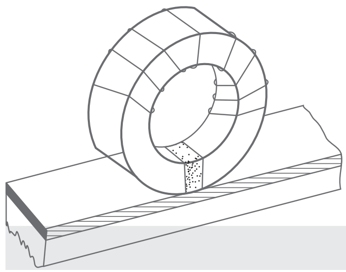

CHAPTER 0

Introduction

0.1 ROLE OF MEASUREMENT SYSTEMS

Measurement provides us with an understanding of physical phenomena and hence the ability to make measurements is vital to an understanding of the physical world in which we live. In the fields of modern research there is a greater and greater requirement for precise measurements as more and more complex experiments are carried out. The development of science and technology is, therefore, very much dependent upon a parallel development of measurement techniques. Knowledge largely depends on measurement, and the technology of measurement, called instrumentation, serves not only science but all branches of engineering, medicine and almost every spheres of human life. Measuring instruments may be used in monitoring of processes and operations. Another important type of application for measuring instruments is in control of processes and operations where instrument serves as a component of an automatic control system. Most specialized instruments are used in experimental scientific and engineering work.

Because of complex experiments needed to be carried out in the fields of modern research and automation in many areas of industry, there is a need for more precise measurements. This has been largely responsible for the increased use of electronic instruments. An electrical signal is a versatile quantity because of the fact that it can be easily amplified, attenuated, measured, rectified, modified, modulated, transmitted, and controlled. This fact created the interest to use electrical methods to measure non-electrical quantities. For this purpose a device known as a *transducer* is used to convert the non-electrical quantities into electrical quantities. Then the quantities are indirectly manipulated with speed and versatility, found in electrical measurement systems. Further higher speed and versatility found in electronic instruments make them more popular. The electronic instruments, nowadays, are used for computing, manipulating, and processing information in much the same way as the mind.

Increase in availability and types of computer facilities, and decrease in the cost of various modules required for digital systems are accelerating the development of digital instrumentation for the measurement. The digital form of measurement is also used to display the measured quantity in readable numbers instead of a deflection of a pointer on a scale which completely eliminates a number of human errors. The inclusion of a topic on digital measurement systems, therefore, becomes a must.

0.2. PRELIMINARIES OF MEASUREMENTS

Though, today, we have very sophisticated measurement systems, we cannot think of a measurement without error. The error can be reduced by selecting a proper method of measurement and by taking some necessary precautions at the time of measurement. Recording the measured data also plays an important role.

Choice of Measurement Method. It is an important work to select a suitable method of measurement before starting it. At the time of selection, the following points should be kept in mind :

1. Apparatus available
2. Accuracy desired
3. Time required
4. Difficulties in measurement
5. Necessary conditions of measurement.

A method must be selected that makes use of available apparatus to obtain the desired result without sacrifying the desired accuracy. During the time of selection it should be kept in mind that the difficulties faced during measurement can be easily overcome. At the same time necessary conditions must be fulfilled. It is not wise to choose a method giving higher accuracy than desired one at the cost of time and money. The method should be as simple as possible and consistent with requirements of the task. It is always beneficial to study carefully different apparatus before starting the actual measurement. Before starting measurement with a particular method, it is advisable to look whether there is a better and simpler method. Wide experiences in the field of measurement are very helpful in selecting a method.

After selection of the method and the apparatus, it is important that they be intelligently used. For this each piece of apparatus and its method of operation should be thoroughly understood. The line diagram of electrical circuits should be drawn in the beginning. It saves time and minimizes the possibilities of wrong connections of apparatus. Before energising, it is necessary to check measuring instruments, other apparatus and electrical circuits. Taking necessary precautions gives better result.

Record Preparation. Record preparation of any experiment is not less important. Therefore the data necessary for preparation of record must be written carefully. These data are recorded mostly in a bound note-book. Sometimes data may be written on loose sheets and after arranging them properly they can be permanently bound together. The habit of memorizing the data or writing them in short is not good because there is every possibility of forgetting some of the data which may cause great inconvenience. Without the line diagram of electrical circuits and specifications of all apparatus, a report can never be said to be complete. Report should be such that the experiment can be repeated with the same method and apparatus at any time. Thus error in the result due to unusual functioning of the instrument or due to any other reason can be removed. Any unusual behaviour of apparatus should be noted on the data sheet, and, if recorded data are rejected or discarded, the reasons for the action should be recorded at the same time. In short, the report must be such that any other person can get every information about the experiment just by going through the record or even can repeat the experiment at any time.

Precautions in Measurement. Certain precautions are essential which must be taken to ensure the safe and efficient use of instruments and also to get better result. There are some precautions that should be taken, in general, regardless of the instruments and the type of measurement undertaken. In making electrical connections, it should be seen that contact surfaces are clean, nuts are firmly tightened, wires and cables have sufficient cross-section for the expected current, and insulation is appropriate for the voltage in use. Sliding contacts should be cleaned occasionally. For the measurement of a large alternating current or voltage, a low range instrument with an instrument transformer should be preferred to a large range instrument. Before energising a circuit all components should be checked to ensure the proper connections, and appropriate range of apparatus. Protective resistors should always be inserted where necessary. At the time of opening a circuit the first break should be made at the terminal nearest the power source. The reversed procedure should be adopted at the time of making the connection, *i.e.* the connection at the power terminal should be made at the last. The operator should be careful where there is a chance of electric shocks.

Other precautions are applicable directly to instruments rather than to the general circuit. The position of the range switch of a multi-range instrument should be checked before closing the circuit. If the initial current is much higher than the steady-state current, the current coils of instruments should be protected against the initial high current by a short-circuiting switch. When delicate instruments, such as micro-ammeters or pivoted galvanometers, are to be moved, they should be protected against mechanical damage by shorting the terminals to provide heavy over damping. Where a coil clamp is provided it should always be set when the instrument is moved. Handling of instruments should be careful giving special attention to laboratory standards. Pivoted instruments should never be placed where they may be exposed to vibration.

0.3. GENERALIZED APPROACH TO MEASUREMENT SYSTEMS

Functional elements. A generalised approach to describe the operation and the performance of measuring instruments and associated equipment is possible and desirable without going for any description of specific physical hardware. The operation can be described in terms of the functional elements of measurement systems, and the performance is defined in terms of the static and dynamic performance characteristics. A measurement system consists of three functional devices :

1. Input device
2. Signal conditioning and processing device
3. Output device.

The operation of the three devices is illustrated in Fig. 0.1. Every instrument and measurement system is composed of one or more of these functional elements.

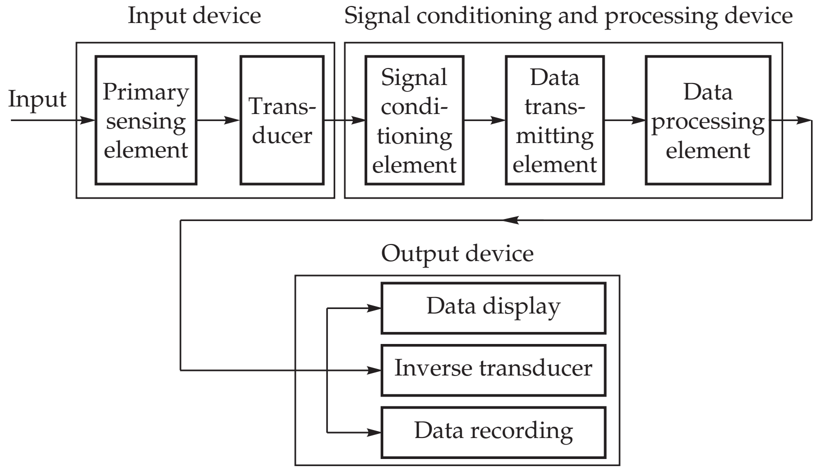

Fig. 0.1. Block diagram illustrating operation of three devices of measurement systems.

Input device. The function of the input device is to sense the quantity under measurement (measurand) and to change it to a proportional electrical quantity, if the measurand is a non-electrical quantity. The non-electrical signal is converted to an electrical signal by means of a transducer ; for example, a strain gauge, thermistor and thermocouple. Thus, the input device consists of two elements : a primary sensing element and a transducer. The primary sensing element can have non-electrical input and output, for example, a manometer, Bourdon tube and diaphram, or they may have electrical input and output, for example, a filter and rectifier. In first case there will be a transducer alongwith the primary sensing element, while in second case there is no need of a transducer and the output of the primary sensing element directly goes to the next block. It is important to note that an instrument always extracts some energy from the measured medium, disturbing the measured quantity. This makes a perfect measurement (theoretically) impossible. The primary job of the input device is, therefore, to select the best sensing element which minimizes this effect.

Signal conditioning and processing device. This functional block consists of three functional elements, namely, signal conditioning element, data transmitting element and data processing element. The output of the input device may not be suitable for the indicating instrument. So, in performing the task of measurement, an instrument may require that the signal output of the input device be manipulated in some way. For this the signal may be changed in numerical value according to some definite rule by preserving the physical nature of the signal. For example, an electronic amplifier receives a small voltage signal as input and delivers an output signal that is also a voltage but is some constant times the input. Another example may be an A/D converter which converts an analogue signal into a digital signal which is essential for measuring an analogue quantity by a digital instrument. An element that performs such a function is called signal manipulating or signal conditioning element. The functional elements may be separated and in that case it becomes necessary to transmit the data from one group of elements to another by using a telemetry system. This telemetry system is called a data transmitting element. The data may be processed, recorded and compared with the input *via* an inverse transducer before being communicated to human observer. The element performing these task is called a data processing element.

Output device. Finally, the information about the measured quantity is to be communicated to a human being for monitoring, control, or analysis purposes. So, it is essential to put the information into a form recognizable by one of the human senses. An element that does this work of translation is called a data presentation element or an output element. This function includes the simple display system and the recording system.

Fig. 0.1 shows a measurement system clearly separated into different functional blocks. This may lead the reader to conclude that the physical apparatus are precisely separable into subassemblies performing some specific functions. This is not the case in general. A specific piece of hardware may perform several of the basic functions. It is also important to note that the sequence of the functional elements shown in Fig. 0.1 is not a rigid one. For example, in the figure the signal conditioning element has been shown to follow a transducer (a variable conversion element), but this is not a rule. The signal conditioning element may precede the transducer.

Input-output configuration. Before we discuss about the performance characteristics of an instrument in the next chapter, it is desirable to study a generalized configuration giving the significant input-output relations of the instrument. There are three types of inputs present in any instrument : *desired inputs, interfering inputs,* and *modifying inputs*. As it is evident from the name, desired inputs represent the quantity that the instrument is specifically intended to measure. Any quantity to which the instrument is unintentionally sensitive is known as interfering input. The quantities that cause a change in the input-output relations for the desired and interfering inputs are termed as modifying inputs. We see from the above definitions of the three inputs that the desired input and the interfering input may be considered as two independent inputs having two separate input-output relations (transfer functions, see Section 1.3) as given by two independent blocks G_D and G_I as shown in the generalized configuration of Fig. 0.2.

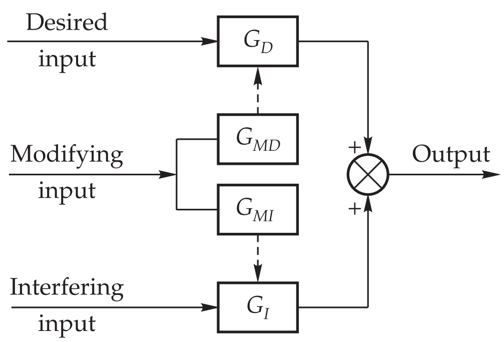

Fig. 0.2. A generalized configuration of a measurement system giving significant input-output relations.

