1

Field Astronomy

1.1. INTRODUCTION

The science of field astronomy offers to surveyors a means of determining the absolute location of any point or absolute location and direction of any line-on the surface of the earth, by making astronomical observations to celestial bodies. The celestial bodies *i.e.,* stars, sun, planets and moon appear to lie on the surface of a very large sphere which appears to move around the earth. To understand the real and apparent motions of these celestial bodies, surveyors must be familiar with the geometry of a sphere and spherical triangle.

Field astronomy has a wide scope in geodetic surveying for determination of true meridian, latitude, longitude and time.

1.2. PURPOSE OF FIELD ASTRONOMY

The application of field astronomy is usually done for the following purposes :

1. To determine the azimuth of the starting base of a triangulation series.
2. To determine the azimuth of starting and closing sides of precise traverses.
3. To determine the latitude and longitude of at least one of the triangulation stations so as to locate its position on the earth surface.
4. To check the accuracy of triangulation series at suitable intervals independently.

5. To carry out exploratory triangulations.
6. To demarcate the international boundaries.

1.3. GEOMETRY OF A SPHERE

A sphere is a solid bounded by a surface whose every point is equidistant from a fixed point called the *Centre* of the sphere. A sphere may be formed by revolving a semi-circle about its diameter.

The important properties of a sphere are as under :

1. A section of a sphere by any plane is a circle whose radius is inversely proportional to the perpendicular distance of the plane from the centre of the sphere. (Fig. 1.1)

Let O and O' be the centres of the great circle and any circle PQS respectively. The perpendicular distance OO' be x.

If R is the radius of the sphere, the radius of the circle PQS

$$O'P = \sqrt{R^2 - x^2}. \quad ...(1.1)$$

In equation (1.1), the value of $O'P$ depends upon the value of x. When $x = R$, the circle reduces to a point and if $x = 0$, the circle attains maximum radius equal to R and its area equals πR^2.

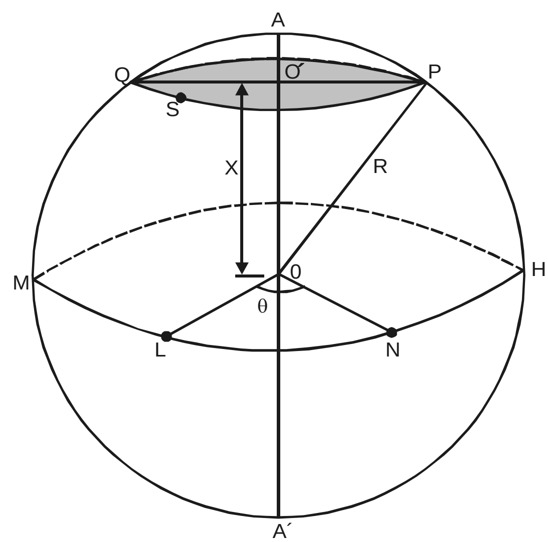


Fig. 1.1. A sphere.

2. Great circle. A section of a sphere is called a great circle if the section plane passes through the centre of the sphere.

Let *MLNH* be one of the great circles which can be obtained by different planes passing through the centre of the sphere *O*. The radius of great circle is equal to the radius of the sphere.

3. Small circle. *A* section of a sphere is called a small circle when the plane cutting the sphere does not pass through the centre of the sphere.

Let *PQS is* a small circle. The radius of a small circle is always less than the radius of the sphere.

4. A diameter of a sphere perpendicular to a great circle is called the axis of the great circle.

The ends *A* and *A´* are called the poles of the axis of the great circle.

5. The area of small circles of the same sphere are proportional to the angle they subtend at the centre of the sphere.

6. The great circles which pass through the poles of any other great circle, are called secondaries and the given great circle is called the primary.

7. The angle between the planes of two secondaries to another great circle, is equal to the arc they intercept on their primary.

8. If the poles of a great circle lie on another great circle, the poles of the latter will lie on the former. The two circles are mutual secondaries.

9. The shortest distance between any two points on the surface of a sphere is along the arc of a great circle passing through the given points.

It may be noted that there can be one and only one such arc.

10. The distance of any point on a small circle from its nearer pole, is called the angular radius or the polar distance of the small circle.

11. The length of an arc of a great circle is equal to the angle it subtends at the centre of the sphere of a unit radius.

Proof (Fig. 1.1.)

Let *LN* be an arc of the great circle *MLNHM*

A, A´ be the poles of its axis *AOA´*.

R be the radius of the sphere

θ be the angle subtended by the arc LN at the centre of the sphere, expressed in circular measure.

Then, arc $LN = R\theta = \theta$ if R is equal to unity.

12. The angular distance from the pole of a great circle to any point on that great circle, is a right angle.

13. The arc of a small circle is equal to the corresponding arc of the great circle multiplied by either the cosine of the distance between the two circles, or the sine of the angular radius of the small circle.

Proof. (Fig. 1.2)

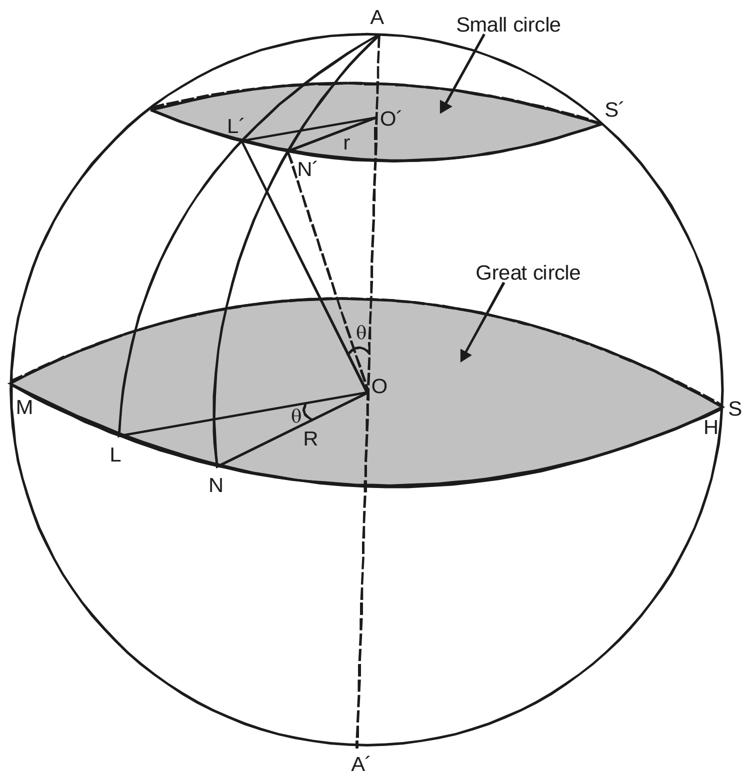


Fig. 1.2. Arcs of small and great circles.

Let S be a great circle whose poles are A and A'.

S' be a parallel small circle.

AL and *AN* are two secondaries to the great circle S cutting the small circle at *S´* at *L´* and *N´* respectively.

Join *L´O´* and *N´O´, O´* being the centre of the small circle *S´*. Join *LO* and *NO*.

As *L´O´* and *N´O´* are parallel to *LO* and *NO* respectively, the angles *L´O´N´* and *LON* are equal.

Let the radius of the small circle *S´* be *r,* and that of the great circle be *R*.

Join *L´* and *O* and denote the angle *L´OO´* as θ

From the right angled triangle *L´O´O* at *O´,* we get

Sin $L'OO' = L'O' / L'O = r / R$ (*OL´* being radius of the sphere)

Similarly, it can be shown that

$$\sin\ N'OO' = \frac{N'O'}{N'O} = \frac{r}{R}$$

i.e. the angle subtended by the radial distance of any point on a small circle is constant.

Again, $$\frac{\text{arc } L'N'}{\text{arc } LN} = \frac{r}{R} = \sin L'OO' = \sin\theta$$

or $$L'N' = \text{LN} \sin\theta = LN \sin AL' \qquad ...(1.2)$$

But $L'L = N'N$ (distance between the two circles)

∴ $L'L = 90° - \theta$

Substituting the values in eqn. (1.2) we get

$$L'N' = LN \cos LL' \qquad ...(1.3)$$

1.4. ARC OF A SMALL CIRCLE

To compare the arc of a small circle on a sphere subtending any angle at the centre of the circle within the arc of a great circle subtending the same angle at its centre.

Let *ab* be the ac of a small circle having its centre *C* and pole *P*. Let *O* be the centre of the sphere.

Construction (Fig. 1.3)

Joint *Ca*, *Cb*, *OA* and *OB*.

Since, *OP* is perpendicular to the plane *Cab* and *OAB,* it is perpendicular to *OA*, *OB*, *Ca* and *Cb*.

Hence, the angle *aCb* or *AOB* measures the angle between the plane *POA* and *POB*.

$\therefore \qquad \angle aCb = \angle AOB$

Hence, $$\frac{\text{arc } ab}{\text{radius } Ca} = \frac{\text{arc } AB}{\text{radius } OA} \qquad ...(i)$$

From eqn. (*i*), we get

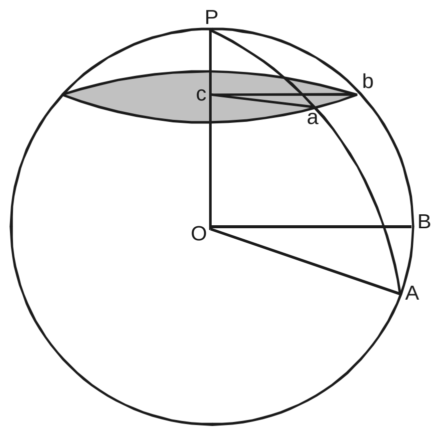


Fig. 1.3.

$$\frac{\text{arc } ab}{\text{radius } AB} = \frac{Ca}{OA} = \frac{Ca}{Oa} = \sin COa = \cos AOa$$

i.e., $$\frac{\text{arc } ab}{\text{radius } AB} = AB \cos Aa \text{ or } ab = AB \sin Pa$$

Also, angle $\quad bPa$ = arc AB is given by

Angle $\quad bPa = ab \sec Aa$ or ab cosec Pa

1.5. A LUNE

The portion of the surface of a sphere enclosed by two great semi-circles is called the *lune*. In the adjoining figure ABC and ADC are two semicircles. The enclosed area ABCDA is a lune shown shaded. (Fig. 1.4).

The angle *BAD* is called the *angle of the lune*. The two triangles such as ABD and CBD which divide the lune in two portions are called the *colunar triangles*.

The area of Lune.

Let *r* be the radius of the sphere.

A be the circular measure of the angle BAC (Fig.1.4)

$$\frac{\text{Area of lune}}{\text{Area of sphere}} = \frac{\text{Angle of the lune}}{2\pi}$$

where 2π is the circumference of the great circle.

$$\therefore \text{Area of lune} = 4\pi r^2 \times \frac{A}{2\pi} = 2Ar^2 \qquad \ldots(1.4)$$

1.6. ANTI-PODAL TRIANGLES

The triangles whose vertices are diametrically opposite to each other are called the antipodal triangles. In (Fig. 1.5) the triangle ABC and DEF are apparently antipodal.

Congruent triangles. When two spherical triangles can be superimposed one over the other, they are called the congruent (or identically congruent).

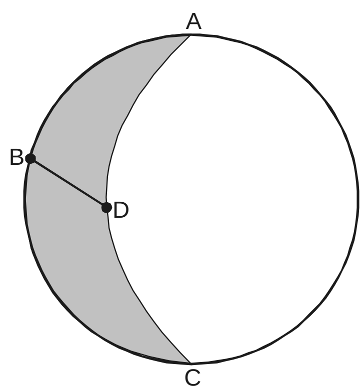


Fig. 1.4. A lune

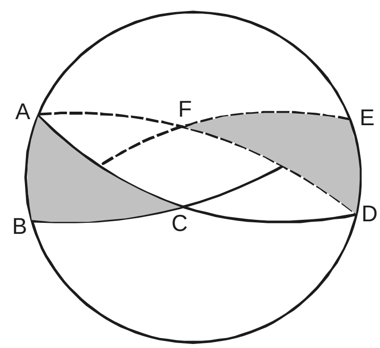


Fig. 1.5. Anti podal triangles

Symmetrical equal triangles. The triangles say ABC and DEC whose six elements of the one are equal to the corresponding elements of the other, but can not be superimposed on each other due to their convexity, are called symmetrical equal (Fig.1.5).

1.7. SPHERICAL TRIANGLE

The triangle which is formed upon the surface of a sphere by the intersections of three great circles, is called a *spherical triangle.*

Let *AB, BC* and *CA* be the arcs of three great circles of the same sphere having its centre at *O*. (Fig. 1.6)

The portion of the surface of the sphere bounded by these three arcs (shown shaded) is a spherical triangle *ABC*.

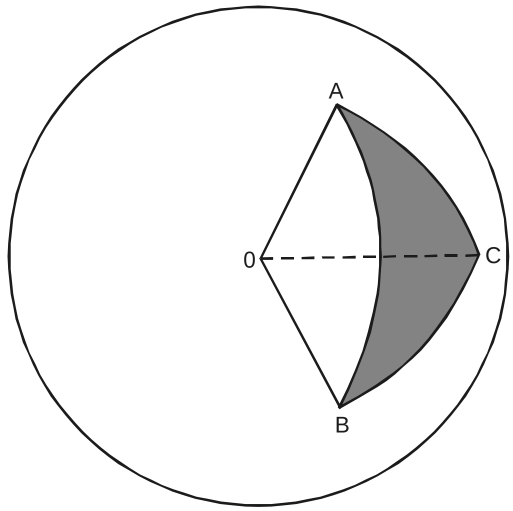


Fig. 1.6. A spherical triangle.

1.8. ELEMENTS OF A SPHERICAL TRIANGLE

A spherical triangle consists of six elements, *i.e.* three arc sides and three spherical angles. The sides of the triangle are the arcs *AB, BC* and *CA* which are generally represented by small letters *a, b* and c respectively.

All the six elements of the spherical triangle *ABC, i.e.* three arc sides and three angles, are expressed as angles.

1.9. SPHERICAL ANGLES

A spherical angle is formed by the intersection of two great circles. It may be defined by the plane angle between tangents to the circles at the point of intersection.

The spherical angles of the triangle *ABC* are the angles between the planes *CAO, AOB* and *BOC,* and are generally represented by capital letters *A*, *B* and C. (Fig. 1.7).

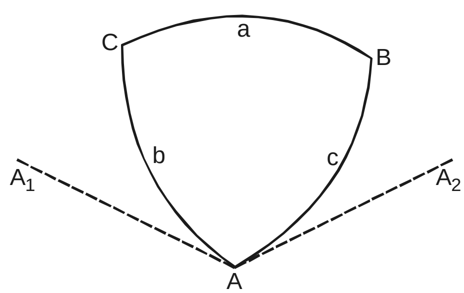


Fig. 1.7. A spherical angle.

1.10. PROPERTIES OF A SPHERICAL TRIANGLE

The important properties of any spherical triangle are as under:

1. *Angles opposite to equal sides are equal and vice versa.*
2. *Any angle is less than two right angles.*
3. *The sum of any two sides is greater than the third.*
4. *The difference between two sides is less than the third.*
5. *The greater angle is opposite the greater side and vice versa*
6. *The sum of the three angles is always greater than two right angles but less than six right angles.*

Proof. Let *ABC* be spherical triangle having *A, B, C* as its angles.

Let a', b', c' be the sides of the polar triangle.

Since, each of the angles *A, B, C* of triangle *ABC* is less than, π

$$A + B + C = \angle 3\pi$$

and $$a' + b' + c' = \angle 2\pi$$

i.e., $$\pi - A + \pi - B + \pi - C < 2\pi$$

$\therefore$ $$A + B + C > \pi$$

Hence, $\pi < A + B + C < 3\pi$ Proved.

1.11. SOLUTION OF A SPHERICAL TRIANGLE

Knowing any three of six elements *a, b, c, A, B* and C of a spherical triangle *ABC,* the remaining three elements may be computed by the following formulae:

Let *A, B* and C be the spherical angles and *a, b, c* the sides opposite them in the spherical triangle ABC (Fig. 1.8).

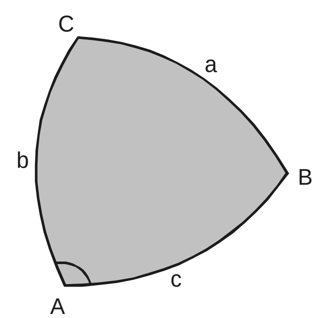


Fig. 1.8.

1. Sine formulae :

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C} \qquad ...(1.5)$$

Proof.

We know that $$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \,.\, \sin c}$$

$$\therefore \quad \sin^2 A = 1 - \cos^2 A = 1 - \frac{(\cos a - \cos b \cos c)^2}{\sin^2 b \,.\, \sin^2 c}$$

$$\text{or } \sin^2 A = \frac{\sin^2 b . \sin^2 c - (\cos a - \cos b \cos c)^2}{\sin^2 b \,.\, \sin^2 c}$$

$$\text{or } \sin^2 A = \frac{(1-\cos^2 b)(1-\cos^2 c) - (\cos^2 a + \cos^2 b \cos^2 c - 2\cos a \cos b \cos c)}{\sin^2 b \,.\, \sin^2 c}$$

$$\text{or } \sin^2 A = \frac{\begin{matrix}(1-\cos^2 b - \cos^2 c + (\cos^2 b . \cos^2 c) - \\ (2\cos^2 a + \cos^2 b \cos^2 c - 2\cos a \cos b \cos c)\end{matrix}}{\sin^2 b \,.\, \sin^2 c}$$

$$\text{or } \sin A = \frac{(1-\cos^2 a - \cos^2 b - \cos^2 c + 2\cos a \cos b \cos c)^{1/2}}{\sin b \,.\, \sin c} \qquad ...(i)$$

Considering the fact that the sides of a spherical triangle are each less than two right angles, we may assume positive values of sin A, sin b, and sin c.

Dividing the eqn. (i) by sin a, we get,

$$\frac{\sin A}{\sin a} = \frac{(1-\cos^2 a - \cos^2 b - \cos^2 c + 2\cos a \cos b \cos c)^{1/2}}{\sin a \sin b \sin c}$$

$$= \frac{[1-(\cos^2 a + \cos^2 b + \cos^2 c) + 2\cos a \cos b \cos c)]^{1/2}}{\sin a \,.\, \sin b \,.\, \sin c} \qquad ...(A)$$

The symmetry of eqn (A) suggests that

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

$$= \frac{[1-(\cos^2 a + \cos^2 b + \cos^2 c) + 2\cos a \cos b \cos c)]}{\sin a \,.\, \sin b \,.\, \sin c}$$

Hence, $\frac{\sin A}{\sin a} \cdot \frac{\sin B}{\sin b} \cdot \frac{\sin C}{\sin c}$ **Proved.**

2. Cosine formula

$$\cos a = \cos b . \cos c + \sin b \sin c \cos A \qquad ...(1.6)$$

Proof. Let AB, BC, CA represent the arcs of a great circle having O as its centre.

ABC is a spherical triangle.

$\therefore \angle BOC = a, \ \angle AOB = c$ and $\angle COA = b$

We know that the angle between two curves is equal to the angle between the tangents to them at their common point of intersection.

Construction. Draw AE and AD tangents to the great circle arcs AC and AB containing the angle A and meeting OC and OB produced respectively at E and D.

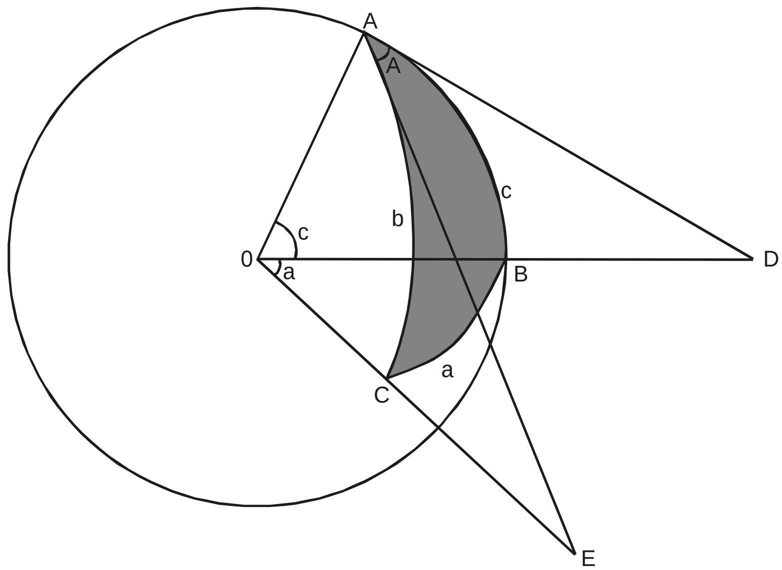


Fig. 1.9.

Assuming the sides containing the angle A, each less than $\pi/2$, we get

$$\angle DAE = \angle A$$

$$\therefore \quad \angle EOD = \angle BOC = a$$

$$\angle AOD = \angle AOB = c$$

$$\angle AOE = \angle AOC = b$$

By applying the cosine rule to plane triangles EOD and EAD, we get,

$$a^2 = b^2 + c^2 + 2bc \cos A$$

Thus, from ΔEOD, we get

$$DE^2 = OD^2 + OE^2 - 2OD.\ OE \cos a \qquad ...(i)$$

and

$$DE^2 = AD^2 + AE^2 - 2AD.\ AE \cos A \qquad ...(ii)$$

We also know that an angle between the tangent and the radius is a right angle

$$\therefore \quad \angle OAD = \pi/2 \quad \text{and} \quad \angle OAE = \pi/2$$

By applying the pythagorus theorem to Δs *OAD* and *OAE*, we get

$$OD^2 = OA^2 + AD^2 \quad ...(iii)$$

$$OE^2 = OA^2 + AE^2 \quad ...(iv)$$

By subtracting eqn (*i*) from eqn (*ii*), we get

$$O = (OD^2 - AD^2) + (OE^2 - AE^2) - 2OD \,.\, OE \cos a + 2AD \,.\, AE \cos A$$

By using the eqns (*iii*) and (*iv*), we get

or $$2OD \,.\, OE \cos a = OA^2 + OA^2 + 2\, ADAE \cos A.$$

or $$OD \,.\, OE \cos a = OA^2 + AD \,.\, AE \cos A$$

Dividing by *OD* . *OE*, we get

$$\cos a = \frac{OA}{OD} \cdot \frac{OA}{OE} + \frac{AD}{OD} \cdot \frac{AE}{OE} \,.\, \cos \text{A} \quad ...(v)$$

From right angled triangle *OAD*, Fig. 1.9, we get

$$\frac{OA}{OD} = \cos c,$$

$$\frac{AD}{OD} = \sin c,$$

$$\frac{OA}{OE} = \cos b,$$

$$\frac{AE}{OE} = \sin b,$$

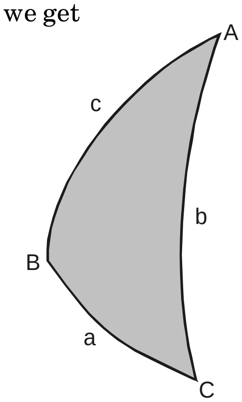


Fig. 1.10.

Substituting these values in eqn. (*v*), we get

$$\cos a = \cos c \cos b + \sin c \sin b \,.\, \cos A \quad (A)$$

Similalry, the other formulae are :

$$\cos b = \cos c \cos a + \sin c \sin a \,.\, \cos B \quad (B)$$

$$\cos b = \cos a \cos b + \sin a \sin b \,.\, \cos C \quad (C)$$

Formulae *A, B* and *C* are very widely used both in spherical trigonometry and astronomy.

$$\cos A = \frac{\cos a - \cos b \,.\, \cos c}{\sin b \,.\, \sin c} \quad ...(1.7)$$

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a ...(1.8)$$

$$\sin\frac{A}{2} = \sqrt{\frac{\sin(S-b)\sin(S-c)}{\sin b \,.\, \sin c}} \qquad ...(1.9)$$

$$\cos\frac{A}{2} = \sqrt{\frac{\sin s \sin(S-a)}{\sin b \,.\, \sin c}} \qquad ...(1.10)$$

$$\tan\frac{A}{2} = \sqrt{\frac{\sin(S-b)\sin(S-c)}{\sin s\ \sin(S-a)}} \qquad ...(1.11)$$

where $a + b + c = 2S$

If two spherical angles A and B and the side c opposite to the third angle C are known, then

$$\tan\frac{1}{2}(a+b) = \frac{\cos\frac{1}{2}(A-B)}{\cos\frac{1}{2}(A+B)}\tan\frac{c}{2} \qquad ...(1.12)$$

$$\tan\frac{1}{2}(a-b) = \frac{\sin\frac{1}{2}(A-B)}{\sin\frac{1}{2}(A+B)}\tan\frac{c}{2} \qquad ...(1.13)$$

If two sides a and b and the spherical angle opposite the third side c are known, then

$$\tan\frac{(A+B)}{2} = \frac{\cos\frac{1}{2}(a-b)}{\cos\frac{1}{2}(a+b)}\cot\frac{C}{2} \qquad ...(1.14)$$

$$\tan\frac{(A-B)}{2} = \frac{\sin\frac{1}{2}(a-b)}{\sin\frac{1}{2}(a+b)}\cot\frac{C}{2} \qquad ...(1.15)$$

If c, B, a and C are any four elements of a spherical triangle, the side a which lies between B and C is called the *inner side,* and c is called the *other side.* Similarly, B is called the *inner angle* and C is called the *other angle.* The relation between four adjacent elements may be stated as under :

(cosine of inner side) × (cosine of inner angle)

= (sine of inner side) × (cotangent of other side)

– (sine of inner angle) × (cotangent of other angle)

i.e. $\cos a \cos B = \sin a \cot c - \sin B \cot c.$ ...(1.16)

1.12. SOLUTION OF A RIGHT ANGLED SPHERICAL TRIANGLE BY NAPIER´S RULE

Solution of a right angled spherical triangle by Napier´s Rule, may be made as under :

(*i*) Draw a circle and divide it into five parts.

(*ii*) Enter two sides containing the right angle, *i.e.* c and a where B is a right angle.

