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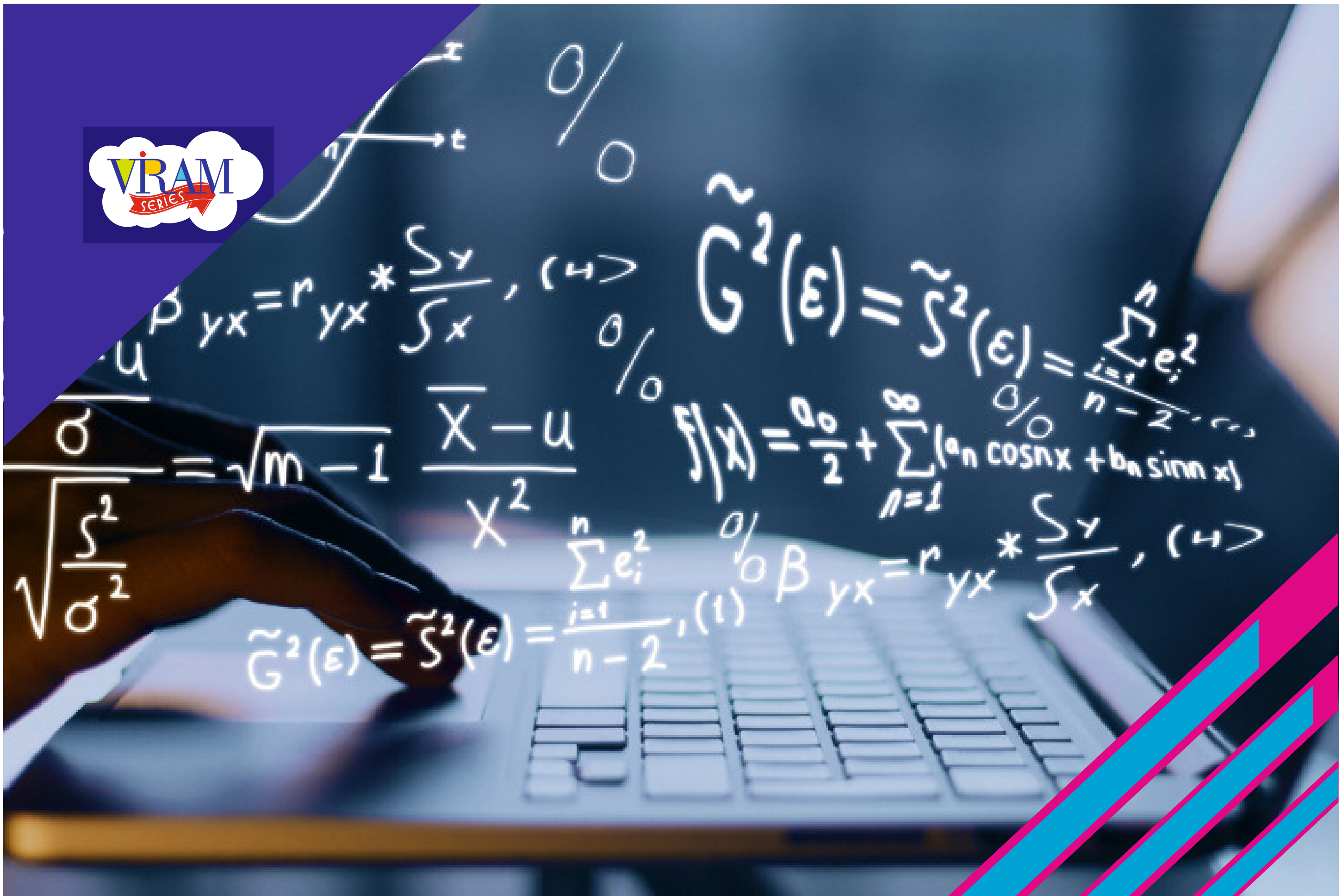

A Textbook of ENGINEERING MATHEMATICS-II

As per the Latest Syllabus of Diploma in Engineering Courses Under Jharkhand University of Technology, Ranchi

Syed Ali Hussain

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Engineering Mathematics-II

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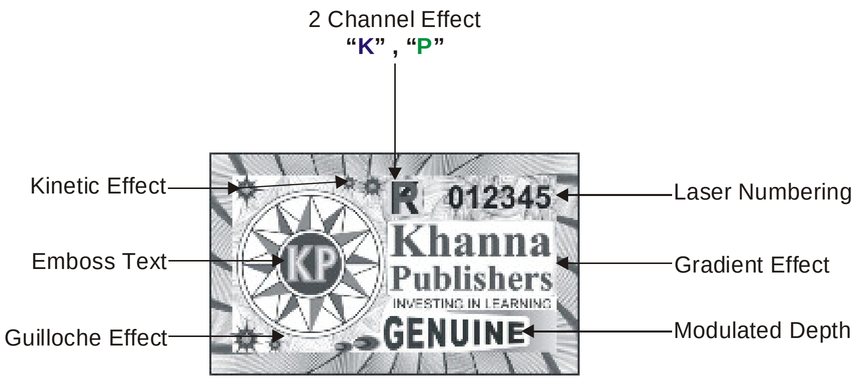

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Preface

This book presents the subject matter in full conformity with the syllabi prescribed by Jharkhand Technical University, Jharkhand. To keep pace with changing trends in technical education at state level, the whole text has been arranged strictly according to diploma engineering pattern.

