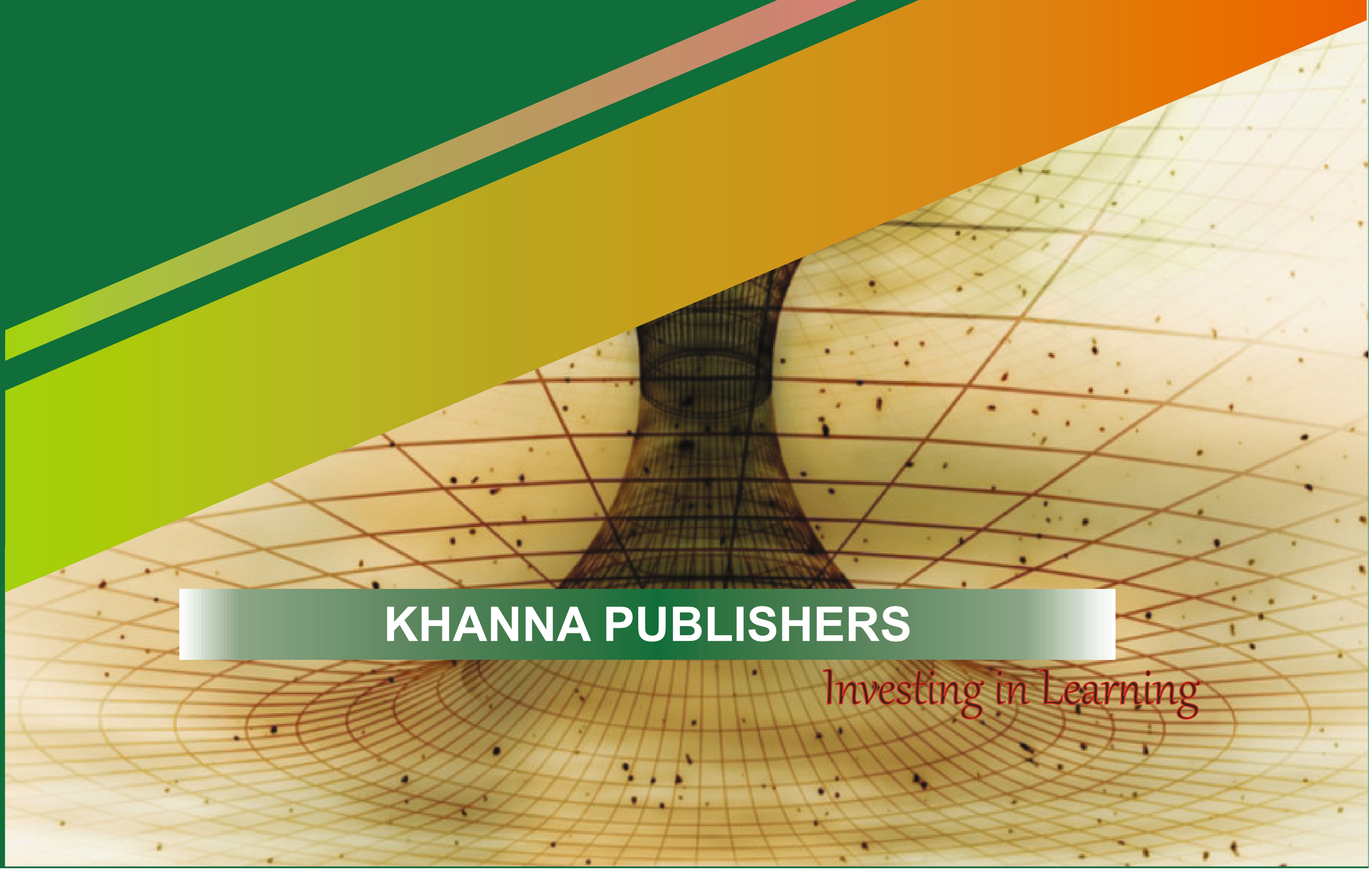
VIRAM SERIES

APPLIED MATHEMATICS-II

For Second Semester Diploma Students

As per AICTE Curriculum for Diploma Students

M.K. Kanyal



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Applied Mathematics-II

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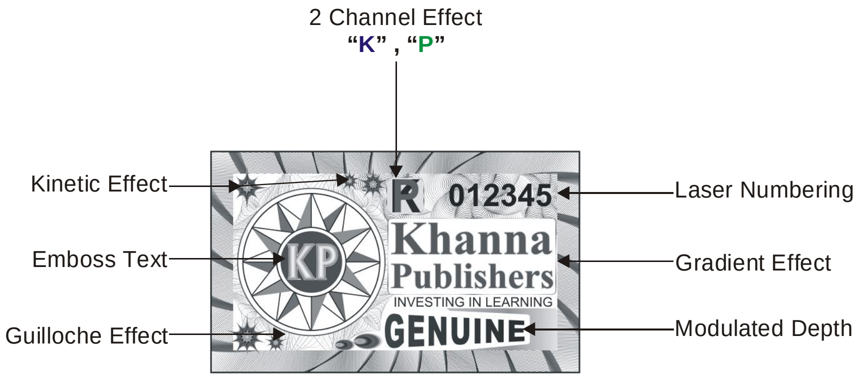

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Preface

This book presents the subject matter in full conformity with the syllabi prescribed by AICTE. To keep pace with changing trends in technical education at state level, the whole text has been arranged strictly according to diploma engineering pattern.

The book provides a result oriented training to young students. The text has been studded with simple self-study questions, so as to provide an insight and proper grip over the topic, as one learns it. The article work in each chapter of a unit is coupled with well graded and carefully selected solved numerical problems for easy comprehension for the beginners. These numericals have been focussed on board pattern.

My special thanks to all my colleagues and friends for their creative suggestions.

In spite of all precautions and care taken to produce a clear and accurate book, some errors and misprints might have been left inadvertently. The author will welcome comments and corrections with gratitude.

— M.K. Kanyal

Contents

UNIT 1: Coordinate-Geometry

1.1 EQUATION OF STRAIGHT LINE

In a straight line is curve such that every point on the line segment joining and two points on it lies on it:

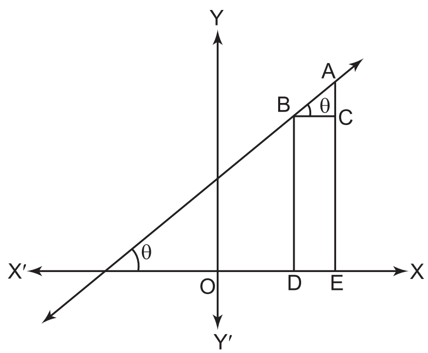