The modifying inputs cause a change in G_D and G_I. The symbols G_{MD} and G_{MI} represent (in the appropriate form) the specific manners in which the modifying inputs affect G_D and G_I.

To illustrate the concept of various inputs and their effects on the output, let us consider a resistance strain gage method of measuring strain in a beam. A scheme is shown in Fig. 0.3.

The strain gage (for detail see chapters 9 and 10) consists of a fine wire grid of resistance R_s cemented firmly to the beam at the point where the strain is to be measured. The strain gage is connected in one arm of the wheatstone bridge. The change in resistance ΔR_s is proportional to the strain, e, *i.e.*

$$\Delta R_s = G_f R_s e$$

where G_f is called the gage factor. When the beam is unstrained, the bridge is balanced by adjusting R_3, *i.e.,* under this condition,

$$R_s = \frac{R_1}{R_2} R_3 \qquad \text{...(0.3-1)}$$

and

$$E_0 = 0 \qquad \text{...(0.3-2)}$$

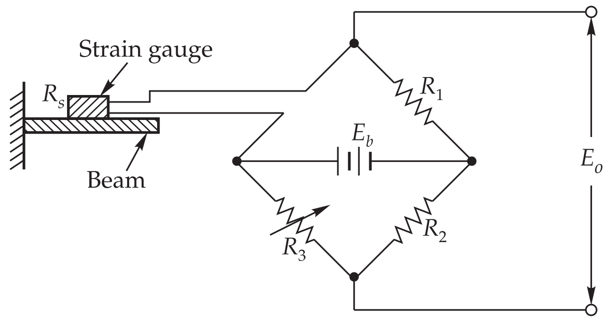

Fig. 0.3. A scheme for measuring strain by using a resistance strain gage.

When the beam is strained and e is the strain in the beam, the bridge output E_0 is given by

$$E_0 = -\,G_f R_s e \,.\, \frac{E_b R_1}{(R_s + R_1)^2} \qquad ...(0.3\text{-}3)$$

The desired input is evidently the strain e, giving a proportional output voltage E_0 assuming G_f and E_b constant. The interfering input is the gage temperature. A change in the gage temperature causes a change in the gage resistance which causes in turn a voltage output even if there is no strain in the beam. Another interfering effect of the temperature is to cause a differential expansion of the gage and the beam. This gives rise to strain and hence a voltage output even though there is no strain in the beam due to force applied to it. Temperature also acts as a modifying input since it also changes the gage factor. Another modifying input is the battery voltage E_b. Both these are modifying inputs as they tend to change the proportionally constant between e (desired input) and E_0 [see Eq. (0.3-3)].

As the spurious (or noise) inputs (interfering and modifying) affect the output for the desired input, it is always tried to reduce their effects. For this it is desirable to make G_{MD} and G_{MI} as small as possible. Negative feedback can be used to reduce the effects of interfering and modifying inputs. The effects of noise inputs can also be reduced by signal filtering, correct grounding, proper screening and shielding. If the magnitude of these inputs and their effects are known, then it is possible to calculate corrections which may be added to or subtracted from the indicated output so that there leaves (ideally) only the component associated with the desired input.

0.4. UNITS

In the first stage of development of the technique of measurement, specific physical standards entirely unrelated to each other gave varying degrees of satisfaction. But with the development of science and technology of measurement a system of units with a logical and simple relation became desirable. A number of systems of electrical units have been used in electrical measurements at various times. Some of them are only of historical interest, some are used chiefly in theoretical discussions, and some others are or have been employed in measurement and have been accepted for this purpose on either a national or international basis. So, it is in our interest to acquaint ourselves with some of important and commonly used systems. Before examining the systems of units it is essential to discuss in short some of the general considerations involved in defining or constructing any system of units.

Fundamental and Derived Quantities. Fundamentally, the measurement of a quantity means the comparison of the quantity with a standard of same kind of quantity. The magnitude of the quantity being measured can be expressed in terms of the chosen unit and a numerical multiplier. But, it is never convenient to have a series of standards, one each for every quantities and also we cannot choose freely the unit of a quantity, because physical quantities are not independent of each other, but they are related by some physical equations. However, it is fortunate that if there are P kinds of quantities to be evaluated and Q independent physical equations expressing relations between them, the sizes of units of only $(P - Q)$ of the quantities can be chosen, and then the Q physical equations can be used to fix the sizes of units of the remaining quantities so that numerical multipliers are usually unity. The $(P - Q)$ quantities, units of which are independently chosen, are called *fundamental quantities* and remaining Q quantities are called *derived quantities*. The units of fundamental quantities are called *fundamental units* and the units of derived quantities are called *derived units*. Surprisingly, in mechanics there are only three fundamental quantities, namely, Length, Mass and Time. Electrical and magnetic quantities involve, in addition, the properties of the media in which the electrical and magnetic actions take place, *i.e.* the permittivity in the case of electrostatic forces and the permeability in the case of magnetic forces.

System of units. A complete set of units, both fundamental and derived, for all kinds of quantities is called a system of units. Some systems of units which were in common use before SI units and are still in use to some extend are briefly introduced here.

F.P.S. system. F.P.S. system or British system of units has foot, pound and second as fundamental units for length, mass and time respectively. This system of units was most commonly used in mechanical work.

C.G.S. system. Before the M.K.S. system of units came in existence the most commonly used system of units in electrical works was the C.G.S. system of units. In this system the fundamental units of length, mass and time are centimetre, gram and second respectively. To define electrical units in terms of mechanical units, there are two convenient starting points. The expression for mechanical force between charges at rest (Coulomb's law) may be used to develop an electrostatic system of units, while the force equation between charges in motion (Ampere's law) may be used to formulate an electromagnetic system of units. The electrostatic system of units for electrical quantities involves (in the addition to fundamental units of length, mass and time) permittivity of the medium. The electromagnetic system of units for electric works involves unit of permeability of the medium as the fourth fundamental unit.

M.K.S. system (Giorgi system). This system was first suggested by the Italian physicist, Prof. Giorgi, in 1901 and is known as the Giorgi-M.K.S. system. This system used the metre, kilogram and second as fundamental mechanical units instead of centimetre, gram, and second of C.G.S. system. This system was adopted by IEC (International Electrical Commission) at its meeting in 1938. The commission recommended the permeability of free space with the value of $\mu_0 = 10^{-7}$ as the fourth unit connecting the electrical to the mechanical units. The advantages of the M.K.S. system over the C.G.S. system are :

1. Its units are identical with the practical units.

2. Its units are same whether build up from the electromagnetic or electrostatic theory.

3. The rather cumbersome conversions necessary to relate the units of the e.m. and e.s. C.G.S. systems to those of the practical system are avoided in this system.

Rationalized M.K.S. system. In some of the physical equations, which must be used in electrical theories and engineering, a numerical factor 4π appears. This appearance can never be prevented as it is fundamental in the geometry of the sphere, but by changing the sizes of certain of the units, it can be removed from equations which are frequently used and shifted to occur in equations which are less common. (See Table 0.1). The M.K.S. system with above changes in physical equations is called rationalized M.K.S. system. The basis of rationalization is the concept of unit flux. In unrationalized system the unit of magnetic flux (or electrostatic flux) is defined as the quantity of flux passing through a unit area on a sphere of unit radius at the centre of which is a unit magnetic pole (or unit charge). Since the area of such a sphere is 4π, the total flux emitted by a unit pole (or charge) is 4π unit of unrationalized system. In the rationalized system, the unit of flux is taken as the total flux emitted by a unit pole (or charge). So one unit of flux in rationalized system is equal to 4π units of flux in unrationalized system.

Table 0.1. Unrationalized and Rationalized form of Electrical and Magnetic Equations

Equations	*Unrationalized*	*Rationalized*
Force F between parallel conductors carrying currents i_1 and i_2 and separated by distance d on a length l.	$F = \frac{2\mu i_1 i_2 \, . \, l}{d}$	$F = \frac{\mu i_1 i_2 l}{2\pi d}$
Force F between two charges Q_1 and Q_2 separated by distance d.	$F = \frac{1}{\varepsilon}\frac{Q_1 Q_2}{d^2}$	$F = \frac{1}{\varepsilon}\frac{Q_1 Q_2}{4\pi d^2}$
Electric flux density D at a distance d from a point charge Q.	$D = \frac{Q}{d^2}$	$D = \frac{Q}{4\pi d^2}$
Electric field strength E near a uniformly charged plane surface of charge density σ.	$E = \frac{4\pi\sigma}{\varepsilon}$	$E = \frac{\sigma}{\varepsilon}$
Capacitance of concentric capacitor of radii a and b, and length l.	$C = \frac{\varepsilon l}{2 \log_e (b/a)}$	$C = \frac{2\pi\varepsilon l}{\log_e (b/a)}$
Capacitance of parallel plate capacitor of plate area A and separation d.	$C = \frac{\varepsilon A}{4\pi d}$	$C = \frac{\varepsilon A}{d}$
Capacitance of concentric sphere capacitor of radii a and b.	$C = \frac{\varepsilon ab}{b - a}$	$C = \frac{4\pi\varepsilon ab}{b - a}$
Force F between small magnetic poles separated by distance d.	$F = \frac{1}{\mu}\frac{m_1 m_2}{d^2}$	$F = \frac{1}{\mu}\frac{m_1 m_2}{4\pi d^2}$
Magnetic flux density B at distance d from a point magnetic pole m.	$B = \frac{m}{d^2}$	$B = \frac{m}{4\pi d^2}$
Inductance L of a single turn solenoid of cross-sectional area A and length d.	$L = \frac{4\pi\mu A}{d}$	$L = \frac{\mu A}{d}$

This view-point certainly changes the magnitudes of some units. However the change in the magnitude of the unit affected depends upon some arbitrary choice. If the sizes of the units of mass, length, time and permeability are left unchanged, then practically all the other units are changed in size. This approach was not accepted as it changes the units of all the six electrical quantities of our interest (such as, charge, current, e.m.f., resistance, capacitance and inductance). In the accepted choice the permeability (μ) of free space was taken as $4\pi \times 10^{-7}$. Hence the permittivity (ε) of free space is $\frac{10^{-9}}{36\pi}$, since $\varepsilon = \frac{1}{c\mu}$, where c is the velocity of light. In this case the units of the six electrical quantities are left unchanged.

In July 1950 the International Electro-technical Commission recommended the ampere as the fourth fundamental unit of rationalized M.K.S. system. This is the reason that this system is sometimes called as rationalized M.K.S.A. system.