The book provides a result oriented training to young students. The text has been studded with simple self-study questions, so as to provide an insight and proper grip over the topic, as one learns it. The article work in each chapter of a unit is coupled with well graded and carefully selected solved numerical problems for easy comprehension for the beginners. These numericals have been focussed on board pattern.

My special thanks to all my colleagues and friends for their creative suggestions.

In spite of all precautions and care taken to produce a clear and accurate book, some errors and misprints might have been left inadvertently. The author will welcome comments and corrections with gratitude.

— Syed Ali Hussain

Contents

1

UNIT

Function, Limit and Continuity

DEFINITION OF VARIABLE

A variable is a symbol which takes a number of values. Thus the symbol which denotes any arbitrary element of a set S is said to be a variable over the set S. Variables are usually denoted by the symbol x, y, u, v, etc.

Independent Variable

That symbol which can take an arbitrary value from a given set is called an independent variable.

Dependent Variable

That symbol whose value depends on independent variable is called a dependent variable.

Constant

A constant is a symbol which remains the same value throughout a set of operation. Thus the symbol which denotes a particular number is a constant. Constant are usually denoted by the symbol a, b, c, p, q, etc.

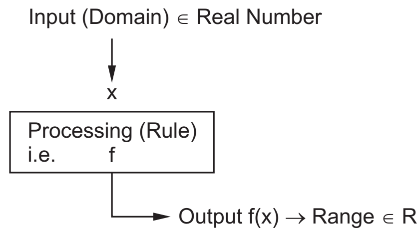
Intervals

Intervals are certain sets of real numbers. They appear with great frequency in calculus. They can be grouped into nine categories.

S. No.	***Name***	***Notation***	***Description***
1.	Open intervals	(a, b) or $]a, b[$	Set of all x such that $a < x < b$.
2.	Closed intervals	$[a, b]$	Set of all x such that $a \le x \le b$.
3.	Left open and right closed	$(a, b]$	Set of all x such that $a < x \le b$.
4.	Right open and left closed	$[a, b)$	Set of all x such that $a \le x < b$.
5.	Open interval	$[a, \infty)$	Set of all x such that $a < x$.
6.	Open interval	$(-\infty, a]$	Set of all x such that $x < a$.
7.	Closed interval	$[a, \infty)$	Set of all x such that $a \le x$.
8.	Closed interval	$(-\infty, a]$	Set of all x such that $x \le a$.
9.	The real line	$(-\infty, \infty)$	Set of real number

FUNCTION

A function is just like a machine that takes the members of the domain as input and applies the rule to each to produce the members of the range as output



Function as a Special Kind of Relation

Definition:

1. Let A and B be two non-empty sets. A relation f from A to B, *i.e.*, a sub-set of $A \times B$ is called a function (or a mapping or a rule) from A to B, if

   (i) for each $a \in A$, there exists $b \in B$ such that $(a, b) \in f$.

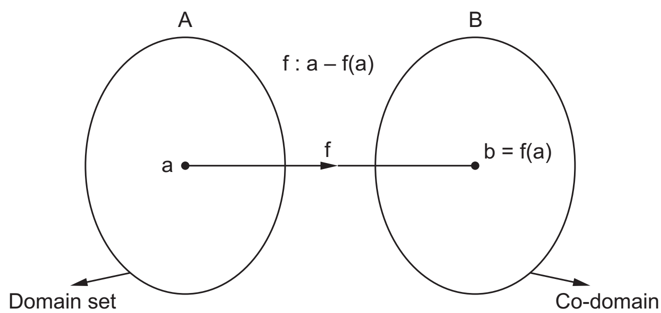
   (ii) $(a, b) \in f$ and $(a, c) \in f \quad \Rightarrow \quad b = c$.

   i.e., $f : A \to B$ or $b = f(a) \;\forall\; a \in A$.

2. If 'f' is a function, the domain of f is the set of all a for which there is some b such that (a, b) is in f. If a is in domain of f, it follows from the definitions of a function that there is a unique number b such that (a, b) is in f.

   i.e., $b = f(a)$

3. A function f in $A \times B$ is a relation in $A \times B$ such that each element a in A is a member of almost one ordered pair, (a, b) in f where b belongs to B.



Function as a Correspondence

Let A and B be two non-empty sets. Then a function 'f' from a set A to a set B is a rule or correspondence which associates each and every elements of set A to elements of B such that

(i) all elements of set A are associated to elements in set B.

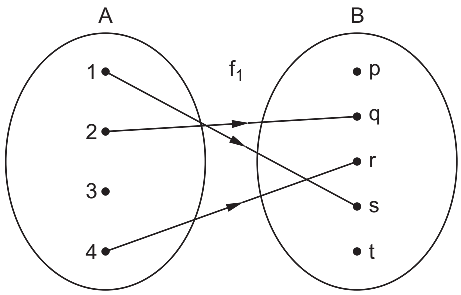
(ii) an elements of set A is associated to a unique element in set B.

i.e., in any ordered pair, first component cannot be repeated.

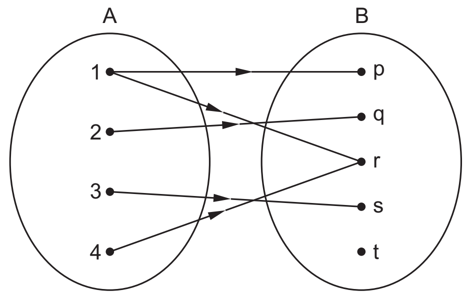
We write $f : A \to B$

Which is read as 'f' is a function from A to B or f maps A to B.