When a point moves on a plane in a given direction then its locus is called straight line. Every first degree equation in x, y represents a straight line. Thus a line is also defined as the locus of a point satisfying the condition $ax + by + c = 0$, where a, b, c are constants. In facts these are the co ordinates of any point on the line and are known as the current co ordinates.

1.1.1 Slope of a Line through Point Slope Form

Let (x_1, y_1) and (x_2, y_2) be two given cartesian co ordinates of the points A and B respectively referred to rectangular co ordinate axes XOX′ and YOY′.

Again let the straight line AB makes an angle θ with the positive x-axis in the anticlockwise direction.

Now by definition, the slope of the line AB is tan θ. Therefore, we have to find the value of $m = \tan\theta$.



Slope of a Line through Two Given Points

DO YOU KNOW?

1. Slope of two parallel lines are equal.
2. Slope of x-axis or slope of a straight line parallel to x-axis is zero, since we know that $\tan\theta° = 0$.
3. Slop of y-axis or slope of a straight line parallel to y-axis is undefined, since we know that tan 90° is undefined.
4. We know that coordinate of the origin is (0, 0). If O be the origin and $M(x, y)$ be a given point, then the slope of the line OM is $\frac{y}{x}$.
5. The slope of the line is the change in the value of ordinate of any point on the line for unit change in the value of abscissa.

Draw AE and BD perpendiculars on x-axis and from B draw BC perpendiculars on AE. Then,

$$AE = y_1,\ BD = y_2,\ OE = x_1 \text{ and } OD = x_2$$

Therefore, $$BC = DE = OE - OD = x_1 - x_2$$

Again, $$AC = AE - CE = AE - BD = y_1 - y_2$$

$\angle ABC = \theta$, since, BC parallel to x-axis.

