

ELECTROMAGNETIC FIELDS WAVES AND ANTENNAS

D.V. PRASAD,

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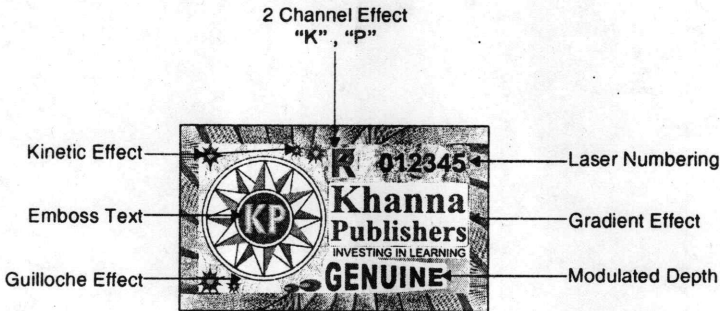
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PREFACE

Electromagnetic Fields and Waves is a subject of importance offered at the second year level of the courses leading to Bachelor's degree in Electrical Engineering and Electronics and Communication Engineering. Out of the experience of teachers teaching this subject, the need of a comprehensive book covering the aspects of fields, Waves and Antennas was felt, the result is this book.

This book will serve as a text book for under-graduate courses of universities and Engineering Colleges offering Electrical Engineering and Electronics and Communication Engineering.

Chapter 1 gives a brief mathematical introduction needed for the course namely the vector analysis. Chapters 2 and 3 give a description of the static electric and magnetic fields. The chapter 4 introduces to the reader the concept of time varying fields and Maxwell's equations. Chapters 5, 6, 7 and 8 discuss in detail the behaviour of an Electromagnetic Wave. The Propagation of Wave in wave guides is understood in chapter 9. The reader is introduced to Transmission Lines in chapter 10. Chapters 11, 12 and 13 give an overall view of the antennas, the basic element to transmit and receive the electromagnetic waves. The last chapter discusses the various means of the wave propagation.

Efforts are taken to present the beginner the concepts of Fields, Waves and Antennas as simply as possible. The author will be grateful for comments, suggestions and criticism for improving the standard of the book.

I take this opportunity to thank my wife Smt. Giri Kumari who helped me at every stage of this book.

D.V. PRASAD

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Vector Analysis

Vector analysis is a shortcut mathematical tool and results in a real economy of time and thought. The field concept applied to understand the strange behaviour of an electromagnetic wave involving multi-variables and especially the fact that the propagation of a wave is more complex in different mediums can be easily understood by applying the concept of vectors which give a clear insight in understanding the physical laws that mathematics describe.

A quantity whose only measurable attribute is its magnitude is called a scalar quantity. The temperature at any point, mass, density, pressure are examples of scalar quantities. A quantity with two measurable attributes namely magnitude and direction is referred to as a vector quantity. The velocity of a moving object, the force applied to a body, the gravitational and magnetic fields of the earth, the voltage gradient, the temperature gradient are examples of vectors. A vector quantity is specified by a directed line in the direction of the vector quantity.

Two vectors A and B satisfy the rules of ordinary algebra namely

$$\bar{A} + \bar{B} = \bar{B} + \bar{A} \quad \text{Commutation in addition} \quad \dots(1.1)$$

$$\bar{A} + (\bar{B} + \bar{C}) = (\bar{A} + \bar{B}) + \bar{C} \quad \text{Association in addition} \quad \dots(1.2)$$

The sum of two vectors \bar{A} and \bar{B} results in a third vector $\bar{C} = \bar{A} + \bar{B}$. There is always a vector \bar{x} such that

$$\bar{A} + \bar{x} = \bar{B}$$

and \bar{x} is called the vector difference between \bar{B} and \bar{A} .