SI Units. At the Eleventh General Conference on Weights and Measures (La Conference Generale des Poids et Measures, CGPM) held in Paris in 1960, it was agreed to adopt the International System of Units (Systeme Internationale d'Unites, abbreviated as SI). Previously, the different systems of units had been in use in various countries causing great inconvenience in international uses. Fundamentally, SI units is an absolute M.K.S. system of units with addition of three more Basic (fundamental) units, namely, ampere (A) for current as fourth basic unit of electrical system, kelvin (K) for temperature as fourth basic unit of thermodynamic system and candela (cd) for luminous intensity as fourth basic unit of illumination system. Thus, this system was having six basic units. In addition, there were three supplementary units, namely, radian (rad) for plane angle, steradian (sr) for solid angle and mol (n) for quantity of substance. For example, the angle subtended by a circle at its centre is 2π rad, the solid angle of a sphere is 4π sr and the quantity of substance in 24 g of carbon is 24/12 = 2 mol and in 18 g of H_2O is 1 mol (*i.e.*, 1 mol means 1 molecular weight of a substance). Later mol was included as a basic unit. Thus, there are seven basic units and two supplementary units. The basic units are defined in Table 0.2. The derived units are expressed in terms of these basic and supplementary units. For example, the units of charge (coulomb) is ampere second (As) and the unit for luminous flux (lumen) is candela-steradian (cdsr). However, special, names and symbols are used for various derived units. Table 0.3 lists the dimensions of various derived quantities with their names and symbols of their units approved by CGPM.

Table 0.2. Basic Units SI System

Physical quantity	*Name of units*	*Unit symbol*	*Definition*
Length	metre	m	1 650 763.73 wavelength, in vacuum, of the radiation corresponding to the transition between the energy levels $2p_{10}$ and $5d_5$ of the krypton 86 atom. Accuracy is 3 parts in 10^{11}.
Mass	Kilo gramme	kg	Mass of the international prototype which is in the custody of the Bureau Internationals des Poids et Measures (BIPM) at Severes near Paris, France. Accuracy is 1 part in 10^9.
Time*	second	s	Duration of 9192631770 periods of the radiation corresponding to the transition between the two energy levels of the caesum 133 atom. Accuracy is 2 parts in 10^{13}.
Electric current	ampere	A	The constant current which, if maintained in two parallel rectilinear conductors of infinite length and of negligible circular cross- section, would produce a force equal to 2×10^{-7} newton per metre length between the two conductors placed 1 metre apart in vacuum. Accuracy is 2 parts in 10^6.
Thermo-dynamic temperature	kelvin	K	The interval of the thermo-dynamic scale on which the temperature of the triple point of water is 273.16 Kelvin. Accuracy is 1 part in 10^4.

*See also Standards of time and frequency in Section-0.6.

Table 0.2. Continue

Luminous intensity	candela	cd	It is the luminous intensity, in the perpendicular direction, of a surface of $(1/6) \times 10^{-5}$ of a black body at the tem-perature of freezing platinum under a pressure of 101 325 newton per metre. Accuracy is 5 parts in 10^3.
Amount of substance	mol	n	The amount of substance which contains as many elementary entities as there are atoms in 0.012 kg of carbon 12. Accuracy is 1 part in 10^6.

Sometimes, SI units may be too large or too small depending upon the magnitude of the quantity *e.g.*, the metre, for instance, is convenient unit for such things as site plans but is too large for precision engineering purposes and too small for the expression of large distances between towns. Hence, multiples and sub-multiples of SI units are formed by means of prefixes ranging from 10^{-18} to 10^{+12} as given in Table 0.4. However, the preference has been expressed for multiples separated by the factor 10^3 that is of the form $10^{\pm 3n}$, where n is an integer. But, in some cases, for the sake of convenience of expressing the unit of a quantity, we may have to deviate from this rule. There are some other rules which should also be followed :

(1) Compound prefixes are not to be used. Thus use nm (nano-meter) for 10^{-9} m and not mμm (milli-micrometer).

(2) It is best to use unit resulting in numerical values ranging from 0.1 to 1000, *e.g.* it is preferred to expressed a length of 0.00254 m as 2.54 mm.

3. To express any measurement, it is unnecessary to use several different sized units. In SI units, a length may be expressed as 1.35 m or 1350 mm instead of 1 m 35 cm or 1 m 350 mm.

4. No abbreviation for a unit should be followed by a full stop.

5. After 3 digits on either side of decimal, there should be a gap instead of a comma *e.g.*, 1 000 is correct but not 1,000.

Table 0.3. Derived SI Units

Quantity	*Name of SI Units*	*Unit Abbreviation*
SPACE AND TIME		
Area	square metre	m^2
Volume	cubic metre	m^3
Velocity	metre per second	m/s
Acceleration	metre per second square	m/s^2
PERIODIC AND RELATED PHENOMENA		
Frequency	hertz	Hz
Rotational frequency	hertz	Hz
Wavelength	metre	m
MECHANICS		
Density	kilogram per cubic metre	kg/m^3
Moment of inertia (dynamic moment of inertia)	kilogramme metre square	$kg\ m^2$
Force	newton	$N = kg\ m/s^2$
Specific weight (specific density)	newton per cubic metre	N/m^3
Moment of force, bending moment, torque, moment of couple	newton metre	Nm
Pressure, Normal stress, Shear stress, Young's modulus, Shear modulus, Bulk modulus	newton per square metre	N/m^2
Energy	joule	J = Nm
Power	watt	W = J/s
HEAT		
Temperature	kelvin	K
Heat	joule	J = Nm
Heat flow rate	watt	W = J/s
Thermal conductivity	watt per metre kelvin	W/m K
coefficient of heat transfer	watt per square metre kelvin	$W/m^2\ K$
Specific heat capacity	joule per kilogramme kelvin	J/kg K

Table 0.3. Continue

LIGHT		
Luminous flux	lumen	lm = cd sr
Illumination	lux	$lx = lm/m^2$
Brightness	candela per square metre	cd/m^2
ELECTRICAL		
Power	watt	W = J/s
Charge	coulomb	C = As
Electric Potential, Potential difference, e.m.f.	volt	V = W/A
Capacitance	farad	F = C/V
Resistance	ohm	Ω = V/A
Magnetic flux	weber	Wb = Vs
Magnetic flux density	tesla	$T = Wb/m^2$
Inductance	henry	H = Wb/A
Electric field strength	volt per metre	V/m
Magnetic field strength	ampere per metre	A/m

Table 0.4. Prefixes and Corresponding Symbols Used in SI Units

Unit multiplier	*Prefix*	*Symbol*
1 000 000 000 000 = 10^{12}	tera	T
1 000 000 000 = 10^9	giga	G
1 000 000 = 10^6	mega	M
1 000 = 10^3	kilo	k
100 = 10^2	hecta	h
10 = 10	deca	de
0.1 = 10^{-1}	deci	d
0.01 = 10^{-2}	centi	c
0.001 = 10^{-3}	milli	m
0.000 001 = 10^{-6}	micro	μ
0.000 000 001 = 10^{-9}	nano	n
0.000 000 000 001 = 10^{-12}	pico	p
0.000 000 000 000 001 = 10^{-15}	femto	f
0.000 000 000 000 000 001 = 10^{-18}	atto	a

0.5. DIMENSIONS OF MECHANICAL AND ELECTRICAL QUANTITIES

It is evident that, disregarding the problem of measurement and the concept of magnitude, length has a quality distinguishing it from all other quantities such as mass, area, or time. This unique quality is called dimensions and is written in a characteristic notation, as for example $[L]$ for length, $[M]$ for mass and $[A]$ for area. However, a separate notation for each quantity is not needed. For example, area is basically length multiplied by length and its dimensions may be expressed by a dimensional equation $[A] = [L^2]$. From this equation it is clear that if the dimensions of length $[L]$ are accepted then there is no need of assigning separate dimensions for area as it is composed of length, $[L^2]$. Like fundamental units, if the dimensions of only fundamental quantities are recognised, then the dimensions of other derived quantities can be expressed in terms of dimensions of fundamental quantities. The dimensions of fundamental quantities are fundamental dimensions while all other dimensions are derived dimensions.

Dimensions of Mechanical Quantities

(1) $$\text{Velocity} = \frac{\text{Length}}{\text{Time}}$$

$$\therefore \quad [V] = [LT^{-1}] \qquad \text{...(0.5-1)}$$

(2) $$\text{Acceleration} = \frac{\text{Velocity}}{\text{Time}} = \frac{\text{Length}}{\text{Time} \times \text{Time}}$$

$$\therefore \quad [a] = [LT^{-2}] \qquad \text{...(0.5-2)}$$

(3) $$\text{Force} = \text{Mass} \times \text{Acceleration}$$

$$= \text{Mass} \times \frac{\text{Length}}{\text{Time}^2}$$

$$\therefore \quad [F] = \frac{[M][L]}{[T^2]}$$

$$= [MLT^{-2}] \qquad \text{...(0.5-3)}$$

(4) $$\text{Work} = \text{Force} \times \text{Distance}$$

$$= \frac{\text{Mass} \times \text{Length} \times \text{Length}}{\text{Time}^2}$$

$$\therefore \quad [W] = [ML^2T^{-2}] \qquad \text{...(0.5-4)}$$

(5) $$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{\text{Mass} \times (\text{Length})^2}{(\text{Time})^3}$$

$$\therefore \quad [P] = [ML^2T^{-3}] \qquad \text{...(0.5-5)}$$

Dimensions of Electrical Quantities

There are two systems of electrical units and, therefore, dimensions of electrical quantities will either be given in e.s. system or be given in e.m. system.

Dimensions in e.s. System

(1) *Charge*. By Coulomb's law, the force exerted between charges Q_1 and Q_2 separated by a distance r is given by

$$F = \frac{Q_1 Q_2}{\varepsilon r^2}$$

$$\text{Dimensionally, } [F] = \frac{[Q^2]}{[\varepsilon][L^2]}$$

or $$[MLT^{-2}] = \frac{[Q^2]}{[\varepsilon L^2]}$$

$$\therefore \quad [Q] = [\varepsilon^{1/2}M^{1/2}L^{3/2}T^{-1}] \qquad \text{...(0.5-6)}$$

(2) *Current*. Current is charge per unit time,

i.e. $$\text{Current} = \frac{\text{Charge}}{\text{Time}}$$

$$\text{Dimensionally, } [I] = \frac{[Q]}{[T]} = \frac{[\varepsilon^{1/2}M^{1/2}L^{3/2}T^{-1}]}{[T]}$$

$$= [\varepsilon^{1/2}M^{1/2}L^{3/2}T^{-2}] \qquad \text{...(0.5-7)}$$

(3) *Potential differences or e.m.f.* Potential difference is work done per unit charge, *i.e.*

$$\text{Potential difference} = \frac{\text{Work}}{\text{Charge}}$$

or $$V = \frac{W}{Q}$$

Dimensionally, $$[V] = \frac{[W]}{[Q]} = \frac{[ML^2T^{-2}]}{[\varepsilon^{1/2}M^{1/2}L^{3/2}T^{-1}]}$$

$$= [\varepsilon^{-1/2}M^{1/2}L^{1/2}T^{-1}] \qquad \text{...(0.5-8)}$$

(4) *Capacitance*. It is given by

$$\text{Capacitance} = \frac{\text{Charge}}{\text{Potential Difference}}$$

or

$$C = \frac{Q}{V}$$

$$\text{Dimensionally, } [C] = \frac{[Q]}{[V]} = \frac{[\varepsilon^{1/2}M^{1/2}L^{3/2}T^{-1}]}{[\varepsilon^{-1/2}M^{1/2}L^{1/2}T^{-1}]}$$

$$= [\varepsilon L] \qquad \text{...(0.5-9)}$$

(5) *Resistance*

$$\text{Resistance} = \frac{\text{Potential difference}}{\text{Current}}$$

or

$$R = \frac{V}{I}$$

$$\text{Dimensionally, } [R] = \frac{[V]}{[I]} = \frac{[\varepsilon^{-1/2}M^{1/2}L^{1/2}T^{-1}]}{[\varepsilon^{1/2}M^{1/2}L^{3/2}T^{-2}]}$$

$$= [\varepsilon^{-1}L^{-1}T] \qquad \text{...(0.5-10)}$$

(6) *Inductance*

$$\text{Inductance} = \frac{\text{e.m.f.}}{\text{Rate of change in current}}$$

or

$$\mathcal{L} = \frac{V}{dI/dT}$$

$$\text{Dimensionally, } [\mathcal{L}] = \frac{[V]}{[I]/[T]} = \frac{[\varepsilon^{-1/2}M^{1/2}L^{1/2}T^{-1}][T]}{[\varepsilon^{1/2}M^{1/2}L^{3/2}T^{-2}]}$$

$$= [\varepsilon^{-1}L^{-1}T^{2}] \qquad \text{...(0.5-11)}$$

Dimensions in e.m. System

(1) *Pole strength*. By Coulomb's law of magnetic poles,

$$F = \frac{m_1 m_2}{\mu r^2}$$

where m_1, m_2 = pole strength

r = distance of separation

μ = permeability.