(*iii*) Enter the complements of the remaining three elements, i.e. (90°— C), (90°— b), (90°— A) either clockwise or anti-clockwise.

Fig. 1.11. Napier´s rule.

Considering any part as a middle part, the two parts adjacent to it, are called, *adjacent parts* and the remaining two parts are called, *opposite parts*. (Fig. 1.11)

Napier´s Rules of Circular Parts may be stated as under :

1. Sine of the middle part = Product of the tangents of adjacent parts.
2. Sine of the middle part = Product of the cosines of opposite parts.

i.e.
$$\sin a = \tan c \,.\, \tan (90° - C)$$
$$= \tan c \,.\, \cot C \qquad ...(1.17)$$

and
$$\sin a = \cos (90° - A) \cos (90° - b)$$
$$= \sin A \sin b \qquad ...(1.18)$$

Note. The following points may be noted :

(*i*) By convention, each side of a spherical triangle is taken to be less than 90°.

(*ii*) The value of the sine or cosine of a side or of an angle obtained from the given data, must be less than unity to achieve a solution.

1.13. SPHERICAL EXCESS

The three angles of a spherical triangle do not sum up exactly 180°, their sum always exceeds two right angles by an amount which is known as the *spherical excess*. The magnitude of the spherical excess of any spherical triangle, is directly proportional to its area.

Proof. Let *ABA´DA* and *ACA´EA* be two intersecting great circles which are intersected by a third great circle *BCDEB* orthogonally (Fig. 1.12).

Then, the arc *BC* is a measure of the spherical angles at *A* and *A´*.

If *R* is the radius of the sphere, the total surface area *S´* of the sphere is $4\pi R^2$.

$$\text{Area of the portion } ABA'CA = \frac{S' \times BC}{2\pi R} = \frac{S' \times R \,.\, A}{2\pi R}$$

$$= \frac{A}{2\pi} S' \text{ if angle } A \text{ is in radians}$$

$$= \frac{A}{360^\circ} S' \text{ if angle } A \text{ is in degrees.}$$

Again, let three great circles *BCEDB, ABFEA* and *ACFDA* intersect each other to form a spherical triangle *ABC* (Fig. 1.13).

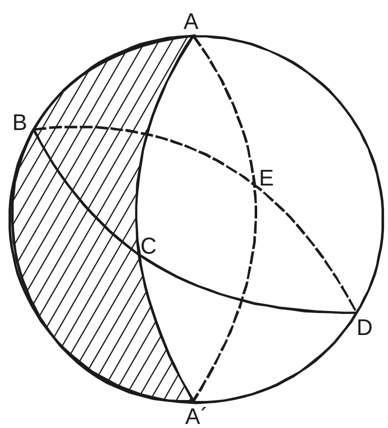


Fig. 1.12

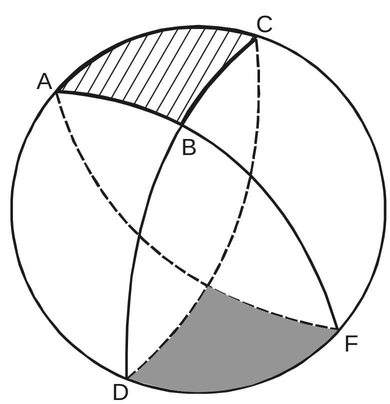


Fig. 1.13

From the symmetry of the Fig. 1.13,

Surface area of the triangle *ABC* = Surface area of the triangle *DEF* = *S*

Surface area of the portion *ABD* = Surface area of the portion *EFC* = β

Surface area of the portion *BCF* = Surface area of the portion *DEA* = γ

Surface area of the portion BDF = Surface area of the portion $EAC = \delta$

Total surface area of the portion $CADBC$

$$= S + \beta = \frac{C}{360^\circ} \times S' \quad ...(1.19)$$

Total surface area of the portion $ACFBA$

$$= S + \gamma = \frac{A}{360^\circ} \times S' \quad ...(1.20)$$

Total surface are of the portion $BAECB$

$$= \text{S} + \delta = \frac{B}{360^\circ} \times S' \quad ...(1.21)$$

By adding equations (1.19), (1.20) and (1.21), we get

$$3\text{S} + \beta + \gamma + \delta = \frac{S'}{360^\circ} (A + B + C)$$

or
$$2\text{S} + (S + \beta + \gamma + \delta) = \frac{S'}{360^\circ} (A + B + C)$$

But, $S + \beta + \gamma + \delta$ = surface area of the hemisphere $ACFDA = \frac{S'}{2}$

$$\therefore \quad 2S + \frac{S'}{2} = \frac{S'}{360^\circ} (A + B + C)$$

or
$$2\text{S} = \frac{S'}{360^\circ} (A + B + C - 180^\circ)$$

But, by definition A + B + C – 180° = spherical excess *(e)* of the spherical triangle ABC.

$$\therefore \quad e = \frac{2 \times 360^\circ}{S'} = \frac{S \times 180^\circ}{\frac{1}{4} S'}$$

$$= \frac{\text{Area of spherical triangle}}{\text{One-fourth area of sphere}} \times 180^\circ$$

or
$$e = \frac{\text{Area of triangle}}{\pi R^2} \times 180^\circ \quad ...(1.22)$$

1.14. THE AREA OF A SPHERICAL TRIANGLE

If A, B and C be the spherical angles of a triangle, the area of the spherical triangle ABC from eqn. (1.22) i.e. $$\text{Area } \Delta = \frac{\pi R^2 (A + B + C - 180°)}{180°}$$

$$= \frac{\pi R^2 e}{180°} \qquad \text{...(1.23)}$$

where e is the spherical excess.

1.15. CELESTIAL SPHERE AND RELATED ASTRONOMICAL TERMS (FIG. 1.14)

The following technical terms may be clearly understood :

1. The Celestial Sphere. The imaginary sphere on which heavenly bodies, *i.e.* stars, sun, moon, etc. appear to lie, is known as the *celestial sphere.* As stars are at vast distances attached to the surface of the imaginary celestial sphere, the centre of the earth may be assumed as the centre of the celestial sphere.

2. The Zenith. The point on the celestial sphere, exactly above the observer's head, is known as *the zenith.* It may be obtained by the prolongation of the plumb line upward up to the celestial sphere.

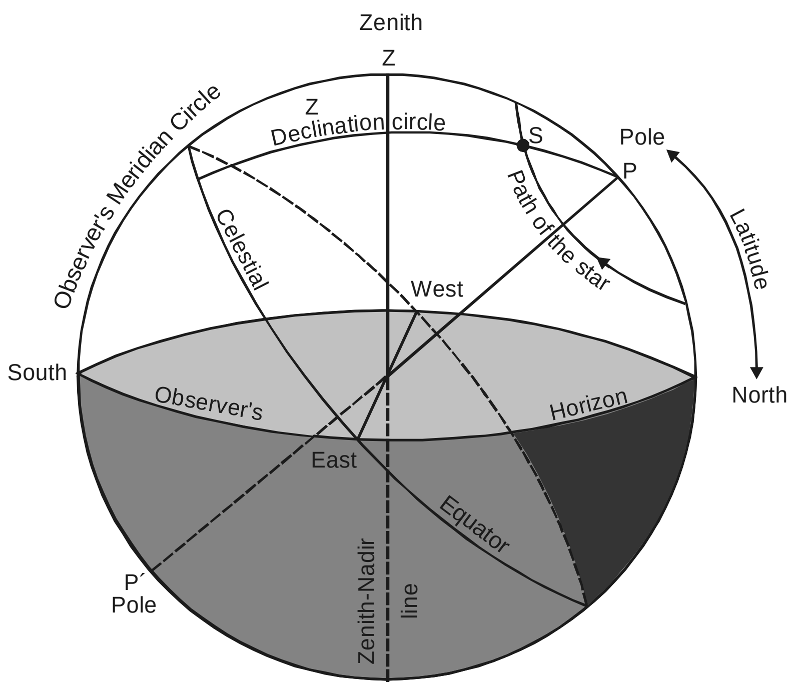


Fig. 1.14. Celestial sphere.

3. The Nadir. The point on the celestial sphere exactly below the observer´s station, is known as the *Nadir.* It may be obtained by the prolongation of the plumb line downward through the earth up to the celestial sphere.

4. The Zenith-Nadir Line. The zenith, the observer's station, the centre of the earth and the Nadir, all lie on a line which is known as *Zenith-Nadir line.*

5. The Celestial Horizon. The great circle of the celestial sphere obtained by a plane passing through the centre of the earth and perpendicular to the Zenith-Nadir line, is known as the *celestial horizon.* The celestial horizon is also sometimes known as *true, rational* or *geocentric horizon.*

6. The Visible Horizon. The small circle of the earth which is obtained by visual rays passing through the point of observation, is known as the *visible horizon.* Its radius depends on the altitude of the point of observation.

7. The Sensible Horizon. The small circle which is obtained by a plane passing through the observer´s station and being tangential to the earth´s surface and perpendicular to the Zenith-Nadir line at the point of observation, is known as the *sensible horizon.*

8. The Terrestrial Equator. The great circle of the earth, the plane of which is perpendicular to its axis of rotation, is known as the *terrestrial equator.*

9. The celestial equator. The great circle of the celestial sphere, the plane of which is perpendicular to the axis of rotation of the earth and its continuation, is known as the *celestial equator.* It may be obtained by projecting the terrestrial equator on the celestial sphere.

10. The terrestrial poles. The points at which the earth's axis of rotation meets the earth's surface, are known as the *terrestrial poles.*

11. The celestial poles. The points at which the earth's axis of rotation, on prolongation on either side, meets the surface of the celestial sphere, are known as *celestial poles.*

12. Vertical circles. The great circles of the celestial sphere, which pass through the Zenith and Nadir, are known as *vertical circles.*

13. The observer´s Meridian. The vertical circle which passes through the Zenith and Nadir of the station of observation as well as through the poles, is known as observer's meridian or celestial meridian.

14. The prime vertical. The vertical circle which is perpendicular to the observer's meridian and which passes through the east and west points of the horizon, is known as *the prime vertical.*

15. North and South Points. The projected points of the elevated north pole and depressed south poles, on the observer's horizon, are known as *north* and *south points* respectively.

16. East and West points. The points at which the prime vertical meets the horizon, are known as *east* and *west points.* These points may also be obtained by the intersection of the equator and horizon.

17. Ecliptic. The great circle of the celestial sphere which the sun appears to describe with earth as centre during a period of one year, is known as *ecliptic.* The angle between the plane of ecliptic and the plane of the equator, is known at *obliquity* and its value is 23° 27′.

18. First Points of Aries and Libra. The first point of Aries (γ) is the point where the sun crosses the equator from south to north on or about 21st March, when day and night are of equal duration. The first point of Libra (♎) is the point where the sun crosses the equator from north to south.

19. Equinoxes. The first point of Aries and the first point of Libra, which are six months apart in time, are generally known as *vernal Equinox and Autumnal Equinox* respectively.

20. Solstices. The points on the ecliptic at which the north or south declination is maximum, are known as the *solstices.*

21. Summer Solstice. The time at which the sun is farthest from the equator is called the *summer solstice.*

22. Winter Solstice. The time at which the sun is farthest south from the equator is called winter solstice.

The traces of ecliptic and equators are shown by two great circles in (Fig. 1.15).

AB is the trace of the equator

CD is the trace of the ecliptic

γ is the first point of Aeries

♎ is the first point of Libra.

D is summer solstice for northern hemisphere.

C is winter solstice for northern hemisphere.

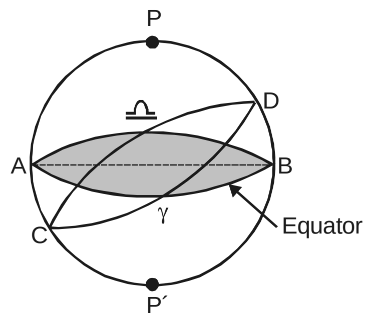


Fig. 1.15. Ecliptic and Equator

23. Motion of sun. The motion of sun during a year is as under:

On 21st March, the sun is at First point of Aries (vernal equinix) and its right ascension and declination both are zero.

On 21st June, the sun is at its summer solstice and its right ascension is 90° (6*h*) and declination is 23°27´ *N*.

On 21st/22nd September, the sun is at First point of Libra (Autumnal Equinox) and its right ascension is 180° (12*h*) and declination is zero.

On 22nd/21st December, the sun is at its winter solstice and its right ascension is 270° *(18h)* and declination is 23° 27´ *S*.

From March 21 to September 22, the sun's declination is north whereas from September 22 to March 21, sun's declination is South.

Days and nights are of equal duration on March 21 and September 22 all over the world.

1.16. DIURNAL MOTION AND SIDEREAL DAY

Due to earth's rotation about its axis *PP´* from west to east, the whole celestial sphere appears to rotate about *PP´* in the opposite direction and all the celestial objects, the stars, the sun, the moon and the planets appears to go round the earth, in the direction of the rotation. That is why, they seem to go round the axis *PP´*, all in the same direction which is opposite to that of the earth's rotation and thus the time taken to go once round the earth is the same for all of them namely the period of the earth's rotation once about its axis. This fixed period is called a ***sidereal day.*** And the apparent motion of the stars is called their ***diurnal motion.*** The angular distances of the stars from each other remains unaltered and the distances of any star S from the poles P and P' will remain constant for all times. This constant distance of a star from P is called its north polar distance and the diurnal (or daily) path of the star in the sky is therefore a small circle having P, P' as poles, and hence remain parallel to the celestial equator. The equator defines itself as a diurnal path of a star having its polar distance as 90°.

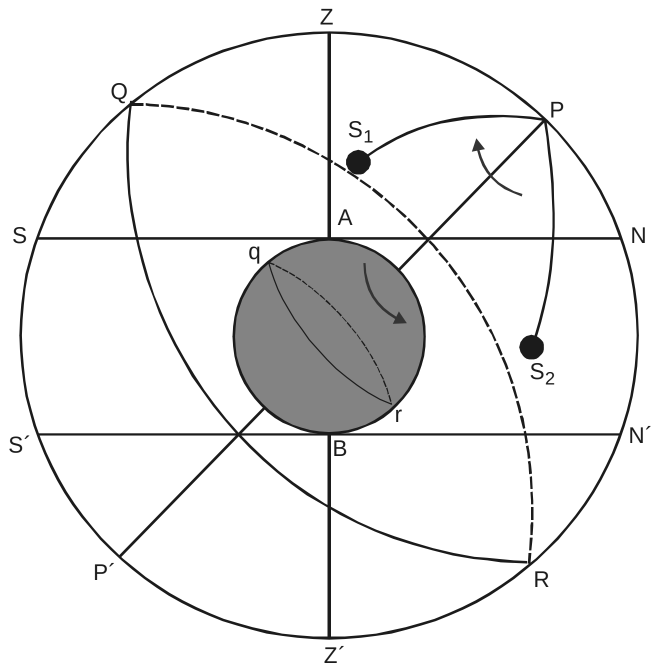


Fig. 1.16. Diurnal motion of a celestial body.

In the above discussion, the poles P and P' are imaginary points, but this position of the north pole P is indicated by a star called pole star which is very near to it. It has been observed that any star seen in the eastern horizon the sun rises gradually up to the observer's meridian and thereafter descends to the western horizon and disappears. It is seen again in the eastern horizon, the next night. Moreover the interval between two consecutive risings of a star on the eastern horizon is found to be the same for all stars at all time. This interval is defined as a sidereal day. Moreover, the distance of any star from the pole star remains unaltered, and thus, it shows that star's polar distance is constant.

Considering the diurnal path of a star S to be a small circle,

S_1A, S_2B parallel to the equator QR cutting the horizon at S_1 and S_2 and meridian at A and B. The transit at A above the pole P is called upper transit and the other at B is called lower transit. the sidereal day can be defined as the interval between two consecutive transits of a star across the same meridian, transits being either both upper or both lower. A sidereal day is further divided into 24 sidereal hours, each hour into 60 minutes and each minute divided into 60 seconds.

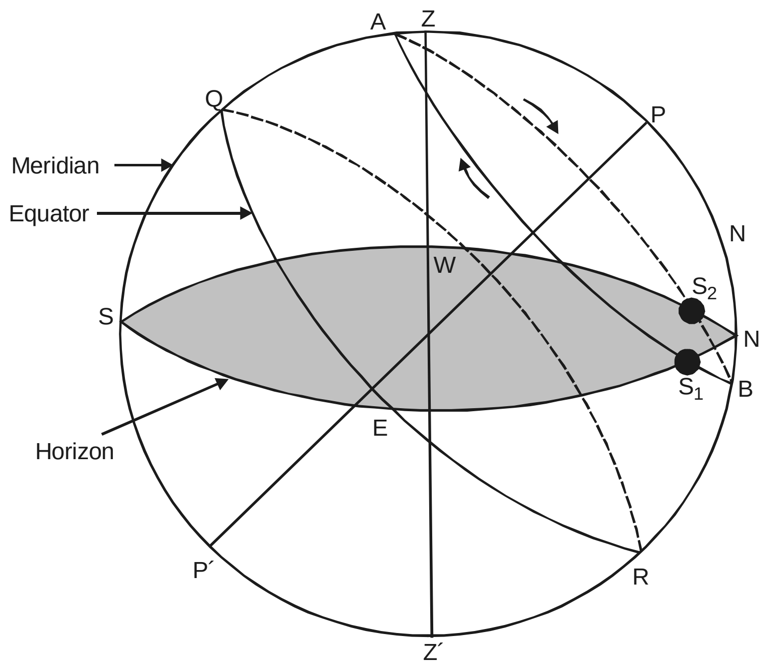


Fig. 1.17.

1.17. ASTRONOMICAL CO-ORDINATE SYSTEMS

In order to locate the position of heavenly bodies on the celes-´ tial sphere, at any moment, the following system of co-ordinates ‘ are used :

1. Right ascension and declination system
2. Altitude and Azimuth system
3. Declination and hour angle system
4. Celestial latitude and longitude system.

1. The right ascension and declination system. (Fig. 1.18). This system is generally used for publication of the star almanacs in which location of heavenly bodies is referred to by spherical co-ordinates *i.e.* Right Ascension and Declination. These coordinates are independent of the observer's position.

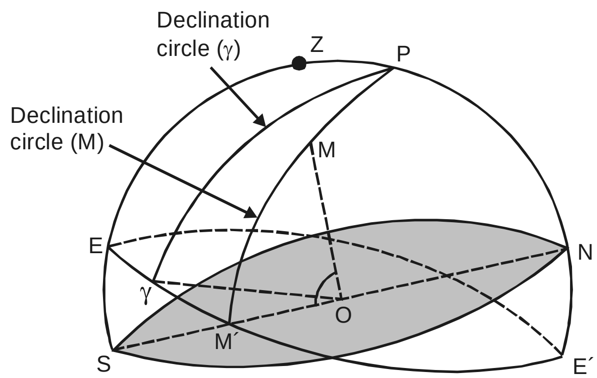


Fig. 1.18. Right ascension and declination.

(*i*) **Declination** (δ). The angular distance of the celestial body from the celestial equator along the great circle passing through the celestial poles and the celestial body, is known as *the declination of* the celestial body. Declination varies from 0° to + 90° if the heavenly body is north of the equator and 0° to – 90° if it is south of the equator. The great circle is called the *declination circle* and the angular distance of the celestial body from the nearer pole is known as the *co-declination* or *polar distance* of the celestial body.

Declination and polar distance of any celestial body are complementary to each other.

(*ii*) **Right Ascension.** The equatorial angular distance measured eastward from the declination circle of the First Point of Aries to the declination circle of the celestial body, is called *Right Ascension*

The spherical co-ordinates of the celestial body M are :

Right ascension $\gamma M'$ measured along the equator eastward.

Declination $M'M$ is measured along declination circle of the celestial body from the equator.

2. Altitude and Azimuth system (Fig. 1.19). This system of coordinates is necessiated to make direct observations with the help of a theodolite. In this system, the horizon is the plane of reference of spherical co-ordinates. The location of the heavenly body is referred to by the azimuth (A) and the altitude (α) of the celestial body.

(*i*) **The Azimuth** (A). The angle between the observer´s meridian and the vertical circle passing through the celestial body and the zenith, is known as the *azimuth*.

(*ii*) **The Altitude** (α). The angular distance of a heavenly body above the horizon, measured on the vertical circle passing through it, is called its *altitude*.

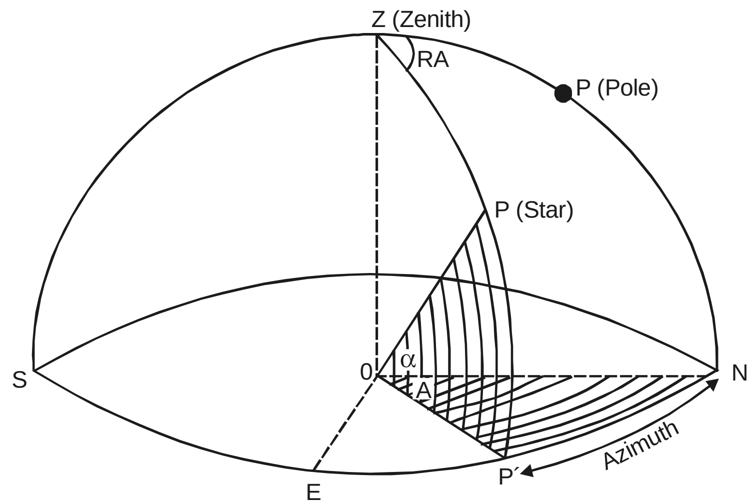


Fig. 1.19. Azimuth and altitude system.

Let P be a heavenly body and P´ be the point where the vertical circle passing through P and Z meets the horizon. The first co-ordinate of P is the azimuth (A), the angle between the observer's meridian and the vertical circle through it. The other co-ordinate of P is the altitude (α), the angular distance measured above or below the horizon along the vertical circle through the body. The azimuth is always measured from north either eastward or westward in northern hemisphere and from south eastward or westward in southern hemisphere The angular distance between the body and the zenith along the vertical circle is known as *zenith distance* or co-altitude of the body. If α is the altitude, the co-ordinates of body are: (*i*) Azimuth (A) and Zenith distance $Z = 90° - \alpha$.

3. Declination and hour angle system (Fig. 1.20).

In this system the location of the heavenly body is referred to by spherical co-ordinates *i.e.* the hour angle and declination of the body.

(*i*) **Hour angle (HA).** The angular distance along the arc of the horizon measured from the observer's meridian westward to the declination circle of the body, is known as the *hour angle*.

Or

The angle between the observer's meridian and the declination circle of the body, is known as the *hour angle* of the body.

The hour angles of the celestial bodies in the northern hemisphere are always measured from the south along horizon and towards west. Its value varies from 0° to 360°. If the hour angle is either zero or 180°, the body lies on the observer´s meridian, if its value varies from 0° to 180°, the body is in the western hemisphere and if its value varies from 180° to 360°, the body is in the eastern hemisphere.

The spherical co-ordinates of the celestial body *M* on the celestial sphere are : Hour angle *EPM´* measured from the south westward ; Declination *M´M* measured from the equator upward.

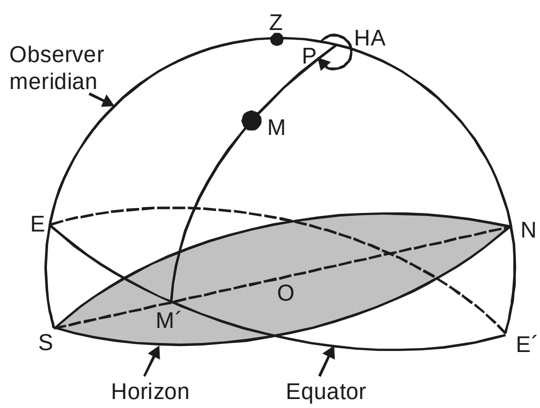


Fig. 1.20. Declination and hour angle System.

In Fig. 1.20 celestial body *M* is seen in the eastern hemisphere.

4. The celestial latitude and longitude system (Fig. 1.21). In this system location of a heavenly body is referred to by spherical the co-ordinates : celestial latitude and celestial longitude.

(*i*) **The celestial latitude.** The arc of a great circle perpendicular to the ecliptic, intercepted between the celestial body and the ecliptic, is known as the *celestial latitude* of the body. Its value varies from zero to 90°.

(*ii*) **The celestial longitude.** The arc of an ecliptic intercepted between the great circle passing through the First point of Aries (γ) and the circle of the celestial latitude passing through the body. Its value varies from 0° to 360°.

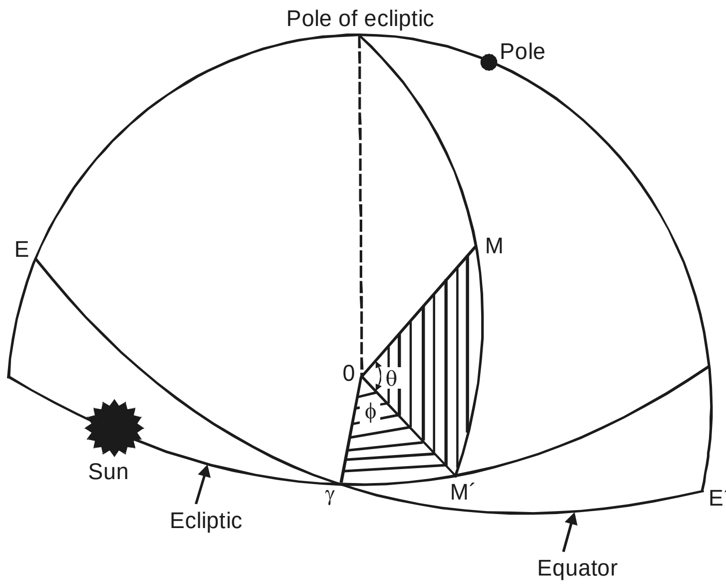


Fig. 1.21. Celestial latitude and longitude System.

In Fig. 1.21 *MM´* is the celestial latitude and γ*M´* is the celestial longitude of the heavenly body M .