1.1. Dot and Cross Products

The dot product of two vectors \bar{A} and \bar{B} is defined as

$$\bar{A} \cdot \bar{B} = AB \cos \theta \quad \dots(1.3)$$

where θ is the angle between the two vectors. In rectangular coordinates the vectors can be expressed as

$$\bar{A} = i_x A_x + i_y A_y + i_z A_z$$

$$\bar{B} = i_x B_x + i_y B_y + i_z B_z \quad \dots(1.4)$$

where i_x, i_y, i_z are the unit vectors in the different axes. The dot product of these two vectors can be expressed as

$$\begin{aligned} \bar{A} \cdot \bar{B} &= (i_x A_x + i_y A_y + i_z A_z) \cdot (i_x B_x + i_y B_y + i_z B_z) \\ &= A_x B_x + A_y B_y + A_z B_z \end{aligned} \quad \dots(1.5)$$

The cross product of two vectors \bar{A} and \bar{B} is defined as

$$\bar{A} \times \bar{B} = AB \cos \theta \quad \dots(1.6)$$

Expressing the vectors as in the form of equation (1.4), the cross product of the two vectors can be evaluated by the determinant.

$$\bar{A} \times \bar{B} = \begin{vmatrix} i_x & i_y & i_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \quad \dots(1.7)$$

Example 1.1. Given $\bar{A} = 2i_x - 5i_y + 3i_z$ and $\bar{B} = 3i_x + B_y i_y + B_z i_z$, find B_y and B_z such that \bar{A} and \bar{B} are parallel.

$$\begin{aligned} \text{Solution. } \bar{A} \times \bar{B} &= \begin{vmatrix} i_x & i_y & i_z \\ 2 & -5 & 3 \\ 3 & B_y & B_z \end{vmatrix} \\ &= i_x (-5B_z - 3B_y) + i_y (9 - 2B_z) + i_z (2B_y + 15) \end{aligned}$$

To satisfy the condition $\bar{A} \times \bar{B} = 0$, each of its components must be individually zero. Thus

$$-5B_z - 3B_y = 0$$

$$9 - 2B_z = 0$$

and

$$2B_y + 15 = 0$$

Since the three equations are not independent of each other, any one of them can be obtained from the other two. If a third order determinant is expanded about a row or a column and two of the cofactors are zero, the third cofactor is also zero.

1.2. Triple Scalar Product

Consider the triple scalar product of three vectors $\bar{D} \cdot \bar{A} \times \bar{B}$. This can be written in its components as

$$\bar{D} \cdot \bar{A} \times \bar{B} = (i_x D_x + i_y D_y + i_z D_z) \cdot \begin{vmatrix} i_x & i_y & i_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \quad \dots(1.8)$$

Since the scalar product of two vectors is the sum of the products of similar components, equation (1.8) can be written as

$$\vec{D} \cdot \vec{A} \times \vec{B} = \begin{vmatrix} D_x & D_y & D_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \quad \dots(1.9)$$

Interchanging the rows, it is clear that

$$\vec{D} \cdot \vec{A} \times \vec{B} = \vec{A} \cdot \vec{B} \times \vec{D} = \vec{B} \cdot \vec{D} \times \vec{A} \quad \dots(1.10)$$

It can also be shown that

$$\vec{A} \times \vec{B} \times \vec{C} = \vec{B} (\vec{A} \cdot \vec{C}) - \vec{C} (\vec{A} \cdot \vec{B}) \quad \dots(1.11)$$

1.3. Representation of Surfaces

A surface of any shape with an area can be represented by a vector normal to the surface, the length corresponding to the area of the surface. A plane surface can be simply be represented by a vector normal to the surface directed outwards as shown in Fig. 1.1.

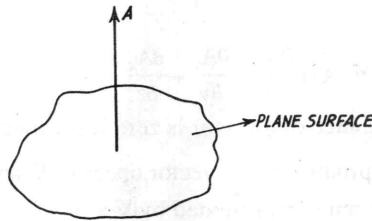


Fig. 1.1. Vector representation of a plane surface.

If the surface is a closed surface, a single vector representation does not hold good and the surface considered has to be split into a large number of plane surfaces, which in turn can be represented by individual vectors as shown in Fig. 1.2.

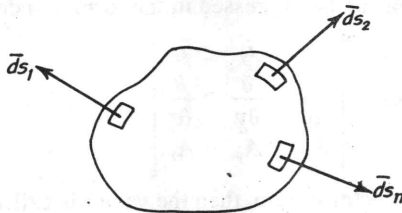


Fig. 1.2. General vector representation.