$$\text{Dimensionally, } [F] = \frac{[m^2]}{[\mu][L^2]}$$

or
$$[MLT^{-2}] = \frac{[m^2]}{[\mu L^2]}$$

$\therefore$
$$[m] = [\mu^{1/2}M^{1/2}L^{3/2}T^{-1}] \qquad \text{...(0.5-12)}$$

(2) *Current*. We know that the force exerted upon a magnetic pole, of strength m units and placed at the centre of a circular wire of radius r, due to a current I flowing in an arc of the circle of length l is given by

$$F = \frac{mIl}{r^2}$$

or
$$I = \frac{Fr^2}{ml}$$

$$\text{Dimensionally, } [I] = \frac{[F][L^2]}{[m][L]}$$

$$= \frac{[MLT^{-2}][L^2]}{[\mu^{1/2}M^{1/2}L^{3/2}T^{-1}][L]}$$

$$= [\mu^{-1/2}M^{1/2}L^{1/2}T^{-1}] \qquad \text{...(0.5-13)}$$

(3) *Magnetic field strength*. It is defined by,

$$\text{Magnetic field strength} = \frac{\text{Force}}{\text{Magnetic pole}}$$

or
$$H = \frac{F}{m}$$

$$\text{Dimensionally, } [H] = \frac{[F]}{[m]} = \frac{[MLT^{-2}]}{[\mu^{1/2}M^{1/2}L^{3/2}T^{-1}]}$$

$$= [\mu^{-1/2}M^{1/2}L^{-1/2}T^{-1}] \qquad \text{...(0.5-14)}$$

(4) *Charge*

$$\text{Charge} = \text{Current} \times \text{Time}$$

or
$$Q = I \times T$$

$$\text{Dimensionally, } [Q] = [I]\,[T]$$
$$= [\mu^{-1/2}M^{1/2}L^{1/2}T^{-1}]\,[T]$$
$$= [\mu^{-1/2}M^{1/2}L^{1/2}] \qquad \text{...(0.5-15)}$$

(5) *Potential difference*

$$\text{Potential difference} = \frac{\text{Work}}{\text{Charge}}$$

or $$V = \frac{W}{Q}$$

$$\text{Dimensionally, } [V] = \frac{[W]}{[Q]} = \frac{[ML^2T^{-2}]}{[\mu^{-1/2}M^{1/2}L^{1/2}]}$$

$$= [\mu^{1/2}M^{1/2}L^{3/2}T^{-2}] \qquad \text{...(0.5-16)}$$

(6) *Capacitance*

$$\text{Capacitance} = \frac{\text{Charge}}{\text{Potential difference}}$$

or $$C = \frac{Q}{V}$$

$$\text{Dimensionally, } [C] = \frac{[Q]}{[V]}$$

$$= \frac{[\mu^{-1/2}M^{1/2}L^{1/2}]}{[\mu^{1/2}M^{1/2}L^{3/2}T^{-2}]}$$

$$= [\mu^{-1}L^{-1}T^2] \qquad \text{...(0.5-17)}$$

(7) *Resistance*

$$\text{Resistance} = \frac{\text{Potential difference}}{\text{Current}}$$

or $$R = \frac{V}{I}$$

$$\text{Dimensionally, } [R] = \frac{[V]}{[I]}$$

$$= \frac{[\mu^{1/2}M^{1/2}L^{3/2}T^{-2}]}{[\mu^{-1/2}M^{1/2}L^{1/2}T^{-1}]}$$

$$= [\mu LT^{-1}] \qquad \text{...(0.5-18)}$$

(8) *Inductance*

$$\text{Inductance} = \frac{\text{E.m.f.}}{\text{Rate of change of current}}$$

or $$\mathcal{L} = \frac{V}{dI/dT}$$

$$\text{Dimensionally, } [\mathcal{L}] = \frac{[V][T]}{[I]} = \frac{[\mu^{1/2}M^{1/2}L^{3/2}T^{-2}][T]}{[\mu^{-1/2}M^{1/2}L^{1/2}T^{-1}]}$$

$$= [\mu L] \qquad \text{...(0.5-19)}$$

The dimensional equations of some electrical quantities in rationalized MKSA system are given in Table 0.5.

Table 0.5. Dimensional equations of some electrical quantities in MKSA system

Quantity	*Symbol*	*Equations from which the dimensions are derived*	*Dimensions*
Charge	Q, q	$Q = IT$	$[TI]$
E.M.F.	V, E, e	$V = \frac{\text{Work}}{Q}$	$[ML^2T^{-3}I^{-1}]$
Resistance	R	$R = \frac{V}{I}$	$[ML^2T^{-3}I^{-2}]$
Capacitance	C	$C = \frac{Q}{V}$	$[M^{-1}L^{-2}T^4I^2]$
Inductance	$\mathcal{L}$	$e = \mathcal{L}\frac{di}{dt}$	$[ML^2T^{-2}I^{-2}]$
Magnetic flux	ϕ	$e = N\frac{d\phi}{dt}$	$[ML^2T^{-2}I^{-1}]$
Flux density	B	$B = \frac{\text{Flux}}{\text{Area}} = \frac{\phi}{A}$	$[MT^{-2}I^{-1}]$
M.M.F.	AT	$AT = NI$	$[I]$
Magnetizing force	H, at	$H = \frac{\text{m.m.f}}{\text{Length}} = \frac{AT}{L}$	$[L^{-1}I]$
Reluctance	$\mathcal{R}$	$\mathcal{R} = \frac{AT}{\phi}$	$[M^{-1}L^{-2}T^2I^2]$
Electric flux	ψ	$\psi = Q$	$[TI]$
Electric flux density	D	$D = \frac{\text{Electric flux}}{\text{Area}} = \frac{\psi}{A}$	$[L^{-2}TI]$
Electric flux strength	E	$E = \frac{dV}{dt}$	$[ML^2T^{-4}I^{-1}]$

Relationship between Electrostatic and Electromagnetic Units. From equation (0.5-6) the dimensions of charge in e.s. system is

$$[Q] = [\varepsilon^{1/2}M^{1/2}L^{3/2}T^{-1}]$$

From Eq. (0.5-15) the dimensions of charge in e.m. system is

$$[Q] = [\mu^{-1/2} M^{1/2} L^{1/2}]$$

Since the dimensions of Q in either system must be same, hence

$$[\varepsilon^{1/2} M^{1/2} L^{3/2} T^{-1}] = [\mu^{-1/2} M^{1/2} L^{1/2}].$$

or $$[\varepsilon^{1/2} L T^{-1}] = [\mu^{-1/2}]$$

$\therefore$ $$[\mu^{-1/2} \varepsilon^{-1/2}] = [L T^{-1}]$$

That is, $$\frac{1}{[\mu\varepsilon]^{1/2}} = \text{dimensions of velocity.}$$

In any system of units, therefore, the permeability of free space (μ_0) and the permittivity of free space (ε_0) are, therefore, related by the equation,

$$\mu_0^{-1/2}\, \varepsilon_0^{-1/2} = \text{a velocity} = c \qquad \text{...(0.5-20)*}$$

This velocity can be shown experimentally to be equal to that of light, *i.e.* 2.998×10^{10} ($\approx 3 \times 10^{10}$) cm per second. To determine this velocity, the ratio of e.m. and e.s. values of some electrical quantities, such as capacitance, resistance, e.m.f. and current must be measured. If we choose capacitance, then the capacitance of some simple form of capacitor can be calculated in e.s. unit. The capacitance in e.m. unit can be measured by Maxwell's bridge method. The resistances in the bridge must be expressed in e.m. unit of resistance.

Let C_{ES} = Calculated value of capacitance in e.s.u.

C_{EM} = Measured value of capacitance in e.m.u.

From Table 0.6

$$\frac{\text{1 e.s.u. of capacitance}}{\text{1 e.m.u. of capacitance}} = \mu\varepsilon \quad \text{(Neglecting the dimensions of } L, M, T\text{)}$$

$$= \frac{1}{c^2} \quad \text{(if medium is free space or air)}$$

or $$\text{1 e.s.u. of capacitance} = \frac{1}{c^2} \text{ of 1 e.m.u. of capacitance}$$

* The permittivity and permeability of a medium in general are

$$\varepsilon = \varepsilon_r \varepsilon_0 \text{ and } \mu = \mu_r \mu_0$$

where ε_0 and μ_0 are permittivity and permeability of free space (or air) respectively and ε_r and μ_r are numeric and are relative permittivity and permeability.

or 1 e.m.u. of capacitance = c^2 of 1 e.s.u. of capacitance

$\therefore$ Number of e.s.u. of capacitance in 1 e.m.u. = c^2

Now, if a length (say of 2 m) is measured both in cm and m and are L_c and L_m respectively, then

$$\frac{L_c}{L_m} = \frac{200}{2} = 100 = \text{No. of cm in 1 m.}$$

By analogy, $$\frac{C_{ES}}{C_{EM}} = \text{No. of e.s.u. of capacitance in 1 e.m.u.}$$
$$= c^2$$

$$\therefore \quad \sqrt{\frac{C_{ES}}{C_{EM}}} = c \qquad \text{...(0.5-21)}$$

The result shows that c is equal to 3×10^{10} cm/s which is velocity of light.

Thus, the velocity of light is equal to the square root of the ratio of measured values of a capacitor in e.s.u. and e.m.u.

Eq. (0.5-20) gives the relationship between μ and ε and therefore, the dimensions of any quantity in e.s. system can be converted into the dimension in e.m. system simply by substituting the value of ε in terms of μ. For example, the dimensions of charge in e.s. system is

$$[Q] = [\varepsilon^{1/2}M^{1/2}L^{3/2}T^{-1}]$$

From Eq. (0.5-20)

$$[\varepsilon^{1/2}] = [\mu^{-1/2}L^{-1}T]$$

Substituting in above equation

$$[Q] = [\mu^{-1/2}M^{1/2}L^{1/2}]$$

The same result was also obtained in Eq. (0.5-15).

The procedure of determining the number of e.m.u. in one e.s.u. can be illustrated by following examples.

