)^{n-1}, 2^n \quad \text{where, } n = 1, 2, 3, 4, 5$$

We get,

$$\begin{aligned} a_1 &= (-1)^{1-1} \cdot 2^1 = 2. \\ a_2 &= (-1)^{2-1} \cdot 2^2 = -4 \\ a_3 &= (-1)^{3-1} \cdot 2^3 = 8 \\ a_4 &= (-1)^{4-1} \cdot 2^4 = -16 \\ a_5 &= (-1)^{5-1} \cdot 2^5 = 32. \end{aligned}$$

Example 1.2. *Write three terms in each of the sequence defined by the following:*

(i) $a_n = \dfrac{n(n-2)}{2}$ *(ii)* $a_n = \dfrac{2n-3}{6}$

(iii) $a_n = n^2 - n + 1$ *(iv)* $a_n = 2n^2 - 3n + 1$

Solution:

(*i*) Given:

$$\begin{aligned} a_n &= \frac{n(n-2)}{2}; \ n = 1, 2, 3 \\ a_1 &= \frac{1 \cdot (1-2)}{2} = -\frac{1}{2} \\ a_2 &= \frac{2 \cdot (2-2)}{2} = 0 \\ a_3 &= \frac{3 \cdot (3-2)}{2} = \frac{3}{2} \end{aligned}$$

(*ii*) Given:

$$\begin{aligned} a_n &= \frac{2n-3}{6}; \quad n = 1, 2, 3 \\ a_1 &= \frac{2 \cdot 1 - 3}{6} = -\frac{1}{6} \\ a_2 &= \frac{2 \cdot 2 - 3}{6} = \frac{1}{6} \\ a_3 &= \frac{2 \cdot 3 - 3}{6} = \frac{1}{2} \end{aligned}$$

(*iii*) Given:

$$a_n = n^2 - n + 1 \qquad n = 1, 2, 3.$$
$$a_1 = 1^2 - 1 + 1 = 1$$
$$a_2 = 2^2 - 2 + 1 = 3$$
$$a_3 = 3^2 - 3 + 1 = 7.$$

(*iv*) Given:

$$a_n = 2n^2 - 3n + 1; \quad n = 1, 2, 3.$$
$$a_1 = 2 \cdot 1^2 - 3 \cdot 1 + 1 = 0$$
$$a_2 = 2 \cdot 2^2 - 3 \cdot 2 + 1 = 3$$
$$a_3 = 2 \cdot 3^2 - 3 \cdot 3 + 1 = 10.$$

Example 1.3. *What is the 18^{th} term of the sequence defined by*

$$a_n = \frac{n(n-3)}{n+4}.$$

Solution: Given:

$$a_n = \frac{n(n-3)}{n+4}.$$

Here $n = 18$

$$a_{18} = \frac{18(18-3)}{18+4} = \frac{18 \times 15}{22} = \frac{135}{11}$$

Example 1.4. *Find the indicated terms in each of the following sequences:*

(*i*) $a_n = n(n-1)(n-2)$; a_5 and a_8. (*ii*) $a_n = 5n - 4$; a_{12} and a_{15}

Solution:

(*i*) Given:

$$a_n = n \cdot (n-1) \cdot (n-2)$$

Here $n = 5$ and 8

$\therefore$ $a_5 = 5 \cdot (5-1) \cdot (5-2) = 5 \cdot 4 \cdot 3 = 60$

$$a_8 = 8(8-1) \cdot (8-2) = 8 \cdot 7 \cdot 6 = 336$$

(*ii*) Given:

$$a_n = 5n - 4.$$

Here, $n = 12$ and 15

$$a_{12} = 5 \times 12 - 4 = 56$$
$$a_{15} = 5 \times 15 - 4 = 71$$

Example 1.5. *Let a sequence be defined by $a_1 = 3$; $a_n = 3 \cdot a_{n-1} + 1$ for all $n > 1$. Find the first four terms of the sequence.*

Solution: Here $a_1 = 3$

and $a_n = 3 \cdot a_{n-1} + 1 \quad n > 1.$

Put $n = 2, 3, 4$

$$a_2 = 3 \cdot a_{2-1} + 1 = 3 \cdot a_1 + 1 = 3 \cdot 3 + 1 = 10$$
$$a_3 = 3 \cdot a_{3-1} + 1 = 3 \cdot a_2 + 1 = 3 \cdot 10 + 1 = 31.$$
$$a_4 = 3 \cdot a_{4-1} + 1 = 3 \cdot a_3 - 1 = 3 \cdot 31 - 1 = 90$$