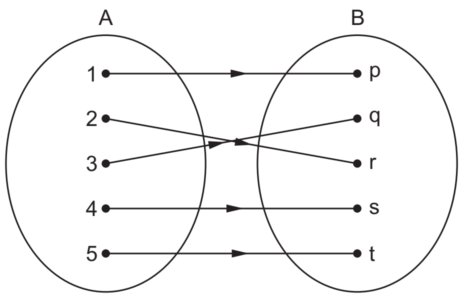
If $a \in A$ is associated to an element $b \in B$, then b is called 'the f-image of a'. Also, a is called the pre-image of b under the function f.



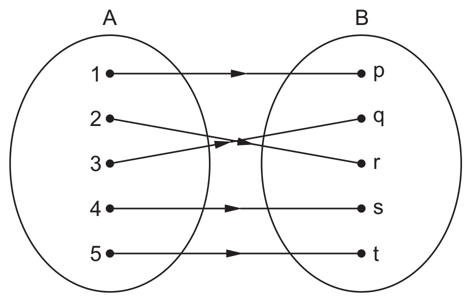
Not a function



Not a function



Is a function



Is a function

Domain, Co-Domain and Range of a Functions

If $$f: A \rightarrow B.$$

Then the set A is called the domain set and the set B is called as the co-domain of f. The set of images of all the elements of A under the mapping f is called the range of f and is denoted as $f(A) \subseteq B$.

Real Valued Function of a Real Variable

If the domain and range of f are a subset of set of real number, *i.e.*, R is called a real valued function of a real variable.

Some Important Real Functions

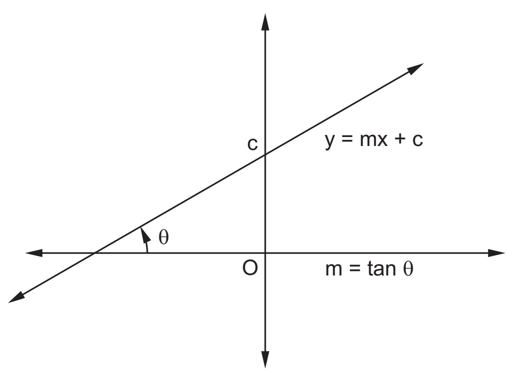
1. Constant function: A function $f : A \rightarrow B$ is a constant function if range of f is the singleton set.
Clearly, domain of $f = R$ and range of $f = \{c\}$.

2. Linear function: Linear function is a polynomial of degree-1.

$$f(x) = ax + b.$$

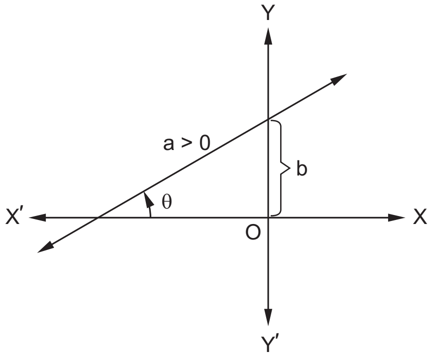
It is also expressed as

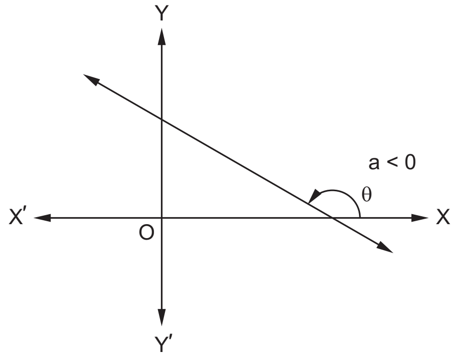
$$f(x) = mx + c.$$



The graph of a linear function is a straight line. The co-efficient of x is a slope or gradient of straight line and 'c' is the intercept made by Y-axis. Its domain and range is the set of real numbers *i.e.*, R.

The graph of $y = ax + b$ when $b > 0$ and 'a' changes its sign.



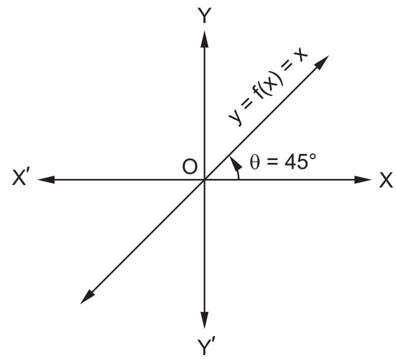


Identity Function

The dependent variable y and independent variable x have same value *i.e.*, domain and range are same. It is a linear function in which $m = 1$ *i.e.*, slope = 1 and $c = 0$. *i.e.*, it passes through the origin. An identity function is represented as

$$f: A \to A.$$

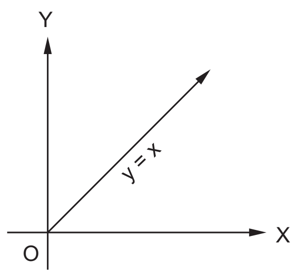
It is a straight line passing through origin and bisecting the 1st co-ordinate system or 1st quadrant.



Example: $f: R \to R$

$f(x) = \ln e^x = x$, is an identity function for $x \in R$.

However, $y = f(x) = e^{\ln x} = x$, is an identity function for $x \in R^+$.



Quadratic Function

The second degree polynomial function $f(x) = ax^2 + bx + c; a \neq 0$ is called a quadratic function. The domain of the function is $x \in R$. The graph of the function is a parabola with vertical axis.