The total vector representation of such a surface is the sum of individual vectors and leads to the surface integral. If F_1, F_2, \dots, F_n are the

values of the vectors of the surfaces ds_1, ds_2, \dots, ds_n , the total summation can be written as

$$\sum_{r=1}^n \bar{F}_r \cdot \bar{ds}_r = \int_S \bar{F} \cdot \bar{ds} \quad \dots(1.12)$$

This integral is usually known as surface integral.

1.4. Gradient, Divergence and Curl of a Vector

The gradient of a scalar V is defined as

$$\text{grad } V = \frac{\partial V}{\partial x} i_x + \frac{\partial V}{\partial y} i_y + \frac{\partial V}{\partial z} i_z \quad \dots(1.13)$$

The scalar product of the vector operator ∇ and the vector \bar{A} is called the divergence of A represented as $\nabla \cdot \bar{A}$

$$\nabla \cdot \bar{A} = \text{div } \bar{A} = \left(i_x \frac{\partial}{\partial x} + i_y \frac{\partial}{\partial y} + i_z \frac{\partial}{\partial z} \right) \cdot (A_x i_x + A_y i_y + A_z i_z) \quad \dots(1.14)$$

$$\therefore \nabla \cdot A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \quad \dots(1.15)$$

If the divergence of a vector is zero it is called solenoidal.

The vector product of the vector operator ∇ and the vector \bar{A} is called the curl of a vector and is represented by $\nabla \times \bar{A}$

$$\begin{aligned} \text{Curl } A &= \nabla \times \bar{A} \\ &= \left(i_x \frac{\partial}{\partial x} + i_y \frac{\partial}{\partial y} + i_z \frac{\partial}{\partial z} \right) \times (A_x i_x + A_y i_y + A_z i_z) \\ &= \left(\frac{\partial A_x}{\partial y} - \frac{\partial A_y}{\partial x} \right) i_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) i_y + \left(\frac{\partial A_y}{\partial z} - \frac{\partial A_z}{\partial y} \right) i_z \quad \dots(1.16) \end{aligned}$$

This expression can be expressed in the form of a determinant as

$$\nabla \times \bar{A} = \begin{vmatrix} i_x & i_y & i_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} \quad \dots(1.17)$$

If the curl of a vector is zero, then the vector is called irrotational.

Example 1.2. Show that the vector $yz i_x + xz i_y + xy i_z$ is solenoidal and irrotational.

Solution. Let the vector

$$A = yz i_x + xz i_y + xy i_z$$

To show that $\nabla \cdot A = 0$ (Solenoidal)
 and $\nabla \times A = 0$ (Irrotational)

$$\begin{aligned} \nabla \cdot A &= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \\ &= \frac{\partial}{\partial x} (yz) + \frac{\partial}{\partial y} (xz) + \frac{\partial}{\partial z} (xy) \\ &= 0 \end{aligned}$$

$\therefore \nabla \cdot A = 0$

The vector is solenoidal.

$$\begin{aligned} \nabla \times A &= \begin{vmatrix} i_x & i_y & i_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} \\ &= \begin{vmatrix} i_x & i_y & i_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz & xy \end{vmatrix} \\ &= i_x(1-1) - i_y(1-1) + i_z(1-1) = 0 \\ \nabla \times A &= 0 \end{aligned}$$

The vector is Irrotational.

1.5. Laplacian

The second order differential operator is called Laplacian defined as the divergence of a gradient of a scalar.

Laplacian $f = \nabla^2 f = \nabla \cdot \nabla f$

$$= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \quad \dots(1.18)$$

$\therefore \nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right)$

is called the Laplacian.

1.6. Identities

The different identities which are widely used in deriving field equations are given below :

- (a) $\nabla \cdot (\nabla \times \bar{A}) = 0$
- (b) $\nabla \times (\nabla f) = 0$
- (c) $\nabla \times \nabla \times \bar{A} = \nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A}$
- (d) $\nabla \cdot \bar{A} \times \bar{B} = \bar{B} \cdot \nabla \times \bar{A} - \bar{A} \cdot \nabla \times \bar{B}$

$$(e) \quad \nabla(ab) = a\nabla b + b\nabla a$$

$$(f) \quad \nabla(\bar{A} \cdot \bar{B}) = (\bar{A} \cdot \nabla)\bar{B} + (\bar{B} \cdot \nabla)\bar{A} + \bar{A} \times (\nabla \times \bar{B}) + \bar{B} \times (\nabla \times \bar{A})$$

$$(g) \quad \nabla \times (\bar{A} \times \bar{B}) = \bar{A} \nabla \cdot \bar{B} - \bar{B} \nabla \cdot \bar{A} + (\bar{B} \cdot \nabla)\bar{A} - (\bar{A} \cdot \nabla)\bar{B}$$