Practice Problem–01

1. Write the first 5-terms in each of the following sequence:
   (*i*) $a_n = (-1)^n.2^n$ **Ans.** $a_1 = -2;\quad a_2 = 4\quad a_3 = -8;\quad a_4 = 16; a_5 = -32.$
   (*ii*) $a_n = 2^n - 2.$ **Ans.** $a_1 = 0;\quad a_2 = 2;\quad a_3 = 6\quad a_4 = 14; a_5 = 30.$
2. Find the indicated terms in each of the following sequences:
   (*i*) $a_n = (-1)^n \cdot (2n-1);\quad a_{13}$ and a_{17}. **Ans.** $a_{13} = -25; a_{17} = -33.$
   (*ii*) $a_n = \dfrac{3n-2}{4n+5},\ a_7$ and a_8. **Ans.** $a_7 = \dfrac{19}{33}; a_8 = \dfrac{22}{37}$
3. Let a sequence be defined by $a_1 = 1; a_2 = 1$ and $a_n = a_{n-1} + a_{n-2}$ for all $a_n > 2$.
   Find $\dfrac{a_{n+1}}{a_n}$ for $n = 1, 2, 3, 4$. **Ans.** $\dfrac{a_2}{a_1} = 1;\ \dfrac{a_3}{a_2} = 2;\ \dfrac{a_4}{a_3} = \dfrac{3}{2};\ \dfrac{a_5}{a_4} = \dfrac{5}{3}.$

Worked Out Examples Based on Arithmetical Progression

Example 1.6. *Show that the progression 16, 11, 6, 1, –4... is an A.P. Write down its first-term and common difference.*

Solution: 16, 11, 6, 1, –4, ...

$$11 - 16 = -5$$
$$6 - 11 = -5$$

i.e., difference between any two consecutive terms is always –5.

∴ The Sequence is in A.P.

$$1^{st}\text{ term} = 16$$
$$c \cdot d = -5.$$

Example 1.7. *Which term of the A.P 5, 9, 13, 17... is 81?*

Solution: Given: 5, 9, 13, 17, ... in A.P.

$a = 5;\quad d = 9 - 5 = 4.$

Let the r^{th} term be 81.

Then,
$$t_r = a + (r-1) \cdot d$$
$$81 = 5 + (r-1) \cdot 4 = 1 + 4r$$
$$4r = 80$$
$$r = 20.$$

Hence, 20^{th} term is 81.

Example 1.8. *Which term of the A.P. 3, 15, 27, 39, ... will be 120 more than its 21st term?*

Solution:
$$a = 3; d = 12.$$
∴
$$21^{st}\text{ term} = 3 + 20 \times 12 = 243$$
$$\text{Required term} = 243 + 120$$
$$T_r = 363$$
$$363 = 3 + (r-1) \cdot 12 = 12r - 9$$
$$12r = 363 + 9 = 372$$
$$r = \frac{372}{12} = 31$$

Hence, 31^{st} term is the required term.

Practice Problems–02

1. Find the value of x for which $(5x + 2)$, $(4x – 1)$ and $(x + 2)$ are in A.P. **Ans.** $x = 3$
2. The 4th and 10th terms of an A.P. are 13 and 25 respectively. Find the first term and the common difference of the A.P. Also find its 17th term. **Ans.** $a = 17$, $d = 2$ $T_{17} = 39$.
3. The 7th term of an A.P is –4 and its 13th term is –16. Find the A.P. **Ans.** 8, 6, 4, 2, 0...
4. Which term of the A.P. 5, 15, 25, ... will be 130 more than its 31st term. **Ans.** 44th
5. If the pth term of an A.P. q and its q^{th} term is p then show that its $(p + q)^{th}$ term is zero.
6. For what values of n, the n^{th} terms of the arithmetic progression 63, 65, 67, ... and 3, 10, 17, ...are equal? **Ans.** $n = 13$.
7. How many 3-digit nos. are divisible by 7? **Ans.** 128
8. Find the 8th term from the end of the 7, 10, 13, ... 184. **Ans.** 135
9. Find the 11th term from the last term toward the first term of the A.P. 10, 7, 4, ..., (–62). **Ans.** – 32
10. The 4th term of an A.P. is zero. Prove that its 25th term is triple its 11th term.

1.4.3 Arithmetic Mean

Definition: If a, A, b are in A.P., then A is the arithmetic mean between a and b and is denoted as A.M.

To find the arithmetic mean between two nos. a and b.

Let a and b be any two given nos.