Therefore, from the right angle ΔABC, We get

$$\tan\theta = \frac{AC}{BC} = \frac{y_1 - y_2}{x_1 - x_2} \Rightarrow \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}.$$

Therefore, the required slope of the line passing through the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Difference of ordinates of the given point}}{\text{Difference of abscissa of the given point}}.$$

Example 1. *Find the slope of a straight line which passes through points (–5, 7) and (–4, 8).*

Solution: We know that the slope of a straight line passes through two points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$. Here the straight line passes through (–5, 7) and (–4, 8). Therefore, the slope of the straight line is given by $m = \frac{8-7}{-4-(-5)} = \frac{1}{-4+5} = \frac{1}{1} = 1$.

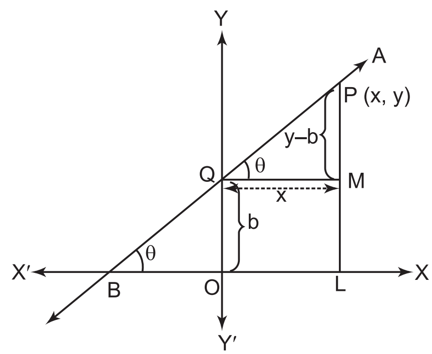
1.1.2 Slope Intercept Form

The equation of a straight line with slope m and making an intercept b on y-axis is $y = mx + b$.

Let a line AB intersects the y-axis at Q and makes an angle θ with the positive direction of x-axis in anticlockwise sense and $OQ = b$.

Now we have to find the equation of the straight line AB.

Let $P(x, y)$ be any point on the line AB. Draw PL perpendicular to x-axis and CM perpendicular on PL.



Slope-intercept Form

Clearly, $< PQM = \theta$. Since, QM parallel to x-axis.

Since the co ordinate of P is (x, y) hence, $PL = y$

$$PM = PL - ML = PL - OQ = y - b$$

Again, $$QM = OL = x$$

Now form the right angle ΔPQM, we get

$$\tan\theta = PM/QM = y - b/x \Rightarrow \tan\theta = y - b/x$$

If $\tan\theta = m$, then we have.

$$m = y - b/x$$

$\Rightarrow y = mx + b$, which is the required equation of the line and satisfied by the co ordinates of all points on the line AB.

Example 2. *Find the equation of a straight line whose slope = –7 and which intersects the y-axis at a distance of 2 units from the origin.*

Solution: Here $m = -7$ and $b = 2$. Therefore, the equation of the straight line is $y = mx + b \Rightarrow y = -7x + 2 \Rightarrow 7x + y - 2 = 0$.

Example 3. *Find the slope and y-intercept of the straight-line $4x - 7y + 1 = 0$.*

Solution: The equation of the given straight line is

$$4x - 7y + 1 = 0$$

$$\Rightarrow \quad 7y = 4x + 1 \Rightarrow y = \frac{4}{7}x + \frac{1}{7}$$

Now, compare the above equation with the equation $y = mx + b$, we get

$$m = 4/7 \text{ and } b = 1/7.$$

Therefore, the slope of the given straight line is 4/7 and its y-intercept = 1/7 units.

1.1.3 Straight Line in Two-point-Form

The equation of a line passing through two points (x_1, y_1) and (x_2, y_2) is $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

Let the two given points be (x_1, y_1) and (x_2, y_2).

We have to find the equation of the straight line joining the above two points.

Let the given points be $A\ (x_1, y_1)$, $B\ (x_2, y_2)$ and $P\ (x, y)$ be point on the straight line joining the points A and B.

Now, the slope of the line AB is $\frac{y_1 - y_2}{x_1 - x_2}$

And the slope of the line AP is $\frac{y - y_1}{x - x_1}$

But the three points A, B and P are collinear.

Therefore, slope of the line AP = slope of the line AB

$$\Rightarrow \quad \frac{y - y_1}{x - x_1} = \frac{y_1 - y_2}{x_1 - x_2} \Rightarrow y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

The above equation is satisfied by the co-ordinates of any point P lying on the line AB and hence, represents the equation of the straight line AB.