Some of the identities are proved and the rest are left to the student as exercise.

$$\begin{aligned} (a) \quad \nabla \cdot (\nabla \times \bar{A}) &= \sum i_x \frac{\partial}{\partial x} \left[i_x \times \frac{\partial \bar{A}}{\partial x} + i_y \times \frac{\partial \bar{A}}{\partial y} + i_z \times \frac{\partial \bar{A}}{\partial z} \right] \\ &= \sum i_x \cdot \left[i_x \times \frac{\partial^2 \bar{A}}{\partial x^2} + i_y \times \frac{\partial^2 \bar{A}}{\partial x \partial y} + i_z \times \frac{\partial^2 \bar{A}}{\partial x \partial z} \right] \\ &= \sum \left(i_x \cdot i_x \times \frac{\partial^2 \bar{A}}{\partial x^2} \right) + \sum \left(i_x \cdot i_z \times \frac{\partial^2 \bar{A}}{\partial x \partial y} \right) \\ &\quad + \sum \left(i_x \cdot i_z \times \frac{\partial^2 \bar{A}}{\partial x \partial z} \right) \\ &= \sum i_x \times i_x \cdot \frac{\partial^2 \bar{A}}{\partial x^2} + \sum i_x \times i_y \cdot \frac{\partial^2 \bar{A}}{\partial x \partial y} \\ &\quad + \sum i_x \times i_z \cdot \frac{\partial^2 \bar{A}}{\partial x \partial z} \\ &= 0 + \sum \left[i_z \cdot \frac{\partial^2 \bar{A}}{\partial x \partial y} - i_y \cdot \frac{\partial^2 \bar{A}}{\partial x \partial z} \right] \\ &= 0 \end{aligned}$$

$$\begin{aligned} (d) \quad \nabla \cdot \bar{A} \times \bar{B} &= \sum i_x \frac{\partial}{\partial x} (\bar{A} \times \bar{B}) \\ &= \sum i_x \cdot \left(\frac{\partial \bar{A}}{\partial x} \times \bar{B} + \bar{A} \times \frac{\partial \bar{B}}{\partial x} \right) \\ &= \sum i_x \cdot \left(\frac{\partial \bar{A}}{\partial x} + \bar{B} \right) + \sum i_x \cdot \left(\bar{A} \times \frac{\partial \bar{B}}{\partial x} \right) \\ &= \bar{B} \cdot \left(\sum i_x \times \frac{\partial \bar{A}}{\partial x} \right) - \bar{A} \cdot \left(\sum i_x \times \frac{\partial \bar{B}}{\partial x} \right) \\ &= \bar{B} \cdot \nabla \times \bar{A} - \bar{A} \cdot \nabla \times \bar{B} \end{aligned}$$

$$(g) \quad \nabla \times (\bar{A} \times \bar{B}) = \sum i_x \times \frac{\partial}{\partial x} (\bar{A} \times \bar{B})$$

$$\begin{aligned}
 &= \sum i_x \times \left(\frac{\partial \bar{A}}{\partial x} \times \bar{B} + \bar{A} \times \frac{\partial \bar{B}}{\partial x} \right) \\
 &= \sum i_x \times \left(\frac{\partial \bar{A}}{\partial x} \times \bar{B} \right) + \sum i_x \times \left(\bar{A} \times \frac{\partial \bar{B}}{\partial x} \right) \\
 &= \sum \left[\left(i_x \cdot \bar{B} \right) \frac{\partial \bar{A}}{\partial x} - \left(i_x \cdot \frac{\partial \bar{A}}{\partial x} \right) \bar{B} \right] \\
 &\quad + \sum \left[\left(i_x \cdot \frac{\partial \bar{B}}{\partial x} \right) \bar{A} - \left(i_x \cdot \bar{A} \right) \frac{\partial \bar{B}}{\partial x} \right] \\
 &= \sum \left(i_x \cdot \bar{B} \right) \frac{\partial \bar{A}}{\partial x} - \bar{B} \left(\sum i_x \cdot \frac{\partial \bar{A}}{\partial x} \right) \\
 &\quad + \bar{A} \left(\sum i_x \cdot \frac{\partial \bar{B}}{\partial x} \right) - \sum \left(\bar{A} \cdot i_x \right) \frac{\partial \bar{B}}{\partial x} \\
 &= \bar{A} (\nabla \cdot \bar{B}) - (\nabla \cdot \bar{A}) \bar{B} + (\bar{B} \cdot \nabla) \bar{A} - (\bar{A} \cdot \nabla) \bar{B}.
 \end{aligned}$$