1.18. THE RELATIVE ADVANTAGES OF DIFFERENT SYSTEMS OF CO-ORDINATES

The horizon, meridian, zenith, north and south points for a station of observation do not vary whereas the altitude, zenith distance, and azimuth of celestial bodies change from place to place and from time to time. The equator, ecliptic, the First point of Aries (γ) and the poles of the equator are common to all observers. The pole *P* in the northern hemisphere can be easily fixed by knowing the latitude of station of observation and consequently the equator can be imagined with a fair accuracy. But, it is difficult to imagine the positions of the First point of Aries and First point of Libra at any instant as they move on the equator due to diurnal motion. It is comparatively more difficult to imagine the ecliptic and the path traverses by the sun on the celestial sphere.

From the above discussion, it is evident that it is not easy to identify celestial bodies (particularly stars) in the sky by their Right Ascension and Declination or by their Latitudes and Longitudes. The latitude of the sun is always zero. The latitude and longitude co-ordinates are therefore very useful to fix the positions of the sun. As the right ascension and the declination of heavenly bodies do not vary, these are very useful to prepare star almanacs. Though, azimuth and altitude vary from place to place and time to time, these co-ordinates can be directly measured with the help of a theodolite.

1.19. TERRESTRIAL CO-ORDINATES OF POINTS ON THE EARTH'S SURFACE (Fig. 1.22)

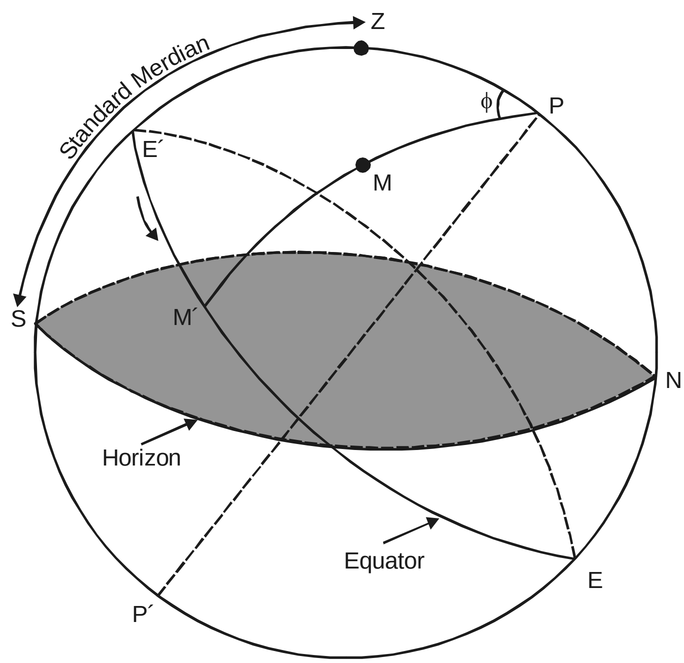


Fig. 1.22. Terrestrial latitude and longitude System.

To locate the position of various points on the surface of the earth, a system of terrestrial latitudes and longitudes is used.

(1) **The terrestrial meridian.** The great circle whose plane passes through the axis of earth *i.e.* through the north and south poles of the earth, is known as *terrestrial meridian.*

(2) **The terrestrial equator.** The trace of the great circle whose plane is perpendicular to the axis of earth's rotation, is known as the *terrestrial equator.*

(3) **The terrestrial latitude.** The angle subtended at the centre of the earth by the arc of meridian intercepted between the station and the equator, is known as the *terrestrial latitude.* Its value varies from 0° to 90°. It is said to be positive if measured above the equator and negative if measured below the equator. The latitude of the equator is zero and that of north and south poles are + 90° and – 90° respectively. The complement of the latitude which is the angular distance between the station and the nearer pole measured along the meridian, is termed *co-latitude.*

(4) **The terrestrial longitude.** The arc of the equator intercepted between the meridian plane of the observer's station and some other arbitrarily chosen fixed meridian plane is known as the *terrestrial longitude* of the place. The standard meridian universally adopted is that of Greenwich, a place west of London in United Kingdom. The value of longitudes varies from 0° to 180° and is said to be east or west of Greenwich. The points on any meridian, have the same longitude.

In Fig. 1.22 if *NS* and *EE´* are the great circles of the horizon and the equator, respectively *P* is the north pole, and *ZE´SP´ENPZ* is the great circle representing the standard meridian. The terrestrial co-ordinates of the place *M* are :

Latitude (θ) = angular distance *M´M* measured along *M´P* above the equator *E´E.*

Longitude (ϕ) = angular distance *E´M´* measured along the equator from the standad meridian.

1.20. FIXING A CELESTIAL BODY ON THE CELESTIAL SPHERE

The celestial bodies remain relatively fixed on the celestial sphere. A diagram illustrating their relative positions on the celestial sphere may be drawn as explained below :

1. Assume the plane of the paper to represent the plane of the observer's meridian.
2. Draw a circle of suitable radius to represent the observer's meridian *NZSZ´* where *NS* is its horizontal diameter and ZZ´ is vertical diameter. *NS* therefore presents the horizon and ZZ´ as the Zenith-Nadir line. Let *O* be the centre of the circle.

3. Draw *OP* making angle *PON* = latitude of the place. Produce *PO* to cut the circle at *P´*. *P* and P´ are the elevated and depressed poles.
4. Draw a line *EE´* passing through the centre of the circle and perpendicular to *PP´*. *EE´ is* the trace of the celestial equator.
5. Draw a line *w´w* making an angle 23° 27´. *ww´* represents the ecliptic of the sun´s path. The points of intersection of ecliptic and equator are First points of Aeries and Libra. The points where sun crosses the equator from south to north is called the First point of Aeries (γ) and the point where sun crosses the equator from north to south, is called the First point of Libra (♎).
6. Measure *EM* on the equator equal to the given right ascension of the star along the equator, taking γ as zero and measuring eastward.
7. Draw a great circle *PM* through *P* and *M* to represent the declination circle of the given celestial body if it is north of equator otherwise join *P´M* for the south declination. To locate the celestial body on the declination circle, measure an arc along the declination circle equal to the given declination with proper sign.
8. If the given celestial body is sun, its position on the celestial sphere will be at the point of intersection of declination and ecliptic already fixed.

Example 1.1. *Draw a diagram of the celestial sphere as seen at a place Latitude 28° 42´ on 15th April at 4 p.m., showing therein the positions of the sun and a star, RA is 6° 36´ and declination 24° 24´ N.*

Solution. (Fig. 1.23).

Draw the celestial sphere as explained here under :

1. Draw a circle to prepresent the observer's meridian.
2. Assume Z, *N as* the zenith and nadir, *H* and *H´* as the North and South points on the horizon.
3. Mark off *HP* and *H´P* equal to 28° 42´ to fix the poistions of north and south poles *P* and *P´* respectively.
4. Locate *Q* and *Q´* to define the equator such that *PQ* = *P´Q* = 90°.

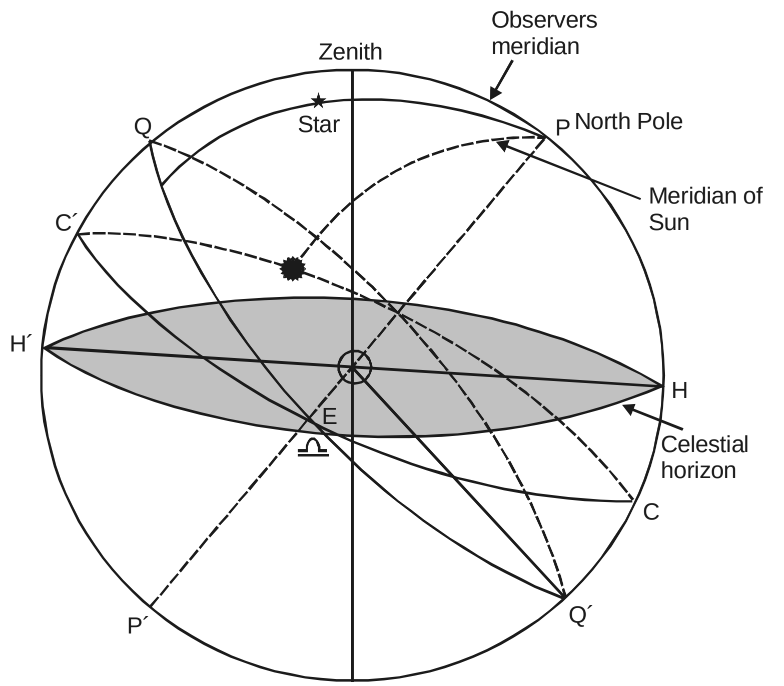


Fig. 1.23.

5. The points of intersection of the horizon and the equator give the locations of the east and west points.
6. As the sun is west of maridian, its hour angle is evidently equal to 4 × 15 = 60°. Mark off *C´M* on the ecliptic equal to 60° measured from the meridian westward. Thus, *PM* is the sun´s declination circle.
7. On 15th April, the sun's *RA* is about 25° so that sun is 25° west of *M* on the equator or 90° – (60° + 25°) from west point. Similarly *E* is 5° from (♎) where (♎) is the First point of Libra.
8. Draw a great circle so that it makes an angle 6° 36´ with meridian on the eastern side, to define the declination circle. Plot an arc equal to 24° 24´ above the equator to locate the star.

1.21. THE PARALLEL OF LATITUDE

A small circle through a point perpendicular to the axis of rotation of the earth, is known as the *parallel of the point.* The latitudes of different points on a parallel are the same. As the latitude of various parallels increases, the radius of the parallels decreases. The parallels of the poles having + 90° or – 90° latitudes are circles of zero radius.

1.22. THE VALUE OF ONE DEGREE LATITUDE

The circles of various parallels are parallel to each other. The entire meridian of a place, which is a great circle, is divided into 4 × 90° equal parts. The value of one degree of latitude is, therefore, equal to the circumference of the earth divided by 360°. *i.e.* $\frac{2\pi \times 6370}{360}$ = 111.12 km if the mean radius of the earth is assumed as 6370 km. The value of a degree of latitude is a constant value every where.

1.23. THE VALUE OF ONE DEGREE LONGITUDE

The meridians of different places are the great circles which converage to the poles. Due to this reason, the value of a degree of longitude has different values at different latitudes. At the equator the value of a degree of longitude is maximum, *i.e.* 111.12 km. This value decreases as the latitude increases and finally its value becomes zero at the north and south poles.

1.24. THE NAUTICAL MILE

The angular distance along the great circle corresponding to an angle of one minute arc subtended at the centre of the earth, is known as the *Nautical Mile.* Its value is equal to total circumference of the great circle divided *by* number of minutes subtended by the total circumference at the centre of the earth.

$\therefore$ Nautical Mile = $\frac{2\pi \times 6370}{360 \times 60}$ = 1.853 km, assuming the mean radius of earth as 6370 km.

Departure. The distance between two points in nautical miles measured along the parallel of latitude is called the departure.

$\therefore$ Departure = difference of longitude in minutes × cos latitude.

The shortest distance measured along the surface of the earth between two places is the length of the arc of the great circle passing through them

Note. The following points may be noted.

(*i*) Write the two points lie in the same hemisphere either western or eastern, the difference between their longitudes is obtained by subtracting one longitude from the other.

(*ii*) When the two points are in different hemisphere, the difference between their longitudes is obtained by the sum of their longitudes.

(*iii*) When the sum of longitudes of two points lying in different hemispheres exceeds 180° it should be subtracted from 360° to obtain the required difference of longitude of the two points.

Example 1.2. *Find the difference of longitude between two places A and B when their longitudes are as under :*

(*i*) *Longitude of A = 45°30′W; Longitude of B = 67°35′E*

(*ii*) *Longitude of A = 63°25′E; Longitude of B = 127°46′W;*

Solution. (*i*) The difference of longitude between *A* and *B*

= 45°30′ + 67°35′ = 113°05′ **Ans.**

(*ii*) The sum of the longitudes of two stations.

= 63°25′ + 127°06′ = 190°31′ **Ans.**

Since the sum of the two longitudes is greater than 180°, the difference of longitude beween *A* and *B*.

= 360° – the sum = 360° – 190°31′

= 169°29′ **Ans.**

Example 1.3. *Determine the distance in nautical miles between Hazaribagh (Jharkhand) and Bahrampur (West Bengal) along the parallel of latitude 24° and having longitude 85°16′ E and 88°22′E respectively.*

Solution.

Distance in nautical miles between two points along the parallel of latitude = departure = difference of longitudes in mintues × cos latitudes.

Here, the difference of longitude between Hazaribagh and Bahrampur

= 88°22′ – 85°16′ = 3°06′

= 3 × 60 + 6 = 186 minutes

∴ Departure = 186 cos 24° = 169.92 nautical miles. **Ans.**

1.25. THE SHORTEST DISTANCE BETWEEN TWO POINTS ON THE EARTH (FIG. 1.24)

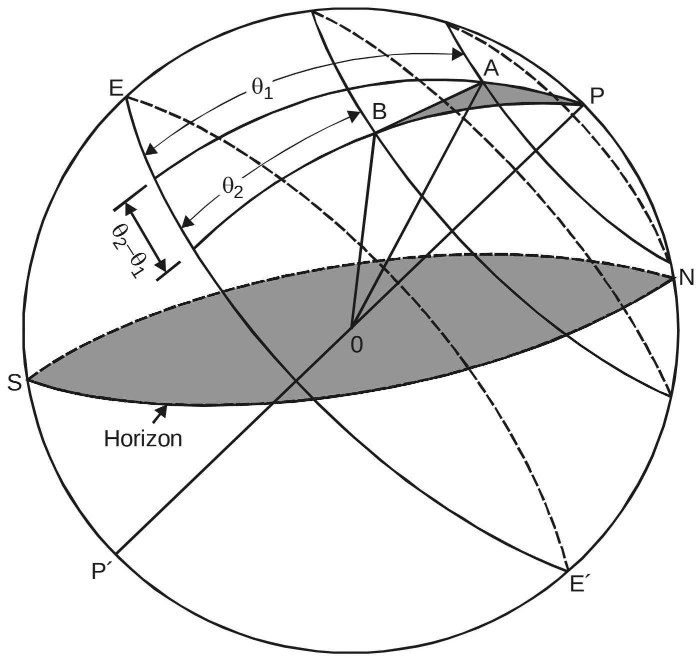


Fig. 1.24

The shortest distance between two points on the surface of the earth is the angular distance measured along the great circle passing through the given points. The value of the shortest distance may be obtained by multiplying the radius of the earth by the angle subtended by the arc of the great circle passing through the points at the centre of the earth in radians.

Let A and *B* be the given points.

θ_1 and θ_2 be the latitudes $(\theta_1 > \theta_2)$

ϕ_1 and ϕ_2 be longitudes $(\phi_2 > \phi_1)$

R is the radius of earth whose centre is at *O*.

In spherical triangle *ABP*, side $AP = 90° - \theta_1$; side $BP = 90° - \theta_2$

Angle $APB = \phi_2 - \phi_1$ = difference in longitudes of the points

Applying cosine rule to the triangle *ABP*, we get

$$\cos AB = \cos AP \cdot \cos BP + \sin AP \cdot \sin BP \cdot \cos APB \quad \text{...(1.24)}$$

Knowing the value of the angle *AB*, the value of the arc *AB* is equal to $R \times$ central angle *AOB* in radians.

$$\therefore \text{ The shortest distance } AB = \frac{R \times \text{angle } AOB \times \pi}{180°} \qquad ...(1.25)$$

Example 1.4. *Determine the shortest distance between Nasik (Maharashtra) having coordinates latitude 20°00´N, Long. 73°45´E and Warangal (Andhra Pradesh) having coordinates latitude 18°00´ Langitude 79°34´E.*

Solution. (1.25)

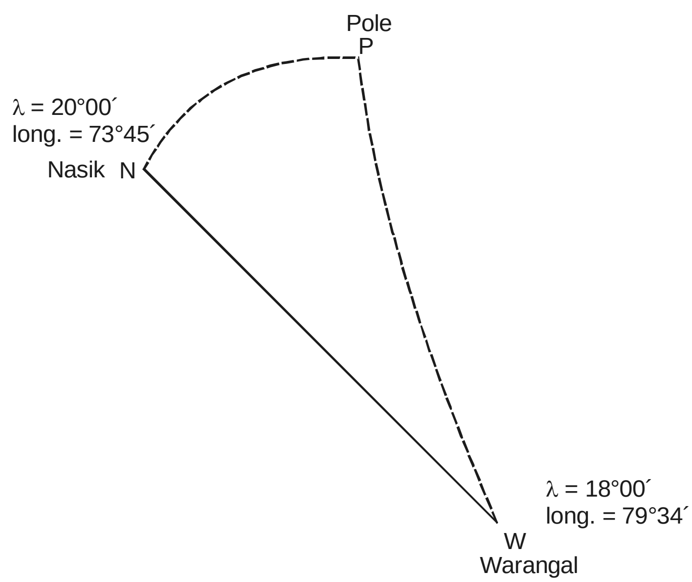


Fig. 1.25.

In spherical triangle PNW, we have

$$PN = 90° - \text{ Latitude of } N$$
$$= 90° - 20°00´ = 70°00´$$
$$PW = 90° - \text{Latitude of } W$$
$$= 90° - 18°00´ = 72°00´$$

The spherical angle *NPW* = difference of longitudes of Narangal and Nasik

$$= 79°34´ - 73°45´ = 5°49´$$

Using the cosine formula for spherical triangle *NPW*, we get

cos NW = cos PN cos PW + sin PN sin PW cos N PW

= cos 70°00´ cos 72°00´ + sin 70°00´ sin 72°00´ cos 5°49´

= 0.34202 × 0.309017 × 0.939693 × 0.951057 × 0.994851

NW = 0.10569 + 0.8891 = 0.99479

$NW = \cos^{-1}$ 0.99479 = 5.851204 = 5°51´33´´

Arc = R × central angle

where R is the earth radius = 6372 km

$$\therefore \qquad NW = \frac{6372 \times 5.851204 \times \pi}{180°}$$

Hence, = 650.30 km **Ans.**

1.26. RELATIONSHIPS BETWEEN VARIOUS CO-ORDINATES

The following relationships are very important in field astronomy.

(1) **Latitude of a place and altitude of the pole** (Fig. 1.26)

Let NS be the horizontal plane passing through the centre of the earth and EE' be the equatorial plane.

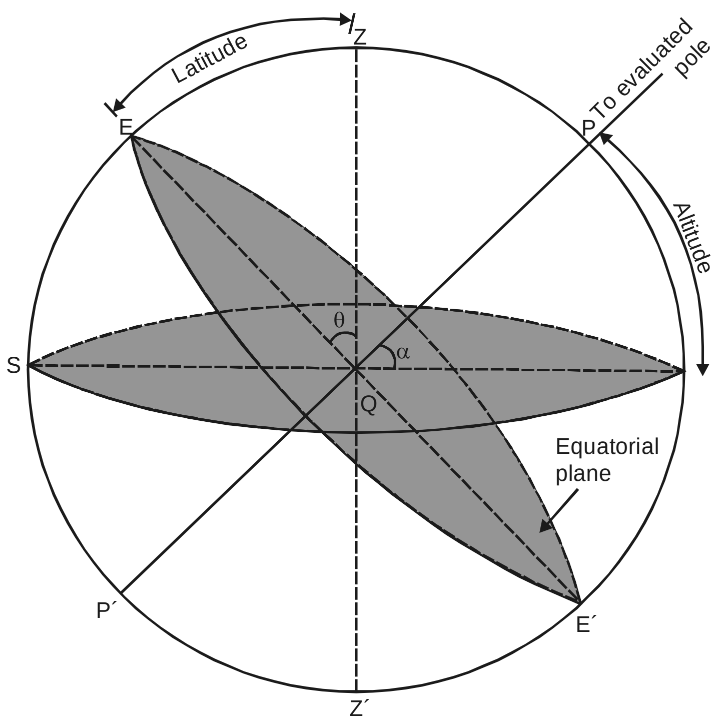


Fig. 1.26. Latitude and altitude of pole.

Now *ZESP´Z´E´NPZ* represents the observer's meridian on the surface of the earth with *Z* as the observer's position.

By definition EZ = latitude = θ

NP = altitude of the elevated pole = α

$$\angle EOZ + \angle ZOP = \angle ZOP + \angle PON = 90°$$

But, $\angle EOZ = \theta$ and $\angle PON = \alpha$

$$\therefore \quad \theta + \angle ZOP = \angle ZOP + \alpha$$

or $$\theta = \alpha$$

i.e. the altitude of the pole is always equal to the latitude of the observer´s position.

(2) **Latitude of the place, the declination and altitude of a celestial body on the observer's meridian. (Fig. 1.27)**

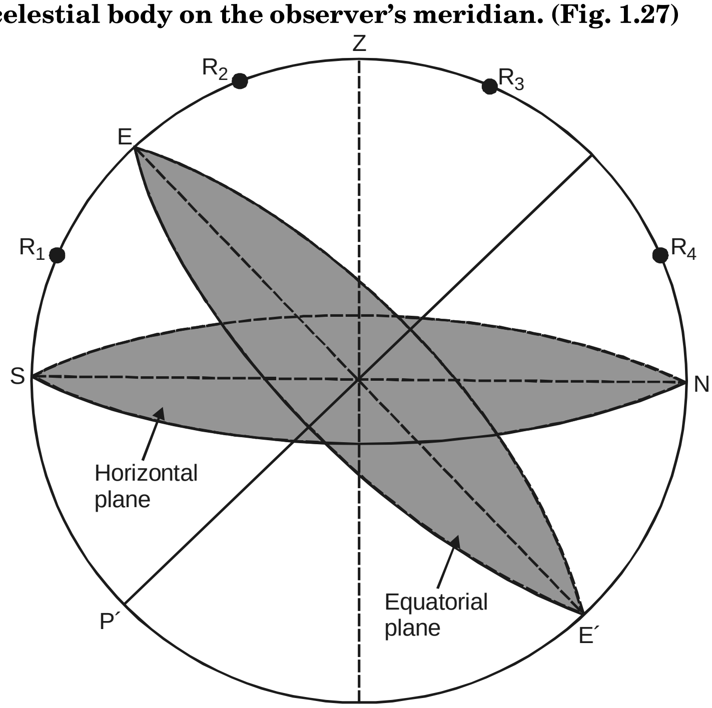


Fig. 1.27. Latitude, declination and altitude of star.

The celestial body may occupy the positions on the observer's meridian as under :

(*i*) The celestial body south of the equator, *i.e.* R_1

(*ii*) The celestial body south of zenith but north of the equator *i.e.* R_2.

(*iii*) The celestial body north of zenith but above the pole, *i.e.* R_3.

(*iv*) The celestial body below the pole but above the horizon, *i.e.* R_4.

The above four positions of the celestial body R are shown in Fig. 1.19.

Let θ be the latitude of the observer's station.

δ and α be the declination and altitude of the celestial body respectively

z be meridian zenith distance.

Evidently $EZ = ER_2 + R_2Z$ in case opposition (*ii*)

But, by definition EZ is the latitude, ER_2 is the declination and R_2Z is the zenith distance of the celestial body.

By giving proper signs to the declinations and the zenith distances, the above equation holds good for all the four positions. If the celestial body is south of the equator, its declination value is negative. Similarly, if the celestial body is north of zenith, the zenith ordinate is negative.

Again, for the position of the celestial body shown at R_3, we get $ZP = ZR_3 + R_3P$.

But, from definition, ZP = colatitude $(90° - \theta)$, ZR_3 is the coaltitude $(90° - \alpha)$ and R_3P is the co-declination $(90° - \delta)$, the polar distance p.

$$\therefore \quad (90° - \theta) = (90° - \alpha) + p$$

or
$$\theta = \alpha - p \qquad ...(1.26)$$

Similarly, for the position of celestial body shown at R_4, we get

$$ZR_4 = ZP + PR_4.$$

But, from definition ZR_4 in the co-altitude ; ZP is the colatitude and PR_4 is the polar distance = p

$$\therefore \quad (90° - \alpha) = (90° - \alpha) + p$$

or
$$\theta = \alpha + p \qquad ...(1.27)$$

From equations (1.25) and (1.26) if the altitudes of a circumpolar star on the observer's meridian when it is north of the pole and south of the pole are known, the latitude of the observer's position may be computed by taking the mean of the altitudes of the star.

1.27. GEOMETRY OF AN ASTRONOMICAL TRIANGLE (Fig. 1.28)

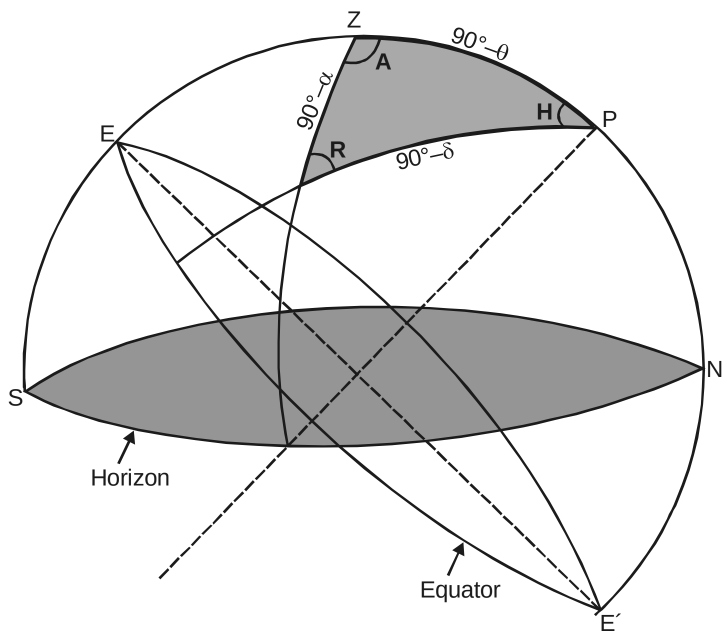


Fig. 1.28. Astronomical triangle.

The spherical triangle obtained by joining the pole, the zenith and the celestial body on the celestial sphere by three great circles, is known as an *astronomical triangle*.

Let α = Altitude of the celestial body

δ = Declination of the celestial body

θ = Latitude of the observer.

Then ZP = Co-latitude of the observer

$= 90° - \theta = \zeta$

ZR = Zenith distance or co-altitude

$= 90° - \alpha = z$

PR = Polar distance or co-declination

$= 90° - \delta = p$

The angle ZPR = hour angle ($H.A.$) of the celestial body.