Let A be the A.M. between a and b. a, A, b are in A.P.

$$A - a = b - A$$

$$2A = a + b$$

$$\therefore \quad A = \frac{a+b}{2}$$

i.e.,

$$\boxed{A = \frac{a+b}{2}}$$

To insert 'n' arithmetic mean between two given nos. a and b.

Let a and b be two given nos. and $A_1, A_2, A_3, \ldots A_n$, be '$n$' arithmetic means.

Then, clearly $a, A_1, A_2, A_3, \ldots A_n, b$ are in A.P.

No. of terms = $n + 2$.

Last term = b = $(n + 2)^{th}$ term

$$\therefore \quad t_{n+2} = a + (n + 1) \cdot d.$$

$$b = a + (n + 1) \cdot d$$

$$d = \frac{b-a}{n+1}$$

$$\therefore \quad t_2 = a + d = A_1$$

$$A_1 = a + \frac{b-a}{n+1} \quad \frac{an+b}{n+1}$$

$$t_3 = A_2 = a + 2d$$

$$= a + 2 \cdot \frac{(b-a)}{n+1} = \frac{a(n-1)+2b}{n+1}$$

$$t_4 = A_3 = a + 3d$$

$$= a + 3 \cdot \frac{(b-a)}{x+1} = \frac{a.(n-2)+3b}{n+1} \text{ and so on.}$$

$$A_n = a + (n-1)\frac{(b-a)}{n-1} = \frac{a+(n-1).b}{n+1}$$

An important result.

It is always a convenient to make a choice of.

1. 3 nos. in AP; $a - d, a, a + d$.
2. 4 nos. in AP; $a - 3d, a - d, a + d, a + 3d$.
3. 5 nos. in AP; $(a - 2d), (a - d), a, (a + d) (a + 2d)$.

1.4.4 Sum of n-terms of an A.P.

To find the sum of n terms of an A.P.

$$S_n = \frac{n}{2}(a+l) \quad \text{and} \quad S_n = \frac{n}{2}\{2a+(n-1)d\}$$

Proof: Let

$$S_n = a + (a + d) + (a + 2d) + (a + 3d) + ... + (a + (n - 1) \cdot d) = l \qquad ...(i)$$

Again re-writing the series in reverse order.

$$S_n = l + (l - d) + (l - 2d), ..., a \qquad ...(ii)$$

Adding (i) and (ii); We get,

$S_n = (a + l) + (a + l) + (a + l) + ...$ up to n terms.

$$= n \cdot (a + l)$$

$$\boxed{S_n = \frac{n}{2}[a+l]}$$

But $l = a + (n - 1) \cdot d$

$$\therefore \qquad S_n = \frac{n}{2}\ [a + a + (n - 1) \cdot d]$$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

i.e.,
$$S_n = \frac{n}{2}[2a+(n-1)d]$$

1.4.5 Summation of Series

1. Σn
2. Σn^2
3. Σn^3.

1. Sum of the 1st 'n' natural number.

   Let $S = 1 + 2 + 3 + ... + n$.

   The terms of the above series are in. A.P.

   Here, $a = 1; d = 1; l = n$

   No. of terms $= n$.

$$S = \frac{n}{2}(1+n)$$

$$\boxed{S = \frac{n(n+1)}{2}}$$

2. Sum of 1^{st} n odd natural nos.

Let $S = 1 + 3 + 5 + \ldots$ upto n-terms.

$a = 1;\ d = 2\ ;\ l = 2n - 1.$

No. of terms $= n$

$$S = \frac{n}{2}(1+2n-1)$$

$$\boxed{S = n^2}$$

3. Sum of 1^{st} n even natural nos.

Let $S = 2 + 4 + 6 + 8 + \ldots$ up to n terms

Here $a = 2;\ l = 2n$

No. of terms $= n$

$$S = \frac{n}{2}(2+2n)$$

$$\boxed{S = n(n+1)}$$

4. Sum of squares of 1^{st} n natural nos.

Let $S = 1^2 + 2^2 + 3^2 + \ldots + n^2.$

We have

$$r^3 - (r-1)^3 = 3r^2 - 3r + 1. \qquad \ldots(i)$$

Putting $r = 1, 2, 3, \ldots n.$

$$1^3 - 0^3 = 3 \cdot 1^2 - 3 \cdot 1 + 1$$
$$2^3 - 1^3 = 3 \cdot 2^2 - 3 \cdot 2 + 1$$
$$3^3 - 2^3 = 3 \cdot 3^2 - 3 \cdot 3 + 1$$

....