Example 4. *Find the equation of the straight line passing through the points (2, 3) and (6, – 5).*

Solution: The equation of the straight line passing through the points (2, 3) and (6, – 5) is

$$\frac{y - 3}{x + 2} = \frac{3 + 5}{2 - 6}, \qquad \left[\text{Using the form, } \frac{y - y_1}{x - x_1} = \frac{y_1 - y_2}{x_1 - x_2}\right]$$

$$\Rightarrow \quad \frac{y - 3}{x + 2} = \frac{8}{-4} \Rightarrow \frac{y - 3}{x + 2} = -2$$

$$\Rightarrow \quad y - 3 = -2x - 4$$

$$\Rightarrow \quad 2x + y + 1 = 0, \text{ which is the required equation.}$$

Example 5. *Find the equation of the straight line joining the points (–3, 4) and (5, – 2).*

Solution: Here the given two points are $(x_1, y_1) = (-3, 4)$ and $(x_2, y_2) = (5, -2)$.

The equation of a line passing through two points (x_1, y_1) and (x_2, y_2) is $y - y_1 = \left[\frac{y_2 - y_1}{x_2 - x_1}\right](x - x_1)$.

So the equation of the straight line in two points form is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow \quad y - 4 = \frac{-2 - 4}{5 - (-3)}[x - (-3)] \Rightarrow y - 4 = \frac{-6}{8}(x + 3)$$

$$\Rightarrow \quad y - 4 = \frac{-3}{4}(x + 3) \Rightarrow 4(y - 4) = -3(x + 3)$$

$$\Rightarrow \quad 4y - 16 = -3x - 9$$

$$\Rightarrow \quad 3x + 4y - 7 = 0, \text{ which is the required equation.}$$

1.1.4 Straight Line in Intercept Form

The equation of a line which cuts off intercepts a and b respectively from the x and y axes is $\frac{x}{a} + \frac{y}{b} = 1$.

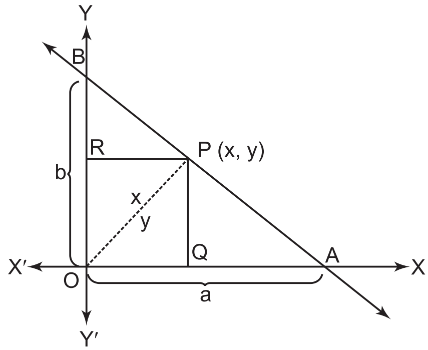
Let the straight line AB intersects the x-axis at A and the y-axis at B where $OA = a$ and $OB = b$.

Now we have to find the equation of the straight line AB.

Let $P(x, y)$ be any point on the line AB. Draw PQ perpendicular on OX and PR perpendicular on OX. Then, join the points O and P. Now, $PQ = y$, $OQ = x$.

Clearly, we see that

Area of the ΔOAB = Area of the ΔOPA + Area of the ΔOPB



Straight Line in Intercept Form

$$\Rightarrow \quad \tfrac{1}{2} OA \,.\, OB = \tfrac{1}{2} \,.\, OA \,.\, PQ + \tfrac{1}{2} \,.\, OB \,.\, PR$$

$$\Rightarrow \quad \tfrac{1}{2} a.\, b = \tfrac{1}{2} \,.\, a \,.\, y + \tfrac{1}{2} \,.\, b \,.\, x$$

$$\Rightarrow \quad ab = ay + bx$$

$$\Rightarrow \quad \frac{ab}{ab} = \frac{ay + bx}{ab}, \text{ dividing both sides by } ab$$

$$\Rightarrow \quad 1 = \frac{ay}{ab} + \frac{bx}{ab} \Rightarrow 1 = \frac{y}{b} + \frac{x}{a}$$

$\Rightarrow \frac{x}{a} + \frac{y}{b} = 1$, which is the equation of the line in the form.

The equation $\frac{x}{a} + \frac{y}{b} = 1$ is the satisfied by the co ordinates of any point P lying on the line AB.

Therefore, $\frac{x}{a} + \frac{y}{b} = 1$ represent the equation of the straight line AB.

Example 6. *Find the equation of the line which cuts off an intercept 3 on the positive direction of x-axis and an intercept 5 on the negative direction of y-axis.*

Solution: The equation of a line which cuts of intercepts a and b respectively from the x and y axes is $\frac{x}{a} + \frac{y}{b} = 1$.

Here, $a = 3$ and $b = -5$.