The angle PZR = azimuth (A) of the celestial body.

The angle ZRP = the parallactic angle.

By making astronomical observations, if the three sides *ZR, RP* and *PZ* of the astronomical triangle *ZPR* are known, the hour angle (*H*) and the azimuth (*A*) of the celestial body, may be computed from the formulae of the spherical trigonometry given in article 1.8.

1.28. DIFFERENT POSITIONS OF THE STAR WITH RESPECT TO THE OBSERVER´S MERIDIAN

Every star appears to move from east to west about the axis of the earth, whose elevated end is known as the *north pole* or simply *a pole.* For the calculation of the azimuth of the star at the time of observation, the following positions of every star in the heaven are important to a surveyor,

(*i*) Star at elongation (*ii*) Star at culmination

(*iii*) Star at prime vertical (*iv*) Star at horizon.

1. Star at Elongation (Fig. 1.29)

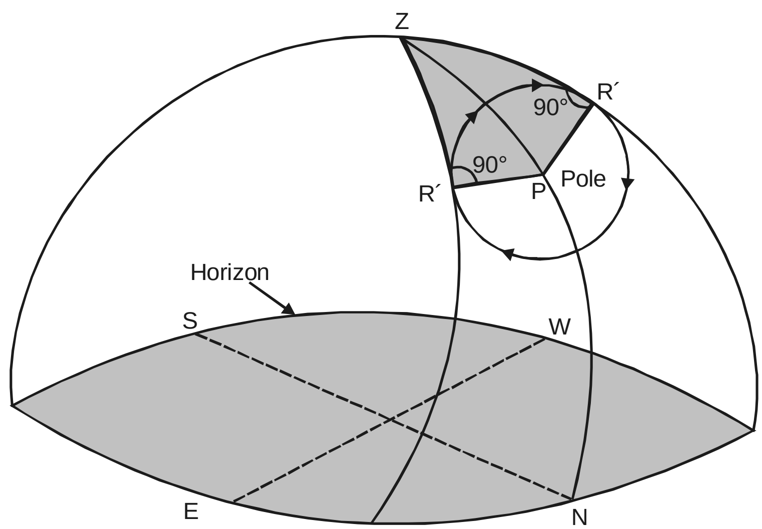


Fig. 1.29. A star at elongation.

A star is said to be at elongation when its distance east or west of the observer´s meridian is the greatest. At elongation, the star does not move in azimuth, its motion being entirely in altitude and as such the azimuth of the star is a maximum. At elongation, the diurnal circle, the path of star and the vertical circle through the star are tangential to each other. The spherical triangles *PRZ* and *PR´Z* are thus right angled triangles at *R* and *R´*.

(*i*) **Star at eastern elongation.** A star is said to be at eastern elongation when it is at its greatest distance to the east of the observer´s meridian, such as *R´* in Fig. 1.21.

(*ii*) **Star at western elongation.** A star is said to be at western elongation when it is at its greatest distance to the west of the observer's meridian, such as *R* in Fig. 1.21.

Knowing the declination (δ) of the star and the latitude (θ) of the place of observation, the azimuth (*A*), the hour angle *(H)* and the altitude (α) of the star, may be easily calculated by applying Napier´s rule as under :

In a right angled triangle *PRZ* [Fig. 1.30].

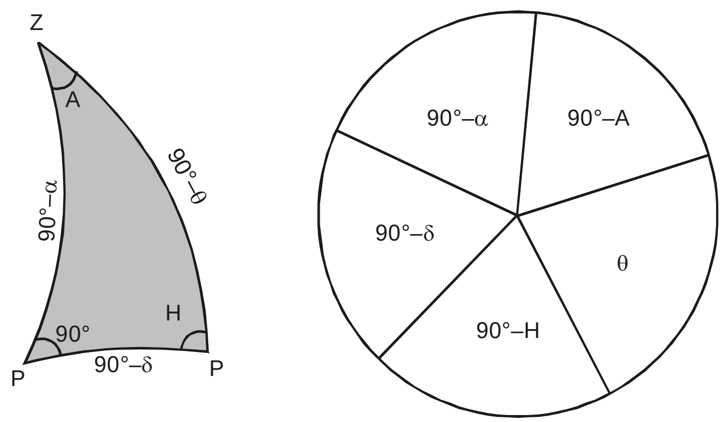


Fig. 1.30.

Let $RP = 90° - \delta$ where δ is declination of star

$PZ = 90° - \theta$ where δ is latitude of place

$ZR = 90° - \alpha$ where α is altitude of star

The five parts of the Napier's circle are entered as $90° - \alpha$, $90° - \delta$, $90° - H$, $90° - (90° - \theta)$ and $90° - A$ in Fig. 1.22(*b*).

(*i*) **Calculation of the hour angle (*H*)**

Sine of middle part = Product of tangents of adjacent parts

i.e., $\sin (90° - H) = \tan (90° - \delta) \,.\, \tan \theta$

or $\cos H = \tan \theta \,.\, \cot \delta.$ ...(1.28)

(ii) **Calculation of the altitude (α)**

Sine of middle part = Product of cosine of opposite parts.

i.e., $\sin \theta = \cos (90° - \delta) \cos (90° - \alpha)$

or $\sin \theta = \sin \delta \cdot \sin \alpha$

or $\sin \alpha = \sin \theta \cdot \text{cosec } \delta.$ ...(1.29)

(*iii*) **Calculation of the azimuth (*A*)**

Sine of middle part = Product of cosine of opposite parts

$$\sin (90° - \delta) = \cos (90° - A) \cos \theta$$

or $\cos \delta = \sin A \cos \theta$

or $\sin A = \cos \delta \cdot \sec \theta.$ ...(1.30)

2. Star at Culmination (Fig. 1.31)

The diurnal circle or the path of the star crosses the observer's meridian twice during one revolution around the pole.

A star is said to be at culmination, when it crosses the observer's meridian. At culmination, the astronomical triangle reduces to an arc of the meridian. There are two culminations of a star.

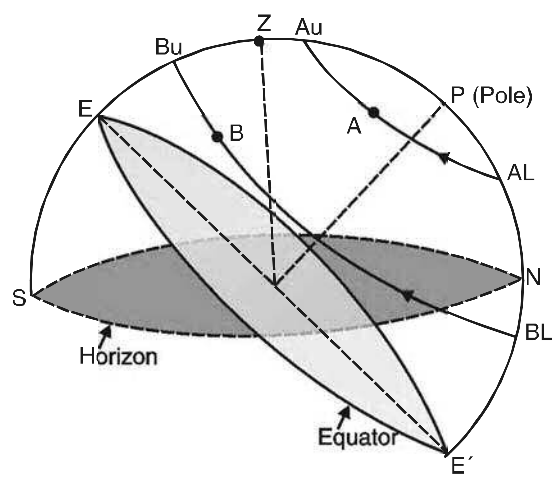


Fig. 1.31. Star at culmination.

(*i*) **Star at upper culmination.** A star is said to be at upper culmination when it crosses the observer's meridian above the celestial pole. At upper culmination the star attains the maximum altitude. In Fig. 1.23, A_u and B_u are the positions of upper culmination of the stars A and *B* respectively. Their paths of revolution are indicated by arrow heads. At upper culmination, the star moves from east to west in azimuth only.

(*ii*) **Star at lower culmination.** A star is said to be at lower culmination when it crosses the observer's meridian below the celestial pole. At lower culmination the star attains the. minimum altitude. In Fig. 1.31. A_L and B_L are the positions of the lower culmination of the stars A and B respectively. At lower culmination, the star moves from west to east in azimuth only.

The upper culmination of a star may occur to the north or south of the zenith, depending upon the declination of the star and the latitude of the observer's position.

(*iii*) **Upper culmination of star *A*.**

$$\text{The zenith distance } (z) = ZA_U = ZP - PA_U$$
$$= (90° - \theta) - (90° - \delta)$$
$$= (\delta - \theta) \quad ...(1.31)$$

At culmination the declination circle of the star coincides with the meridian of the observer

(*iv*) **Upper culmination of star *B*.**

$$\text{The zenith distance } (z) = ZB_U = PB_U - ZP$$
$$= (90° - \delta) - (90° - \theta)$$
$$= (\theta - \delta) \quad ...(1.32)$$

(*v*) **Lower culmination of star *A*.**

$$\text{The zenith distance } (z) = ZA_L = ZP + PA_L$$
$$= (90° - \theta) + (90° - \delta) = 180° - (\delta + \theta)$$

(*vi*) **Lower culmination of star B.**

$$\text{The zenith distance } (z) = ZB_L = ZP + PB_L$$
$$= (90° - \theta) + (90° - \delta) = 180° - (\theta + \delta)$$

Note. The following points may be noted :

(*i*) *The upper culmination occurs on the north side of the zenith if declination (δ) is greater than altitude (θ)*

(*ii*) *The upper culmination occurs on the south side of the zenith if declination (δ) of the star is less than latitude (θ)*

(*iii*) *If declination of the star is equal to the latitude of the observer, the culmination of the star is in the zenith.*

(*iv*) *Hour angle of the star at upper culmination is zero hour*

(*v*) *Hour angle of the star at western elongation is 6 hours*

(*vi*) *Hour angle of the star at lower culmination is 12 hours*

(*vii*) *Hour angle of the star at eastern elongation is 18 hours.*

3. Star at prime vertical (Fig. 1.32)

A star is 'said to be at prime vertical when it occupies a position on the prime vertical. When the star is on prime vertical, the astronomical triangle *PZR* becomes a right angled triangle at *Z*.

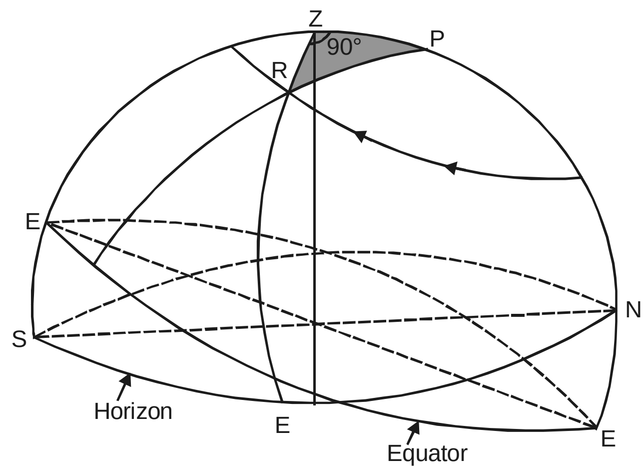


Fig. 1.32. Star at prime vertical.

Knowing the declination (δ) of the star and the latitude (θ) of the observer, the altitude (α) of the star and hour angle *(H)* of the star may be easily calculated by applying the Napier's rule as under:

In the right angled spherical triangle *RZP,* shown in Fig. 1.33,

Let $RP = 90° - \delta$ = co-decliriatipn

$RZ = 90° - \alpha$ = zenith distance

$PZ = 90° - \theta$ = co-latitude.

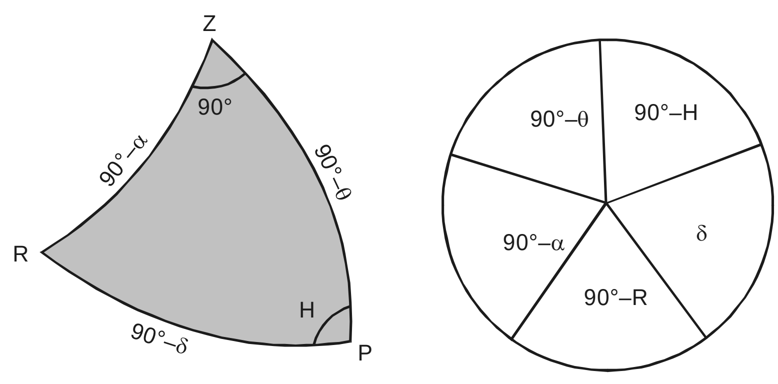


Fig. 1.33.

The five parts of the Napier circle are entered as 90° – θ, 90° – α, 90° – *R,* 90° – (90° – δ), (90° – *H) as* shown in Fig. 1.25(*b*).

(*i*) **Calculation of the hour angle (*H*)**

The sine of the middle part,

= product of the tangents of the adjacent parts

or $\sin(90° - H) = \tan(90° - \theta) \cdot \tan\delta$

or $\cos H = \tan\delta \cdot \cot\theta$ ...(1.33)

(*ii*) **Calculation of the altitude (α)**

The sine of middle part = product of the cosine of the opposite parts

i.e., $\sin\delta = \cos(90° - \alpha)\cos(90° - \theta)$

$= \sin\alpha \cdot \sin\theta$

∴ $\sin\alpha = \sin\delta \operatorname{cosec}\theta.$ ...(1.34)

4. Star at horizon (Fig. 1.34)

A star is said to be at horizon when its altitude is zero. Hence star's zenith distance at horizon is 90°. Knowing the declination (δ) of the star and the latitude (θ) of the observer, the azimuth (*A*) and the hour angle *(H)* of the star, may be calculated by applying the formula

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}$$

Here $a = (90° - \delta)$

$b = (90° - \alpha)$

$c = (90° - \theta)$

$$\cos A = \frac{\cos(90° - \delta) - \cos(90° - \alpha) - \cos(90° - \theta)}{\sin(90° - \alpha)\sin(90° - \theta)}$$

∴ $$\cos A = \frac{\sin\delta}{\cos\alpha\cos\theta} - \tan\alpha \cdot \tan\theta$$

But, when the star is at horizon, α = 0°

$$\cos A = \frac{\sin\delta}{\cos\theta} = \sin\delta\sec\theta \quad \text{...(1.35)}$$

Similarly the value of hour angle *H* may be calculated.

$$\therefore \cos H = \frac{\cos(90° - \alpha)}{\sin(90° - \delta)\sin(90° - \theta)} - \frac{\cos(90° - \delta)\cos(90° - \theta)}{\sin(90° - \delta)\sin(90° - \theta)}$$

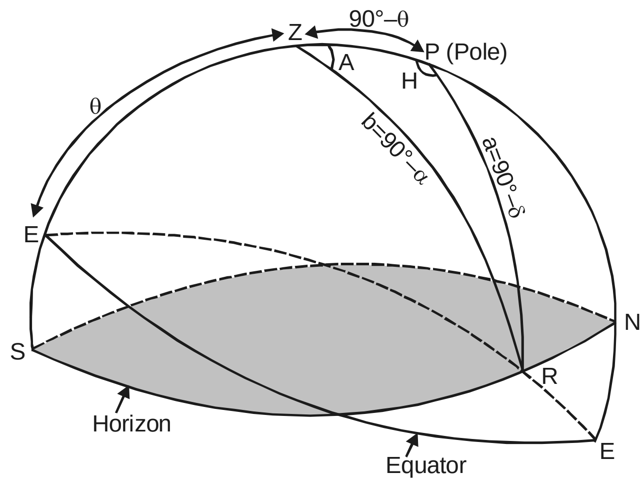


Fig. 1.34. Star at horizon.

$$\cos H = \frac{\sin\alpha}{\cos\delta\cos\theta} = \tan\delta\tan\theta$$

Putting $\alpha = 0$, we get

i.e. $\cos H = -\tan\delta\tan\theta.$ ...(1.36)

5. Circumpolar Stars (Fig. 1.35.)

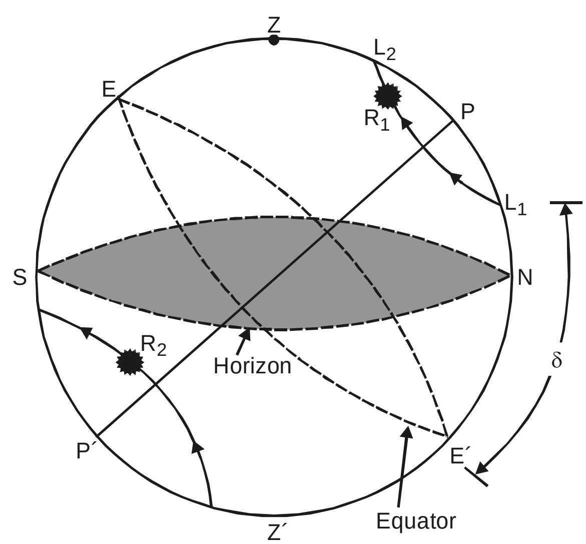


Fig. 1.35. Circumpolar stars.

The stars which remain always above the horizon of the observer's position and do not set at any time, are known as *circumpolar stars.* Such stars appear to the observer to describe circles about the pole. The number of circumpolar stars increases with the increase of latitude of the observer's position. For an observer at the equator the number of circumpolar stars, is zero whereas for an observer at the north pole all stars in the northern hemisphere are circumpolar. Similarly, for an observer at south pole all the stars in the southern hemisphere, are circumpolar.

R_1 and R_2 are the circumpolar stars in the northern and southern hemispheres where Z and Z´ represent Zenith and Nadir respectively. The path of the circumpolar star R_1 is along $L_1R_1L_2$.

Here $\qquad PL_1 < PN$

i.e. $\qquad (90° - \delta) < PN$

But, altitude of the pole star = latitude of the observer's position

$\therefore \qquad (90° - \delta) < \theta$

i.e. $\qquad \delta > (90° - \theta) \qquad$ ...(1.37)

i.e. a star will be circumpolar star if its declination is greater than the co-latitude of the observer's position.

Example 1.5. *Calculate the shortest distance between two places A and B, given that the latitudes of A and B are 28° 30´ N and 32° 42´ N and their longitudes are 76°18´ and 82°54´ E respec-tively. (Assume the radius of the earth as 6370 km.)*

Solution. (Fig. 1.36.)

In the spherical triangle *ABP* we get

$$AP = 90° - 28°\ 30' = 61°\ 30'$$

$$BP = 90° - 32°42' = 57°18'$$

Angle $\qquad APB$ = Longitude of B – Longitude of A

$$= 82°54' - 76°18' = 6°36'$$

Applying the cosine rule to the spherical triangle *ABP,* we get

$\cos AB = \cos AP \,.\, \cos BP + \sin \text{AP} \,.\, \sin BP \cos APB$

$= \cos 61°30' \,.\, \cos 57°18' + \sin 61°30' .\ \sin 57°18' \times \cos 6°\ 36'$

$= 0.47716 \times 0.54024 + 0.87882 \times 0\text{-}841511 \times 0.993373$

$= 0.25778037 + 0.73463327 = 0.99241357$

$\therefore AB = 7°03'43''.3$

and arc $AB = R \times$ central angle θ

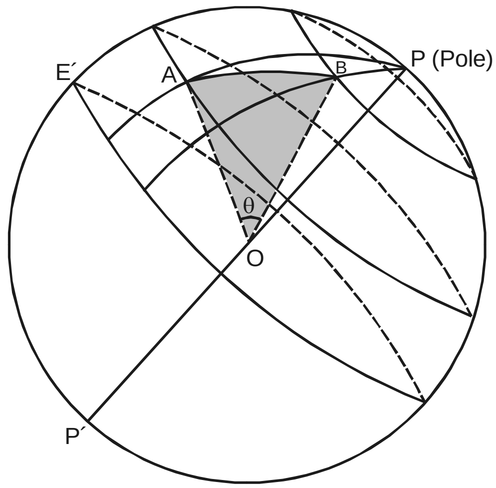


Fig. 1.36

$$= \frac{6370 \times 7.0620277 \times \pi}{180°}$$

= 785.138 km. **Ans.**

Example 1.6. *Determine the hour angle and declination of a star from the following data :*

Latitude of the place = *48°30´ N*

Azimuth of the star = *50° W*

Altitude of the star = 28°*24´*

Solution.

In the astronomical triangle *PSZ,* we have

Colatitude $PZ = 90° - 48°30´ = 41°30´$

Coaltitude $SZ = 90° - 28°24´ = 61°36´$

Azimuth $A = 50°00´$

(1) **Calculation of the declination**

Applying cosine formula (1.5) to ΔPSZ, we get

$$\cos PS = \cos SZ \,.\, \cos PZ + \sin SZ \,.\, \sin PZ \,.\, \cos A$$

$$\therefore \quad \cos (90° - \delta) = \cos 61°36´ \cos 41°30´ + \sin 61°36´ \sin 41°30´ \cos 50°$$

$$= 0.475624 \times 0.748956 + 0.479649 \times 0.662620 \times 0.642788$$

$$= 0.35622144 + 0.37466378$$

$$= 0.73088522$$

$$90° - \delta = 43°02' 21''.5$$

Declination $\delta - 90° - 43° 02' 21''.5 = 46° 57' 38''.5$. **Ans.**

(2) Calculation of the hour angle *(H)*

Applying sine rule to ΔPSZ, we get

$$\sin H = \frac{\sin A \sin SZ}{\sin PS}$$

$$= \frac{\sin 50° \sin 61°36'}{\sin 43°02'21''.5}$$

$$= \frac{0.766044 \times 0.879646}{0.682500} = 0.98732575$$

$\therefore$ Hour angle *(H)* $= 80° 52' 05''.5$. **Ans.**

Example 1.7. *Find the azimuth and the hour angle of the sun at sunset at a place of latitude 49°, its declination being given to be 19° S.*

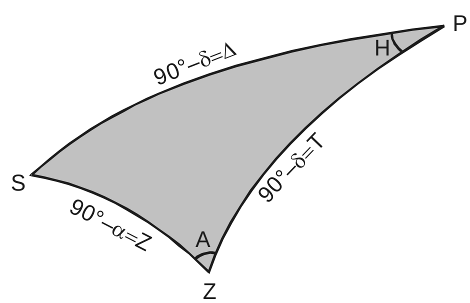


Fig. 1.37.

Solution. (Fig. 1.37)

In the spherical triangle *PZS*,

PZ = colatitude = 90° – 49° = 41°

ZS = colatitude = 90° – 0° = 90°

PS = codeclination 90° – (– 19°) = 109°

Applying cosine formula

$$\cos A = \frac{\cos PS - \cos PZ \cos ZS}{\sin PZ \sin SZ}$$

$$= \frac{\cos 109° - \cos 41° \cos \cos 90°}{\sin 41° \sin 90°}$$

$$= \frac{-0.325568 - 0.754710 \times 0}{0.656059 \times 1}$$

or $$\cos A = \frac{-0.325568}{0.656059} = -\,0.49624805$$

As the value of $\cos A$ is negative, it lies between 90° and 180°,

$$180° - A = 60°\ 14'52.8''$$

or $$A = 180° - 60°14'\ 52''.8 = 119°\ 45'\ 07''.2$$

Azimuth of the Sun at sunset = 119°45´ 07´´.2 w **Ans.**

Again, applying the cosine formula we get

$$\cos H = \frac{\cos SZ - \cos PZ \cos PS}{\sin PZ\ .\ \sin PS}$$

$$= \frac{\cos 90° - \cos 41° \cos 109°}{\sin 41°\ .\ \sin 109°}$$

$$= \frac{0 + 0.754710 \times 0.395568}{0.656059 \times 0.944519}$$

$\therefore$ Hour angle $H = 66°\ 39'\ 54''$ **Ans.**

Example 1.8. *Find the shortest distance between two places K and L, given that the latitudes at K and L are 19°00´ N and 13°04´ N and their longitudes are 72°30´ E and 80°12´ E respectively.*

Take radius of earth = 6370 km.

Solution. (Fig. 1.38)

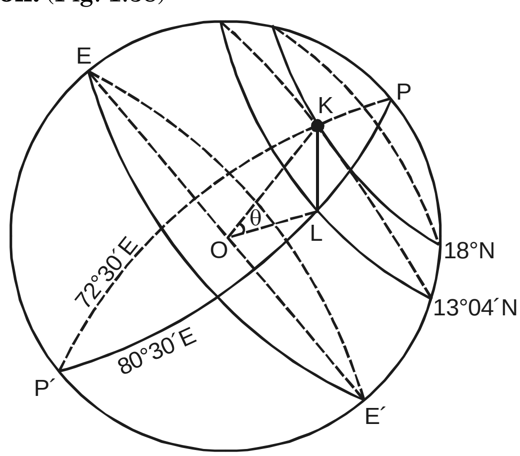


Fig. 1.38

Let K and L be the given places.

θ be the angle subtended by the arc KL at the centre of earth. In the spherical triangle KPL, we get

$$KP = 90° - 19° = 71°$$

$$PL = 90° - 13° 04' = 76° 56'$$

Angle $KPL = 80° 12' - 72° 30' = 7° 42'$

Applying the cosine rule to spherical triangle PKL, we get

$$\cos KL = \cos KP \cos PL + \sin KP \sin PL \cos KPL.$$

Substituting the values we get

$$= \cos 71° \cos 76° 56' + \sin 71° \sin 76° 56' \cos 7° 42'$$

$$= 0.073606041 + 0.91273165$$

or $\cos KL = 0\text{-}986313805$

or $KL = 9° 28' 54''.4$

$\therefore$ Shortest distance between the places K and L

$$= \frac{6370 \times 9°.4818 \times \pi}{180°}$$

or $= 1054.16$ km. **Ans.**

Example 1.9. *If the latitude of the observer's station is 28° 30′ N and declination of the star is 62° 35′ N, calculate the zenith distances of the star at its upper and lower culminations.*

Solution.

Here $\theta = 28° 30' N$

$$\delta = 62° 35' N.$$

The star's declination δ being greater than latitude (θ), its upper culmination is on the north side of the zenith.

$\therefore$ Zenith distance of star at its upper culmination

$$= \delta - \theta$$

$$= 62° 35' - 28° 30' = 34° 05'$$ **Ans.**

The zenith distance of star at its lower culmination

$$= 180° - (\theta + \delta)$$

$$= 180° - (28°30' + 62°35')$$

$$= 88° 55'$$ **Ans.**

As the declination of the star is greater than $90° - \theta$, it is a circumpolar star.

Example 1.10. *Both culminations of a star occur on the north-side of the zenith and its observed altitudes at a place at upper and lower culminations are 56°30´ and 10°30´ respectively. Find the latitude of the place and declination of the star.*

Solution.