....

$$(n-1)^3 - (n-2)^3 = 3(n-1)^2 - 3(n-1) + 1$$
$$n^3 - (n-1)^3 = 3n^2 - 3n + 1$$

By adding

$$n^3 = 3 \cdot (1^2 + 2^2 + 3^2 + \ldots + n^2) - 3\,(1 + 2 + 3 + \ldots n) + (1 + 1 + 1 \ldots \text{ up to } n \text{ terms})$$

$$= 3 \cdot S - 3 \cdot \frac{n \cdot (n+1)}{2} + n$$

$$3 \cdot S = n^3 + \frac{3 \cdot n(n+1)}{2} - n = n^3 - n + \frac{3}{2} n \cdot (n+1)$$

$$= n \cdot (n-1)(n+1) + \frac{3}{2} n(n+1) = n \cdot (n+1) \cdot \left(n - 1 + \frac{3}{2}\right)$$

$$= \frac{n(n+1)(2n+1)}{2}$$

$$\therefore \qquad S = \frac{n(n+1)\cdot(2n+1)}{6}$$

5. Sum of the cubes of the first n natural nos.

Let $\qquad S = 1^3 + 2^3 + 3^3 + ... + n^3.$

We have,

$$r^4 - (r-1)^4 = 4r^3 - 6r^2 + 4r - 1 \qquad ...(i)$$

Putting $\qquad r = 1, 2, 3, ... n.$

$$1^4 - 0^4 = 4\cdot 1^3 - 6\cdot 1^2 + 4\cdot 1 - 1,$$
$$2^4 - 1^4 = 4\cdot 2^3 - 6\cdot 2^2 + 4\cdot 2 - 1$$
$$3^4 - 2^4 = 4\cdot 3^3 - 6\cdot 3^2 + 4\cdot 3 - 1$$

....

....

$$(n-1)^4 - (n-2)^4 = 4\cdot(n-1)^3 - 6\cdot(n-1)^2 + 4\cdot(n-1) - 1$$
$$n^4 - (n-1)^4 = 4n^3 - 6n^2 + 4\cdot n - 1.$$

By adding

$$n^4 = 4\cdot S - 6\cdot(1^2 + 2^2 + 3^2 + ... + n^2) + 4(1 + 2 + 3 + ... n) - (1 + 1 + 1 + ... \text{ up to } n \text{ terms})$$
$$= 4\cdot S - \frac{6n(n+1)(2n+1)}{6} + \frac{4n(n+1)}{2} - n$$
$$4\cdot S = n^4 + n(n+1)(2n+1) - 2n(n+1) + n.$$
$$= n^4 + n + n(n+1)(2n+1) - 2n(n+1).$$
$$= n(n+1)(n^2 - n + 1) + n(n+1)(2n+1) - 2n(n+1).$$
$$= n(n+1)\{n^2 - n + 1 + 2n + 1 - 2\}$$
$$= n(n+1).\ n(n+1)$$

$$\therefore \qquad S = \left\{\frac{n(n+1)}{2}\right\}^2$$

1.4.6 The Σ–notation

The sum of a series can be conveniently expressed by writing the Greek letter Σ, pronounced as Sigma, just before the general term of the Series.

1. $\Sigma n = \dfrac{n(n+1)}{2}$
2. $\Sigma(2n-1) = n^2$
3. $\Sigma(2n) = n(n+1)$
4. $\Sigma n^2 = \dfrac{n(n+1)(2n+1)}{6}$
5. $\Sigma n^3 = \left\{\dfrac{n(n+1)}{2}\right\}^2$

1.5 THE GEOMETRIC PROGRESSION

A sequence of no., whose first term is different from zero and each term beginning with the second, is equal to its predecessor multiplied by one and the same no. is called a geometric progression.

If a sequence (a_n) is a geometric, then by definition,

$$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \ldots = \frac{a_n}{a_{n-1}} = \ldots$$

That is the ratio of any term to its predecessor is equal to one and the same number. This no. is called the common ratio of a geometric progression and is denoted by r.

Ex. 3, 6, 12, 24, ...

4, 8, 16, 32, ... and so on.

Thus, a geometric progression (a_n) is defined by the following conditions.

(*i*) $a_1 = a$, where a is called the first term ($a \neq 0$).

(*ii*) $a_{n+1} = a_n \cdot r$ for any $n \geq 1$.

If a is the 1st term and r the *c.r.* then G.P can be written, as $a, ar, ar^2, ar^3, \ldots, ar^{n-1}, \ldots$

1.5.1 General Term of G.P.

Let (a_n) be a geometric progression and r the *c.r.* of the progression.