Therefore, the equation of the straight line is $\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow \frac{x}{3} + \frac{y}{-5} = 1$

$\Rightarrow \frac{x}{3} - \frac{y}{5} = 1 \Rightarrow 5x - 3y = 15 \Rightarrow 5x - 3y - 15 = 0$ ·

Example 7. *Find the intercepts of the straight line 4x + 3y = 24 on the co ordinate axes.*

Solution: Given equation $4x + 3y = 24$.

Now convert the given equation into intercept form.

$$4x + 3y = 24$$

$$\Rightarrow \quad \frac{4x + 3y}{24} = \frac{24}{24}, \text{ Dividing both sides by 24}$$

$$\Rightarrow \quad \frac{4x}{24} + \frac{3y}{24} = 1 \Rightarrow \frac{x}{6} + \frac{y}{8} = 1, \text{ which is the intercept form.}$$

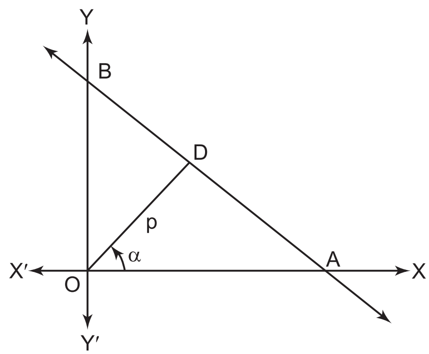
Therefore, x-intercept = 6 and y-intercept = 8.

1.1.5 Straight Line in Normal Form

The equation of the straight line upon which the length of the perpendicular from the origin is p and this perpendicular makes an angle a with x-axis is $x \cos a + y \sin a = p$.

If the line length of the perpendicular draw from the origin upon a line and the angle that the perpendicular makes with the positive direction of x-axis be given then to find the equation of the line.

Suppose the line AB intersects the x-axis at A and the y-axis at B. Now from the origin O draw OD perpendicular to AB.



Straight Line in Normal Form

The length of the perpendicular OD from the origin = p and $\angle XOD = a$, $(0 \le a \le 2n)$.

Again, from the right-angled ΔODB, we get

$$\angle OBD = \frac{\pi}{2} - \angle BOD = \angle DOX = \alpha$$

Therefore, $$\frac{OD}{OB} = \sin\alpha$$

or $$\frac{p}{OB} = \sin\alpha \quad \text{or} \quad OB = \frac{p}{\sin\alpha}$$

Since the intercepts of the line AB on x-axis and y-axis are OA and OB respectively, hence the required

$$\frac{x}{OA} + \frac{y}{OB} = 1$$

$$\Rightarrow \quad \frac{x}{\frac{p}{\cos\alpha}} + \frac{y}{\frac{p}{\sin\alpha}} = 1 \quad \Rightarrow \quad \frac{x\cos\alpha}{p} + \frac{y\sin\alpha}{p} = 1$$

$\Rightarrow x\cos\alpha + y\sin\alpha = p$, which is the required form.

Example 8. *Find the equation of the straight line which is at a of distance 7 units from the origin and the perpendicular from the origin to the line makes an angle 45° with the positive direction of x-axis.*

Solution: We know that the equation of the straight line upon which the length of the perpendicular from the origin is p and this perpendicular makes an angle a with x-axis is $x \cos \alpha + y \sin \alpha = p$.

Here $p = 7$ and $\alpha = 45°$

Therefore, the equation of the straight line in normal form is

$x \cos 45° + y \sin 45° = 7$

$$\Rightarrow \quad x.\frac{1}{\sqrt{2}} + y.\frac{1}{\sqrt{2}} = 7 \quad \Rightarrow \quad \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = 7$$

$\Rightarrow \quad x + y = 7\sqrt{2}$, which is the required equation.

DO YOU KNOW?

1. The equation of straight line in the form of $x \cos \alpha + y \sin \alpha = p$ is called its normal form.
2. In equation $x \cos \alpha + y \sin \alpha = p$, the value of p is always positive and $0 \le \alpha \le 360°$.

1.1.6 Intersection of Two Straight Lines

Let the equations of two intersecting straight lines be

$$a_1x + b_1y + c_1 = 0 \quad ...(i)$$

and

$$a_2x_1 + b_2y_1 + c_2 = 0 \quad ...(ii)$$

Suppose the above equations of two intersecting lines intersect at $p(x_1, y_1)$.