Given : Both culminations are on the north side of the zenith. Zenith distance of the star at its upper culmination

$$= \delta - \theta = 90° - \text{altitude}$$

Zenith distance of the star at its lower culmination

$$= 180° - (\theta + \delta) = 90° - \text{altitude}$$

$$\delta - \theta = 90° - 56°30' = 33°\ 30' \quad ...(i)$$

$$180° - (\theta + \delta) = 90° - 10°\ 30' = 79°\ 30'$$

or $$\delta + \theta = 180° - 79°\ 30' = 100°\ 30' \quad ...(ii)$$

Solving eqns. (*i*) and (*ii*) we get

$$\delta = 67°\ 00'$$

and $$\theta = 333°\ 0'$$ **Ans.**

Example 1.11. *If the upper culmination of a star (declination 48°38´ N) is in the zenith of the observer's place, find the latitude of the place and altitude of the star at its lower culmination.*

Solution. As the star culminates in the zenith, polar distance of the star = Co-latitude.

i.e. $$90° - \delta = 90° - \theta$$

or $$\delta = \theta$$

$\therefore$ Latitude $(\theta) = 48°\ 38'$ *N*. **Ans.**

The zenith distance at lower culmination

$$= 180° - (\theta + \delta)$$

$$= 180° - (48°\ 38' + 48°\ 38')$$

$$= 180° - 97°\ 16' = 82°\ 44'$$

$\therefore$ Altitude of star $= 90° - 82°\ 44'$

$= \mathbf{7°\ 16'}$. **Ans.**

Example 1.12. *Calculate the declination of the sun at a place of latitude 28° 30´ if it rises on prime vertical.*

Solution.

Let *P* be the celestial pole

Z be the zenith of the place

S be the sun at rise.

Apparently PZS is a right angled triangle at Z, the sun being on prime vertical at sunrise.

Here, colatitude $PZ = 90° - 28°\ 30' = 61°\ 30'$

Coaltitude $ZS = 90°$

Azimuth $= 90°$

$$\cos PS = \cos 61°\ 30' \cos 90° + \sin 61°\ 30' \sin 90° \cos 90°$$

$$\cos PS = 0$$

$$PS = 90°$$

i.e. Declination of sun $= 90° - 90°$

$= \mathbf{0°}$. **Ans.**

Example 1.13. *Find the azimuth and hour angle of the sun at sun rise at a place of latitude 28° 30′ N, its declination being given as 23° 27′ N.*

Solution. In astronomical triangle PZS we have

$$PZ = 90° - 28°\ 30' = 61°\ 30'$$
$$PS = 93° - 23°\ 27' = 66°\ 33'$$
$$ZS = 90°\text{, at sun rise.}$$

Let A be the azimuth and H be the hour angle of the sun. Applying the cosine formula we get

$$\cos ZS = \cos PZ \cos \text{PS} + \sin \text{PZ} \sin \text{PS} \cos H.$$

or
$$\cos H = \frac{\cos ZS - \cos PZ \cos PS}{\sin PZ \sin PS}$$

$$= \frac{0 - \cos 61°30' \cos 66°33'}{\sin 61°30' \sin 66°33'}$$

$$= \frac{0 - 0.477159 \times 0.397949}{0.878817 \times 0.917408}$$

$$= \frac{-0.18988494}{0.80623374} = -0.23552094$$

$$H = 76°\ 22'\ 40''$$ **Ans.**

Applying the sine rule to ΔPZS we get

$$\sin A = \frac{\sin 76°22'40'' \sin 66°33'}{\sin 90'} = 0.89160131$$

$$180° - A = 63°04'30''.7$$

$$A = 180° - 63°\ 04''\ 30''.7$$

or Azimuth $A = 116° 55' 29''.3$ **Ans.**

Example 1.14. *Determine the hour angle and declination of a star from the following data :*

Altitude = 22° *36´*

Azimuth = *42° W*

Latitude of the place = *40° N.*

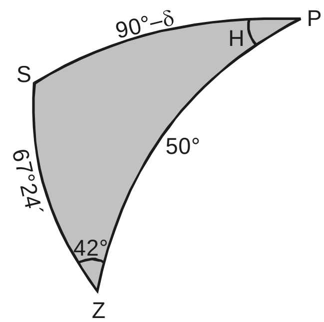


Fig. 1.39.

Solution. (Fig. 1.39)

Given : Altitude = 22° 36´

Azimuth = 42° *W*

Latitude = 40° *N*

∴ Colatitude $ZS = 90° - 22° 36'$

$= 67° 24'$

Colatitude $PZ = 90° - 40° = 50°$

Azimuth $SZP = 42° W.$

Let δ be the declination and *H* be the hour angle of the star.

From the astronomical triangle *SZP,* we get

$$\cos (90° - 6) = \cos 67° 24' \cos 50° + \sin 67° 24' \sin 50° \cos 42° \quad ...(i)$$

Substituting the values in Eqn. (i) we get

$$\cos (90° - \delta) = 0.384296 \times 0.642788 + 0.923210 \times 0.766044 \times 0.743145$$

$$= 0.24702085 + 0.52556662$$

$$= 0.77258742$$

∴ $90° - \delta = 39°.4132$

$= 39° 24' 47''.52$

∴ Declination $\delta = 90° - 39° 24' 47''.52$

$= 50° 35' 12''.48$ **Ans.**

Again, applying sine rule to the spherical triangle, we get

$$\frac{\sin H}{\sin 67°24'} = \frac{\sin 42°}{\sin 39°.4132}$$

$$\sin H = \frac{\sin 42° \sin 67°24'}{\sin 39°.4132} = 0.9729716$$

∴ Hour angle $H = 76° 38' 54''.6.$ **Ans.**

Example 1.15. *Find the azimuth and the hour angle of the sun at sun set for a place of latitude 49°, its declination being given to be 19° S.*

Solution.

Given : Altitude $\alpha = 0$

or zenith distance $z = 90°$

declination $\delta = 19°$ S

or Co-declination $= 109°$

Latitude $\theta = 49°\ N$

or Co-latitude $= 41°$

Let H be the hour angle and A be the azimuth

From the astronomical triangle PZS we get

Applying cosine formula to ΔPZS we get

$$\cos H = \frac{\cos 90° - \cos 109° \cos 41°}{\sin 109 \sin 41°}$$

$$= \frac{0.325568 \times 0.754710}{0.945519 \times 0.656059} = 0.39610343$$

$$H = 66°\ 39'\ 55''$$

Hour angle $H = 66°\ 39'\ 55''$ **Ans.**

$$\cos A = \frac{\cos 109° - \cos 90° \cos 41°}{\sin 90° \sin 41°}$$

$$= \frac{\cos 109°}{\sin 41°} = \frac{-0.325568}{0.656059} = -0.49624805$$

or $A = 119°45'7''.2$ **Ans.**

Alternately, applying the sine formula to ΔPZS we get

$$\frac{\sin A}{\sin 109°} = \frac{\sin 66°39'55''}{\sin 90°}$$

or $\sin A = \sin 66°\ 39'\ 55''\ \sin 109°$

$= 0.918206 \times 0.945519 = 0.86818121$

$A = 60°14'52''.8$ or $119°45'\ 7''.2$

i.e. $A = 119°\ 45'\ 7''.2.$ **Ans.**

Example 1.16. *Calculate the latitude of the place where a given star at its lower culmination remains at the horizon and its upper culmination occurs in zenith.*

Solution. As the upper culmination of the star occurs in zenith, the latitude of the place is equal to the declination of the star

i.e. $\theta = \delta.$

Zenith distance at lower transit

$$= 180° - (\theta + \delta) = 90°$$

or $$90° = 180° - (\theta + \delta)$$

or $$90° = 180° - 2\theta \quad ...(\because \quad \theta = \delta)$$

$$2\theta = 90°$$

$$\theta = 45°$$

$\therefore$ Latitude of the place = 45° *N*. **Ans.**

Example 1.17. *What is the geodetic area enclosed by the spherical triangle ABP on the earth's surface when the coordinates of the stations are as follows ?*

Coordinate of A = 30° *N*, 45° *E*

Coordinate of B = *50° N, 60° E*

Coordinate of P = Pole

Assume radius of the earth

= 6378 km and π = 3.1415927

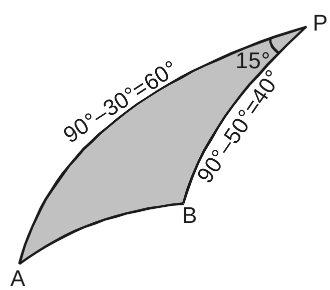


Fig. 1.40.

Solution. (Fig. 1.40)

In spherical triangle *APB,*

Side $AP = 90° - 30° = 60°$

Side $BP = 90° - 50° = 40°$

Cos AB = cos60° cos 40° + sin 60° sin 40° cos 15°

$$= 0.5 \times 0.7660444 + 0.8660254 \times 0.6427876 \times 0\text{-}9659258$$

$$= 0.3830222 + 0.5377023 = 0.9207245$$

$$AB = 22°.967768.$$

Applying sine rule to ΔAPB we get

$$\sin A = \frac{\sin 15° \times \sin 40°}{\sin 22.967768} = \frac{0.258819 \times 0.6427876}{0.3902132} = 0.4263455$$

$\therefore$ Angle A =25.235863

Applying sin rule to ΔAPB we get

$$\sin B = \frac{\sin 15° \times \sin 60°}{\sin 22°.967768} = \frac{0.258819 \times 0.8660254}{0.3902132} = 0.5744138$$

$\therefore$ Angle B = 35.058592 or 144°.941408

$\therefore$ Sum of angles of spherical triangle APB

= 25.235863 + 144°.941408 + 15° = 185.177271

$\therefore$ Spherical excess, E = 185.177271 – 180° = 5°.177271

And Area of spherical triangle APB

$$= \frac{\pi R^2 \times e}{180°} = \frac{3.1415927 \times 6378^2 \times 5°.177271}{180°}$$

= 36,87,461.6 sq. km. Ans.

1.29. CORRECTIONS TO THE OBSERVED ALTITUDE OF A CELESTIAL BODY

The following corrections are generally applied to the observed alti-tudes of the celestial bodies for deducing their true altitudes at the time of observation.

(*i*) Refraction correction. (*ii*) Dip correction.
(*iii*) Parallax correction. (*iv*) Semi-diameter correction.
(*v*) Index error correction. (*vi*) Bubble error correction.
(*vii*) Azimuth correction.

Correction nos. (*i*) to (*iv*) are observational corrections whereas correction Nos. (*v*) to (*vii*) are instrumental corrections.

1. **Refraction correction** (Fig. 1.41.)

It is a well established fact that the density of the air decreases as the distance from the earth surface increases. We also know that rays of light passing through layers of air of different densities get bent and thus their path is along a curve. To an observer A, the star S appears to be situated at S', higher than its real position. Due to refraction, the observed altitude of a heavenly body appears greater than what it really is.

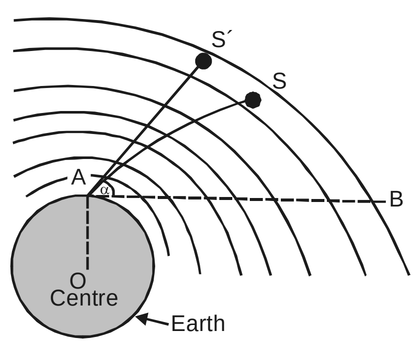


Fig. 1.41. Refraction Correction.

The angle $S'AS$ is known

as the correction for refraction. Correction for refraction is always *subtracted* from the observed altitude.

Let S be the actual position of the celestial body.

S' be the apparent position of the celestial body due to refraction.

A is the observer's position.

α is the apparent altitude of the celestial body.

The angle $S'AS$ *is* the required refraction correction.

$\therefore$ The true altitude = $\angle SAB = \angle S'AB - \angle S'AS$

= observed altitude–refraction correction

The magnitude of refraction correction depends upon the following factors :

(*i*) Density of the air.

(*ii*) Temperature of the air.

(*iii*) Barometric pressure of the air.

(*iv*) Altitude of the celestial body.

For apparent altitudes greater than 20°, the value of correction for refraction, may be calculated from the following formula.

Correction for refraction in seconds $= 58'' \cot \alpha$ ...(1.38)

or " " $= 58'' \tan z$ ...(1.39)

where α and z are the apparent altitude and zenith distance of the celestial body.

1. Tangent formula for refraction correction (Fig. 1.42). The effective atmosphere extends only upto about 160 km from the

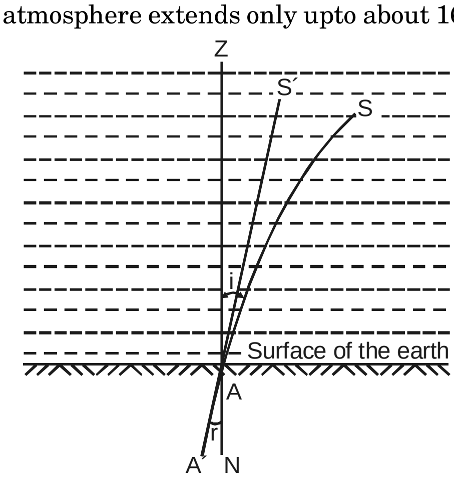


Fig. 1.42. Tangent formula.

surface of the earth. The curvature of the earth if neglected, its surface may be assumed flat.

Let *ZAN* be Zenith-Nadir line.

S be the actual position of the star.

S´ be its apparent position as seen by the observer *A* on the surface of the earth.

The ray of light *SA* is curved whereas the ray *S´A* and its continuity is straight.

S´A is a tangent to the curved line *SA* at *A*.

SA, *S´A*, *ZA* and *A´A* all lie in one plane.

$\angle ZAS$ and $\angle A'AN$ may be assumed as the angles of incidence and refraction respectively.

Hence, $\dfrac{\sin ZAS}{\sin A'AN} = \mu$ = coefficient of refraction

But, $\angle ZAS' = \angle A'AN$, being opposite angles

$\therefore \quad \mu \sin ZAS' = \sin ZAS = \sin (ZAS' + S'AS)$

$= \sin ZAS' \cos S'AS + \cos ZAS' \sin S'AS$

But, $\angle S'AS$ being very small, $\cos S'AS = 1$ and $\sin S'AS = S'AS$ in radian measure.

$\therefore \quad \mu \sin ZAS' = \sin ZAS' + \cos ZAS' \,.\, S'AS$

or $\quad \sin ZAS' (\mu - 1) = \cos ZAS' \,.\, S'AS$

or $\quad S'AS = (\mu - 1) \tan ZAS' = (\mu - 1) \tan z$

where z is apparent zenith distance.

i.e. the refraction is proportional to the tangent of the apparent zenith distance.

2. Effect of astronomical refraction on the Coordinates of Celestial bodies. (Fig. 1.43)

Let *S* be the actual position of the celestial body on the celestial sphere.

Z be the zenith of observer.

S´ be the displaced position of the celestial body *S* due to refraction towards the zenith on the great circle *ZS*.

1. Effect on observed altitude (Fig. 1.43)

The actual observed zenith distance $ZS' = z$

$\therefore$ SS' is the refraction correction $= K \tan z$.

i.e. effect of refraction is to decrease the zenith distance of a *celestial body or to increase the observed altitude of the body.*

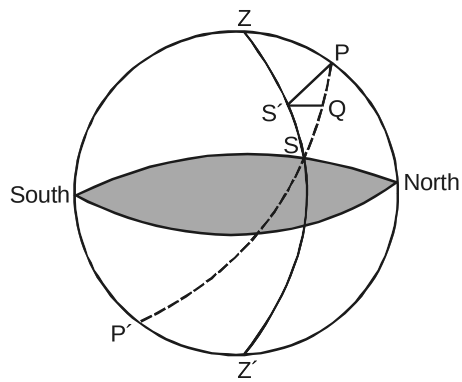


Fig. 1.43.

(2) **Effect on declination :**

Construction : Drop a perpendicular $S'Q$ to PS where PS is the declination circle of the celestial body S.

As the sides of the triangle $SS'Q$ are small, it can be assumed a plane triangle.

Let angle $S'SQ$ be θ

$$SQ = SS' \cos\theta \qquad ...(i)$$
$$= K \tan z \,.\, \cos\theta.$$

From Fig. (1.43) we note

$$PS = PQ + QS = PS' + QS$$

i.e. True co-declination $= 90° - \delta + K \tan z \cos\theta$.

or True declination $= 90° - (90° - \delta + K \tan z \cos\theta)$

$$= \delta - K \tan z \cos\theta.$$

i.e. effect of the refraction is to decrease the declination of the body.

(3) **Effect on the hour angle**

Let PS be the hour circle for true position of the body.

PS' is the hour circle for the displaced position of the body.

$S'Q$ is the decrease in the body's hour angle.

Now $S'Q = SS' \sin\theta = k \tan z \sin\theta$

i.e. the decrease in the hour angle is $S'PQ$ equal to $k \tan z \sin\theta \sec\delta$.

Note. The following points may be noted :

(*i*) *Astronomical refraction is large and not reliable, if the altitude of the body is less than 20°.*

(*ii*) *Astronomical refraction decreases as the altitude of the star increases.*

(*iii*) *Astronomical refraction is zero for celestial bodies exactly over the head of the observer, whereas for a star at horizon its value is 33°.*

(*iv*) *Astronomical refraction is same for all bodies for a particular altitude irrespective of their distances from the earth's surface.*

(*v*) *For accurate work, the astronomical refraction may be obtained from Bessal's Refraction Tables.*

(*vi*) *The astronomical refraction is always subtracted from observed altitude.*

2. Dip Correction. Altitudes of celestial bodies are observed either by a theodolite or by an astronomical sextant. In case of theodolite, the horizontal line passing through the trunnion axis is defined and the observed altitude does not require any correction for dip. But in case of a sextant, the vertical angle is observed between the visible horizon and the celestial body in vertical plane.

Apparently, the observed altitude by sextant is always more than true altitude. The angle between the sensible horizon and the visible horizon is called and *angle of dip*. Magnitude of dip depends upon the altitude of the observer's position above M.S.L.

Derivation of Dip correction (Fig. 1.44).

Let *S* be the position of the body

AH be the sensible horizon of the observer's place

AB be the visible horizon of the observer's place

SAB be the observed altitude duly corrected for refraction by a sextant

h be the height of the observer's position in metres above M.S.L.

R be the radius of the earth in metres.

Hence, α is the corrected altitude of the body.

β is the angle of the dip.

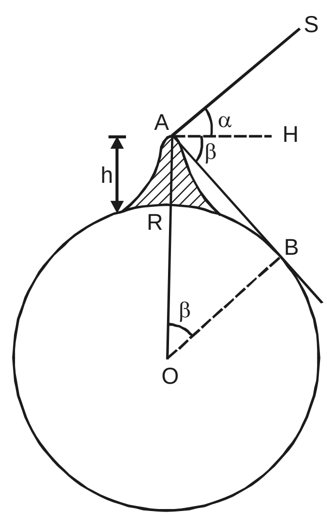


Fig. 1.44. Dip correction.

From the Fig. 1.36

$\angle AOB = 90° - \angle AOB = \beta$

and $$AB = \sqrt{(R+h)^2 - R^2}$$

Again, $$\tan \beta = \frac{AB}{OB}$$

$$= \frac{\sqrt{(R+h)^2 - R^2}}{R}$$

$$= \sqrt{\frac{h(2R+h)}{R^2}} \quad ...(1.40)$$

or $$\tan \beta = \sqrt{\frac{2h}{R}} \text{ (Appx.)} \quad ...(1.41)$$

$\therefore$ The true altitude = observed altitude – dip correction

i.e. $$\angle SAH = \angle SAB - \angle HAB$$

Note. The following points may be noted :

(*i*) *The correction for dip is always negative. It is subtracted from the observed altitude duly corrected for refraction.*

(*ii*) *The magnitude of the dip correction varies with height of the observer' station above M.S.L.*

3. Parallax Correction. (Applicable to sun only). As the stars are assumed to be projected on a celestial sphere of infinite radius, their altitudes above the sensible horizon and above the horizon passing through the centre of the earth, are practically the same. The sun being comparatively nearer to earth, its altitude when measured at any point on the surface of the earth, considerably differs from that deduced at the centre of the earth.

The difference of altitudes of the sun at a point on the surface of the earth and at the centre of the earth, is known as *sun's Parallax* in altitude.

Hence, the sun's parallax in altitude may be defined as the angle subtended at the centre of the sun by the line joning the centre of the earth to the place of observation (Fig. 1.45).

Let *S* be the position of the sun.

Angle *SAH* be the apparent altitude above the sensible horizontal *AH*.

The angle *SOB* is the altitude of the sun deduced at the centre of the earth where *OB* is the true horizon.

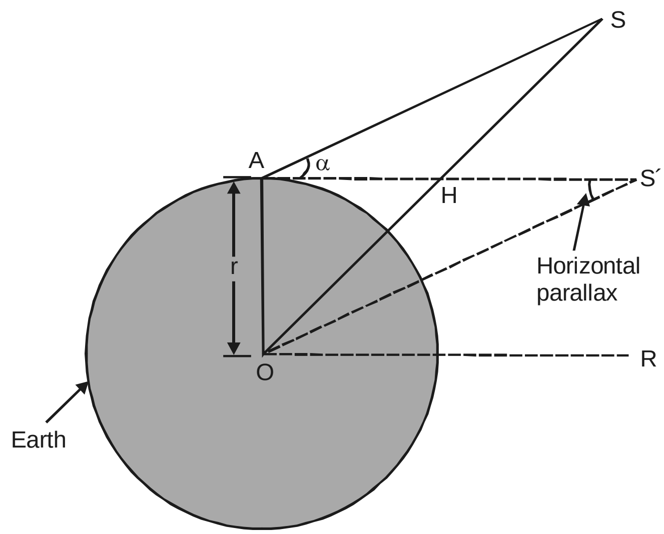


Fig. 1.45. Sun's parallax correction.

When the sun is on the sensible horizon, the apparent altitude of the sun *S´* is zero and then the angle of parallax *AS´O,* is known as the sun's *horizontal parallax.* The horizontal parallax varies inversely to the distance of the sun from the earth.

The maximum horizontal parallax is 8´´.95 on 31st December. The minimum horizontal parallax is 8´´.66 on 1st July.

The average value of the horizontal parallax to be used for computation is assumed to be 8´´.8.

Derivation of sun's parallax formula.

From Fig. (1.45) we know that

$$\angle ASO = \text{parallax in altitude}$$

$$\angle AS'O = \text{horizontal parallax.}$$

$\because$ *AS´* and *OB are* parallel, being sensible and true horizons.

$$\therefore \quad \angle SOB = \angle SHS' = \angle HAS + \angle ASH$$

or True altitude = observed altitude + parallax in altitude.

In $\angle ASO$, $\angle ASO$ = parallax in altitude

$$\angle SAO = 90° + \alpha$$

where α is the observed altitude

Hence, applying the sine rule, we get

$$\frac{\sin ASO}{AO} = \frac{\sin OAS}{OS}$$

or
$$\sin ASO = \frac{AO \sin OAS}{OS} = \frac{AO \cos \alpha}{OS}$$

But
$$\frac{OA}{OS} = \frac{OA}{OS'} = \sin AS'O$$

$\therefore$
$$\sin ASO = \frac{OS . \sin AS'O}{OS} . \cos \alpha$$

or $\angle ASO = \angle AS'O \cos \alpha \qquad (\sin \theta = \theta)$

or Parallax in altitude = horizontal parallax × cos α ...(1.42)

= 8″.8 cos θ. ...(1.43)

Note. The correction for parallax is always *positive.*

4. Sun's semi-diameter Correction. (Fig. 1.46).

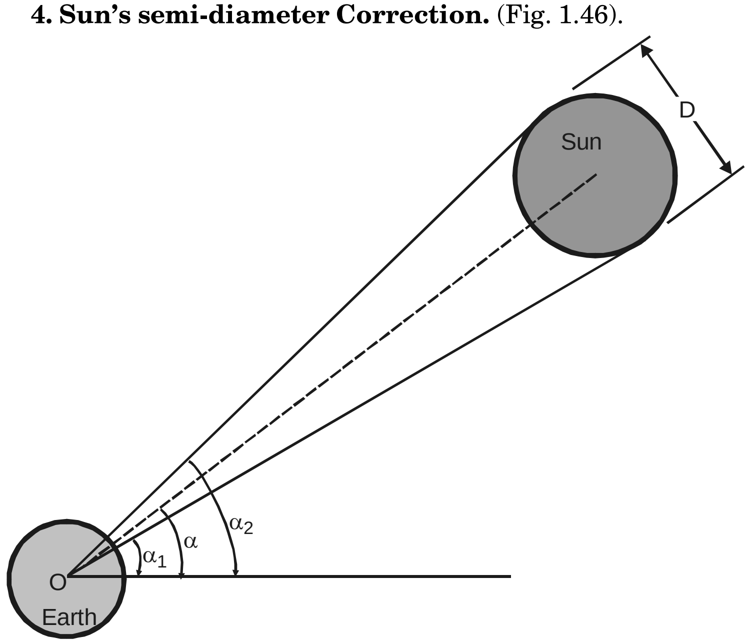


Fig. 1.46. Semi-diameter correction.

While making observations to the sun, it is very difficult to set the cross hairs at its centre. To overcome this difficulty, the observations are generally made either to its upper limb or lower limb. The altitude of the sun's centre may then be obtained by applying the correction for the semi-diameter to the observed altitude algebriacally. The correction for semi-diameter is positive if the lower limb is observed and negative if the upper limb is observed.

If D is the angle subtended by the sun at the centre of the earth, the correction for semi-diameter is $\frac{D}{2}$. The altitude of the sun at the centre is $\alpha_1 + \frac{D}{2}$ and $\alpha_2 - \frac{D}{2}$ where α_1 and α_2 are the observed altitudes to the lower and upper limbs of the sun respectively.

The semi-diameter correction varies from about 15´ 46´´ in July to about 16´ 18´´ in January. For rough calculation, its average value may be taken as 16´.

Derivation of Sun's semi-diameter correction (Fig. 1.47.)