Example 0.5-1. *From Eqs. (0.5-7) and (0.5-13) the dimensions of current in e.s. and e.m. systems are respectively*

$$[I] = [\varepsilon^{1/2}M^{1/2}L^{3/2}T^{-2}]$$

and $$[I] = [\mu^{-1/2}M^{1/2}L^{1/2}T^{-1}].$$

Solution. Hence,

$$\left[\frac{1\,\text{e.s.u.}}{1\,\text{e.m.u.}}\right] = \left[\frac{\varepsilon^{1/2}M^{1/2}L^{3/2}T^{-2}}{\mu^{-1/2}M^{1/2}L^{1/2}T^{-1}}\right]$$

Neglecting the dimensions of L, M and T

$$\frac{1\,\text{e.s.u.}}{1\,\text{e.m.u.}} = \varepsilon^{1/2}\,\mu^{1/2} = \frac{1}{c} = \frac{1}{2.998 \times 10^{10}}$$

$$\approx \frac{1}{3 \times 10^{10}}$$

$\therefore$ 1 e.s.u. of current = $\dfrac{1}{3 \times 10^{10}}$ of e.m.u. of current ...(0.5-22)

Example 0.5-2. *From Eqs. (0.5-9) and (0.5-17) the dimensions of capacitance in e.s and e.m. system are respectively*

$$[C] = [\varepsilon L]$$

and $$[C] = [\mu^{-1}L^{-1}T^{2}].$$

Solution. Hence,

$$\left[\frac{1\,\text{e.s.u.}}{1\,\text{e.m.u.}}\right] = \left[\frac{\varepsilon L}{\mu^{-1}L^{-1}T^{2}}\right]$$

Neglecting the dimension of L and T

$$\frac{1\,\text{e.s.u.}}{1\,\text{e.m.u.}} = \varepsilon\mu = \frac{1}{c^2} = \frac{1}{9 \times 10^{20}}$$

$\therefore$ 1 e.s.u. of capacitance = $\dfrac{1}{9 \times 10^{20}}$ of 1 e.m.u. of capacitance ...(0.5-23)

The number of e.m.u. in one e.s.u. of other electrical quantities is given in the Table 0.6.

Dimensional Equations. Let

$$x \propto y^m z^n w^p$$

where x, y, z and w are physical quantities the dimensions of which are known and m, n and p are unknown numerical constants. These unknown constants can be determined by substituting the dimensions of x, y, z and w, and equating the corresponding indices of L, M, T, μ and ε. The following example will illustrate the above statement.

Example 0.5-3. *The electrical power in a circuit is proportional to the current and resistance of the circuit, each raised to some power. Determine these powers by the use of dimensions of quantities involved.*

Solution. Let $$P \propto I^m R^n$$

or $$P = K I^m R^n$$

where K is a number which has no dimensions. Then, substituting the dimensions of P, I and R from Table 0.6

Table 0.6. Dimensions of Electrical Quantities

Quantity	*Symbol*	*Equations from which the dimensions are derived*	*Dimensions*		*Practical Unit*	*No. of e.m. units in one practical unit*	*Ratio : 1 e.s.u. / 1 e.m.u. (Neglecting L, M, T)*	*No. of e.s.u. in one practical unit*
			e.m. system	*e.s. system*				
Charge	Q, q	$F = \frac{Q_1Q_2}{\varepsilon r^2}$	$\mu^{-1/2} M^{1/2} L^{1/2}$	$\varepsilon^{1/2} M^{1/2} L^{3/2} T^{-1}$	coulomb	10^{-1}	$\varepsilon^{1/2} \mu^{1/2} = \frac{1}{3 \times 10^{10}}$	3×10^9
Current	I, i	$I = \frac{Q}{T}$	$\mu^{-1/2} M^{1/2} L^{1/2} T^{-1}$	$\varepsilon^{1/2} M^{1/2} L^{3/2} T^{-2}$	ampere	10^{-1}	$\varepsilon^{1/2} \mu^{1/2} = \frac{1}{3 \times 10^{10}}$	3×10^9
e.m.f. or Potential	V, e, E	$V = \frac{\text{Work}}{Q} = \frac{W}{Q}$	$\mu^{1/2} M^{1/2} L^{3/2} T^{-2}$	$\varepsilon^{-1/2} M^{1/2} L^{1/2} T^{-1}$	volt	10^8	$\varepsilon^{-1/2} \mu^{-1/2} = 3 \times 10^{10}$	$\frac{1}{3 \times 10^2}$
Resistance	R	$R = \frac{V}{I}$	$\mu L T^{-1}$	$\varepsilon^{-1} L^{-1} T$	ohm	10^9	$\varepsilon^{-1} \mu^{-1} = 9 \times 10^{20}$	$\frac{1}{9 \times 10^{11}}$
Capacitance	C	$C = \frac{Q}{V}$	$\mu^{-1} L^{-1} T^2$	εL	farad*	10^{-9}	$\varepsilon\mu = \frac{1}{9 \times 10^{20}}$	9×10^{11}
Inductance	$\mathcal{L}$	$E = \mathcal{L}\frac{di}{dt}$	μL	$\varepsilon^{-1} L^{-1} T^2$	henry**	10^9	$\varepsilon^{-1} \mu^{-1} = 9 \times 10^{20}$	$\frac{1}{9 \times 10^{11}}$
Impedance	Z	$Z = \frac{V}{I}$	$\mu L T^{-1}$	$\varepsilon^{-1} L^{-1} T$	ohm	10^9	$\varepsilon^{-1} \mu^{-1} = 9 \times 10^{20}$	$\frac{1}{9 \times 10^{11}}$
Electric Power	P	$P = \frac{\text{Work}}{\text{Time}} = VI$	$ML^2 T^{-3}$	$ML^2 T^{-3}$	watt	10^7	1	10^7
Electrical energy	W	$W = PT$	$ML^2 T^{-2}$	$ML^2 T^{-2}$	joule	10^7	1	10^7

* Mostly used practical unit is μF (micro-farad).

** Mostly used practical unit is mH (milli-henry).

$$[ML^2T^{-3}] = K\ [(\mu^{-1/2}\ M^{1/2}\ L^{1/2}\ T^{-1})^m \times (\mu LT^{-1})^n]$$

$$= K\left[\mu^{\left(n-\frac{m}{2}\right)} M^{\frac{m}{2}} L^{n+\frac{m}{2}} T^{-(m+n)}\right]$$

Equating powers, we have

For M : $\quad 1 = \dfrac{m}{2}$

For L : $\quad 2 = n + \dfrac{m}{2}$

or $\quad m = 2$

and $\quad n = 1.$

For T : $\quad -3 = -(m + n)$

which is satisfied by $\quad m = 2,\ n = 1.$

For μ : $\quad 0 = n - \dfrac{m}{2}$

which is also satisfied by $m = 2$, $n = 1$.

$\therefore \quad P \propto I^2R$

The dimensional equations of physical quantities can also be used to check possible errors in equations which have been derived, sometimes, from somewhat complicated theory. The following example will illustrate the statement.

Example 0.5-4. *In the course of deriving an expression, the following form was arrived at*

$$I = V\left\{\frac{1}{Z} + \frac{1}{R} + \frac{C}{L}\right\}$$

Show that there must have been an algebraical error, and find out the terms which require correction. All parameters are connected with an electrical circuit.

Solution. Now, substituting the dimensions of various quantities in e.m. system, we have

Left-hand side : $[\mu^{-1/2}M^{1/2}L^{1/2}T^{-1}]$

Right hand side :

$$[\mu^{1/2}M^{1/2}L^{1/2}T^{-2}]\left[\frac{1}{[\mu LT^{-1}]} + \frac{1}{[\mu LT^{-1}]} + \frac{[\mu^{-1}L^{-1}T^2]}{[\mu L]}\right]$$

$$= \frac{[\mu^{1/2}M^{1/2}L^{3/2}T^{-2}]}{[\mu LT^{-1}]}[1 + 1 + (\mu^{-1}L^{-1}T)]$$

$$= [\mu^{-1/2}M^{1/2}L^{1/2}T^{-1}]\ [2 + (\mu^{-1}L^{-1}T)]$$

In order that the dimensions of this expression should be same as those of the left-hand side, the terms under second bracket should be dimensionless. For this, the last term in the bracket should be multiplied by μLT^{-1} which is the dimension of resistance or impedance. The expression, therefore, must be as

$$I = V\left(\frac{1}{Z} + \frac{1}{R} + \frac{C}{L}R_1\right)$$

where R_1 is a resistance.

0.6. STANDARDS OF MEASUREMENT

It is essential to provide means of establishing and maintaining the magnitude of the various units. This is generally done by means of standards. The simplest kind of standards is a physical object having the desired property. Sometimes, it is realized by reference to natural phenomena including physical and atomic constants. The various types of standards of measurements are classified in the following categories :

International standards. These standards are one and only one for one unit kept at the International Bureau of Weights and Measures, and are internationally acceptable. These are defined with the greatest possible accuracy. International standards are periodically evaluated and checked by absolute measurements. These standards are not used by the ordinary user for the purpose of calibration or comparison.

Primary standards. These standards are maintained by national laboratories of standards in different part of the world. All the primary standards are independently calibrated by absolute measurement in absolute units at each of the national laboratories. The primary standards are mainly used for the verification and calibration of secondary standards. One cannot use these standards outside the national laboratories.

Secondary standards. These standards are realized by comparison with the primary standards with the greatest possible accuracy. These standards are maintained by various laboratories and industrial organisations of national importance. Secondary standards are periodically sent to national laboratories for calibration and comparison against the primary standards. National laboratories give a certificate for that to the user.

Sub-standards. The sub-standards (or working standards) are calibrated in comparison with secondary standards with a little lesser accuracy. These standards are used for practical comparison and checking calibrations of instruments. These are maintained by various laboratories or manufacturers.

Standards for mass. The fundamental unit of mass in the international system (SI) is the kilogram (kg). It is the mass of the international prototype (Table 0.2) which is in the custody of the International Bureau of Weights and Measures at Severes, near Paris, France. The international prototype consists of a platinum-irridum alloy cylinder, the mass of which is equal to the mass of a cubic decimeter of water at its temperature of 4°C at maximum density. The primary standard of mass is calibrated to an accuracy of the range of 1 part in 10^9. The secondary standard of mass generally have an accuracy of the range of 1 ppm (part per million). The accuracy of the sub-standard is of the range of 5 ppm and sub-standards are made in a wide range of values to suit almost any application.

Standards of length. The meter is the SI unit of length and as defined in Table 0.2 is equal to 1 650 763.73 wavelengths in vacuum of the radiation corresponding to the transition between the energy levels 2 p 10 and 5 d5 of the Krypton 86 atom. The prototype representation is the distance between two lines engraved on a platinum irridium bar preserved at the International Bureau of Weights and Measures. The sub-standards most widely used in industries are precision gage-blocks of steed. These blocks have two parallel surfaces placed at a specified distance apart with accuracy tolerances in the range of 0.5–0.25 micron.

Standards of time and frequency. The SI unit of time as defined in Table 0.2 is the fraction 1/31 556 925.974 7 of the tropical year* for 1900 January 0 at 12 h ephemeris time**. The disadvantage of this unit of time is that it takes several years to determine it. Even then it is not error free. Now for physical measurements, the second has been defined in terms of an atomic standard. However, for navigation, geodetic surveys and celestrial mechanics, the SI unit of time is still continuing to be used. The development and refinement of atomic resonator have made it possible to control the frequency of an oscillator. Hence atomic clock has come into existence. The transition between two energy

*The tropical year is the time between consecutive passages, in the same direction, of the sun through the earth's in auditorial plane.