Then (x_1, y_1) will satisfy both equations (i) and (ii).

Therefore, $\quad a_1x_1 + b_1y_1 + c_1 = 0$ and $a_2x_1 + b_2y_1 + c_2 = 0$

Solving the above two equations by using the method of cross-multiplication, we get,

$$\frac{x_1}{b_1c_2 - b_2c_1} = \frac{y_1}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Therefore, $$x_1 = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

and $$y_1 = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}, a_1b_2 - a_2b_1 \neq 0$$

Therefore, the required co-ordinates of the point of intersection of the lines (i) and (ii) are

$$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}\right), \left(\frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}\right), a_1b_2 - a_2b_1 \neq 0$$

Example 9. *Find the co ordinates of the point of intersection of the lines 2x – y + 3 = 0 and x + 2y – 4 = 0.*

Solution: We know that the co-ordinates of the point of intersection of the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are

$$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}\right), \left(\frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}\right), a_1b_2 - a_2b_1 \neq 0$$

DO YOU KNOW?

To find the coordinates of the point of intersection of two non-parallel lines, we solve the given equations simultaneously and the values of x and y so obtained determine the coordinates of the point of intersection.

If $a_1b_2 - a_2b_1 = 0$, then $a_1b_2 = a_2b_1$

$$\Rightarrow \quad \frac{a_1}{b_1} = -\frac{a_2}{b_2}$$

$$\Rightarrow \quad -\frac{a_1}{b_1} = -\frac{a_2}{b_2}$$

Therefore, in this case the straight lines are parallel and hence they do not intersect at any real point.

Given equations are

$$2x - y + 3 = 0 \quad ...(i)$$

$$x + 2y - 4 = 0 \quad ...(ii)$$

Here $a_1 = 2, b_1 = -1, c_1 = 3, a_2 = 1, b_2 = 2$ and $c_2 = -4$.

$$\left(\frac{(-1).(-4)-(2).(3)}{(2).(2)-(1).(-1)}, \frac{(3).(1)-(-4).(2)}{(2).(2)-(1).(-1)}\right)$$

$$\Rightarrow \quad \left(\frac{4-6}{4+1}, \frac{3+8}{4+1}\right) \quad \Rightarrow \quad \left(\frac{11}{5}, \frac{-2}{5}\right).$$

Therefore, the co ordinates of the point of intersection of the lines $2x - y + 3 = 0$ and $x + 2y - 4 = 0$ are $\left(\frac{11}{5}, \frac{-2}{5}\right)$.

1.1.7 Perpendicular Distance Formula

If two lines of slopes m_1 and m_2 are perpendicular, then the angle between the lines θ is of 90°.

Therefore, $\cot\theta = 0$

$$\Rightarrow \quad \frac{1 + m_1m_2}{m_2 - m_1} = 0 \quad \Rightarrow \quad 1 + m_1m_2 = 0$$

$$\Rightarrow \quad m_1m_2 = -1.$$

Thus when two lines are perpendicular, the product of their slope is –1. If m is the slope of a line, then the slope of a line perpendicular to it is $-1/m$.

Let us assume that the lines $y = m_1x + c_1$ and $y = m_2x + c_2$ make angles α and β respectively with the positive direction of the x-axis and θ be the angle between them.

Therefore, $\alpha = \theta + \beta = 90° + \beta$ [Since, $\theta = 90°$]

Now taking tan on both sides we get,

$$\tan\alpha = \tan(\alpha + \beta) \quad \Rightarrow \quad \tan\alpha = -\cot\beta$$

$$\tan\alpha = -\frac{1}{\tan\beta}$$

$$\text{or} \quad m_1 = -\frac{1}{m_2} \quad \text{or} \quad m_1m_2 = -1.$$

Therefore, the condition of perpendicularity of the lines $y = m_1x + c_1$, and $y = m_2x + c_2$ is $m_1m_2 = -1$.

Conversely, if $m_1m_2 = -1$, then

$$\tan\alpha.\tan\beta = -1$$

$$\frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta} = -1 \quad \Rightarrow \quad \sin\alpha\sin\beta = -\cos\alpha\cos\beta$$

$$\cos\alpha\cos\beta + \sin\alpha\sin\beta = 0 \quad \Rightarrow \cos(\alpha - \beta) = 0$$

Therefore, $\alpha - \beta = 90°$

Therefore, $\theta = \alpha - \beta = 90°$

Thus, the straight lines AB and CD are perpendicular to each other.