1.7. Stokes' Theorem

The Stokes' theorem states that the surface integral of a vector field \bar{A} over a surface s is equal to the line integral of \bar{A} around contour c .

i.e.
$$\int_s (\nabla \times \bar{A}) \cdot ds = \oint_c \bar{A} \cdot dL \quad \dots(1.19)$$

Example 1.3. A circle of radius 2 with centre at the origin rests in the yz plane. If $\bar{A} = i_x 3y^2 + i_y 4z + i_z 6y$ find $\oint_c \bar{A} \cdot dL$ where the contour is the circumference of the circle.

Solution. To evaluate the integral $\oint_c \bar{A} \cdot dL$ it requires that an expression for dL on the circumference of the circle in terms of x, y, z be found. In order to lessen the process of evaluation, the Stoke's theorem is directly applied i.e.

$$\begin{aligned}
 \oint_c \bar{A} \cdot dL &= \int_s \nabla \times \bar{A} \cdot ds \\
 \text{Curl } A &= \nabla \times A = \begin{vmatrix} i_x & i_y & i_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3y^2 & 4z & 6y \end{vmatrix} \\
 &= i_x (6 - 4) + i_y (0) + i_z (-6y) = 2i_x - 6yi_z.
 \end{aligned}$$

Since the plane of the circle is normal to the x axis, the unit vector is in x direction.

$$\begin{aligned} \therefore \oint_s \nabla \times \bar{A} \cdot ds &= \int_s (2i_x - 6yi_z) \cdot i_x ds \\ &= 2 \int_s ds = 2\pi r^2 \\ &= 2\pi (2^2) \quad [\because r = 2] \\ &= 8\pi. \end{aligned}$$

1.8. Divergence Theorem

This theorem states that the integral of a divergence of a vector over a volume is equal to the surface integral of the vector over a closed surface, i.e.

$$\int_s \bar{A} \cdot ds = \int_V \nabla \cdot A dV \quad \dots(1.20)$$

1.9. Helmholtz's Theorem

It states that a vector field is completely specified by its divergence and curl. If the vector field F is irrotational i.e. $\nabla \times F = 0$, then F can be obtained from a scalar function as

$$F = -\nabla \phi \quad \dots(1.21)$$

If the divergence of the field is identically zero the field is solenoidal i.e. $\nabla \cdot F = 0$ and can be obtained from a vector quantity as

$$F = \nabla \times A \quad \dots(1.22)$$

A general vector can now be expressed as

$$F = -\nabla \phi + \nabla \times A \quad \dots(1.23)$$

The other forms like cylindrical and spherical co-ordinate systems are also widely used and hence the relations and formulas for conversion are presented below.

<i>Cartesian to Cylindrical</i>	<i>Cylindrical to Cartesian</i>
$x = r \cos \theta$	$r = \sqrt{x^2 + y^2}$
$y = r \sin \theta$	$\theta = \tan^{-1} y/x$
$z = z$	$z = z$
$A_r = A_x \cos \theta + A_y \sin \theta$	$A_x = A_r \cdot \frac{x}{\sqrt{x^2 + y^2}} - A_\theta \cdot \frac{y}{\sqrt{x^2 + y^2}}$
$A_\theta = -A_x \sin \theta + A_y \cos \theta$	$A_y = A_r \cdot \frac{y}{\sqrt{x^2 + y^2}} + A_\theta \cdot \frac{x}{\sqrt{x^2 + y^2}}$
$A_z = A_z$	$A_z = A_z$