**Ephemeris time is based on astronomical observations of the motion of the moon about the earth.

levels E_1 and E_2 of an atom is accompanied by the emission or absorption of radiation with a frequency (f) given by

$$f = \frac{E_2 - E_1}{h} \qquad ...(0.6\text{-}1)$$

where h is Planck's constant.

This equation is valid if the energy states are not affected by external conditions, such as magnetic fields. Then, the frequency is a physical constant and depends only on the internal structure of the atom. The inverse of frequency, thus, gives a *constant time interval*. The first atomic clock, based on the energy transition of cesium 133 atom, came into existence in 1955. The time interval given by the cesium 133 atomic clock is more accurate than that given by a clock calibrated by SI unit. The fundamental unit of time (SI second) is defined as the duration of 9 192 631 770 Hz of the radiation corresponding to the transition between two energy level of the cesium 133 atom. The atomic time standard is more precise than that of ephemeris time. In this case, the determinations of time intervals can be made in a few minutes to greater accuracy than it is possible in astronomical measurements that take many years to complete.

Voltage Standards. To make electrical measurements, it is desirable that we should have a cell whose e.m.f. remains constant and is accurately known. It must be made of chemicals which can be purified and reproduced easily. No chemical reaction must take place inside the cell except when the current is drawn from it. Such a cell is called a *standard cell*.

Weston and Clark cells were used as standard cells. In year 1908 the Conference on Electrical Units and Standards suggested to prefer the Weston cell as it has some advantages over the Clark cell. The advantages may be listed as follows :

(1) Its e.m.f. does not change as much with temperature.

(2) It has small hysteresis effects attending temperature variations.

(3) It has longer life.

(4) It is free from formation of layer of gas which forms sometimes in Clark cell interrupting the circuit.

Both Clark and Weston cells are essentially voltaic cells of special forms. The metals used in them—mercury, cadmium and zinc—can be found in pure form easily. This is essential for maintaining the e.m.f. constant.

Weston Standard cell. This cell was patented by Weston in 1892. This has now replaced the Clark cell.

The positive electrode consists of mercury, covered on its surface with a paste of mercurous sulphate crystals and mercury acting as depolaizer, is put in one limb of an H-shaped vessel. The negative electrode is a 12% amalgam of mercury and cadmium in another limb (Fig. 0.4).

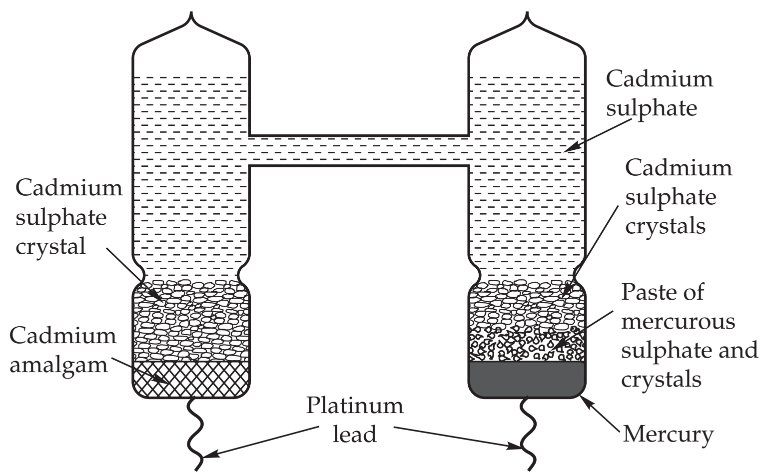

Fig. 0.4. Weston standard cell.

The electrolyte is a saturated solution of cadmium sulphate. Saturation is being maintained by adding crystals of cadmium sulphate.

The e.m.f. of the cell is 1.0183 volt at 20°C and its value at any other temperature is given by

$$E_t = 1.0183 - 0.0000406\,(t - 20) \qquad \text{...(0.6-1)}$$

The following precautions must be taken when using a standard cell :

(1) It must be kept in mind that an extremely feeble current should be taken from a standard cell or it will be ruined. Hence, a standard cell is, generally, used in null methods of measurement such as in measurements by potentiometers. A high resistance should, therefore, be connected in series when putting it in a circuit.

(2) To avoid hysteresis effects due to variations in temperature the cells should be stored in a dry position having a fairly uniform temperature of about 15°C to 20°C.

(3) Care should be taken in handling the cell as any appreciable shaking up of the chemicals in the cell tends to produce variations of e.m.f.

Current standards. It is not possible to set up a physical standard of current. The current is defined in terms of electro-chemical equivalent of silver. The international ampere is the unvarying current which, when paesed through a solution of silver nitrate in water, deposits silver at the rate of 0.001118 gm per second. It is also defined in terms of the force exerted between the fixed and moving coils carrying the current. A current balance may be used for weighing the force. Such standards are, practically, not used. We can easily measure the current by comparing the drop in a standard resistance with a standard cell with the help of a potentiometer. This is the reason for advocating for volt as primary international unit in place of ampere.

Standard Resistors. Measuring the value of a resistance by comparing with a standard resistance is considerably easier than to determine its value by absolute measurement. Also, for general purposes, measurement of resistances by comparison can be made with sufficient accuracy. Therefore, it is convenient to have available standard resistors which can be used as reference standard.

Bifilar winding is used for standard resistors. The commonly used metal is manganin. The coil is insulated from the metal former (in case of D.C. standard resistors) by a layer of shellaced silk which is baked before the wire is wound on. For heavy current rated resistors adequate cooling must be provided.

0.7. APPLICATIONS OF MEASUREMENT SYSTEMS

As pointed out in section 0.1, the applications of measurement systems may be classified according to the following scheme :

1. Monitoring of processes and operations.
2. Control of processes and operations.
3. Experimental engineering analysis.

Monitoring of processes and operations. Some applications of measuring instruments may be characterized as having essentially a monitoring function. Panel meters used in power stations serve in such capacity. They simply indicate the working condition (frequency, voltage and load) of the power system, and their readings do not serve any control function in the ordinary sense. Similarly, electric meters in the home keep track of the quantity of the electricity used so that the cost to the user may be computed. A speedometer in an automobile serves to monitor the speed of the vehicle.

Control of processes and operations. Another extremely important application of measuring instruments is that in which the instrument serve as a component of an automatic control system. A functional block diagram illustrating the operation of a feedback system is shown in Fig. 0.5.

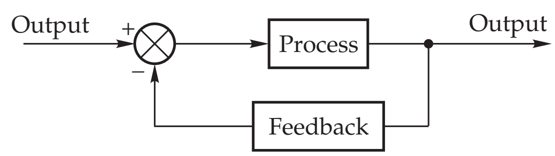

Fig. 0.5. A feedback system.

The feedback block consists of a measuring instrument and a control element. For controlling any variable by a feedback scheme it is first necessary to measure it. All such control systems, therefore, must have at least one measuring instrument.

For an example, let us consider a speed control system shown in Fig. 0.6. The tachogenerator senses the speed of the motor and converts the speed to a proportional voltage which is essential to compare the input voltage with the output speed. This provides the information necessary for proper functioning of the control system. Another example is the typical home-heating system shown in Fig. 0.7. This scheme employs a thermocouple which senses the room temperature and provides the information necessary for the control. Much more sophisticated examples are found among the air-craft and missile control where a single control system may require information from many measuring instruments such as pilot static tube, angle of attack sensors, thermocouples, accelerometers, and gyroscope.

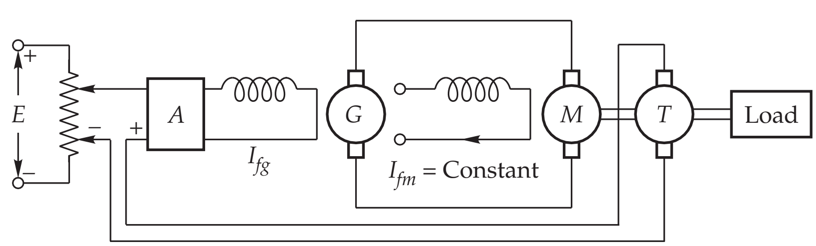

Fig. 0.6. A speed control system.

Experimental engineering analysis. Engineering problems are solved either theoretically or experimentally. Some problems require the application of both theoretical and experimental methods. In theoretical method of solving engineering problems, accurate measurements are necessary to give a true result. This may require expensive and complicated equipment.

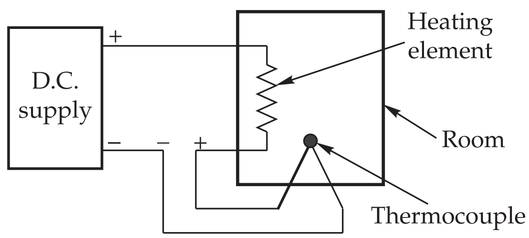

Fig. 0.7. A home-heating system.

Though we classified applications into three categories, one may find instances where the distinction between monitoring, control, and analysis functions is not clear cut. For instance, the data obtained in a power station by panel meters serve mainly in a monitoring function. In power generation station these data may be used for the control of power generation. The data recorded for a long time in a power station may be used for short-term and long-term load forecasting.

0.8. ILLUSTRATIVE EXAMPLES

Example 1. *Derive the dimensions of* (*a*) *voltage gradient*, (*b*) *current density*, (*c*) *charge*, (*d*) *magnetic flux density*, (*e*) *force*, (*f*) *permeability*, (*g*) *magnetic pole*, (*h*) *resistivity*, (*i*) *permittivity in LTVI system*.

Solution. (*a*) Voltage gradient $= \dfrac{\text{Voltage}}{\text{Distance}}$

$$[g] = [L^{-1}V]$$

(*b*) Current density $= \dfrac{\text{Current}}{\text{Area}}$

$$[\delta] = [L^{-2}I]$$

(*c*) Charge = Current × Time

$$[Q] = [TI]$$

(*d*) Magnetic flux density

$$B = \frac{\text{Flux}}{\text{Area}} = \frac{\phi}{A}$$

$$[B] = [\phi L^{-2}]$$

But $$V = \frac{d\phi}{dt}$$

or $$[\phi] = [TV]$$

Hence, $$[B] = [L^{-2}TV]$$

(*e*) Force $$= \frac{\text{Work}}{\text{Distance}} = \frac{\text{Voltage} \times \text{Charge}}{\text{Distance}}$$

$$[F] = [L^{-1}TVI]$$

(*f*) Permeability, from Table 0.2, is given by

$$F = \frac{2\mu i_1 i_2 l}{d}$$

or Permeability $$\mu = \frac{Fd}{2i_1 i_2 l}$$

$$\therefore \quad [\mu] = [FI^{-2}] = [L^{-1}TVI^{-1}]$$

(*g*) Magnetic pole,

Since, $$F = \frac{1}{\mu}\frac{m_1 m_2}{d^2}$$

$$\therefore \quad [F] = \frac{[m^2]}{[\mu][L^2]}$$

$$\therefore \quad [m] = [F^{1/2}\mu^{1/2}L] = [TV]$$

(*h*) Resistivity $$\rho = \frac{\text{Resistance} \times \text{Area}}{\text{Length}} = \frac{\text{Voltage}}{\text{Current}} \times \text{Length}$$

$$\therefore \quad [\rho] = [LVI^{-1}]$$

(*i*) Permittivity is given by

$$F = \frac{Q_1 Q_2}{\varepsilon d^2}$$

$\therefore$ Permittivity $$\varepsilon = \frac{Q^2}{Fd^2}$$

$$\therefore \quad [\varepsilon] = [L^{-1}TV^{-1}I]$$

Example 2. *Derive the dimensions of* (*i*) *magnetic flux*, (*ii*) *magnetic flux density*, (*iii*) *mmf*, (*iv*) *magnetising force*, (*v*) *reluctance*, (*vi*) *permeance in the e.m. system of unit.*