Example 10. *Let P (6, 4) and Q (2, 12) be the two points. Find the slope of a line perpendicular to PQ.*

Solution: Let m be the slope of PQ.

Them $$m = \frac{12-4}{2-6} = \frac{8}{-4} = -2$$

Therefore the slope of the line perpendicular to $PQ = -\frac{1}{m} = \frac{1}{2}$.

Example 11. *Without using the Pythagoras theorem, show that P (4, 4), Q (3, 5) and R (–1, – 1) are the vertices of a right angled triangle.*

Solution: In ΔABC, we have:

m_1 = Slope of the side $PQ = \frac{4-5}{4-3} = -1$

m_2 = Slope of the side $PR = \frac{4-(-1)}{4-(-1)} = 1$

Now clearly we see that $m_1 m_2 = 1 \times (-1) = -1$.

Therefore, the side PQ perpendicular to PR that is $\angle RPQ = 90°$.

Therefore, the given points P (4, 4), Q (3, 5) and R (–1, –1) are the vertices of a right angled triangle.

Example 12. *Find the ortho-centre of the triangle formed by joining the points P (–2, – 3), Q (6, 1) and R (1, 6).*

Solution: The slope of the side QR of the ΔPQR is $\frac{6-1}{1-6} = \frac{5}{-5} = -1$.

Let PS be the perpendicular from P on QR; hence, if the slope of the line PS be m then,

$$m \times (-1) = -1 \quad \text{or} \quad m = 1.$$

Therefore, the equation of the straight line PS is

$$y + 3 = 1\,(x + 2) \quad \text{or} \quad x - y = 1 \qquad \text{...(1)}$$

Again, the slope of the side RP of the ΔPQR is $\frac{6+1}{1+2} = 3$.

Example 13. *Find the slope and y-intercept of the straight line 5x – 3y + 15 = 0. Find also the length of the portion of the straight line intercepted between the co ordinate axes.*

Solution: The equation of the given straight line is,

$$5x - 3y + 15 = 0$$

$$\Rightarrow \quad 3y = 5x + 15 \Rightarrow y = \frac{5}{3}x + 5$$

Now, comparing equation $y = \frac{5}{3}x + 5$ with the equation $y = mx + c$ we get, $m = \frac{5}{3}$ and $c = 5$.

Therefore, the slope of the given straight line is $\frac{5}{3}$ and its y-intercept = 5 units.

Again the intercept form of the equation of the given straight line is,

$$5x - 3y + 15 = 0 \quad \Rightarrow \quad 5x - 3y = -15$$

$$\Rightarrow \quad \frac{5x}{-15} - \frac{3y}{-15} = \frac{-15}{-15} \quad \Rightarrow \quad \frac{x}{-3} + \frac{y}{5} = 1$$

Clearly, the given line intersects the x-axis at A (–3, 0) and the y-axis at B (0, 5)

$$\text{distance} = AB$$

$$\sqrt{(-3)^2 + (5)^2} = \sqrt{9 + 25} = \sqrt{34} \text{ units.}$$

Example 14. *Find the equation of the straight line passes through the point (2, 3) so that the line segment intercepted between the axes is bisected at this point.*

Solution: Let the equation of the straight line be $\frac{x}{a} + \frac{y}{b} = 1$, which meets the x and y axes at A (a, 0) and B (0, b) respectively. The coordinates of the mid-point of AB are $\left(\frac{a}{2}, \frac{b}{2}\right)$. Since the point (2, 3) bisects AB, therefore,

$$\frac{a}{2} = 2 \text{ and } \frac{b}{2} = 3 \quad \Rightarrow a = 4 \text{ and } b = 6.$$

Therefore, the equation of the required straight line is $\frac{x}{4} + \frac{y}{6} = 1$ or $3x + 2y = 12$.

More examples to solve the problems on slope and intercept.