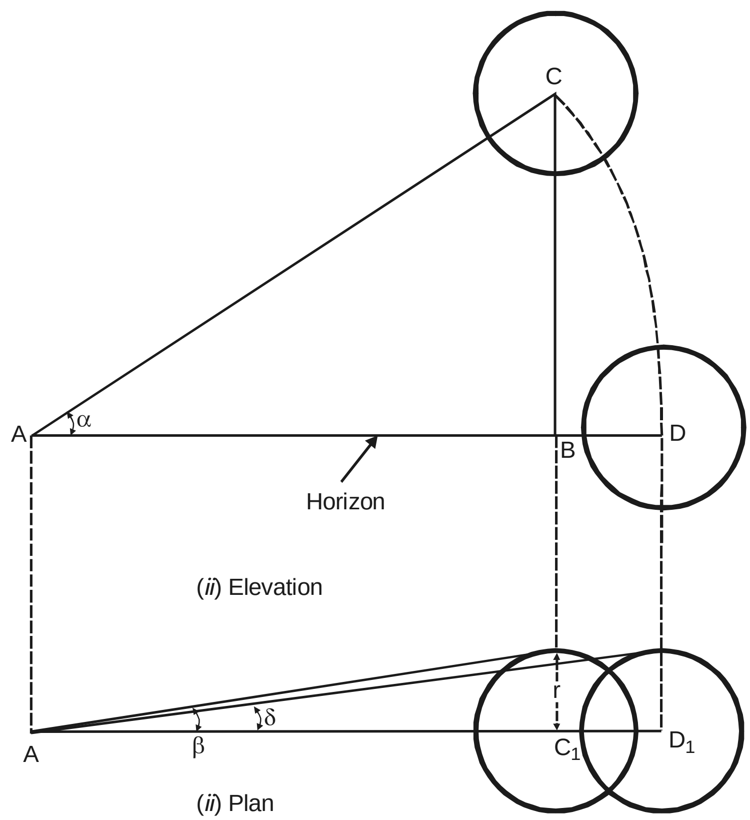


Fig. 1.47.

Let A be the observer's position

D be the sun's position on the horizon

C be the sun at, an altitude d

r be the radius of the sun

δ be the semi-diameter correction when the sun is at horizon

β be the semi-diameter correction when the sun is at C.

Evidently, $r = AD_1 . \delta = AC_1 . \beta$

$$\beta = \frac{AD_1.\delta}{AC_1} \qquad ...(1.44)$$

But, from the elevation (Fig. 1.39) we get

$$AD = AC = AD_1 \text{ and } \frac{AC}{AB} = \sec \alpha = \frac{AC}{AC_1}$$

where AB and AC_1 are the projections of AC.

$$AC = AC_1 \sec . \alpha \qquad ...(1.45)$$

Substituting the value of $AC = AD_1$ in equation (1.43), we get

$$\beta = \frac{AC_1}{AC_1} \delta \sec \alpha = \delta . \sec \alpha \qquad ...(1.46)$$

i.e. semi-diameter correction for horizontal angles is equal to the sun's semidiameter correction multiplied by the secant of the altitude.

Note. The following points may be noted :

(*i*) *The correction to the horizontal angles for the sun at horizon is 16′ approximately.*

(*ii*) *The correction to the horizontal angles for the sun at zenith is infinite.*

(*iii*) *To avoid semi-diameter correction for azimuth, an equal number of sights are taken to opposite limbs of the sun. The mean of the horizontal angles and the mean of the vertical angles at the mean of the times, is used for calculation.*

5. Index error Correction.

In a perfectly adjusted theodo-lite with the line of sight horizontal, the verniers of the vertical circle should read zero. If they do not, the vertical angles measured with such a theodolite, will be incorrect. The reading on the vertical circle with horizontal line of sight, is known as the *Index Error.* Though, the index error can be eliminated completely by making observations on both faces, but due to rapid change of the altitude of celestial bodies, it becomes difficult to make observations on both faces. This is why it becomes necessary to ascertain the magnitude of the index error of the transit before hand so that the observations made on one face only, can be corrected.

Determination of index error

To determine the index error the following steps are followed :

1. Set up the theodolite on firm ground and level it accurately by using the altitude bubble.
2. On face left with telescope normal, bisect a well defined point (say a church spire) and observe its vertical angle α_1 after bringing the altitude bubble central of its run.
3. On face right with telescope inverted, bisect the same point and observe its vertical angle α_2 after bringing the altitude bubble central of its run.

Let the index error of the vertical circle be e

Correct vertical angle (α) on face left = $\alpha_1 + e$

Correct vertical angle (α) on face right = $\alpha_2 - e$

$$\alpha = \frac{(\alpha_1 + e) + (\alpha_2 + e)}{2} = \frac{\alpha_1 + \alpha_2}{2}$$

i.e. the correct vertical angle (α) of a point is the mean of its observed angles α_1 and α_2 observed on both faces.

Index error $\quad e = \alpha - \alpha_1 = \alpha_2 - \alpha$

Illustration. Let $\quad \alpha_1 = 5°18'\,35''$ on face left

$\alpha_2 = 5°\,18'\,55''$ on face right

$$\therefore \text{ Correct vertical angle } \alpha = \frac{5°18'35''+5°18'55''}{2} = 5°\,18'\,45''$$

∴ The index error correction for face left observations

$$= 5°\,18'\,45'' - 5°\,18'\,35'' = +\,10''$$

Similarly, index error correction for face right observations

$$= 5°\,18'\,55'' - 5°\,18'\,45'' = -\,10''$$

Note. The index error correction is *positive* for any face observation if the observed apparent vertical angle is less than the true vertical angle, otherwise it is *negative.*

6. Bubble Error correction. If the altitude bubble does not occupy its central position while making observation of vertical angles, a correction known as *bubble correction* is found necessary.

If ΣE = the sum of readings of the end of the bubble towards the eye piece.

$\Sigma 0$ = the sum of readings of the end of the bubble towards the objective.

n = the number of ends of the bubble readings.

v = the angular value of one division of the bubble in seconds.

then C = correction for the bubble error

$$= + \frac{\Sigma 0 - \Sigma E}{n} \times v \text{ seconds} \qquad ...(1.48)$$

Note. The following points may be noted :

(*i*) If $\Sigma 0$ is greater than ΣE, the bubble error correction is positive.

(*ii*) If ΣE is greater than $\Sigma 0$, the bubble error correction is negative.

(*iii*) If single face observations are taken, the value of n is 2 and if both face observations are made, the value of n is 4.

7. Azimuth Correction. In astronomical observations, vertical angles are generally large. It is, therefore, very important that the instrument must be in perfect adjustment, *i.e.*

(1) the transit is accurately levelled to make its vertical axis truly vertical

(2) the horizontal axis (trunnion axis) is exactly perpendicular to the vertical axis. The altitude bubble may be accurately adjusted and hence error due to this, can be eliminated. However, the error due to inclination

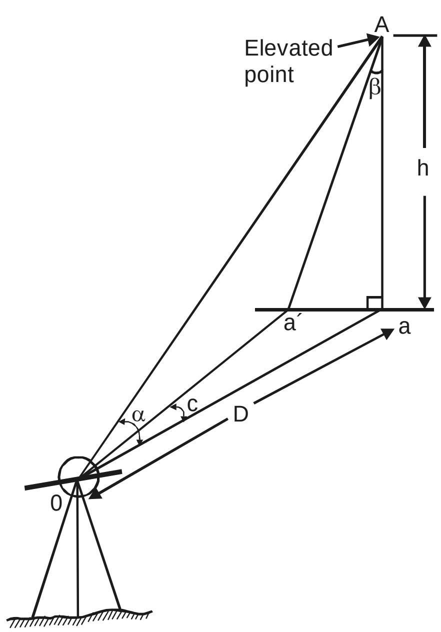


Fig. 1.48. Azimuth correction

of the trunnion axis with horizontal cannot be eliminated. However, Inclination of the trunnion axis can be determined by means of a striding level fitted with a very sensitive level tube.

If β is the inclination of the horizontal axis of the transit with respect to horizontal in seconds and α is the vertical angle of the object sighted, the azimuth correction in seconds

$$C = \beta \tan \alpha \text{ seconds} \qquad \text{...(1.48)}$$

Derivation of the formula (Fig. 1.48).

Let the elevated point sighted be A, the angle of elevation be α, height of point A above horizontal plane be h. Let, the horizontal axis be inclined through an angle β to the horizontal.

When the telescope is transited, the inclination of the horizontal axis to the horizontal will cause the line of sight to move in an inclined plane. Let the trace of such an inclined plane be Aa'. The error in the azimuth is the angle $a'oa$. Again, the angle of inclination β is equal to the angle $a'Aa$.

From $\Delta a'Aa$, we get, $aa' = h \tan \beta = h\beta$

Again $aa' = D \tan c = D.c.$

or
$$C = \frac{aa'}{D} = \frac{h\beta}{D}$$

where D is the horizontal distance between transit station and the point sighted.

But
$$h = D \tan \alpha$$

∴
$$c = \frac{h\beta}{D} = \frac{D \tan \alpha \,.\, \beta}{D} = \beta \tan \alpha$$

i.e. azimuth correction is proportional to the tangent of the altitude.

Determination of the value of β. Let l_1 and r_1 be the readings of the left hand end and right hand end of the striding bubble in first position ; l_2 and r_2 be the readings of the left hand end and right hand end of the striding bubble in second position.

First position. Deviation of the centre of the bubble from the centre of the striding level = $\dfrac{l_1 - r_1}{2}$

Second Position. Deviation of the centre of the bubble from the centre of the striding level = $\dfrac{l_2 - r_2}{2}$

i.e. The mean deviation = $\dfrac{1}{2}\left[\dfrac{l_1 - r_1}{2} + \dfrac{l_2 - r_2}{2}\right]$

$$= \frac{(l_1 + l_2) - (r_1 + r_2)}{4}$$

$$= \frac{\Sigma l - \Sigma r}{4}$$

∴ Inclination of the trunnion axis

$$\beta = \frac{\Sigma l - \Sigma r}{4} \times v \text{ in seconds}$$

where v *is* the angular value of one division of the striding level

Σl = the sum of the readings of the left hand end of the bubble in both positions,

Σr = the sum of the readings of the right hand end of the bubble in both positions.

Note. The following points may be noted :

(*i*) *If Σl is greater than Σr, left hand end of the horizontal axis is higher.*

(*ii*) *If Σr is greater than Σl, right hand end of the horizontal axis is higher.*

(*iii*) *For angles of elevation, the correction is positive if the left hand end of the horizontal axis is higher and negative if the right hand end is higher.*

(*iv*) *For angles of depression, correction is positive if the right hand end of the horizontal axis is higher and negative if the left hand end is higher.*

(*v*) *Horizontal reading to each direction is corrected separately and the required corrected angle, is obtained by subtraction thereafter.*

Example 1.18. *Find the true altitude of the sun's centre which gave an apparent altitude of 55°34′23′′ to the sun's lower limb.*

Given : The sun′s horizontal parallax is 9′′ and sun's diameter is 31′46′′.

Solution.

1. Correction for refraction

$$r = 58'' \cot \alpha = 58'' \times \cot 55°34'23''$$
$$= 58 \times 0.68540565 = 39''.8. \qquad (-\text{ ve})$$

2. Correction for parallax

$$P = 9'' \cos \alpha$$
$$= 9 \cos 55°34'23'' = 9 \times 0.565355$$

$= 5.1''$ (+ve)

3. Correction for semi-diameter = 1/2 (31′46″)

$D = 15'53''$ (+ve)

∴ Total correction $= -r + p + D$

$= -0'39''.8$

$+ 0.05''.1$

$+ 15'53''.0$

∴ Effective correction = 15′18″.3

Observed altitude of the sun = 55°34′23″

Effective correction = + 15′18″-3

∴ True altitude of the sun

= 55°49′41″.3 **Ans.**

Example 1.19. *The following readings were taken on a reference mark (R.M.) and on a star :*

Object	*Striding level on turnnion aixs*				*Altitude*	*Horizontal Circle*		*Average*
	Direct		*Reversed*			*Vernier*		
	L	*R*	*L*	*R*		*A*	*B*	
R.M.	*12.2*	*2.4*	*8.8*	*5.8*	*3°14′00″*	*66°14′10″*	*246°14′20″*	*66°14′15″*
Star	*1.0*	*2.7*	*9.4*	*6.1*	*60°46′50″*	*112°31′50″*	*292°32′00″*	*112°31′55″*

The value of the bubble division is 12″. The altitude has been corrected for the altitude level. Find the corrected horizontal angle between the reference mark and the star.

Solution.

Observations to reference mark (R.M.) :

$$\Sigma l = 12.2 + 8.8 = 21.0$$

$$\Sigma r = 2.4 + 5.8 = 8.2$$

$$\therefore \quad \beta = \frac{\Sigma l - \Sigma r}{4} \times d = \frac{21.0 - 8.2}{4} \times 12'' = +\,38''.4$$

As Σl is greater than Σr, the left hand end of axis is higher.

∴ The correction $c = \beta \tan \alpha = 38''.4 \times \tan 3°14'0''$

$= 2''.2$ (positive)

Correction is positive because left hand end is higher and the vertical angle is angle of elevation.

∴ Corrected azimuth $= 66°14'15'' + 2''.2 = 66°14'17''.2$

Observation to the star :

$$\Sigma l = 1.0 + 9.4 = 10.4$$

$$\Sigma r = 2.7 + 6.1 = 8.8$$

$$\therefore \quad \beta = \frac{\Sigma l - \Sigma r}{4} \times d = \frac{10.4 - 8.8}{4} \times 12'' = \frac{1.6}{4} \times 12'' = 4''.8$$

As Σl is greater than Σr, the left hand end is higher

∴ The correction $c = \beta \tan \alpha$

$= 4.8 \times \tan 60°46'50''$

$= 4.8 \times 0.178786 = 8''.58$

Corrected azimuth $= 112°31'55'' + 8''.6 = 112°32'03''.6$

∴ Corrected horizontal angle between *R.M.* and star

$= 112°32'03''.6 - 66°14'17''.2$

$= 46°17'46''.4$ **Ans.**

1.30. TIME

The interval which lapses between any two instants, is termed as *time.* The earth revolves about its axis in 24 hours and the measurement of time is based upon the apparent motion of the celestial bodies by their rotation.

1.31. APPARENT MOTION OF THE HEAVENLY BODIES

Due to rotation of the earth about its axis from the west to the east, the celestial bodies *i.e.* stars and sun appear to revolve from the east to the west around the earth. All celestial bodies appear to cross the observer´s meridian twice in 24 hours, *i.e.* once at its upper culmination and again at its lower culmination.

In addition to the rotation of the earth about its own axis, the earth also moves in an eclliptic orbit round the sun and makes one complete revolution in the duration of one year. The apparent movement of the sun relatively to the stars, therefore, appears to be from the west to east.

1.32. CLASSIFICATION OF TIME

The following times are generally used by astronomers :

1. The sidereal time
2. The apparent solar time
3. The mean solar time
4. The standard time.

1. The sidereal time. The hour angle of the First point of Aeries (γ) measured west-ward 0 to 24 hours at any instant, is the *sidereal time* of that instant. The interval of time between two successive upper transits of the First point of Aeries (γ) is called the *sidereal day*. The sidereal time is generally used by astronomers. The sidereal day is one of the principal units of time for astronomers. Further sub-divisions of a side real day are :

A sidereal day is divided into 24 sidereal hours.

A sidereal hour is divided into 60 sidereal minutes.

A sidereal minute is divided into 60 sidereal seconds.

The position of the Vernal Equinox is not at a fixed point. Due to precessional movement of the axis, it moves westward. The actual interval between two culminations of the Equinox differs by about 0.01 second of time.

The Local sidereal Time (L.S.T.). The interval of time which elapses since the upper transit of the First point of Aeries (γ) over observer's meridian, is known as the *local sidereal time* of the place. In other words, it may be said that the local sidereal time is the measure of the angle through which the earth rotates since the Equinox (First point of Aeries) was on the observer's meridian. But, we also know that the equatorial angular distance measured from the First point of Aeries to the hour circle of the body, is known as its *right ascension*. Hence, it may be said that the local sidereal time of any place is equal to the right ascension of the meridian of the place, *i.e.* observer's meridian.

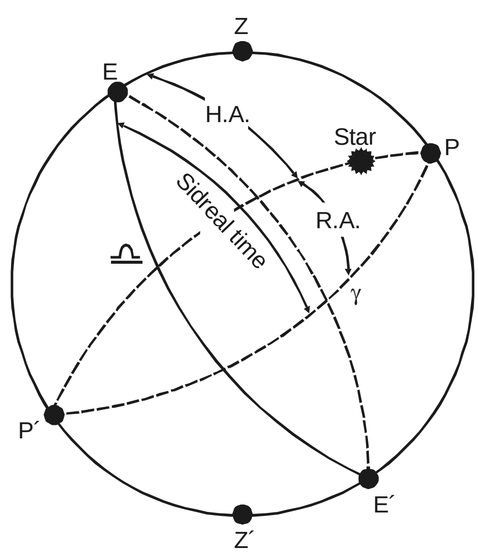


Fig. 1.49. Relation between sidereal time, right ascension and hour angle of start.

The local sidereal time (L.S.T.) = R.A. of a star + Westernly hour angle of the star...(1.49)

Proof. (Fig. 1.49)

EE' be the celestial equator

S be the position of the star

γ be the First point of Aeries

BP be the declination circle of the star

$\angle EPB$ = arc EB = Hour angle of the star S

Arc γB = right ascension of the star

But, L.S.T. = $E\gamma = EB + B\gamma$

L.S.T. = $H.A. + R.A.$

If the sum is greater than 24 hours, deduct 24 hours. If the sum is negative, add 24 hours to get correct local sidereal time.

When a star is at its upper culmination, its hour angle is zero and hence the sidereal time of upper culmination of the star is equal to its right ascension, *i.e.*

Local sidereal time of upper transit of star = R.A. of star. But, we also know that 24 hours of sidereal time correspond to 360°of rotation of the earth. Hence, *the difference between the local sidereal time of two places, is always equal to the difference in their longitudes.*

2. The apparent solar time. The measurement of time based on daily apparent motion of the sun around the earth, is known as the *apparent solar time.* The interval of the time between two successive lower transits (culminations) of the centre of the sun over the meridian of the place is called the *apparent solar day.* The lower transit of the sun is chosen so that the date changes only at midnight and not at noon.

Further sub-divisions of an apparent solar day are :

An apparent solar day is divided into 24 hours

An apparent solar hour is divided into 60 minutes

An apparent solar minute is divided in to 60 seconds.

The sun's apparent daily path is along a great circle (ecliptic) inclined to the equator at an angle of 23°27´. As the rate of movement of the sun along the ecliptic is not uniform, the length of the apparent solar day. throughout the year is also not uniform. This is why apparent-solar time cannot be recorded by a clock having a uniform rate of movement. The apparent solar time can only be recorded with the help of a sun dial.

3. The mean solar time. To overcome the difficulty of recording the variation of apparent solar time by a clock, a fictitious sun is assumed to move at a uniform rate along the equator so as to have a solar day of a uniform duration. The motion of the mean sun is the average of the motion of the true sun in right ascension. The start and arrival of the mean sun and true sun are assumed to be the same at the Vernal Equinox (the First point of Aeries).

The interval of time between two successive lower transits of the mean sun is called *mean solar day* or a *civil day*. The duration of a mean solar day is the average of all the apparent solar days of a year. It may be noted that the rate of increase of right ascension of the true sun is not uniform whereas the rate of increase of right ascension of the assumed mean sun, remains constant throughout the year.

Further sub-divisions of a mean solar day are :

One mean solar day is divided into 24 hours

One mean solar hour is divided into 60 minutes

One mean solar hour is divided into 60 seconds.

The Local mean noon (L.M.N.). The instant when the mean sun crosses the local meridian at its upper transit, is known as the *local mean noon*.

The Local mean time (L.M.T.). The hour angle of the mean sun reckoned west-ward from 0 to 24 hours, is known as the *local mean time*. The mean solar day begins at the mid night and completes at the next mid-night. The zero hour of the mean solar day is at the local mean mid-night. As the duration of a mean solar day is 24 hours, the difference in local mean times of two places, is always equal to the difference of their longitudes.

A civil day is divided into two periods, *i.e.* mid-night to noon and noon to midnight. Each period is of 12 hour duration. The time of an event occurring between mid-night and noon is denoted by A.M. (antemeridian) whereas the time of the event occurring between noon and mid-night is denoted by P.M. (Post-meridian). The astro-nomical day is divided into 24 hours from 0 hour to 24 hours.

Note. The following points may be noted :

(*i*) *Local sidereal time (L.S.T.)*

= R.A. of the sun + H.A. of the sun

= R.A. of the mean sun + H.A. of the mean sun.

(*ii*) *The instant at which the sun crosses the meridian of a place, is called local apparent noon (L.A.N.)*

(*iii*) *The instant at which the mean sun crosses the meridian of the place, is called local mean noon (L.M.N.)*

(*iv*) *The sidereal time of apparent noon is equal to right ascension of the sun.*

(*v*) *The sidereal time of the mean noon is equal to right ascension of mean sun.*

(*vi*) *All places on the same meridian have same local time.*

(*vii*) *The local time of eastern meridians will be later than that of western meridians.*

4. The Standard Time. As the local mean time at any meridian is reckoned from the lower transit of the mean sun at the meridian, the local mean time of each meridian will, therefore, be different. To avoid the confusion arising from the use of different local mean times by the people, the mean time of the central meridian of a country is referred to as the *standard time* of the particular country. The meridian whose local mean time is used as the standard time of the country, is known as the *standard meridian* of the country. Standard meridian of a country is generally selected such that it usually lies an exact number of hours from Greenwich. Of course, India is an exception to this, as its standard meridian is 5½ hours (Long. 82°30´) east of Greenwich. Watches and clocks throughout any country keep the standard time of the country irrespective of their locations in the country.

The difference between the local mean time of any meridian and the standard time is due to the difference of longitudes between the given meridian and the standard meridian. Thus, the local mean time of any place can be easily obtained by adding/subtracting the difference of longitudes between the place and the standard meridian according as the place is east or west of the standard meridian.

i.e. Standard time = L.M.T. ± difference of longitude converted to time, the signs plus and minus in the above equation are used according as the place is west or east of the standard meridian.

Example 1.20. *Calculate the local mean time at a place whose longitude is 92°30´E, when the standard time is 8 hours 40 m 30 s. Assume the standard meridian of the country as 82°30´E.*

Solution.

Difference in longitudes = 92°30´– 82°30´

= 10°= 10 x 4 = 40 minutes

As the place is east of standard meridian,

Standard time = L.M.T. – Difference in longitudes in time

i.e. 8 h 40 m 30 s = L.M.T. – 40 m

or L.M.T. = 8h 40m 30s + 40m

= 9h 20m 30s. **Ans.**

1.33. RELATIONSHIP BETWEEN THE HOUR ANGLE, RIGHT ASCENSION AND TIME

The following relations between the hour angle, right ascension and time of any meridian are noteworthy :

1. Apparent solar time = Hour angle of the sun + 12 hours.
2. Mean solar time = Hour angle of the mean sun + 12 hours.
3. Local mean sidereal time = Right ascension of the mean sun + Hour angle of the mean sun.

The hour angle of the sun at its upper culmination being zero, the sidereal time of apparent noon = right ascension of the sun. Similarly, the hour angle of the mean sun at its upper culmination being zero, the sidereal time of mean noon = right ascension of the mean sun.

1.34. EQUATION OF TIME

The difference between the apparent solar time and the mean solar time at any instant, is known as the *equation of time.* In olden days, the apparent time was first determined be making observations to the sun and then it was reduced to mean time with the help of the equation of time. But, nowadays, mean time is determined from sidereal time obtained by making observations to star or even directly from wireless signals transmitted from Greenwich.

The values of equation of time at 0 hour (mid-night) at Greenwich are tabulated in the Nautical Almanac for every day of the year. *The equation of time is thus treated as a correction which may be applied to the mean time to obtain apparent time.*

Equation of time = Apparent solar time – Mean solar time. The equation of time is positive when the apparent solar time is more than the mean solar time and negative when the apparent solar time is less than the mean solar time. For example if at 0 hour G.M.T. on 15th October, 1975, the equation of time is + 13 m 10 s, it means that the apparent time at 0°h mean time is 0 h 13 m 10 s. In other words, we may say that the true sun is 13 m 10 s ahead of the mean sun. Similarly if the equation of time is –10 m 45 s on 16th January 1975, the apparent time at 0 h mean time will be 23 h 49 m 15 s on earlier day, *i.e.* on 15th January 1975. In this case the true sun is behind the mean sun at that time. The value of equation of time varies from 0 to about 16 minutes at different seasons of the year. It vanishes four times during a year, *i.e.on* or about of April 15, June 14, September 1 and December 25. On these four dates, the mean sun and the true sun are on the same meridian and hence, the apparent time and mean time are also the same.

Reasons of Variation of Equation of Time. The main reasons of the existence of the equation of time and its variation are the following :

(*i*) The path of the earth around the sun is elliptical and not circular. Hence, its motion is not uniform and varies with its distance from the sun.

(*ii*) The movement of the true sun is along the ecliptic which does not correspond to the movement of the mean sun assumed to move along the equator.

Derivation of the Equation of Time. We know that

L.S.T. = R.A. of the mean sun
+ Hour Angle of the mean sun ...(1.50)

L.S.T. = R.A. of the true sun
+ Hour Angle of the true sun ...(1.51)

Subtracting Equation (1.51) from Equation (1.50), we get R.A. of the mean sun – R.A. of the true sun

= H.A. of the true sun – H.A. of the mean sun.

But, the equation of time = Hour Angle of the true sun – Hour Angle of mean sun

= Apparent time – Mean time

∴ Apparent time = Mean time + Equation of time.

1.35. CONVERSION OF TIMES

To convert different times, conversion of the difference in longitudes in time interval may be made as under :

360°of longitudes = 24 hours of time

15°of longitudes = 1 hour of time

1°of longitude = 4 minutes of time

15´ of longitude = 1 minute of time
1´ of longitude = 4 seconds of time
15´´ of longitude = 1 second of time.

Example 1.21. *Convert the following difference in longitudes into interval of time* (*a*) 62°*17´42´´(b) 176°24´57´´.*

Solution.