By definition of the geometric progression.

$$a_2 = ar = a \cdot r^{2-1}.$$
$$a_3 = ar^2 = a \cdot r^{3-1}.$$
$$a_4 = ar^3 = a \cdot r^{4-1}.$$

....

....

$$a_n = a \cdot r^{n-1}.$$

If n be the no. of term, and if l denotes the last term or the n^{th} term we have

We have

$$l = a_n = a \cdot r^{n-1}.$$

1.5.2 Sum of 'n' Terms of a Geometric Progression

Let the Sum of 'n' terms of a G.P. be S_n.

Let a be the 1st term, r the common ratio, n the no. of terms, and S the sum required.

Then,

$$S_n = a + ar + ar^2 + ar^3 + \ldots + ar^{n-2} + ar^{n-1}.$$

Case 1. If c.r. $(r) = 1$. ...(*i*)

Then, $S_n = a_1 \cdot n$

Case 2. $r \neq 1$.

Multiplying on both sides by r.

$$r \cdot S_n = ar + ar^2 + ar^3 + \ldots + ar^{n-1} + ar^n. \quad \ldots(ii)$$

By (*ii*) and (*i*)

$$r \cdot S_n - S_n = ar^n - a.$$
$$(r-1) = a(r^n - 1)$$

$$\therefore \quad S_n = \frac{a(r^n - 1)}{r - 1} \quad \ldots(iii)$$

If $r < 1$

Then

$$S_n = \frac{a(1-r^n)}{1-r} \qquad \ldots(iv)$$

Since, $a \cdot r^{n-1} = l$ then from (*iii*)

$$S_n = \frac{rl-a}{r-1}$$

Note:

(*i*) If a, b, c are in G.P.

$$b^2 = a \cdot c$$

(*ii*) The common difference of a G.P.

$$= \left(\frac{l}{a}\right)^{1/n-1}$$

Where, a = first term of G.P.
l = last term of G.P.
n = No. of terms of G.P.

(*iii*) Three nos. in G.P. can be taken as $\frac{a}{r}, a, ar$.

five terms of G.P. can be written as $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$.

In general $(2m+1)$ nos. in G.P can be written as

$\frac{a}{r^m}, \frac{a}{r^{m-1}}, \frac{a}{r^{m-2}}, \ldots \frac{a}{r}, a, ar, \ldots ar^{m-1}, ar^m$.

(*iv*) Four nos. in G.P. can be taken as $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$; Six nos. in G.P can be taken as $\frac{a}{r^5}, \frac{a}{r^3}, \frac{a}{r}\ an, an^3, an^5$.

In general, $(2m)$ nos. in G.P. can be written as $(m \in N)$

$\frac{a}{r^{2m-1}}, \frac{a}{r^{2m-3}}, \ldots \frac{a}{r^3}, \frac{a}{r}, ar, ar^3, \ldots\ ar^{2m-3}, ar^{2m-1}$.

1.5.3 Sum of Infinite G.P.

Let $S = a + ar + ar^2 + ar^3 + \ldots + ar^{n-1} + \ldots$ up to ∞

$$S_n = \frac{a\cdot(1-r^n)}{1-r}, |r|<1.$$

as $n \to \infty$

$$S_\infty = \lim_{n\to\infty} \frac{a.(1-r^n)}{1-r} = \frac{a}{1-r}.$$

i.e.,

$$\boxed{S_\infty = \frac{a}{1-r}}$$

1.6 HARMONIC PROGRESSION

A sequence is said to H.P. if the reciprocals of its terms are in A.P. If the sequence a_1, a_2, a_3, ..., a_n is an H.P. then

$$\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \dots, \frac{1}{a_n} \text{ is an A.P.}$$

1.6.1 General Term of H.P.

$$a_n = \frac{1}{a+(n-1).d}$$

In case we have to assume a certain no. of terms which are in H.P. then suggested choice is as follows:

(*i*) For three terms $\frac{1}{a-d}, \frac{1}{a}, \frac{1}{a+d}$

(*ii*) For four terms, $\frac{1}{a-3d}, \frac{1}{a-d}, \frac{1}{a+d}, \frac{1}{a+3d}$

Note:

(*i*) There is no general formula for finding out the sum n term of H.P.

(*ii*) No term of H.P. can be zero.