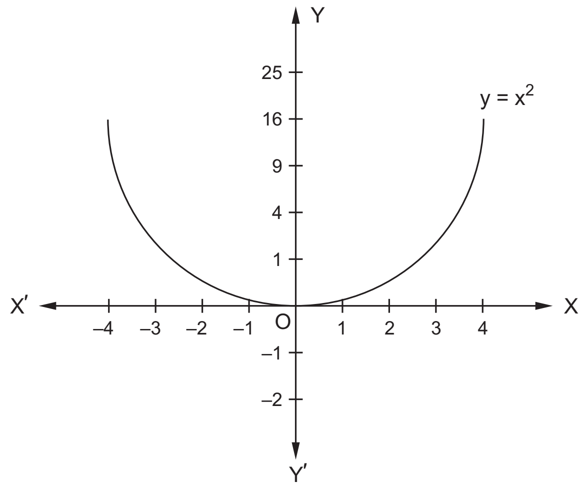
In $$y = ax^2 + bx + c$$

$$\Rightarrow \quad y = a\left\{x^2 + \frac{b}{a}x + \frac{c}{a}\right\} = a\left\{x^2 + 2 \cdot x \cdot \frac{b}{2a} + \frac{b^2}{4a^2}\right\} - \left\{\frac{b^2 - 4ac}{4a}\right\}$$

$$= a\left(x + \frac{b}{2a}\right)^2 - \frac{D}{4a}$$

where, $D = b^2 - 4ac$ = Discriminant.

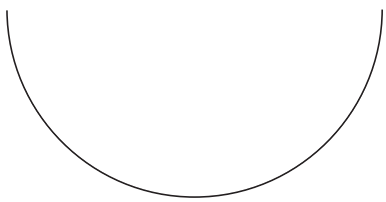
The graph of $y = ax^2 + bx + c$ is a parabola with axis $x = -b/2a$ and vertex $(-b/2a, -D/4a)$, which opens upward if $a > 0$ and downward if $a < 0$.



$y = x^2$. The graph of the quadratic function is symmetric about Y-axis.

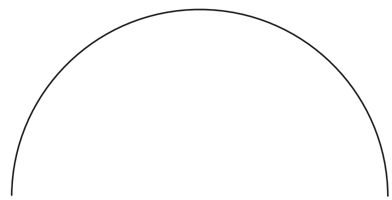
The point of intersection of the parabola with its axis of symmetry is called the vertex of the parabola. The vertex of $y = x^2$ is the origin (0, 0). It increases on either side, $(0, \infty)$ and $(-\infty, 0)$.

Concavity



f(x) = ax^2 + bx + c if a > 0

Concave upward



f(x) = – ax^2 + bx + c

Concave downward

TYPES OF FUNCTION

Algebraic Function

An algebraic function is one which is obtained by performing with the variable and known constants any finite number of operations of addition, subtraction, multiplication, division and extraction of integral roots. Polynomial, rational and irrational functions are algebraic.

The function $y = f(x)$ is an algebraic function of x, if y is the root of an equation of the n^{th} degree in y whose co-efficients are polynomial function of x.

Example: $y = x^2 - 3x + 2$

$$y = \frac{x^2 - 2 \cdot x^{3/2}}{x - 1}$$ and so on are algebraic functions.

Transcendental Functions

All functions which are not algebraic like trigonometric, inverse trigonometric exponential and logarithmic functions are called "transcendental functions".

Example:

$$y = \sin x - \cos x.$$
$$y = \sin^{-1} x + 2$$
$$y = a^x - 1$$
$$y = \log_e (ax + b) \text{ etc.}$$

Piecewise Defined Function

These functions are defined by different formulae in different parts of their domains like, modulus, greatest integer, least integer, fraction parts functions etc.

Example: A function f is defined by

$$f(x) = \begin{cases} 1 - x & \text{if } x \leq 1 \\ x^2 & \text{if } x > 1 \end{cases}$$

Find $f(0)$; $f(1)$ and $f(2)$

$$f(0) = 1 - 0 = 1$$
$$f(1) = 1 - 1 = 0$$
$$f(2) = 2^2 = 4.$$

Polynomial Function

A function of the form $f(x) = a_0 \cdot x^n + a_1 \cdot x^{n-1} + a_2 \cdot x^{n-2} + \ldots + a_{n-1} \cdot x + a_n$ is called a polynomial function in x. If $a \neq 0$, then we say that the polynomial is of degree n. All a's $\in R$ are co-efficients of polynomial. The following notations are used: $P(x)$, $Q(x)$, $R(x)$, $p(x)$, $q(x)$, $r(x)$.

Example: $P(x) = x^4 - 3x^3 + x^2 + 3x - 2.$

Rational Function

Rational function is defined as the ratio of two real polynomial with the condition that polynomial in denominator is not zero polynomial.

$$f(x) = \frac{p(x)}{q(x)}, q(x) \neq 0.$$

Example: $f(x) = \dfrac{2x^2 - x + 1}{2x^2 - 5x - 3}, x \neq -\dfrac{1}{2}, x \neq 3.$

$$p(x) = \frac{2x^4 - x^2 + 1}{x - 1}, x \neq -1.$$

Irrational Function

1. The term radical is the name given to n^{th} root sign, $\sqrt[n]{x}$. A radical number is n^{th} root of a real number. If y is n^{th} root of x *i.e.*, $x = y^n$.

   or $y = x^{1/n}$ or $y = \sqrt[n]{x}$ where, $n \in N \geq 2$.

   For $n = 2$, $y = \sqrt{x}$ *i.e.*, square root of x.

   Irrational functions are those which contain atleast one fractional power of x.

   Note: If n is an even integer, then x cannot be negative.