<i>Cartesian to Spherical</i>	<i>Spherical to Cartesian</i>
$x = r \sin \theta \cos \phi$	$r = \sqrt{x^2 + y^2 + z^2}$
$y = r \sin \theta \sin \phi$	$\theta = \cos^{-1} \frac{z}{\sqrt{x^2 + y^2 + z^2}}$
$z = r \cos \theta$	$\phi = \tan^{-1} y/x$
$A_r = A_x \sin \theta \cos \phi$ $+ A_y \sin \theta \sin \phi$ $+ A_z \cos \theta$	$A_x = \frac{A_r \cdot x}{\sqrt{x^2 + y^2 + z^2}} + \frac{A_\theta xz}{\sqrt{(x^2 + y^2)(x^2 + y^2 + z^2)}} - \frac{A_\phi y}{\sqrt{x^2 + y^2}}$
$A_\theta = A_x \cos \theta \sin \phi$ $+ A_y \cos \theta \sin \phi$ $- A_z \sin \theta$	$A_y = \frac{A_r \cdot y}{\sqrt{x^2 + y^2 + z^2}} + \frac{A_\theta yz}{\sqrt{(x^2 + y^2)(x^2 + y^2 + z^2)}} + \frac{A_\phi y}{\sqrt{x^2 + y^2}}$
$A_\phi = -A_x \sin \phi$ $+ A_y \cos \phi$	$A_z = \frac{A_r z}{\sqrt{x^2 + y^2 + z^2}} - \frac{A_\theta \cdot \sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2 + z^2}}$

Example 1.4. Two vector fields are given by

$$A = 3y^2 i_x + 4zi_y + 6yi_z$$

and

$$B = e^{-x} \cos y i_x + e^{-x} \sin y i_y - \frac{1}{z^2} i_z.$$

Determine whether these vector fields may be gradients of scalar fields.

Solution. For the vectors to be gradients of scalar fields the curl of the vectors should be zero.

$$\begin{aligned} \nabla \times A &= \begin{vmatrix} i_x & i_y & i_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3y^2 & 4z & 6y \end{vmatrix} \\ &= 2i_x - 6yi_z. \end{aligned}$$

Since the curl of A is not zero, A cannot be the gradient of a scalar field.

$$\nabla \times B = \begin{vmatrix} i_x & i_y & i_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^{-x} \cos y & e^{-x} \sin y & \frac{1}{z^2} \end{vmatrix}$$

$$= 0.$$

Therefore B is the gradient of a scalar field.

Example 1.5. Let $r = xi_x + yi_y + zi_z$ be the position vector of a point $P(x, y, z)$. Find $(\text{div } r)$ and show that $\int_s r \cdot ds$ over any closed surface is equal to three times the volume enclosed by that surface.

Solution. $\nabla \cdot r = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(z)$

$$= 3.$$

By Divergence theorem,

$$\int_s r \cdot ds = \int_V \nabla \cdot r dV$$

$$= \int_V 3 dV$$

$$= 3V.$$

Example 1.6. Determine the constant c such that the vector

$$\bar{F} = (x + ay) i_x + (y + bz) i_y + (x + cz) i_z$$

will be solenoidal.

Solution. A vector \bar{F} is solenoidal if

$$\nabla \cdot \bar{F} = 0$$

$$\therefore \nabla \cdot \bar{F} = \frac{\partial}{\partial x}(x + ay) + \frac{\partial}{\partial y}(y + bz) + \frac{\partial}{\partial z}(x + cz)$$

$$= 1 + 1 + c$$

For F to be solenoidal

$$1 + 1 + c = 0$$

$$\therefore c = -2.$$

Example 1.7. Use Stoke's Theorem to prove that the identity

$$\text{Curl}(\text{grad } \phi) = 0.$$

Solution. Stokes Theorem states that

$$\oint_c F \cdot dL = \int_S \text{curl } F \cdot dS$$

This book will serve as a text book for those who are leading to bachelor's degree in Electrical Engineering and Electronics and Communication Engineering. Out of the experience of teachers teaching this subject, the need of a comprehensive book covering the aspects of fields, Waves and Antennas was felt, the result is this book.

- Vector Analysis
- Electrostatic Field
- Magnetic Field
- Time Varying Fields and Maxwell's Equations
- Plane Wave Propagation
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