(*iii*) For H.P. whose first term is a and second term is b, then n^{th} term is

$$t_n = \frac{ab}{b+(n-1).(a-b)}$$

(*iv*) If a, b, c are in H.P.

$$\Rightarrow \qquad b = \frac{2ac}{a+c} \text{ or } \frac{a}{c} = \frac{a-b}{b-c}.$$

(*v*) Questions in H.P. are generally solved by inverting the terms, and making use of the properties of the corresponding A.P.

1.7 RECOGNISATION OF A.P., G.P., H.P.

Let a, b, c be three successive terms of a sequence.

(*i*) if $\frac{a-b}{b-c} = \frac{a}{a}$, then a, b, c are in A.P.

(*ii*) if $\frac{a-b}{b-c} = \frac{a}{b}$, then a, b, c are in G.P.

(*iii*) if $\frac{a-b}{b-c} = \frac{a}{c}$, then a, b, c are in H.P.

Alternatively

(*i*) a, b, c are in A.P. iff $b = \frac{a+c}{2}$.

(*ii*) a, b, c are in G. P. iff and if, $b^2 = ac$.

(*iii*) a, b, c are in H.P. iff and if, $b = \frac{2ac}{a+c}$

In general we can use the following.

If $t_n - t_{n-1}$ = constant for all $n \geq 2$. Then the sequence is in A.P.

If $\dfrac{t_n}{t_{n-1}}$ a constant, for all $n \geq 2$. Then

The sequence is in G.P.

If $\dfrac{1}{t_n} - \dfrac{1}{t_{n-1}} = a$ constant, for all $n \geq 2$.

Then the sequence is in H.P.

1.8 GEOMETRIC MEAN

When three positive quantities are in G.P. the middle one is said to be the geometric mean of the other two. Thus a, b, c are in G.P. Then

$$b = \sqrt{ac}.$$

or if a, G, b be in G.P.

then
$$G = \sqrt{ab}.$$

The geometric mean of a set of any n positive nos. $a_1, a_2, a_3, \ldots a_n$ is

$$G = (a_1 \cdot a_2 \cdot a_3 \ldots a_n)^{1/n}$$

To insert a given number of geometric means between two given positive quantiles.

Let a and b be the given positive quantities, n the no. of means.

Let $G_1, G_2, G_3, \ldots G_n$ be the n G.M.s between a and b.

Then, $a, G_1, G_2, G_3, \ldots, G_n, b$ will be in G.P.

In all there will be $n + 2$ terms; So that we have to find a series of $n + 2$ terms in G.P. of which a is the first and b the last term.

Let 'r' be the $c.r$.

Then, b = the $(n + 2)^{\text{th}}$ term $= a \cdot r^{n+2-1} = ar^{n+1}$

$$r^{n+1} = \frac{b}{a}$$

$$\therefore \quad r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$G_1 = a \cdot r;\ G_2 = ar^2, \ldots G_n = ar^n.$$

$$G_1 = a \cdot \left(\frac{b}{a}\right)^{\frac{1}{n+1}}; \quad G_2 = a \cdot \left(\frac{b}{a}\right)^{\frac{2}{n+1}};$$

....

....

$$G_n = a \cdot \left(\frac{b}{a}\right)^{\frac{n}{n+1}}.$$

1.9 HARMONIC MEAN

When three quantities are in harmonic progression the middle one is said to be the harmonic mean of the other two. If a, b, c are in H.P., b is the H.M. between a and c.

$$\boxed{b = H.M = \frac{2ac}{a+c}}.$$

To find the harmonic mean between two given quantities.

Let a and b be the two quantities and H the harmonic mean.

Since a, H, b are in H.P. we must have

$$\frac{2}{H} = \frac{1}{a} + \frac{1}{b}$$

$$H = \frac{2ab}{a+b}.$$

The harmonic mean of a set of any n nos. $a_1, a_2, a_3, a_4, \ldots a_n$ is

$$H = \left[\left\{\frac{\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \ldots + \frac{1}{a_n}}{n}\right\}\right]^{-1}$$

or,
$$\frac{1}{H} = \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \ldots. + \frac{1}{a_n}}$$

To insert a given no. of harmonic means between two given quantities

Let a and b be two given nos. and $H_1, H_2, H_3, \ldots, H_n$ are H.M.s between them.

Then $a, H_1, H_2, H_3, \ldots, H_n, b$ will be in H.P.