2. If the roots all are even *i.e.*, $\sqrt{\ }$; $\sqrt[4]{\ }$; $\sqrt[6]{\ }$ ... and so on, then it is defined for non-negative real values of the radicand *i.e.*, if the radicand is negative *i.e.*, $x < 0$ the root is

imaginary, if the radicand is zero, then the root is also zero. If the radicand is positive *i.e.*, $x > 0$, then the root is also positive.

3. If 'n' is odd *i.e.*, $\sqrt[3]{n}$; $\sqrt[5]{n}$; $\sqrt[7]{n}$ and so on, then it is defined for all values of the radicand. If the radicand is negative then, the root is negative, if the radicand is zero, the root is zero, and if the radicand is positive, then the root is positive.

Modulus Function

The modulus function returns a non-negative value of a variable. That is why this function is also referred as "absolute value function".

The modulus of a real number x is defined as

$$|x| = \begin{cases} x & \text{when } x \geq 0 \\ -x & \text{when } x < 0 \end{cases}$$

Example:

$$|5| = 5; \left|\frac{\pi}{2}\right| = \frac{\pi}{2}$$

$$\left|\frac{2}{3}\right| = \frac{2}{3}; \; |-2| = -(-2) = 2.$$

Also,

$$|f(x)| = \begin{cases} f(x) & \text{when } f(x) \geq 0 \\ -f(x) & \text{when } f(x) < 0 \end{cases}$$

Example:

$$f(x) = |2x - 1|$$

$$f(x) = 2x - 1, \text{ if } 2x - 1 \geq 0 = -(2x - 1), \text{ if } 2x - 1 < 0$$

This gives,

$$f(x) = \begin{cases} 2x - 1 & \text{if } x \leq \frac{1}{2} \\ 1 - 2x & \text{if } x > \frac{1}{2} \end{cases}$$

Example 1. *Prove that:* $\sqrt{x + 2\sqrt{x-1}} + \sqrt{x - 2\sqrt{x-1}}$ *is equal to 2, if* $1 \leq x \leq 2$, *and to* $2\sqrt{x-1}$, *if* $x > 2$.

Solution:

$$\sqrt{x + 2\sqrt{x-1}} + \sqrt{x - 2\sqrt{x-1}} = \sqrt{(x-1) + 2\sqrt{x-1} + 1} + \sqrt{(x-1) - 2\sqrt{x-1} + 1}$$

$$= |\sqrt{x-1} + 1| + |\sqrt{x-1} - 1|$$

$$= \sqrt{x-1} + 1 + |\sqrt{x-1} - 1|$$

$$= \begin{cases} 2\sqrt{x-1} & \text{if } \sqrt{x-1} > 1 \\ 2 & \text{if } \sqrt{x-1} \leq 2 \end{cases}$$

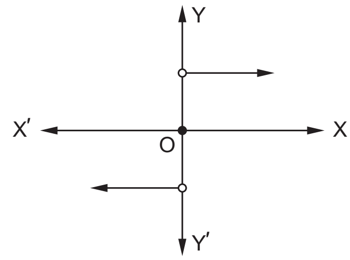
$$= \begin{cases} 2\sqrt{x-1} & \text{if } x > 2 \\ 2 & \text{if } 1 \leq x \leq 2 \end{cases}$$

Example 2. *Simplifying the function* $y = x + |x - 1| - 3$.

Solution:

$$y = \begin{cases} x + (x-1) - 3 & \text{if } x - 1 \geq 0 \\ x - (x-1) - 3 & \text{if } x - 1 < 0 \end{cases} = \begin{cases} 2x - 4 & \text{if } x \geq 1 \\ -2 & \text{if } x < 0 \end{cases}$$

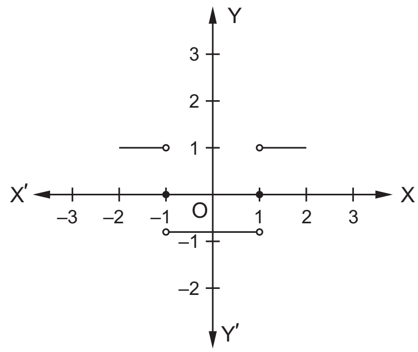
Signum Function



The signum function, denoted as $y = \text{sgn}(x)$, is defined as

$$y = \text{sgn}(x) = \begin{cases} \dfrac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases} = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$$

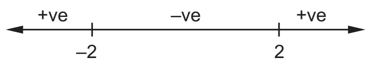
Example: Simplify $y = \text{sgn}(x^2 - 1)$.



$$y = \begin{cases} -1 & \text{if } x^2 - 1 < 0 \\ 0 & \text{if } x^2 - 1 = 0 \\ 1 & \text{if } x^2 - 1 > 0 \end{cases}$$

$$= \begin{cases} -1 & \text{if } -1 < x < 1 \\ 0 & \text{if } \quad x = \pm 1 \\ 1 & \text{if } \quad x > -1 \text{ or } x > 1 \end{cases}$$

Example 3. *Solve the equation:* $x^2 - 5x\ \text{sgn}(x^2 - 4) + 6 = 0$.
Solution: Consider the sign scheme of $x^2 - 4$.



Let $x < -2, x > 2$. Then, $x^2 - 5x \cdot 1 + 6 = 0 \quad x = 2, 3$.
$\Rightarrow \qquad x = 3$
Let $-2 < x < 2$. Then, $x^2 - 5x \cdot (-1) + 6 = 0 \quad x = -2, -3$. No solution
Let $x = \pm 2$. Then $x^2 - 0 + x = 0$. No solution
Finally, the solution is $x = 3$.