Solution. (*i*) Magnetic flux is given by

$$\text{e.m.f.} = -\frac{d\phi}{dt}$$

$$\therefore \quad [\phi] = [VT]$$

Using Eq. (0.5-16)

$$[\phi] = [\mu^{1/2}M^{1/2}L^{3/2}T^{-1}]$$

(*ii*) Magnetic flux density

$$B = \frac{\text{Flux}}{\text{Area}} = \frac{\phi}{A}$$

$$\therefore \quad [B] = [\phi L^{-2}] = [\mu^{1/2}M^{1/2}L^{-1/2}T^{-1}]$$

(*iii*) $$\text{mmf} = \text{ampere} \times \text{turns}$$

$$\therefore \quad [at] = [I] = [\mu^{-1/2}M^{1/2}L^{1/2}T^{-1}]$$

(*iv*) Magnetizing force

$$H = \frac{\text{mmf}}{\text{Length}}$$

$$[H] = \frac{[at]}{[L]} \quad \text{(}a\text{ is ampere – turn which have dimensions of current)}$$

$$= [\mu^{-1/2}M^{1/2}L^{-1/2}T^{-1}]$$

(*v*) Reluctance $$= \frac{\text{mmf}}{\phi}$$

$$[\mathcal{R}] = \frac{[at]}{[\phi]} = \frac{[\mu^{-1/2}M^{1/2}L^{1/2}T^{-1}]}{[\mu^{1/2}M^{1/2}L^{3/2}T^{-1}]} = [\mu^{-1}L^{-1}]$$

(*iv*) Permeance $$= \frac{1}{\text{Reluctance}}$$

$$[\rho] = [\mu L]$$

Example 3. *Find the ratio of the size of M.K.S. rationalized unit to the size of C.G.S. e.s.u. of electric flux density by using LTVI system. It is given that 1 V =* $\frac{1}{300}$ *C.G.S. e.s.u. of e.m.f. and 1 A =* 3×10^9 *C.G.S. e.s.u. of current.*

Solution. Dimensions of electric flux density

$$[D] = \frac{[Q]}{[L^2]} = [ITL^{-2}]$$

$$\frac{1 \text{ M.K.S. unit of } D}{1 \text{ C.G.S. e.s.u. of } D} = \left(\frac{3 \times 10^9}{1}\right)\left(\frac{1}{1}\right)\left(\frac{100}{1}\right)^{-2} = 3 \times 10^5$$

But rationalized system of $D = 4\pi \times$ Unrationalized system of D

$$\therefore \quad \frac{1 \text{ M.K.S. rationalized unit of } D}{1 \text{ C.G.S. e.s.u. of } D} = 12\pi \times 10^5.$$

Example 4. *Given that 1 A = 10^{-1} e.m.u. of current and that 1 V = 10^8 e.m.u. of potential difference, find the number of e.m.u. of inductance in 1 henry by using LTVI system of dimensions.*

Solution. Inductance

$$\mathcal{L} = \frac{emf}{dI/dt}$$

$$\therefore \quad [\mathcal{L}] = \frac{[V][T]}{[I]} = [VI^{-1}T]$$

$$\frac{1 \text{ M.K.S. unit of inductance (henry)}}{1 \text{ e.m.u. of inductance}} = \left(\frac{10^8}{1}\right)\left(\frac{10^{-1}}{1}\right)^{-1}\left(\frac{1}{1}\right) = 10^9$$

$\therefore$ 1 henry = 10^9 e.m.u. of inductance.

Example 5. *The magnetic force F of an electromagnet is given by*

$$F \propto B^m A^n \mu^p$$

where B = *flux density in the air-gap*

A = *cross-sectional area of the air-gap*

μ = *permeability*.

Determine the value of m, n and p.

Solution. From Example 2 of this section

$$[B] = [\mu^{1/2} M^{1/2} L^{-1/2} T^{-1}]$$

Also, $$[F] = [MLT^{-2}]$$

Now, $$F = KB^m A^n \mu^p$$

where K is a constant having no dimensions.

Dimensionally,

$$[F] = [B]^m [A]^n [\mu]^p$$

$$= [\mu^{1/2} M^{1/2} L^{-1/2} T^{-1}]^m [L^2]^n [\mu]^p$$

or $$[MLT^{-2}] = \left[\mu^{\left(p + \frac{m}{2}\right)} M^{\frac{m}{2}} L^{\left(2n - \frac{m}{2}\right)} T^{-m}\right]$$

Equating corresponding indices, we have

For M : $1 = \frac{m}{2}$, *i.e.* $m = 2$

For μ : $0 = p + \frac{m}{2}$ $\therefore$ $p = -1$

For L : $1 = 2n - \frac{m}{2}$ $\therefore$ $n = 1$

For T : $-2 = -m$.

Which is satisfied also by $m = 2$

$$\therefore \qquad F \propto \frac{B^2 A}{\mu}$$

Example 6. *The energy stored per unit area in a parallel plate condenser is given by,* $W = K\varepsilon^a V^b d^c$

where ε = *permittivity of medium*

d = *distance between plates*

V = *potential difference applied*

K = *a dimensionless constant quantity*.

Determine the value of a, b and c.

Solution. Dimensionally,

$$[W] = [\varepsilon]^a [V]^b [L]^c$$

or
$$[ML^2T^{-2}] = [\varepsilon]^a [\varepsilon^{-1/2} M^{1/2} L^{1/2} T^{-1}]^b [L]^c$$

$$= \left[\varepsilon^{\left(a - \frac{b}{2}\right)} M^{\frac{b}{2}} L^{\left(c + \frac{b}{2}\right)} T^{-b}\right]$$

Equating the corresponding indices

For M : $1 = \frac{b}{2}$, *i.e.* $b = 2$

For ε : $0 = a - \frac{b}{2}$ $\therefore$ $a = 1$

For L : $2 = c + \frac{b}{2}$ $\therefore$ $c = 1$

For T : $-2 = -b$. Which is satisfied also by $b = 2$

$$\therefore \qquad W = K\, \varepsilon\, V^2 d$$

Example 7. *The mean deflecting torque of an electrodynamometer type of wattmeter may be written as,* $T_d \propto V^p m^q Z^r$

where V = *voltage applied*

m = *mutual inductance between fixed and moving coils*

Z = *impedance of the load circuit*

Determine the values of p, q and r.

Solution. Given that

$$T_d \propto V^p m^q Z^r$$

or
$$T_d = KV^p m^q Z^r$$

where K is a dimensionless constant quantity.

Dimensions of various quantities are :

$$\text{Torque} = \text{Force} \times \text{Radius}$$

or
$$[T_d] = [MLT^{-2}][L] = [ML^2T^{-2}]$$

From Table 0.6,

$$[m] = [\mu L]$$
$$[Z] = [\mu LT^{-1}]$$
$$[V] = [\mu^{1/2}M^{1/2}L^{3/2}T^{-2}]$$

Hence
$$[ML^2T^{-2}] = [\mu^{1/2}\, M^{1/2}\, L^{3/2}\, T^{-2}]^p[\mu L]^q[\mu LT^{-1}]^r$$
$$= \left[\mu^{\left(\frac{p}{2}+q+r\right)} M^{\frac{p}{2}} L^{\left(\frac{3p}{2}+q+r\right)} T^{-(2p+r)}\right]$$

Equating the corresponding powers,

For M : $\quad 1 = \dfrac{p}{2}$, *i.e.* $p = 2$

For T : $\quad -2 = -2p - r \quad \therefore \quad r = -2$

For L : $\quad 2 = \dfrac{3}{2}p + q + r \quad \therefore \quad q = 1$

For μ : $\quad 0 = \dfrac{p}{2} + q + r$

which is satisfied also by $p = 2$, $q = 1$ and $r = -2$

$$\therefore \qquad T_d \propto \frac{V^2 m}{Z^2}$$

Example 8. *Determine a, b, c and d in the following expression for the eddy current loss W per unit length of a round wire,* $W \propto f^a\, B_{max}{}^b\, d^c\, \rho^d$

where B_{max} = *maximum flux density*

f = *frequency*

d = *diameter of the wire*

ρ = *specific resistance of the material.*

Solution.
$$W \propto f^a\, B_{max}{}^b\, d^c\, \rho^d$$

or
$$W = Kf^a\, B_{max}{}^b\, d^c\, \rho^d$$

where K is a numerical constant.

Dimensions of various quantities are :

$$[W] = \frac{[\text{Power loss}]}{[\text{Length}]} = \frac{[ML^2T^{-3}]}{[L]} = [MLT^{-3}]$$

$$[f] = [T^{-1}]$$

$$[B] = [\mu^{1/2}\ M^{1/2}\ L^{-1/2}\ T^{-1}]$$

$$[d] = [L]$$

$$[\rho] = \frac{[R][L^2]}{[L]} = [\mu L^2 T^{-1}]$$

Hence $\quad [MLT^{-3}] = [T^{-1}]^a[\mu^{1/2}\ M^{1/2}\ L^{-1/2}\ T^{-1}]^b[L]^c[\mu L^2T^{-1}]^d$

$$= \left[\mu^{\left(\frac{b}{2}+d\right)} M^{\frac{b}{2}} L^{\left(-\frac{b}{2}+c+2d\right)} T^{(-a-b-d)}\right]$$

For the above equation to be, dimensionally, balanced, the following equations must be satisfied :

$$\frac{b}{2} = 1, \qquad i.e. \qquad b = 2$$

$$\frac{b}{2} + d = 0 \qquad \therefore \qquad d = -1$$

$$-\frac{b}{2} + c + 2d = 1 \qquad \therefore \qquad c = 4$$

$$-a - b - d = -3 \qquad \therefore \qquad a = 2$$

$$\therefore \qquad W \propto \frac{f^2 {B_{max}}^2 d^4}{\rho}.$$

Example 9. *Some error had been made in the derivation of the expression* $I = \dfrac{V\omega m}{[(\omega^2 m + R_1R_2)^2 + \omega^2 L_1 L_2 R_1^2]^{1/2}}$ *for current in a particular circuit containing mutual-inductance m, self-inductances L_1 and L_2, and resistances R_1 and R_2.* ω *is angular velocity, i.e.* $\omega = 2\pi f$. *Make a correction to ensure that the equation is dimensionally correct.*

Solution. The dimensions of various quantities are :

$$[I] = [\varepsilon^{1/2}\ M^{1/2}\ L^{3/2}\ T^{-2}]$$

$$[V] = [\varepsilon^{-1/2}\ M^{1/2}\ L^{1/2}\ T^{-1}]$$

$$[\omega] = [T^{-1}]$$

$$[R] = [\varepsilon^{-1}\ L^{-1}\ T]$$

$$[m],\ [L] = [\varepsilon^{-1}\ L^{-1}\ T^2].$$

Now, by substituting the dimensions in above expression, we have,

$$\text{L.H.S.} = [\varepsilon^{1/2} M^{1/2} L^{3/2} T^{-2}]$$

$$\text{R.H.S.} = \frac{[\varepsilon^{-1/2} M^{1/2} L^{1/2} T^{-1}][T^{-1}][\varepsilon^{-1} L^{-1} T^2]}{\{([T^{-2}][\varepsilon^{-1} L^{-1} T^2] + [\varepsilon^{-2} L^{-2} T^2])^2 + [T^{-2}][\varepsilon^{-2} L^{-2} T^4][\varepsilon^{-2} L^{-2} T^2]\}^{1/2}}$$

$$= \frac{[\varepsilon^{1/2} M^{1/2} L^{3/2} T^{-2}]}{\{[\varepsilon L T^{-2} + 1]^2 + 1\}^{1/2}}$$