(*a*) $62° = \frac{62}{15}$ h = 4 h 08 m 0s

$17´ = \frac{17}{15}$ m = 0 h 01 m 8s

$42´´ = \frac{42}{15}$ s = 0 h 00 m 2.8s

Total = 4h 09m 10.8s **Ans.**

(*b*) $176° = \frac{176}{15}$ h = 11h 44m 00s

$24´ = \frac{24}{15}$ m = 0h 01m 36s

$57´´ = \frac{57}{15}$ s = 0h 00m 03.8s

Total = 11h 45m 39.8s. **Ans.**

Example 1.22. *Express the following intervals of time into difference in longitudes.*

(a) 5h 300m 45s. *(b) 10h 24m 12s.*

Solution.

(*a*) 5h = 5 × 15° = 75°0´00´´
30m = 30 × 15´ = 7°30´00´´
45s = 45 × 15´´ = 0°11´15´´ **Ans.**
Total = 82°41´15´´long.

(*b*) 10h = 10 × 15° = 150°0´00´´
24m = 24 × 15´ = 6°0´00´´
12s = 12 × 15´´ = 0°3´00´´
Total = 156°03´00´´Long. **Ans.**

1.36. CONVERSION OF STANDARD TIME TO LOCAL MEAN TIME

The following abbreviations are usually referred to in connection with time conversion :

L.M.T. = Local mean time
L.M.M. = Local mean mid-night (0h)
L.M.N. = Local mean noon (12h)
G.M.T. = Greenwich mean noon
G.M.M. = Greenwich mean mid-night
G.M.N. = Greenwich mean noon
L.A.T. = Local apparent time
L.A.N. = Local apparent noon
L.S.T. = Local sidereal time
G.S.T. = Greenwich sidereal time
I.S.T. = Indian standard time.

We know that the difference between the standard time and the local mean time is equal to the difference in longitudes between the meridian of the place and the standard meridian of the country.

If the meridian of the place is east of the standard meridian, the sun moving apparently from east to west will transit the meridian of the place earlier than the standard meridian. Similarly, if the merid-ian of the place is west of the standard meridian, the sun will transit the standard meridian earlier than the meridian of the place. Hence, it is evident that the local time will be more in eastern longitudes and lesser in western longitudes as compared to the standard time.

i.e. L.M.T. = Standard time ± Difference in longitudes ...(1.52)

= Greenwich Mean Time ± Difference in longitudes...(1.53)

In above equations, +ve sign is used if the meridian of the

given place is east of the standard meridian (or Greenwich) and –ve sign signifies west of the Standard meridian (or Greenwich).

Example 1.23. *If the standard time at a place in India is 18 hours, 18 minutes, 18 seconds corresponding to standard meridian (82°30′E), find the local mean time for the places whose longitudes are :*

(*a*) *90°E.* (*b*) 48°*W*.

Solution.

(*a*) The longitude of the place = 90°*E*

The longitude of the standard meridian

= 80°30′*E*

∴ Difference in longitudes = 90°– (82°30′) = 7°30′

The longitude of the meridian of the given place is more than that of the standard meridian, hence, the place is east of the standard meridian.

Conversion of difference in longitudes into time interval

$$7° \text{of longitude} = \frac{7}{15}\text{ h} = 0\text{h } 28\text{m } 0\text{s}$$

$$30' \text{of longitude} = \frac{30}{15}\text{ m} = 0\text{h } 2\text{m } 0\text{s}$$

Total = 0h 30m 0s

∴ L.M.T. = Standard time + Difference in longitudes

= 18h 18m 18s + Oh 30m 0s

= 18h 48m 18s past mid-night

= 6h 48m 18s P.M. **Ans.**

(*b*) The longitude of the place = 48°*W*

The longitude of the Indian standard meridian = 82°30′*E*

∴ Difference in the longitudes = 82°30′– (–48°) = 130°30′

As the meridian of the given place is west of Greenwich and the standard meridian is east of Greenwich, the place is west of the Indian standard meridian.

$$\text{Now } 130° \text{of longitude} = \frac{130}{15}\text{ h} = 8\text{h } 40\text{m } 0\text{s}$$

$$30' \text{of longitude} = \frac{30}{15}\text{ m} = 0\text{h } 02\text{ m } 0\text{s}$$

Total = 8h 42m 0s

Now L.M.T. = Standard Time – Difference in longitudes

= 18h 18m 18s – 8h 42m 0s

= 9h 36m 18s past mid-night

= 9h 36m 18s A.M. **Ans.**

Example 1.24. *Find the G.M.T. corresponding to the following local mean times :*

(*a*) *8h 36m 48s A.M. at a place in longitude 76°45′E*

(*b*) *8h 36m 48s P.M. at a place in longitude 76°45′W.*

Solution.

(*a*) Longitude of the place = 76°45′*E*

$$76° \text{ of longitude} = \frac{76}{15} \text{h} = 5\text{h } 04\text{m } 0\text{s}$$

$$45' \text{ of longitude} = \frac{45}{15} \text{m} = 0\text{h } 03\text{m } 0\text{s}$$

Total = 5h 07m 0s.

As the place is east of Greenwich, the local time will be ahead

i.e. L.M.T. = G.M.T. + Difference in longitude in time

or G.M.T. = L.M.T. – Difference in longitude in time

= 8h 36m 48s – 5h 7m 0s

= 3h 29m 48s. **Ans.**

(*b*) Longitude of the place is 76°45′W

$$76° \text{ of longitude} = \frac{76}{15} \text{h} = 5\text{h } 04\text{m } 0\text{s}$$

$$45' \text{ of longitude} = \frac{45}{15} \text{m} = 0\text{h } 03\text{m } 0\text{s}$$

Total = 5h 07m 0s.

As the place is west of Greenwich, the local mean time will be behind.

i.e. L.M.T. = G.M.T. – Difference in longitude in time

or G.M.T. = L.M.T. + Difference in longitude in time

= 8h 36m 48s P.M. + 5h 7m 0s

= 20h 36m 48s past mid-night + 5h 7m 0s

= 25h 43m 48s

or G.M.T. = 1h 43m 48s A.M. next day. **Ans.**

Example 1.25. *The Greenwich Civil Time (G.C.T.) at the time of astronomical observations was known to be 8h 30m 48s P.M. on January 21, 1980. If the longitude of the place of observation, is 93°45′45′′E, find the L.M.T. of the place.*

Solution.

Longitude of the place = 93°45′45′′*E*

$$93° = \frac{93}{15}\text{ h} = 6\text{h } 12\text{m } 00\text{s}$$

$$45' = \frac{45}{15}\text{ m} = 0\text{h } 03\text{m } 00\text{s}$$

$$45'' = \frac{45}{15} = 0\text{h } 00\text{m } 03\text{s}$$

$$\text{Total} = 6\text{h } 15\text{m } 03\text{s}$$

As the place of observation is east of Greenwich, the local mean time will be ahead of the Greenwich civil time.

Now G.C.T. = 8h 30m 48s P.M.

G.M.T. = 20h 30m 48s past mid-night

L.M.T. = G.M.T. + Difference in longitude

= 20h 30m 48s + 6h 15m 03s

= 26h 45m 51s past mid-night of Jan. 21, 1980

= 2h 45m 51s A.M. on Jan. 22, 1980. Ans.

1.37. CONVERSION OF LOCAL MEAN TIME TO LOCAL APPARENT TIME AND VICE VERSA

We know that

Equation of time = Apparent solar time – Mean solar time

or = Local apparent time – Local mean time

Hence, Local apparent time

= Local mean time + Equation of time.

But, the equation of time varies in magnitude throughout the year. Its correct values are published in Nautical Almanac at the time of lower transit of the sun, for each day of the year. The correct value of equation of time is obtained by adding or subtracting the rate of increase or decrease since mid-night.

Procedure :

1. Convert the Local Mean Time (L.M.T.) to Greenwich Mean Time (G.M.T.).
2. Compute the interval of time before or after Greenwich mean mid-night or Greenwich mean noon in hours and part thereof.

3. Compute the amount of increase or decrease of the equation of time for the interval of time obtained in step (2) above.
4. Calculate the equation of time at the computed Greenwich mean time obtained in step (1) above.
5. Calculate the Greenwich apparent time (G.A.T.) by adding or subtracting the equation of time computed in Step (2) above to the G.M.T.
6. Calculate the local 'apparent time by adding or subtracting the difference in longitude in time to the local mean time.

Example 1.26. *Find the local apparent time of observation of sun at a place in longitude 72°26´E, corresponding to local mean time 9h 25m 20s. The equation of time at G.M.N. is 4m 34.22s addi-tive to the mean time, and decreases at the rate of 0.24s per hour.*

Solution.

Calculation of G.M.T.

Longitude of the place = 72°36´*E* = 4h 50m 24s

G.M.T. of the observations L.M.T. of observation – Difference in longitude

= 9h 25m 20s – 4h 50m 24s

= 4h 34m 56s

Mean time interval before G.M.N.

= 12h – (4h 34m 56s) = 7h 25m 04s

= 7.42 hours.

It is given that equation of time decreases at the rate of 0.24s per hour at G.M.N.

Hence, the increase of equation of time in mean time interval = 7.42 × 0.24 = 1.78s.

∴ Equation of time at G.M.T. = 4m 34.22s + 1.78s = 4m 36s

But G.A.T. = G.M.T. + E.T.

∴ G.A.T. of observation = 4h 34m 56s + 4m 36s

or G.A.T. = 4h 39m 32s

But, L.A.T. = G.A.T. + Difference in longitudes

= 4h 39m 32s + 4h 50m 24s

or L.A.T. of observation = 9h 29m 56s. **Ans.**

Example 1.27. *Find the local mean time of observation of the sun at a place in longitude 72°36′E, corresponding to local apparent time 9h 29m 56s. The equation of time at G.M.N. is 4m 34.22s additive to mean time and increases at the rate of 0.24s per hour.*

Solution.

Longitude of the place = 72°36′*E* = 4h 50m 24s

L.A.T. of observation = 9h 29m 56s (given)

Subtract longitude in time = 4h 50m 24s

∴ G.A.T. of observation = 4h 39m 32s

Time interval before G.M.T. = 12h – 4h 39m 32s

= 7h 20m 28s

Note. (In the absence of G.M.N. of observation, G.A.T. of observation has been used, for calculating the time interval)

= 7.34 hour

∴ Increase for 7.34 @ 0.24s per hour

= 7.34 × 0.24 = 1.76s

But E.T. at G.M.N. = 4m 34.22s

∴ E.T. at G.A.T. of observation = 4m 34.22s = + 1.76s

= 4m 35.98s

But G.A.T. = G.M.T. + E.T.

Now, G.A.T. of observation = 4h 39m 32.00s

E.T. (–ve) = 4m 35.98s

= 4h 34m 56.02s

G.M.T. of observation = 4h 34m 56.02s

Add longitude interval in time = 4h 50m 24.00s

L.M.T. of observation = 9h 25m 20.02s **Ans.**

Example 1.28. *Find the L.M.T. of observation at a place from the following data :*

L.A.T. of observation= 15h 12m 40s

Equation of time at G.M.N. = 5m 10.65s, additive to apparent time and increasing at 0.22 seconds per hour.

Longitude of the place = 20°30′W.

Solution.

Longitude of the place	= 20°30′*W*
Longitude of the place in time	= 1h 22m
	h m s
L.A.T. of observation	= 15h 12m 40s
Add longitude in time	= 1h 22m 00s
G.A.T. of observation	= 16h 34m 40s
E.T. at G.M.N.	= 5m 10.65s (Given)
M.T. interval for G.M.N.	= 4h 34m 40s
	= 4.57778h

Increase in E.T. for 4-57778h @ 0.22 seconds per hour

$= 0.22 \times 4.57778$

$= 1.01$ s

∴ Equation of time at observation = 5m 10.65s + 1.01s

= 5m 11.66s

G.M.T. of observation = G.A.T. of observation + E.T.

= 16h 34m 40s + 5m 11.66s

= 16h 39m 51.66s

Deduct longitude in time	= 1h 22m
∴ L.M.T. of observation	= 15h 17m 51.66s **Ans.**

1.38. CONVERSION OF SIDEREAL TIME INTERVAL TO MEAN TIME INTERVAL

In one tropical year, the mean sun apparently goes around the earth once with respect to the First point of Aeries (γ) in the same direction as that of the earth´s rotation. Let us suppose that earth makes *n* rotations with respect to the First point of Aeries (γ) in one tropical year. As per definition, a sidereal day is the time taken by the earth in one complete rotation with respect to the First point of Aeries (γ). Hence, total number of sidereal days in a tropical year should be equal to *n*. But, actually the earth rotates only $(n - 1)$ times with respect to *n* sidereal days in a tropical year. Hence, corresponding to *n* sidereal days in a tropical year, there will be only $(n - 1)$ mean solar days. According to Bessal, there are 365.2422 mean solar days in a tropical year. Hence, there will be 366.2422 sidereal days in one tropical year.

Equating the number of sidereal days with the mean solar days, we get

366.2422 sidereal days = 365.2422 mean solar days.

1 sidereal day $= \frac{365.2422}{366.2422}$ mean solar day

1 sidereal day $= 1 - \frac{1}{366.2422}$ mean solar day

1 sidereal day = 23h 56m 4.09s mean solar time.

∴ 1h sidereal time = 1h – 9.8296s mean solar time

1m sidereal time = 1m – 0.1638s mean solar time

1s sidereal time = 1s – 0.0027s mean solar time.

Hence, conversion of sidereal time to the mean solar time may be made by simply subtracting 9.8296 seconds per hour from the given sidereal time. As the required correction is always *negative,* it is known as *retardation.*

Again,

365.2422 mean solar days = 366.2422 sidereal days.

1 mean solar day $= 1 + \frac{1}{365.2422}$ sidereal days

1 mean solar day = 24h 3m 56.56s sidereal time.

1 hour mean solar time = 1h + 9.8565s sidereal time.

1 minute mean solar time = 1m + 0.1642s sidereal time.

1 second mean solar time = 1s + 0.0027s sidereal time.

Hence, conversion of mean solar time to the sidereal time may be made by simply adding 9.8565 seconds per hour to the given mean solar time. As the required correction is always *positive,* it is known as *acceleration.*

Note. The following points may be noted :

(*i*) *The mean solar day is 3m 56.56s longer than the sidereal day.*

(*ii*) *The sidereal day is 3m 55.91s shorter than the mean solar day.*

Example 1.29. *Convert 6 hours 30 minutes 40 seconds sidereal time to mean solar time interval.*

Solution.

To convert sidereal time to mean solar time, the retardation @ 9.8296 seconds per hour of sidereal time is applied.

∴ Total retardation @ 9.8296 seconds per sidereal hour.

6h × 9.8296 = 58.9776 seconds
30m × 0.1638 = 4.9140 seconds
40s × 0.0027 = 0.1080 seconds
Total = 63.9996 seconds
= 1m 3.9996 seconds

Now, sidereal time interval = 6h 30m 40.0000s (Given)
Subtract retardation = 1m 3.9996s
∴ Mean solar time interval = 6h 29m 36.0004s. **Ans.**

Example 1.30. *Convert 6h 29m 36s mean solar time to side-real time interval.*

Solution.

To convert mean solar time to sidereal time, the acceleration @9.8565s per hour of mean solar time is applied.

∴ Total acceleration @ 9.8565 seconds per mean solar hour.

6h × 9.8565 = 59.1390 seconds
29m × 0.1642 = 4.7618 seconds
36s × 0.0027 = 0.0972 seconds
Total = 63.9980 seconds
= 1m 3.988s

∴ Mean time interval = 6h 29m 36s
Add acceleration = 1m 3.988
∴ Sidereal time interval = 6h 30m 39.988s. **Ans.**

1.39. CONVERSION OF LOCAL MEAN TIME AT ANY INSTANT TO LOCAL SIDEREAL TIME IF GREENWICH SIDEREAL TIME (G.S.T.) AT GREENWICH MEAN MID-NIGHT (G.M.M.) IS KNOWN

Following steps are involved :

(1) Calculate local sidereal time (L.S.T.) at local mean midnight (L.M.M.) from the given G.S.T. at G.M.M. as under :

(*i*) Convert the longitude of the place to time.

(*ii*) Calculate the total retardation/acceleration for the longitude in time accordingly as the place is east/west of Greenwich, @ 9.8565s per hour of longitude.

(*iii*) Obtain L.S.T. at L.M.M. by subtracting the retardation from the G.S.T. at G.M.M.

(2) Calculate the mean time interval between the local mean mid-night and the given local mean time.

(3) Convert the mean time interval to sidereal time interval by adding the acceleration to or subtracting the retardation from the mean solar interval as the case may be.

(4) Calculate the required local sidereal time by adding or subtracting the sidereal interval obtained in step (3) to the local time at local mid-night.

Note. If the standard time is given instead, convert it into L.M.T. before converting to L.S.T.

Example 1.31. *Find L.S.T. at a place in longitude 90°W of 10 AM if G.S.T. at G.M.M. is 13h 58m 4.1s.*

Solution.

The longitude of the place = 90°W

The longitude in time $= \frac{90}{15}$ = 6h W

Since the place is west of Greenwich, the gain in sidereal time for 6h of longitude is required.

∴ Total acceleration @ 9.8565s per hour = 6 × 9.8565s
= 59.1390S

L.S.T. at G.M.M. = G.S.T. at G.M.M. + Acceleration
= 13h 58m 4.1s + 59.1390s
= 13h 59m 03.2398

Now the given L.M.T. = 10.00h

∴ Mean time interval from L.M.M. = 10h

Total acceleration @ 9.8565s per hour for 10h mean time interval
= 10 × 9.8565 = 98.565s = 1m 38.565s

But, sidereal interval (SI) = Mean time interval + Acceleration
= 10h + 1m 38.565s = 10h 1m 38.565s

∴ L.S.T. at local mean time = L.S.T. at G.M.M. + S.I.
= 13h 59m 3.239s + 10h 01m 38.565s
= 24h 0m 41.804s

or L.S.T. at L.M.T. = 0h 0m 41.804s **Ans.**

Example 1.32. *Find the L.S.T. at a place in longitude 76°30′E at 4h 30 m P.M., G.S.T. at G.M.N. being 4h 36m 18s.*

Solution.

Longitude of the place = 76°30´E

76° of the longitude = $\frac{76}{15}$ h = 5h 4m 0s

30´ of longitude = $\frac{30}{15}$ m = 0h 2m 0s

∴ Longitude of the place in time = 5h 6m 0s

Since the place is east of Greenwich, a loss of sidereal time for 5h 6m has to be calculated, *i.e.*

5h × 9.8565s = 49.2825

6m × 0.1642s = 0.9852

Total = 50.2677

L.S.T. at L.M.N. = G.S.T. at G.M.N. – Retardation

= 4h 36m 18s – 50.2677s

= 4h 35m 27.7323s

Now, given L.M.T. = 4h 30m P.M.

∴ M.T. interval past L.M.N. = 4h 30m

To convert M.T. interval to sidereal time interval an acceleration is added.

Total acceleration for 4h 30m

4h × 9.8565s = 39.4260s

30m × 0.1642 = 4.9260s

Total = 44.3520s

Sidereal time in interval = Mean time interval + Acceleration past L.M.N.

= 4h 30m + 44.352s

= 4h 30m 44.352s

Now L.S.T. at L.M.N.	= 4h 35m	27.7323s	
Add S.I. Past L.M.N.	= 4h 30m	44.3520s	
∴ L.S.T. at L.M.T.	= 9h 06m	12.0843s	
	= 9h 06m	12.084s	**Ans.**

1.40. CONVERSION OF LOCAL SIDEREAL TIME AT ANY INSTANT TO LOCAL MEAN TIME IF GREENWICH SIDEREAL TIME (G.S.T.) AT GREENWICH MID-NIGHT (G.M.M.) OR AT GREENWICH MEAN NOON (G.M.N.) IS KNOWN

Following steps are involved :

1. Calculate the local sidereal time (L.S.T.) at local mean mid-night from the given G.S.T. at G.M.M. as explained earlier.
2. Calculate the sidereal time interval between the local mean mid-night and the given local sidereal time.
3. Convert the sidereal time interval to mean time interval by subtracting the retardation or adding the acceleration from the sidereal time interval as the case may be.
4. Calculate the required local mean time by adding or subtracting the mean time interval obtained in step (3) to the local mean time at local mid-night.

Note. If standard time is required, convert the calculated local mean time to the standard time, as explained earlier.

Example 1.33. *Find L.M.T. at a place in longitude 90°W if L.S.T. of the place is 0h 0m 41.8s and G.S.T. at G.M.M. is 13h 58m 4.1s.*

Solution.

Conversion of 90°longitude into time = $\frac{90°}{15}$ = 6h

Since the place is west of Greenwich

L.S.T. at L.M.M. = G.S.T. at G.M.M. + Acceleration

Now, acceleration for 6h @ 9.8565s per hour

= 6 × 9.8565 = 59.1390s

G.S.T. at G.M.M. = 13h 58m 4.1s

Add acceleration 59.139s

∴ L.S.T. at L.M.M. = 13h 59m 3.239s

Now, given local sidereal time = 0h 0m 41.1s

= 24h 0m 41.800s

Subtract L.S.T. at L.M.M. = 13h 59m 3.239s

∴ S.I. interval = 10h 01m 38.561s

To convert sidereal interval to mean time interval, apply retardation @ 9.8296 per sidereal hour.

10h × 9.8296 = 98.2960s

1m × 0.1638 = 0.1638s

38.561 × 0.0027 = 0.1041s

Total retardation = 98.5639s = 1m 38.5639s

Mean time interval = S.I. – Retardation

= 10h 01m 38.56s – 1m 38.56s

or L.M.T. = 10.00 A.M. **Ans.**

Example 1.34. *Find the L.M.T. at a place in longitude 76°15´E if L.S.T. of the place is 6h 42m 18s and G.S.T. at G.M.M. is 8h 16m 1.5s.*

Solution.

Longitude of the place 76°15´E

Longitude of the place in time

$$76° = \frac{76°}{15} \text{ h} = 5\text{h } 4\text{m } 0\text{s}$$

$$15´ = \frac{15}{15} \text{ m} = 0\text{h } 1\text{m } 0\text{s}$$

Total = 5h 5m 0s

Since the place is east of Greenwich

L.S.T. at L.M.M. = G.S.T. at G.M.M. – Retardation for 5h 5m

5h × 9.8565 = 49.2825s

5m × 0.1642 = 0.8210s

Total retardation = 50.1035s

∴ L.S.T. at L.M.M. = 8h 16m 01.5s – 50.1s

or = 8h 15m 11.4s

Given L.S.T. = 6h 42m 18s

or L.S.T. = 30h 42m 18s

S.I. past L.M.M. = 30h 42m 18s – 8h 15m 11.4s

= 22h 27m 6.6s

To convert S.I. to mean time interval, apply retardation

22h × 9.8296 = 3m . 36.2512s

27m × 0.1638 = 4.4226s

6.6035s × 0.0027 = 0.0178s

Total = 3m 40.69s

L.M.T.= sidereal interval past M.M. – Retardation

= 22h 27m 6.6s – 3m 40.7s

= 22h 23m 25.9s

or L.M.T. = 10h 23m 25.9s P.M. **Ans.**

Example 1.35. *Find sidereal interval, given mean interval as 8h 30m.*

Solution.

To convert mean interval to sidereal interval, the acceleration @ 9.8565 seconds per hour is applied.

∴ Total acceleration @ 9.8565 per mean hour

8h × 9.8565 =78.852s

30m × 0.1642 = 4.926s

Total acceleration = 83.778 = 1m 23.778

Mean Time interval = 8h 30m

Add acceleration = 1m 23.778s

∴ Sidereal time interval = 8h 31m 23.778s **Ans.**

1.41. DETERMINATION OF THE L.M.T. OF THE UPPER TRANSIT OF A KNOWN STAR IF G.S.T. OF G.M.M. IS KNOWN

We know that the right ascension expressed in time of any star at its upper transit is equal to the local sidereal time. The right ascension and declination of important stars are generally published in the Nautical Almanac. Hence, knowing the right ascension of the star, the local sidereal time (L.S.T.) of its upper transit, is known. Now L.S.T. can be easily converted into L.M.T. by the method explained earlier.

Following steps are involved :

(*a*) From the Nautical Almanac, find the right ascension of the given star on the given day.

(*b*) Convert the right ascension (R.A.) to time to obtain local sidereal time {L.S.T.) at the time of upper transit of the star.

(*c*) From the given G.S.T. at G.M.M., calculate the L.S.T. at local mean mid-night.

(*d*) Find the sidereal interval between the L.S.T. of transit of star and the local mean mid-night.

(*e*) Convert the sidereal interval to mean time interval to get L.M.T. of upper transit.

Example 1.36. *Calculate the L.M.T. of upper transit of a star at a place in longitude 82°30´W, whose R.A. is 20h 10m 30s. Given: G.S.T. of previous G.M.N. as 9h 30m 30s.*

Solution. Longitude of the place is 82°30´W

Longitude of the place is 5h 30m in time

Since the place is west of Greenwich, a gain of sidereal time for 5h 30m is to be calculated.

5h × 9.8565s = 49.2825s

30m × 0.1642s = 3.2840

Total acceleration = 52.5665s

G.S.T. of G.M.N. = 9h 30m 30s (given)

Add acceleration = 52.5665

∴ L.S.T. ofL.M.N. = 9h 31m 33.5665s

Now, R.A. of star = L.S.T. = 20h 10m 30s

Subtract L.S.T. of L.M.N. = 9h 31m 22.5665s

∴ S.I. past L.M.N. = 10h 39m 7.4335s

To convert S.I. interval to mean interval, apply a retardation

10h × 9.8296s = 98.2860s

39m × 0.1638 = 6.3882

7.4335 × 0.0027 = 0.0128

Total retardation = 104.6970s = 1m 44.697s

∴ Mean time interval = S.I. – Retardation

= 10h 39m 7.4335S – 1m 44.6970s

= 10h 37m 22.7365s past L.M.N.