Exponential Function

Definition: An exponential function associates every real number x to a function given by $y = f(x) = a^x$.
where,

(*i*) the base a is positive real number but excluding 1, *i.e.*, $a > 0$ and $a \neq 1$.
(*ii*) the exponent x is a real number.
(*iii*) the number y represents the result of exponentiation, a^x and is a positive real number *i.e.*, $y > 0$.

In other words,

If $a > 0$ and $a \neq 1$, then the function defined by $f(x) = a^x, x \in R$ is called an exponential function. With base a, $y = a^x$ is defined for all real x and $0 < y < \infty$.

Laws of Indices

(*i*) $a^0 = 1, a \neq 0$ (*ii*) $a^{-m} = \dfrac{1}{a^m}, a \neq 0$

(*iii*) $a^{m+n} = a^m \cdot a^n$, where m and n are rational numbers.

(*iv*) $a^{m-n} = \dfrac{a^m}{a^n}$, where m and n are rational numbers.

(v) $(a^m)^n = a^{mn}$ (vi) $a^{p/q} = \sqrt[q]{a^p}$.

Composite Exponential Function

A function in which both the base and exponent are functions of x, for instance, x^x, $(\sin x)^x$, $x^{\tan x}$.

In general any function of the form $y = [u(x)]^{v(x)} = u^v$ is a composite function.

This function is also called an exponential power function or a power exponential function.

Hyperbolic Function

1. Hyperbolic sine *i.e.*, $\sinh x = \dfrac{e^x - e^{-x}}{2}$

2. Hyperbolic cosine *i.e.*, $\cosh x = \dfrac{e^x + e^{-x}}{2}$

3. Hyperbolic tangent *i.e.*, $\tanh x = \dfrac{e^x - e^{-x}}{2} \div \dfrac{e^x + e^{-x}}{2} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

4. Hyperbolic cosecant *i.e.*, $\text{cosech}\, x = \dfrac{2}{e^x - e^{-x}}$

5. Hyperbolic secant *i.e.*, $\text{sech}\, x = \dfrac{2}{e^x + e^{-x}}$

6. Hyperbolic contangent *i.e.*, $\coth x = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$.

Logarithmic Function

If $a > 0$, $a \neq 1$, then the function $y = \log_a x$. $x \in R^+$ is called a logarithmic function with base a. If $a = e$ the logarithmic function is denoted by $y = \ln x$.

$$y = \log_a x \qquad x \in R^+ \quad \text{and} \quad -\infty < y < \infty.$$

Domain of logarithmic function = $(0, \infty)$

Range of logarithmic function = R

Example: $y = \log_a (x^2 + 1)$, $x \in R$.

Rational Function

The function which can be written as the quotient of two polynomial function is said to be a rational function.

If

$$P(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n \cdot x^n$$
$$Q(x) = b_0 + b_1 x + b_2 x^2 + \ldots + b_m \cdot x^m$$

be two polynomial functions then a function f defined by

$$f(x) = \frac{P(x)}{Q(x)} \text{ is a rational function of } x.$$

Example: $f(x) = \dfrac{7x^4 - x^2 + 2}{x^2 - 4x + 3}$ is a rational function which is defined for all real values of x except 1 and 3.

Even and Odd Function

A map $f : A \to B$ is said to be an even function iff $f(-x) = f(x)$ for all $x \in A$.

Example: $f(x) = \cos x$ for all $x \in R$.

A map $f : A \to B$ is said to be an odd function iff $f(-x) = -f(x)$ for all $x \in A$.

Example: $f(x) = x^3; \forall x \in R.$

$$f(-x) = (-x)^3 = -x^3 = -f(x).$$

Explicit Function

A function is said to be an explicit function of a variable x, if dependent variable y can be expressed in terms of independent variable x only.

Example: $y = x + \sin x.$

Implicit Function

A function is said to be an implicit function of a variable x, if dependent variable y cannot be expressed in terms of independent variable x only.

Example: $y = \sin(x + y) + 2x + 3.$

DOMAIN OF A FUNCTION

Definition: Domain of a function $y = f(x)$ is defined as the set of all real values of x for which y is defined *i.e.*, for which y is real.

Working Rule

In order to find out the domain of a function $y = f(x)$. We find all those real values of x for which y is defined.

(*i*) **For algebraic function:**

(*a*) Denominator should not be zero.

(*b*) Expression under even root *i.e.*, square root 4^{th} root sixth root etc. should not be negative.

(*ii*) **For logarithmic function:** $\log_b a$ is defined when $a > 0$, $b > 0$ and $b \neq 1$.

(*iii*) **For inverse circular function:**

(*a*) $\sin^{-1} x$ and $\cos^{-1} x$ are defined for $-1 \leq x \leq 1$.

(*b*) $\tan^{-1} x$ and $\cot^{-1} x$ are defined for all $x \in R$

(*c*) $\sec^{-1} x$ and $\operatorname{cosec}^{-1} x$ are defined for $x \leq -1$ or $x \geq 1$ *i.e.*, for all x except those lying between – 1 and 1.

(*iv*) **For exponential function:** a^x is defined for all x, where $a > 0$.

(*v*) **For trigonometrical function:**

(*a*) $\sin x$ and $\cos x$ are defined for all x.

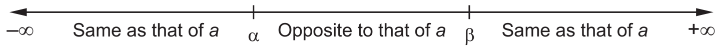
(*b*) $\tan x$ and $\sec x$ are defined for all x except at $x = (2n + 1) \cdot \pi/2$, where $n = 0, \pm 1, \pm 2, \ldots$ *i.e.*, it is not defined for odd multiple of $\pi/2$.