For the dimensions of both sides to be same, the denominator must be dimensionless. For this the first term in the square bracket must be multiplied by $\varepsilon^{-1} L^{-1} T^2$ which are the dimensions of inductance. Therefore, the missing term is a mutual inductance and the correct expression for the current is

$$I = \frac{V\omega m}{[(\omega^2 m^2 + R_1 R_2)^2 + \omega^2 L_1 L_2 R_1^{\,2}]^{1/2}}.$$

Example 10. *In Anderson bridge, the expression for self-inductance has been derived as* $\mathcal{L} = \dfrac{CR_3}{R_4}[r(R_2 + R_4) + R_4]$, *where C is capacitance and* R_2, R_3, R_4 *and r are resistances. It is suspected that the error has been done in the derivation. Find the missing term and correct the expression.*

Solution. The dimensions of various quantities are

$$[\mathcal{L}] = [\mu L]$$
$$[C] = [\mu^{-1} L^{-1} T^2]$$
$$[R] = [\mu L T^{-1}]$$

After substituting the dimensions of various quantities in the expression, we have

$$\text{L.H.S.} = [\mu L]$$

$$\text{R.H.S.} = \frac{[\mu^{-1} L^{-1} T^2][\mu L T^{-1}]}{[\mu L T^{-1}]} \{[\mu L T^{-1}]([\mu L T^{-1}] + [\mu L T^{-1}]) + [\mu L T^{-1}]\}$$

$$= [\mu L][2 + \mu^{-1} L^{-1} T]$$

For dimensions balance, the second factor in R.H.S. must be dimensionless. Hence, the second term in the bracket must be multiplied by $[\mu L T^{-1}]$ which are the dimensions of resistance. Thus, the missing term is R_2 and the correct expression is

$$\mathcal{L} = \frac{CR_3}{R_4}[r\,(R_2 + R_4) + R_2 R_4].$$

Example 11. *Use LTVI system to correct the expression,* $I_e = K\dfrac{BlbA}{(2b+l)\rho}$ *for eddy current in the former of a moving coil instrument in terms of the flux density B, the length l, the breadth 2b, the cross-sectional area of the current path A, the resistivity of the former* ρ *and numerical constant K.*

Solution. Dimensions in *LTVI* system has been derived in Example 1. They are

$$[I_e] = [I]$$
$$[B] = [L^{-2}\,TV]$$
$$[l],\ [b] = [L]$$
$$[A] = [L^2]$$
$$[\rho] = [LVI^{-1}]$$

After substituting the dimensions of various quantities, we have

$$\text{L.H.S.} = [I]$$
$$\text{R.H.S.} = \frac{[L^{-2}TV][L^4]}{[L][LVI^{-1}]} = \frac{[L^2TV]}{[L^2VI^{-1}]} = [IT]$$

So, for dimensional balance, there must be a term of angular velocity ω having dimensions of $[T^{-1}]$ in R.H.S. Therefore, the correct expansion is

$$I_e \propto \frac{Blb\omega A}{(2b+l)\rho}.$$

Example 12. *In a new system of units, kilometre, metric tonne and minute are units of length, mass and time respectively. Determine the number of M.K.S. units (in one new units) of power, current and voltage. The unit of permeability remains same.*

Solution. (*i*) The dimensions of power

$$[P] = [ML^2T^{-3}]$$

$$\frac{\text{One new unit of power}}{\text{One M.K.S. unit of power}} = \left(\frac{1000}{1}\right)\left(\frac{1000}{1}\right)^2\left(\frac{60}{1}\right)^{-3}$$
$$= \frac{1}{2.16} \times 10^4$$

∴ One new unit of power

$$= \frac{1}{2.16} \times 10^4 \text{ M.K.S. unit of power}$$
$$= \frac{1}{2.16} \times 10^4 \text{ watt}$$

(*ii*) The dimensions of current

$$[I] = [\mu^{-1/2} M^{1/2} L^{1/2} T^{-1}]$$

$$\frac{\text{One new unit of current}}{\text{One M.K.S. unit of current}} = \left(\frac{1}{1}\right)^{-1/2} \left(\frac{1000}{1}\right)^{1/2} \left(\frac{1000}{1}\right)^{1/2} \left(\frac{60}{1}\right)^{-1}$$

$$= \frac{1}{6} \times 10^2 \text{ ampere}$$

$\therefore$ One new unit of current

$$= \frac{1}{6} \times 10^2 \text{ M.K.S. unit of current}$$

$$= \frac{1}{6} \times 10^2 \text{ ampere}$$

(*iii*) The dimensions of voltage

$$[V] = [\mu^{1/2} M^{1/2} L^{3/2} T^{-2}]$$

$$\frac{\text{One new unit of voltage}}{\text{One M.K.S. unit of voltage}} = \left(\frac{1}{1}\right)^{1/2} \left(\frac{1000}{1}\right)^{1/2} \left(\frac{1000}{1}\right)^{3/2} \left(\frac{60}{1}\right)^{-2}$$

$$= \frac{1}{36} \times 10^4$$

$\therefore$ One new unit of voltage

$$= \frac{1}{36} \times 10^4 \text{ M.K.S. unit of voltage}$$

$$= \frac{1}{36} \times 10^4 \text{ volt.}$$

OBJECTIVE QUESTIONS

1. The device that converts a physical quantity into an electrical quantity is called

(*a*) sensor (*b*) transducer

(*c*) processor (*d*) converter.

2. In rationalized MKS system of units, the value of permittivity of free space is taken equal to

(*a*) $4\pi \times 10^{-7}$ (*b*) 10^{-7}

(*c*) $\dfrac{10^{-9}}{36\pi}$ (*d*) $36\pi \times 10^{-12}$.

3. Dimensions of power is

(*a*) $[ML^2T^{-3}]$ (*b*) $[ML^2T^{-2}]$

(*c*) $[MLT^{-1}]$ (*d*) $[ML^3T^{-2}]$.

4. Dimensions of charge in e.s.u. is given by

(*a*) $[\varepsilon^{-1/2}ML^{3/2}T^{-1}]$ (*b*) $[\varepsilon^{1/2}M^{-1/2}L^{2}T^{-2}]$

(*c*) $[\varepsilon^{1/2}M^{1/2}L^{3/2}T^{-1}]$ (*d*) $[\varepsilon^{1/2}M^{1/2}L^{1/2}T^{-1}]$.

5. Dimensions of current in e.m.u. is

(*a*) $[\mu^{-1/2}M^{1/2}L^{1/2}T^{-1}]$ (*b*) $[\mu^{1/2}M^{1/2}L^{3/2}T^{-1}]$

(*c*) $[\mu^{-1/2}M^{1/2}L^{1/2}T^{-2}]$ (*d*) $[\mu^{1/2}M^{1/2}L^{1/2}T^{-1}]$.

6. The e.m.f. of Weston standard cell at the temperature of 20°C is

(*a*) 1.1087 V (*b*) 1.0183 V

(*c*) 1.1137 V (*d*) 1.1083 V.

7. The tachometer is used as a transducer to measure

(*a*) angle (*b*) torque

(*c*) angular speed (*d*) linear speed.

Answers

1. (*b*) **2.** (*c*) **3.** (*a*) **4.** (*c*) **5.** (*a*)

6. (*b*) **7.** (*c*)

REVIEW QUESTIONS

1. Write a brief essay on the role of measurement systems.
2. What are the points to be considered while choosing a measurement method? What are important points to be considered while preparing the measurement records? Mention general precautions to be taken in measurement.
3. What are the general functional blocks of a generalized instrumentation system? Draw a neat block diagram. Explain the function and requirement of each block with examples.
4. What are desired input, interfering input and modifying input? How do these inputs affect the output of a measurement system? Explain with an example.
5. Derive relationship between electrostatic and electromagnetic units.
6. Describe a method to show that the product $(\varepsilon_0^{-1/2}\ \mu_0^{-1/2})$ has the dimensions of velocity and the velocity is equal to that of light.
7. What is meant by the dimensions of a quantity? Derive the dimension of (*i*) charge, (*ii*) voltage, (*iii*) resistance, (*iv*) current density and (*v*) magnetic flux in MTI system of units.

8. What are advantages and disadvantages of rationalized MKS system of units? Name and define the basic and secondary units of SI system of units.

9. What are values of dielectric permittivity ε_0 and magnetic permeability μ_0 of free space in (*a*) c.g.s electrostatic system, (*b*) c.g.s. electromagnetic system, (*c*) rationalized MKS system and (*d*) unrationalized MKS system of units?

10. What are different types of standards? Describe a standard cell. What precautions must be taken when using a standard cell.

11. What are different applications of measurement systems? Explain each of them with at least one example.

EXERCISES

1. Derive dimensions of (*a*) current density, (*b*) voltage gradient, (*c*) charge, (*d*) magnetic flux density, (*e*) force, (*f*) permeability, (*g*) magnetic pole, (*h*) resistivity and (*i*) permittivity in LMTA system.

2. Derive dimensions of (*a*) electric flux, (*b*) potential gradient, (*c*) electric flux density, (*d*) electrostatic force, (*e*) resistance, (*f*) conductance and (*g*) potential in electrostatic system of units.

3. It is suspected that an error has been made in the derivation of the expression

$$I = \frac{V}{[(r_p + R)^2 + \omega^2 L_p]^{1/2}}$$

for the current through the pressure coil of a wattmeter, in terms of voltage V, angular frequency ω, self inductance L_p and resistance r_p, and resistance R.

Point out the term or terms missing and write the correct expression. Use any system of units to check the correctness of the expression.

4. The electrical power in a circuit is given by the equation

$$P = KV^m I^n$$

where V = voltage applied to the circuit

I = current drawn by the circuit

and K = numerical constant

Determine the values of m and n. Use LMTQ system of units.

5. There are some terms missing in the following expressions :

$$R_1 = \frac{R_2 R_3 R_4 \omega^2 C}{1 + \omega^2 R_4^2 C^2}$$

and $$L = \frac{R_2 R_3 C}{1 + \omega^2 R_4 C^2}$$

where L = self inductance, C = capacitance.

R_i = resistances (i = 1, 2, 3, 4) and

ω = angular frequency.

Find the term or terms missing and the correct expressions.

6. Prove that the following equations are dimensionally correct.

(*i*) $r = \dfrac{R\omega^2 M_1 M_2}{R^2 + \omega^2 L^2}$

(*ii*) $C = \dfrac{1}{\omega^2 M}$

where r, R = resistances

M_1, M_2, M = mutural inductances

L = self inductance

C = capacitance

$\omega = 2\pi f$ (f = frequency)

7. The capacitance is measured by Combell' method and is given by

$$C = \omega^a M^b$$

where $\omega = 2\omega f$ (f = frequency)

M = mutual inductance

a, b = unknown constants.

Determine the values of a and b.

8. In a new system of units kilometer, metric tonne and minute are units of length, mass and time respectively. Determine the number of M.K.S. units in one new units of force, velocity, resistance and inductance. The unit of permittivity remains same.