Example 15. *Find the equation of the straight line passing through the points (–3, 4) and (5, – 2); find also the co-ordinates of the points where the line cuts the co ordinate axes.*

Solution: The equation of the straight line passing through the points (–3, 4) and (5, – 2) is

$$\frac{y-4}{x+3} = \frac{4+2}{-3-5}, \qquad \left[\text{Using the form, } y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)\right]$$

$$\Rightarrow \frac{y-4}{x+3} = \frac{6}{-8} \qquad \Rightarrow \frac{y-4}{x+3} = \frac{3}{-4}$$

$$\Rightarrow 3x + 9 = -4y + 16 \qquad \Rightarrow 3x + 4y = 7 \qquad \ldots(i)$$

$$\Rightarrow \frac{3x}{7} + \frac{4y}{7} = 1 \qquad \Rightarrow \frac{x}{\frac{7}{3}} + \frac{y}{\frac{7}{4}} = 1$$

Therefore, the straight line (i) cuts the x-axis at $\left(\frac{7}{3}, 0\right)$ and the y-axis at $\left(0, \frac{7}{4}\right)$.

Example 16. *Find the equation of a straight line that has y-intercept 4 and is perpendicular to straight line joining (2, – 3) and (4, 2).*

Solution: Let m be the slope of the required straight line.

Since the required straight line is perpendicular to the line joining P (2, – 3) and Q (4, 2).

Therefore,

$$m \times \text{Slope of } PQ = -1 \qquad \Rightarrow m \times \frac{2+3}{4-2} = -1$$

$$\Rightarrow m \times \frac{5}{2} = -1 \qquad \Rightarrow m = -\frac{2}{5}.$$

The required straight line cut off an intercept of length 4 on y-axis.

Therefore, $b = 4$

Hence, the equation of the required straight line is $y = -\frac{2}{5}x + 4$

$\Rightarrow \quad 2x + 5y - 20 = 0.$

Example 17. *Find the area of triangle formed by the axes of coordinate and the straight line $5x + 7y = 35$.*

Solution: Given straight line $5x + 7y = 35$

The intercept form of the given straight line is

$$5x + 7y = 35$$

$$\Rightarrow \quad \frac{5x}{35} + \frac{7y}{35} = 1$$

$$\frac{x}{7} + \frac{y}{5} = 1$$

Straight line intersects the x-axis at P (7, 0) and the y-axis at Q (0, 5).

Thus if O is origin, $OP = 7$ and $OQ = 5$

So, area $= \frac{1}{2}(OP \times OQ) = \frac{1}{2} \times 7 \times 5 = \frac{35}{2}$ Sq. unit .

Example 18. *Prove that the points (5, 1), (1, – 1) and (11, 4) are collinear. Also find the equation of the straight line on which these points lie.*

Solution: Let the given points be P (5, 1), Q (1, – 1) and R (11, 4). Then the equation of the line passing through P and Q is

$$y - 1 = \frac{-1-1}{1-5}(x - 5) \quad \Rightarrow \quad y - 1 = \frac{-2}{-4}(x - 5)$$

$$\Rightarrow \quad y - 1 = \frac{1}{2}(x - 5) \quad \Rightarrow \quad 2(y - 1) = (x - 5)$$

$$\Rightarrow \quad 2y - 2 = x - 5 \quad \Rightarrow \quad x - 2y - 3 = 0$$

Clearly, the point (11, 4) satisfies the equation $x - 2y - 3 = 0$. Hence the given points lie on the same straight line, whose equation is $x - 2y - 3 = 0$.

Example 19. *Find the angle the straight line perpendicular to the straight line $\sqrt{3x} + y = 1$, makes with the positive direction of the x-axis.*

Solution: The given equation of the straight line $\sqrt{3x} + y = 1$

Convert the above equation into slope-intercept form we get,

$$y = -\sqrt{3x} + 1 \qquad \text{...(i)}$$

Let us assume that the given straight line (i) makes an angle θ with the positive direction of the x-axis.

Then, the slope of the straight line (i) will be tan θ.

Hence, we must have, $\tan\theta = -\sqrt{3}$ [Since, the slope of the straight line $y = -\sqrt{3}x + 1$ is $-\sqrt{3}$]

$\Rightarrow \tan\theta = -\tan 60° = \tan(180° - 60°) = \tan 120°$

$\Rightarrow \tan\theta = \tan 120°$

Since the straight line makes an angle 120° with the positive direction of the x-axis, hence a straight line perpendicular to the line (i) make an angle $120° - 90° = 30°$ with the positive direction of the x-axis.

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