(*c*) $\operatorname{cosec} x$ and $\cot x$ are defined for all except at $x = n\pi$ where $n = 0, \pm 1, \pm 2, \ldots$

(*vi*) **Sign scheme for quadratic expression:** $ax^2 + bx + c$.

(*a*) If the roots of the corresponding equation $ax^2 + bx + c = 0$ are real and equal or imaginary sign of $ax^2 + bx + c$ will be same as that of a for all real x.

(*b*) If roots are real and unequal say α and β. α < β. Then sign scheme for $ax^2 + bx + c$ will be as follows.



(*vii*) **Sign scheme for rational integral functions:** For sign scheme of a rational integral function, equate numerator and denominator to zero *i.e.*, $\dfrac{p(x)}{q(x)}$.

Put $\quad p(x) = 0 \quad$ and $\quad q(x) = 0$

Then find the real values of x. Mark this point on a number line. This will divide the interval]– ∞, ∞[into a number of sub-intervals. Test the sign of given function in any one of these sub-intervals by putting any one value in this sub-intervals. Sign of given expression will be alternate, positive and negative, if no root is repeated even no. of times. But if any value of x say α as found above is repeated even number of times then given expression will have same sign on the two sub-intervals of which α is the dividing point and in other sub-intervals signs will be alternatively positive and negative (imaginary roots will be lefted out).

Examples of sign scheme:

(*i*) $f(x) = \dfrac{x^2 - 3x + 2}{x^2 - 9x + 20}$

Put numerator and denominator = 0

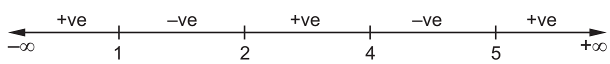
i.e., $\quad x^2 - 3x + 2 = 0 \;\Rightarrow\; x^2 - 2x - x + 2 = 0 \;\Rightarrow\; x(x - 2) - (x - 2) = 0$

$\Rightarrow \quad (x - 1)(x - 2) = 0 \;\Rightarrow\; x = 1, 2$

Again, $\quad x^2 - 9x + 20 = 0 \;\Rightarrow\; x^2 - 5x - 4x + 20 = 0 \;\Rightarrow\; x(x - 5) - 4(x - 5) = 0$

$\Rightarrow \quad (x - 4)(x - 5) = 0 \;\Rightarrow\; x = 4, 5.$

Here no value of x are not repeated even number of times. Therefore, sign scheme of the above roots.



(*ii*) $f(x) = \dfrac{x^2 - 3x + 2}{x^2 - 4x + 3}$.

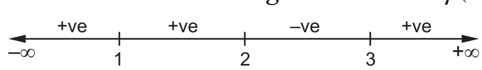
Put $\quad x^2 - 3x + 2 = 0 \;\Rightarrow\; x^2 - 2x - x + 2 = 0 \;\Rightarrow\; x(x - 2) - 1(x - 2) = 0$

$\Rightarrow \quad (x - 1)(x - 2) = 0 \;\Rightarrow\; x = 1, 2$

Again $\quad x^2 - 4x + 3 = 0 \;\Rightarrow\; x^2 - 3x - x + 3 = 0 \;\Rightarrow\; x(x - 3) - 1(x - 3) = 0$

$\Rightarrow \quad (x - 1)(x - 3) = 0 \;\Rightarrow\; x = 1, 3$

Here, 1 is repeated twice therefore sign scheme for $f(x)$ is as follows



]– ∞, 1[and]1, 2[have same sign.

(*iii*) $f(x) = \dfrac{x^2 - 3x + 2}{(x - 3)^2}$.

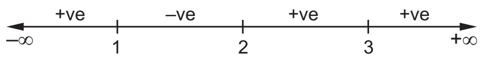
Put $\quad x^2 - 3x + 2 = 0 \;\Rightarrow\; (x - 1)(x - 2) = 0 \;\Rightarrow\; x = 1, 2$

$$(x-3)^2 = 0$$

$\therefore \quad x = 3, 3$

Here 3 is repeated twice.

Putting $x = 4$.



$]2, 3[$ and $]3, \infty[$ have same signs.

(*iv*) $f(x) = \dfrac{x^2 - 3x + 2}{-x^2 + x - 1}$

$\Rightarrow \quad x^2 - 3x + 2 = 0 \Rightarrow (x-1)(x-2) = 0 \Rightarrow x = 1, 2.$

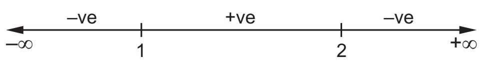
$-x^2 + x - 1 = 0 \Rightarrow -(x^2 - x + 1) = 0$

$\Rightarrow \quad x^2 - x + 1 = 0$

No real value of x is obtained.

Since $\quad D < 0$

Therefore, sign scheme for $f(x)$ is as follows

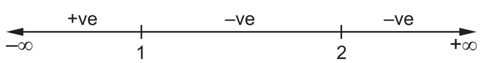


WORKED OUT EXAMPLES

Example 1. *Find the domain definition of the function:* $f(x) = \dfrac{x}{\sqrt{x^2 - 3x + 2}}$.

Solution: Numerator is defined for all real values of x.

Denominator $> 0 \Rightarrow x^2 - 3x + 2 > 0 \Rightarrow (x-1)(x-2) > 0 \Rightarrow x = 1, 2.$



i.e., $\quad x < 1$ and $x > 2$

$\therefore \quad D_f =]-\infty, 1[\cup]2, \infty[.$

Example 2. *Find the domain of the function:* $y = \dfrac{1}{\sqrt{x - |x|}}$.