If 'd' is the $c.d.$ of the corresponding A.P.

$$\therefore \quad b = (n + 2)^{\text{th}} \text{ term of H.P.}$$

$$= \frac{1}{(n+2)^{\text{th}} \text{ term of corresponding A.P.}}$$

$$b = \frac{1}{\frac{1}{a} + (n+2-1)d}$$

$$\frac{1}{b} = \frac{1}{a} + (n+1)d$$

$$d = \frac{a-b}{(n+1)ab}$$

$$\therefore \quad \frac{1}{H_1} = \frac{1}{a} + d, \frac{1}{H_2} = \frac{1}{a} + 2d, \ldots$$

$$\frac{1}{H_n} = \frac{1}{a} + nd.$$

$$\frac{1}{H_1} = \frac{1}{a} + \frac{(a-b)}{(n+1)ab}$$

$$\frac{1}{H_2} = \frac{1}{a} + 2 \cdot \frac{(a-b)}{(n+1)ab}$$

$$\frac{1}{H_3} = \frac{1}{a} + 3 \cdot \frac{(a-b)}{(n+1)ab}$$

....

....

$$\frac{1}{H_n} = \frac{1}{a} + n \cdot \frac{(a-b)}{(n+1) \cdot ab}$$

1.10 QUADRATIC EQUATION

An equation that contains a variable with an exponent of 2, but no higher power, is called a quadratic equation. A quadratic equation in x is an equation that can be written in the standard form.

$$ax^2 + bx + c = 0;$$

Where a, b and c are real nos. with $a \neq 0$.

A quadratic equation in x is also called a second degree polynomial equation in x.

Solving Quadratic equations by factoring:

Factoring is the process of writing a polynomial as the product of two or more polynomials.

Ex.
$$x^2 - 3x + 2 = 0$$
$$(x-1)(x-2) = 0.$$

1.10.1 The Quadratic Formula

Let the standard quadratic equation

$$ax^2 + bx + c = 0 \quad \text{where } a, b, c \text{ are real nos. } a \neq 0.$$

Dividing each term by a, we get,

$$x^2 + \frac{b}{a}.x + \frac{c}{a} = 0.$$

$$\Rightarrow \quad x^2 + 2.x\frac{b}{2a} + \frac{b^2}{4a^2} + \frac{c}{a} - \frac{b^2}{4a^2} = 0.$$

$$\Rightarrow \quad \left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2} = 0$$

$$\Rightarrow \quad \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

1.10.2 Discriminant

The expression under radical in the quadratic formula $b^2 - 4ac = D$ is discriminant. It determines the nature of two zeroes, depending upon whether $D > 0$; $D = 0$ or $D < 0$.

If $D = 0$ both the roots reduces to $\frac{-b}{2a}$, and thus equal to one another also both the roots are equal. When $D = 0$ the expression $ax^2 + bx + c$ is a perfect square of a linear expression in x.

A quadratic equation cannot have more than two roots.

For, if possible, Let the quadratic $ax^2 + bx + c = 0$ have three different roots α, β and γ. Then since each of the three values must satisfy the equation, we have

$$a\alpha^2 + b\alpha + c = 0 \quad \ldots(i)$$

$$a \cdot \beta^2 + b \cdot \beta + c = 0 \quad \ldots(ii)$$

and $$a \cdot \gamma^2 + b \cdot \gamma + c = 0 \quad \ldots(iii)$$

By (*i*) – (*ii*)

$$a(\alpha^2 - \beta^2) + b(\alpha - \beta) = 0$$

$$(\alpha - \beta) \{a(\alpha + \beta) + b\} = 0$$

Since $(\alpha - \beta) \neq 0$

$$\therefore \quad a(\alpha + \beta) + b = 0 \quad \ldots(iv)$$

Similarly by (*ii*) and (*iii*)

$$a(\beta^2 - \gamma^2) + b(\beta - \gamma) = 0$$

$$(\beta - \gamma) \{a(\beta + \gamma) + b\} = 0$$

But $\beta - \gamma \neq 0$

$$a(\beta + \gamma) + b = 0 \quad \ldots(v)$$

By (*iv*) – (*v*)

$$a(\alpha - \gamma) = 0$$

Which is impossible since 'a' is not zero.

Sum and product of roots.

It α and β are the roots of the quadratic equation.

$$ax^2 + bx + c = 0, \quad \text{then}$$

(*i*) $\alpha + \beta = -b/a$

(*ii*) $\alpha \cdot \beta = c/a$.

Formation of quadratic equation

If α and β be the two roots of the quadratic equation. Then, we can easily determines the equation whose roots are α and β.

x^2 – (Sum of roots)x + Product of roots = 0

i.e., $$x^2 - (\alpha + \beta)x + \alpha\beta = 0.$$

1.10.3 Nature of Roots

Nature of roots of a quadratic equation $ax^2 + bx + c = 0$ implies whether the roots are real or imaginary. By analyzing the expression $b^2 - 4ac$, called the discriminant D, we get an idea about the nature of roots.

Consider the quadratic equation $ax^2 + bx + c = 0$, where $a, b, c \in R$ and $a \neq 0$ then.

(*i*) $D > 0 \Leftrightarrow$ roots are real and distinct

(*ii*) $D = 0 \Leftrightarrow$ roots are real and coincident (equal)

(*iii*) $D < 0 \Leftrightarrow$ roots are imaginary.

If $p + iq$ is one root of a quadratic equation with real co-efficients then the other roots must be its conjugates $p - iq$ and *vice-versa*. $p, q \in R$ and $\sqrt{-1} = i$.

1.10.4 Rational Roots

Consider the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$,

Then

(*i*) if $-\dfrac{b}{a} \in Q, \dfrac{c}{a} \in Q$ and

$D = b^2 - 4ac$ is a perfect square of a rational no. Then both roots are rational.

(*ii*) If $\dfrac{-b}{a} \in Q, \dfrac{c}{a} \in Q$ and

$D = b^2 - 4ac > 0$ is not a square of a rational no. Thus both roots are irrational.

If $p + \sqrt{q}$ be one root then the other roots must be its conjugate $p - \sqrt{q}$ and vice-versa.

(*iii*) If $-\dfrac{b}{a} = 1$ and $\dfrac{c}{a} = 1.$ and

$\dfrac{b^2 - 4ac}{4a^2}$ is perfect square of an integer, then both the roots are integers when $a = 1$

$b, c \in I$ and $b^2 - 4ac$ is a perfect square both roots are integer.

1.10.5 Common Roots of Quadratic Equations

Consider the quadratic equation

$$ax^2 + bx + c = 0 \quad \ldots(i)$$

and

$$a_1x^2 + b_1x + c_1 = 0 \quad \ldots(ii)$$

may have two common roots.

The above equation have both roots common if they are identical. They are identical if and only if the co-efficient of similar powers of x in the two equations are proportional.

i.e.,

$$\boxed{\frac{a}{a_1} = \frac{b}{b_1} = \frac{c}{c_1}}$$

To find the condition that the equations $ax^2 + bx + c = 0$ may have at least one root common.

Let $x = \alpha$ be one common root.

$$a\alpha^2 + b\alpha + c = 0$$

and

$$a_1\alpha^2 + b_1\alpha + c_1 = 0.$$

By cross multiplication.

$$\frac{\alpha^2}{ca_1 - b_1c} = \frac{\alpha}{ca_1 - c_1a} = \frac{1}{ab_1 - a_1b}$$

To eliminate α, square the second of these equal ratios and equate it to the product of the other two, thus

$$\frac{\alpha^2}{(ca_1 - c_1a)^2} = \frac{\alpha^2}{(bc_1 - b_1c)} \times \frac{1}{ab_1 - a_1b}$$

$$\therefore \quad (ca_1 - c_1a)^2 = (bc_1 - b_1c)(ab_1 - a_1b).$$

Which is the condition required.

ENGINEERING MATHEMATICS-I

As per the Latest Syllabus of Diploma in Engineering Courses Under
Jharkhand University of Technology, Ranchi

About the Book

This book presents the subject matter in full conformity with the syllabi prescribed by Jharkhand Technical University, Jharkhand. To keep pace with changing trends in technical education at state level, the whole text has been arranged strictly according to diploma engineering pattern.

The book provides a result oriented training to young students. The text has been studded with simple self-study questions, so as to provide an insight and proper grip over the topic, as one learns it. The article work in each chapter of a unit is coupled with well graded and carefully selected solved numerical problems for easy comprehension for beginners. These numericals have been focussed on board pattern.

Some unique features of this book are :

- Detailed theory with worked examples
- Systematic approach
- Large number of graded solved examples
- Large number of unsolved practice problems.

About the Author

Syed Ali Hussain is currently working as Senior Lecturer at Al-Kabir Polytechnic in the department of Engineering Mathematics. He is first class in B.Sc. (Hons) and M.Sc. in Mathematics from Ranchi University Jamshedpur, Jharkhand.

KHANNA PUBLISHERS®
ISO 9001:2015
4575/15, Onkar House, Opp. Happy School,
Ground Floor, Daryaganj, New Delhi-110002
Phones: 011-45033819, 9811541460
E-mail: contactus@khannapublishers.in

Website:
www.khannapublishers.in

9788194538059