Solution: For y to be defined

$$x - |x| > 0$$

When $x > 0$, $\quad |x| = x$

$\therefore \quad x - |x| > 0$

$\Rightarrow \quad x - x > 0$, which is not possible.

When $x < 0$, $\quad |x| = -x$

$\therefore \quad x - |x| > 0 \Rightarrow x - (-x) > 0$

$\Rightarrow \quad 2x > 0 \Rightarrow x > 0$

But in this case $x < 0 \quad \therefore \quad x \not> 0$.

Hence, the given function is not defined for any x.

Example 3. *Find the domain of definition of the function:* $y = \log_{10} \dfrac{x-5}{x^2 - 10x + 24} - \sqrt[3]{x+5}$.

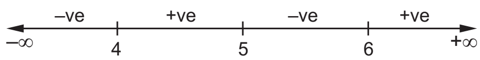
Solution: For y to be defined

(*i*) $\dfrac{x-5}{x^2 - 10x + 24} > 0$

When $x - 5 = 0 \;\Rightarrow\; x = 5$

and when $x^2 - 10x + 24 = 0 \;\Rightarrow\; (x-4)(x-6) = 0 \;\Rightarrow\; x = 4, 6.$

Sign scheme,



$$D_f = \,]4, 5[\cup]6, +\infty[\qquad \text{...(A)}$$

(*ii*) $(x+5)^{1/3}$ is defined for all x. ...(B)

Combining (*A*) and (*B*), we get,

$$D_f = \,]4, 5[\cup]6, \infty[.$$

Example 4. *Find the domain of definition of the function:* $y = \cos^{-1} \dfrac{2}{2 + \sin x}$.

Solution: For y to be defined

$$-1 \le \frac{2}{2+\sin x} \le 1 \;\Rightarrow\; \left|\frac{2}{2+\sin x}\right| \le 1$$

$$\Rightarrow \quad \frac{|2|}{|2+\sin x|} \le 1$$

$$\Rightarrow \quad |2| \le |2 + \sin x| \;\Rightarrow\; 2 \le 2 + \sin x$$

$$\Rightarrow \quad \sin x \ge 0$$

$$\Rightarrow \quad 2n\pi + 0 \le x \le 2n\pi + \pi$$

$$\Rightarrow \quad 2n\pi \le x \le (2n+1)\pi$$

$$D_f = \{x : 2n\pi \le x \le (2n+1)\cdot\pi;\ n = 0; \pm 1, \pm 2, \pm 3, ...\}.$$

Example 5. *Find the domain of definition of the following function:*

(*i*) $y = \sqrt{x^2 - 4x + 3}$ (*ii*) $y = \dfrac{1}{\sqrt{|x| - x}}$

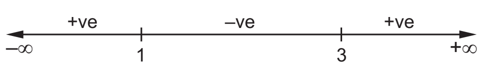
Solution:

(*i*) $y = \sqrt{x^2 - 4x + 3}$

For y to defined,

$$x^2 - 4x + 3 \ge 0 \;\Rightarrow\; (x-1)(x-3) \ge 0 \;\Rightarrow\; x = 1, 3.$$

Sign scheme



$$\therefore \quad D_f = \,]-\infty, 1] \cup [3, \infty[.$$

(ii) $y = \dfrac{1}{\sqrt{|x| - x}}$

For y to be defined,

$$|x| - x > 0$$

When $x > 0$, $\quad |x| = x \Rightarrow |x| - x = x - x > 0$

When $x < 0$, $\quad |x| = -x \Rightarrow |x| - x > 0$

$\Rightarrow \quad -x - x > 0$

i.e., $\quad -2x > 0 \Rightarrow x < 0$

$\therefore \quad D_f =]-\infty, 0[.$

Example 6. *Find the range of the function:* $y = \dfrac{x}{1 + x^2}$.

Solution: Here $y = \dfrac{x}{1+x^2}$ which is defined for all x.

$$D_f =]-\infty, \infty[$$

Given $\quad y = \dfrac{x}{1+x^2} \quad$ or $\quad y(1 + x^2) = x \Rightarrow y + x^2y = x$

$\Rightarrow \quad yx^2 + y - x = 0 \quad$ *i.e.*, $\quad yx^2 - x + y = 0$

For x to be real

$$D \geq 0$$

i.e., $\quad (-1)^2 - 4 \cdot y \cdot y \geq 0 \Rightarrow 1 - 4y^2 \geq 0$

$\Rightarrow \quad (1 - 2y)(1 + 2y) \geq 0 \Rightarrow y = -\dfrac{1}{2}, \dfrac{1}{2}$

$\Rightarrow \quad -\dfrac{1}{2} \leq y \leq \dfrac{1}{2}.$

Thus range of $f = \left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$.

Example 7. *Find the range of the function:* $y = \dfrac{1}{2 - \sin 3x}$.

Solution: Given $\quad y = \dfrac{1}{2 - \sin 3x} \quad$ ...(1)

$$D_f =]-\infty, \infty[$$

From (1), $\quad 2 - \sin 3x = \dfrac{1}{y}$

$\Rightarrow \quad \sin 3x = 2 - \dfrac{1}{y} = \dfrac{2y - 1}{y}$

$\therefore \quad x = \dfrac{1}{3} \cdot \sin^{-1} \dfrac{2y - 1}{y} \quad$ ...(2)

$\because \quad y = \dfrac{1}{2 - \sin 3x}$

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