∴ L.M.T. at upper transit = 10h 37m 22.7365s P.M. **Ans.**

1.42. DETERMINATION OF TIME OF ELONGATION OF A CIRCUMPOLAR-STAR (FIG. 1.50)

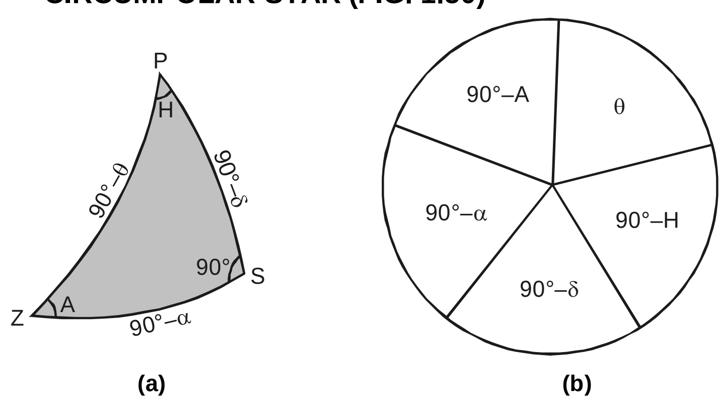


Fig. 1.50.

Let S be the position of the star at elongation.

P and Z be the pole and zenith respectively.

At elongation, angle ZSP is a right angle.

(1) Applying the Napier´s sine rule,

Sine of middle part = product of tangents of adjacent parts

$$\therefore \quad \sin (90° - H) = \tan \theta . \tan (90° - \delta)$$

$$\cos H = \tan \theta . \cot \delta \quad \text{...(1.54)}$$

Knowing the value of latitude (θ) of the place and the declination (δ) of the circumpolar star, the value of the hour angle *(H)* of the star at the instant of elongation can be calculated.

(2) Now, local sidereal time of elongation = Right ascension of the star + Hour angle of the star, where value of

$H = \dfrac{H}{24 - H}$ accordingly as star is $\dfrac{W}{E}$ of the meridian

(3) Convert L.S.T. to L.M.T. as already explained earlier.

(4) Calculate the time of elongation by allowing the chronometer error, if any.

Example 1.37. *Find the L.S.T. of western elongation of Polar is in the evening at a place in latitude 30°22´15´´; given that the R.A. of the star is 1h 52m 12.0s and its declination is + 89°03´46´´.*

Solution.

Latitude of the place θ = 30°22´15´´ (Given)

Declination of the star = + 89°03´46´´ (Given)

We know from Eq. (1.53), that when the star is at elongation,

$$\cos H = \tan 9 . \tan (90° - \delta)$$

$$= \tan 30°22'15'' \times \tan [90° - (89°03'46'')]$$

$$= \tan 30°22'15'' \times \tan 0°56'14''$$

$$= 0.586012 \times 0.0163591 = 0.0095866289$$

$$H = 89°27'02''.52$$

or H in time = 5h 57m 48.2s

But, R.A. of Polaris = 1h 52m 12.0s (Given)

L.S.T. = H.A. + R.A.

∴ L.S.T. of elongation = 7h 50m 00.2s. **Ans.**

1.43. METHODS FOR DETERMINATION OF TIME FROM ASTRONOMICAL OBSERVATIONS

The observations for the determination of time are made only for finding out the error of the chronometer. For the determination of the time it is required to find the hour angle of the heavenly body. Determination may be made from the meridian or ex-meridian observations of the heavenly bodies.

The following five methods are usually employed for the determination of time :

1. By meridian observation of the stars.
2. By ex-meridian observation of the stars.
3. By equal altitudes of the stars.
4. By meridian observation of the sun.
5. By ex-meridian observation of the sun.

1. Determination of Time by making Meridian Observation to heavenly bodies

Given data : (*i*) Local longitude ; (*ii*) Direction of local meridian.

Principle of the Method

At the time of upper transit of a star, its hour angle is zero. Hence, right ascension of star equals the local sidereal time.

The accurate values of right ascension of important stars are tabulated in the Nautical Almanac for each day of the year.

1. Field observations with stars :

Procedure : Following steps are involved :

1. Set up a transit and level it accurately. Set the line of collimation to lie along the direction of the observer's meridian.
2. Note down the chronometer time at the instant the star transits, across the vertical wire of the diaphragm.
3. Compare the right ascension of the star with the observed chronometer time, to obtain the error of the chronometer.

Calculations :

1. If the chronometer is keeping Greenwich sidereal time, add local .longitude in time to the right ascension, to get true Greenwich sidereal time of observation.
2. If the chronometer is keeping local mean time, the local sidereal time determined, is required to be converted first to local mean time to obtain the error of the chronometer.

2. Field observations with Sun

Procedure : Following steps are involved :

1. Set up the theodolite and level it accurately. Set the line of collimation along the pre-determined direction of the meridian.
2. Note down the time at which sun's west and east limb cross the vertical hair by means of a chronometer.
3. Take the mean of the readings, to get the local apparent time.

Calculations :

1. At the upper transit of the sun, the hour angle is zero and local apparent time is 12 hours.
2. Find the G.M.T. of G.A.N. from the Nautical Almanac.
3. Find the L.M.T. of L.A.N. by applying the longitude to G.M.T. of G.A.N.
4. Find the difference of L.M.T. of L.A.N. and the local apparent time of transit, *i.e.* 12 hour, which is the required error of the chronometer.

Example 1.38. *A star at 71°15´E transits at 9h 12m 25s P.M. recorded with a chronometer keeping Indian standard time. Determine the chronometer error if G.S.T. at G.M.M. on the day of observation was 14h 12m 24s. Given : R.A. of the star as 10h 42m 17.25s.*

Solution.

Longitude of the place =71°15´*E*

Longitude of the place in time = 4h 45m

Now G.S.T. at G.M.M. = 14h 12m 24s (Given)

Loss of the sidereal time @ 9-8565s per hour of longitude

$4h \times 9.8565 = 39.43$ seconds

$45m \times 0.1642 = 7.39$ sec.

Total retardation = 46.82 seconds

L.S.T. of L.M.M. = G.S.T. of G.M.M. – Retardation
= 14h 12m 24s – 46.82s
= 14h 11m 37.18s.

Now, L.S.T. of observation
= R.A. of the star

∴ S.I. past mid-night = (10h 42m 17.25s+24h) –14h11m37.18s
= 20h 30m 40.07s

To convert S.I. to mean time interval, apply a retardation 9.8296 seconds per hour of sidereal time

$$20h \times 9.8296 = 196.59 \text{ seconds}$$
$$30m \times 0.1638 = 4.91 \text{ seconds}$$
$$40.07s \times 0.0027 = 0.11 \text{ seconds}$$

Total retardation = 201.61 seconds = 3m 21.61s

Mean time interval past L.M.M.

i.e. Local time of observation = S.I. – Retardation

= 20h 30m 40.07s – 3m 21.61s

= 20h 27m 18.46s.

Standard time as recorded by the chronometer

= 9h 12m 25s P.M.

= 21h 12m 25s past L.M.M.

Local time of the chronometer

= 21h 12m 25s – Difference of longitude

= 21h 12m 25s – (82°30′ – 71°15′)

= 21h 12m 25s – 11°15′

= 21h 12m 25s – 45m = 20h 27m 25s

Now, local time of chronometer = 20h 27m 25s

and Local time of observation = 20h 27m 18.46s

∴ chronometer error = 6.54s fast **Ans.**

2. Determination of time by making Ex-Meridian observation to a star. This is a most convenient and suitable method for the determination of time.

Given data :

(*i*) R.A. of the star (*ii*) Declination of the star

(*iii*) Latitude of the place of observation.

Field observations : The following steps are involved :

1. Set up the theodolite and level it carefully.
2. Observe the altitude (α) of the star when it is out of the meridian of the observer.
3. Observe the time of the chronometer corresponding to the instant of observation.

Calculations :

1. Calculate the components of the spherical triangle *PZS*. (Fig. 1.51).

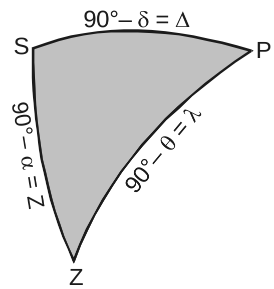


Fig. 1.51.

Co-latitude t»f the place = 90° – θ = λ

Co-declination of the star = 90° – δ = Δ

Co-altitude of the star = 90° – α = Z

2. Solve the spherical triangle PZS to get the value of the hour angle (H) of the star at the time of observation, using the following formula :

$$\tan \frac{H}{2} = \sqrt{\frac{\sin(S-\lambda) \,.\, \sin(S-\Delta)}{\sin S \,.\, \sin(S-Z)}} \qquad ...(1.55)$$

where $2S = Z + X + A$

or

$$\cos H = \frac{\sin\alpha - \sin\theta\sin\delta}{\cos\theta\cos\delta} \qquad ...(1.56)$$

$$= -\tan\theta \,.\, \tan\delta + \frac{\sec\theta}{\cos\delta}\sin\alpha \qquad ...(1.57)$$

3. L.S.T. = R.A. ± $\frac{H}{15}$ using +ve sign if the star is *west* of the 15 meridian and negative if *east* of the meridian.

4. Convert L.S.T. to L.M.T. from the given G.S.T. at G.M.M.

5. The difference in the L.M.T's of observation and the computed value, is the error of chronometer.

Note. The following points may be noted :

1. *The minimum altitude of the star should be 20°, to avoid uncertainities in the refraction as refraction correction is applied to the observed altitudes.*
2. *The star should be observed when it is actually on the prime vertical, to obtain accurate results.*
3. *To increase the accuracy, several observations in quick succession may be made preferably on both the faces of transit.*
4. *To eliminate the instrumental errors, observations to the stars on the east and west of the meridian, should be made.*
5. *If the star is observed on prime vertical, the errors in latitude and elevation produce minimum error in time, as*

$$\cos H = \frac{\tan\delta}{\tan\theta}$$

Example 1.39. *To find the chronometer error on 1-4-1956 ex-meridian observations to star* β *Leo, east of meridian were made and* *the following data recorded :*

Latitude of the place = 30°22´15´´

Corrected altitude of the star = *32°06´41´´*

Right ascension of the star = *11h 46m 51.8s*

Declination of the star = *14°48´47´´*

Sidereal time observed by sidereal chronometer was 7h 57m 30.9s.

Solution.

Given data :

Latitude $\theta = 30°22´15´´$

Declination $\delta = 14°48´47´´$

Altitude $\alpha = 32°06´41´´$

Substituting the values in equation (1.55), we get

$$\cos H = \frac{\sin 32°06´41´´ - \sin 30°22´15´´ \times \sin 14°48´47´´}{\cos 30°22´15´´ \times \cos 14°48´17´´}$$

or $$\cos H = \frac{\sin 32°06´41´´}{\cos 30°22´15´´ \times \cos 14°48´17´´} - \tan 30°22'15´´ \times \tan 14°48´47´´$$

$$= \frac{0.5315669}{0.8627712 \times 0.9667615} - 0.58601200 \times 0.2644550$$

$= 0.63729652 - 0.1549738 = 0.4823227$

$H = 61°09´46´´.2$

or H = 4h 04m 39-07

But R.A. = 11h 46m 51.8s

L.S.T. = R.A. – H (star observed east of meridian)

= 11h 46m 51.8s – 4h 04m 39.1s

or = 7h 42m 12.7s

Observed chronometer time = 7h 57m 30.9s

Chronometer error = 7h 57m 30.9s – 7h 42m 12.7s

= 15m 18.2s (fast). **Ans.**

Example 1.40. *The following notes refer to an observation for time on a star :*

Latitude of the place = *36°30´30´´N*

Mean observed altitude of the star = *30°12´10´´*

R.A. of star = *5h 18m 12.45s*

Declination of star = 16°12′18″.4

The star is to the east of the meridian. Mean sidereal time observed by chronometer = 1h 2m 5.25s.

Find the error of chronometer.

Solution. (Fig. 1.52).

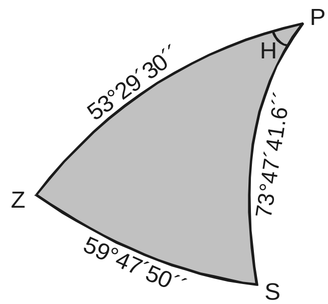


Fig. 1.52.

Given : Latitude of place = 36°30′30″*N*

Co-latitude of place *PZ* = 90° – 36″30′30″ = 53°29′30″

Declination of star = 16°12′18″.4

Co-declination of star *PS* = 90° – 16°12′18″.4 = 73°47′41″.6

Mean observed altitude = 30°12′10″

Zenith distance *ZS* = 90° – 30°12′10″ = 59°47′50″

Let *H* be the hour angle measured eastwardly.

Applying cosine formula to the astronomical triangle *PZS,* we

$$\cos H = \frac{\cos ZS - \cos PZ \,.\, \cos PS}{\sin PZ \,.\, \sin PS}$$

$$= \frac{\cos 56°47'50'' - \cos 53°29'30'' \, \cos 73°47'.6''}{\sin 53°29'33'' \times \sin 73°47'41''.6}$$

$$= \frac{0.503052 - 0.594940 \times 0.279077}{0.80377 \times 0.960269}$$

$$= \frac{0.503052 - 0.1660341}{0.7718354} = 0.43665773$$

$H = 64°06'33''$

∴ Hour angle westwardly

= 360°.H = 360° – 64°06′33″

Hour angle in time

$$= \frac{360° - 64°06'33''}{15} = 19h\ 43m\ 33.80s$$

Now L.S.T. = R.A. + H.A.

= 5h 18m 12.45s + 19h 43m 33.80s

= 25h 01m 46.25s = 1h 01m 46.25s

∴ Correction of chronometer.

= 1h 02m 05.25s – 1h 01m 46.25s

= 19 sec. fast. **Ans.**

3. Determination of time by making observations to a star at equal altitudes. This is a very simple method and is generally used when accurate direction of the observer's meridian is not, known and an accurate result is required.

Principle of the method. When a star is observed at the same altitude on opposite sides of the meridian, the mean of the two chronometer times is evidently the time at which the star transits the observed meridian. When the star crosses the meridian, its hour angle is equal to zero and its right ascension expressed in time, therefore, equals the local sidereal time.

Field Observations : Following steps are followed.

1. Set up the theodolite on firm ground and level it carefully.
2. Sight the star, bring it in the field of view and follow it in azimuth with the help of the horizontal tangent screw.
3. Note down the chronometer time (T_1) at the instant the star crosses the horizontal hair preferably near the centre.
4. Swing the theodolite to the right or left according as the star sighted is south or north of the zenith and follow the motion of the star without disturbing the vertical circle.
5. Note down the chronometer time (T_2) again when the star crosses the horizontal hair preferably at the previous position.

Calculations :

Mean time of the transit of star = ½ ($T_1 + T_2$) = T

∴ L.S.T. = T = Right ascension of the star.

Note. The,following points may be noted :

1. *The face of the transit remains the same during both observations.*

2. *The vertical circle remains clamped during both observations and vertical slow motion screw is not touched.*
3. *The altitude bubble is made central before each observation.*
4. *The observations to the star should be made preferably when it is at prime vertical.*
5. *A series of observations are made on the same star and mean time is accepted for the calculation of the'right ascension.*
6. *To eliminate the uncertainities of refraction, the vertical angle of the star should not be less than 55°.*
7. *The interval of time between two observations can be reduced if the declination of the selected star is nearly equal to the latitude of the observer.*

Effect of the error in the altitude of stars (Fig. 1.53).

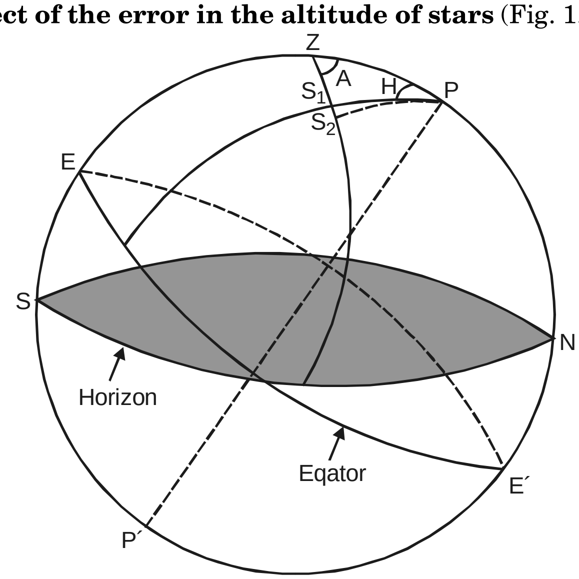


Fig. 1.53.

The effect of a slight error in altitudes may be ascertained as under:

Let $S_1Z = z$, the zenith distance when the star is east of the meridian.

$S_2Z = z + dz$, the zenith distance when the star is east of the meridian. Where dz is error in altitude.

A = the azimuth of star

PZ = the co-latitude (λ)

PS_1 = the polar distance (Δ)

$\angle ZPS_1$ = the hour angle *(H)* of the star east of the meridian.

$\angle ZPS_2$ = the hour angle *(H + h)* of the star west of the meridian

$\angle PZS_2$ = the azimuth (A) of the star west of the meridian. Applying cosine formula to astronomical $\angle PZS_1$, we get

$$\cos z = \cos \lambda \cos \Delta + \sin \lambda \sin \Delta \cos H \qquad ...(1.58)$$

Similarly, from the astronomical $\angle PZS_2$, we get

$$\cos (z + dz) = \cos \lambda \cos \Delta + \sin \lambda \sin \Delta \cos (H + h) \qquad ...(1.59)$$

where h is the error introduced in hour angle due to an error dz in the zenith distance

Subtracting Eqn. (1.58) from Eqn. (1.59), we get

$$\cos (z + dz) - \cos z = \sin \lambda \sin \Delta \{\cos (H + h) - \cos H\}$$

or $$(\cos z \cos dz + \sin z \sin dz) - \cos z = \sin \lambda \sin \lambda [\cos H \cos h + \sin H \sin h - \cos H] \qquad ...(1.60)$$

As dz and h both are nearly zero, cos dz and cos h each may be equated to 1 and sin dz and sin h each equals to dz and h.

The equation (1.60) now reduces to :

$$dz \sin z = h \sin H \sin \lambda \sin \Delta \qquad ...(1.61)$$

Applying sine formula to ΔS_1PZ, we get

$$\frac{\sin z}{\sin H} = \frac{\sin \Delta}{\sin A} \qquad ...(1.62)$$

Substituting the value of sin z from Eqn. (1.62), in Eqn. (1.61), get

$$\therefore \qquad h = \frac{dz \,.\, \sin z}{\sin \lambda \sin \Delta \sin H} = \frac{dz \,.\, \sin H \,.\, \sin \Delta}{\sin \lambda \sin \Delta \sin H \sin A}$$

or $$h = \frac{dz}{\sin \lambda \sin A} \qquad ...(1.63)$$

To have the least value of h, the denominator of R.H.S. of eqn. (1.63) should be maximum, *i.e.* $\sin A = 1$.

or $$A = 90°$$

i.e. the error in the hour angle of the star due to slight error in altitude is minimum when the azimuth of the star is 90°or it is on the prime vertical.

4. Determination of time by making meridian observations to the sun. This method is similar to the one described ear-lier by making meridian observation to the star. The main difference of sun observations is that two times of observations are recorded when the east and west limbs of the sun cross the predetermined direction of the meridian. The mean of the two observed times is the time at which the centre of the sun transits, which corresponds to the local apparent noon (L.A.N.).

Field Observations : The following steps are involved.

1. Set up the theodolite and level it carefully.
2. Sight the telescope along the predetermined direction of the meridian accurately by the vertical hair near the intersection.
3. Fit a dark glass to the eye piece and elevate the telescope to the expected elevation of the sun. Do not move the theodolite in azimuth.
4. Note down the time when the sun's east limb touches the vertical hair and again when it's west limb becomes tangential to the vertical hair.

Calculations :

Let the observed times be T_1 and T_2.

∴ The time at which the sun transits the observer's meridian

$$T = {}^1/_2(T_1 + T_2)$$

or

$$\text{L.A.N.} = T$$

Convert the local apparent noon (L.A.N.) to local mean time to get the chronometer error.

5. Determination of time by making ex-meridian obser-vations to the sun. This method is similar to the one described earlier by making ex-meridian observations to stars.

Field Observations. The following steps are followed :

1. Set up the theodolite on firm ground and level it.
2. Observe the altitude of the lower limb on the face left and the time (T_1) of observation.
3. Change the face and observe the altitude of the upper limb and time (T_2) of observation.

Calculations :

1. Compute the corrected altitude of the sun.
2. Compute the time of observation, *i.e.* $\frac{1}{2}(T_1 + T_2)$
3. Calculate the hour angle of the sun as under :

Let θ be the latitude of the place.

α be the altitude of the sun.

δ be the declination of the sun.

Now, Co-latitude $= 90° - \theta = \lambda$

Zenith distance $= 90° - \alpha = z$

Co-declination $= 90° - \delta = \Delta$

$$\therefore \quad \tan H/2 = \sqrt{\frac{\sin(S-\lambda)\sin(S-\Delta)}{\sin S \,.\, \sin(S-z)}}$$

where $\lambda + z + \Delta = 2S$ and H is the hour angle.

If the sun is east of meridian, L.A.T. of observation

$= (24\text{h} - H/15)$ past local apparent noon.

$= (12\text{h} - H/15)$ past local apparent mid-night.

If the sun is west to meridian, L.A.T. of observation

$= H/15$ past local apparent noon.

$= 12\text{h} + H/15$ past local apparent mid-night

4. Convert L.A.T. to L.M.T. as explained earlier.
5. The difference between the time of observation by chro-nometer and its computed value is the error of the chronometer.

Note. The following points may be noted :

1. *Initial time of observation as recorded by the chronometer may be accepted for the purpose of obtaining the value of the declination of the sun.*
2. *Again, correct the computation for the hour angle by obtaining the value of declination corresponding to the time computed.*
3. *Balancing of observations is affected by making a series of observations on both faces, east and west of meridian.*
4. *The observations should be made preferably between 8 A.M. and 9 A.M. and between 3 P.M. and 4 P.M.*

Example 1.41. *Following observations were made to determine the error of the watch at a place whose latitude is 30°36´20´´N and* *longitude is 76°15´24´´E.*

The mean corrected altitude of the sun = 32°42´35´´
The mean watch time of observation = 15h 57m 36s
The declination of the sun at the time of observation = 16°24´45´´N
G.M.T. of G.A.N. = 11h 52m 23.4s.

Find the correct watch error if the watch is known to be 3m fast on L.M.T.

Solution.

Longitude in time = 5h 05m 1.60s

In the astronomical triangle *PZS, we get*

$$Z = 90° - \alpha = 90° - 32°42´35´´ = 57°17´25´´$$
$$X = 90° - \theta = 90° - 30°36´20´´ = 59°23´40´´$$
$$\Delta = 90° - \delta = 90° - 16°24´45´´ = 73°35´15´´$$
$$2S = 190°16´20´´$$
$$S = 95°08´10´´$$
$$(S–Z) = 95°08´10´´ - 57°17´25´´ = 37°50´45´´$$
$$(S - A.) = 95°08´10´´ - 59°23'40´´ = 35°44'30´´$$
$$(S - \Delta) = 95°08´10´´ - 73°35´15´´ = 21°32´55´´$$

Substituting the above values in the formula

$$\tan \frac{H}{2} = \sqrt{\frac{\sin(S-\lambda)\sin(S-\Delta)}{\sin S \,.\, \sin\,(S-Z)}}\text{, we get}$$
$$= \sqrt{\frac{\sin 35°44´30´´ \,\sin 21°32´55´´}{\sin 95°08´10´´ \,\,\sin 37°50´45´´}}$$
$$= \sqrt{\frac{0.584132 \times 0.367290}{0.955985 + 0.613539}}$$
$$= \sqrt{0.35109538}$$
$$\tan \frac{H}{2} = 0.59253302$$
$$\frac{H}{2} = 30°38´53´´.16$$
$$H = 61°17´46´´.32$$

or H = 4h 05m 11.09s

∴ L.A.T. = 12h + H

= 16h 05m 11.09s

G.A.T. = L.A.T. – Longitude in time

= 16h 05m 11.09s – 5h 05m 1.60s

= 11h 00m 9.49s

Now, G.M.T. of G.A.N. = 11h 52m 23.4s

12h = 11h 52m 23.4s + E.T.

or E.T. = 12h – 11h 52m 23.4s

= 0h 07m 36.6s to be subtracted from the apparent time

or G.M.T. = G.A.T. – E.T.

= 11h 00m 9.49s – 7m 36.64s

= 10h 52m 32.89s

∴ L.M.T. = G.M.T. + Longitude in time

= 10h 52m 32.89s + 5h 05m 1.6s

= 15h 57m 34.49s

∴ Watch error = 15h 57m 36s – 15h 57m 34.49s

= 1.51s fast. **Ans.**

1.44. VARIATIONS IN THE LENGTH OF THE DAY ON THE EARTH'S SURFACE

The variation in the length of the day at any place is due to the sun's annual motion combined with the earth's rotation.

Let θ be the latitude of the observer and ω be the sun's obliquity. We shall discuss the variation at different latitudes.

1. The observer on the equator. To an observer on the equator the celestial poles *P* and *P´* appear on the horizon and coincide with the north and south points. The celostial equator coincides with the prime vertical. On March, 21 and September 23 the sun is on the equator and its diurnal path on these days, is along the equator itself. Apparently the hour angle at sun rise is a right angle. Hence the day is of twelve hours duration. On other days of the year, the sun's diurnal path remains parallel to the prime vertical, that's why the day and night are equal throughout the year but not of 12 hour durations.

2. The observer at a place ($0 < \theta < \omega$). On March 21, the sun rises at east point and his diurnal path is along the equator. The hour angle of the sun rise is 90° and the day and night are of equal duration. From March 21 to June 22, the rising and setting points recede from east and west point towards the north. The sun's hour angle at rising goes on increasing to have its maximum value on June 22. Consequently the days increase in length and nights shorten. The longest day occurs on June 22. The sun's meridian altitude on June 22 is $90° + \theta - \omega$. From September 23 to December 22, the sun's hour angle at rising goes on decreasing so that days shorten and nights lengthen. The shortest day is on December 22. The sun's meridian altitude is minimum on December 22 *i.e.* $90° - \theta - \omega$.

3. The observer in frigid zone [$(90 - \omega) < \theta < 90°$)]. The celestial pole *P* for higher latitudes will be very near the zenith and the angle between the horizon and the equator will be $90° - \theta$. In higher latitudes, the days increase in length at a more rapid rate. The latitude of the place in the frigid zone being more than $90° - \omega$, the colatitude is less than ω. On some day between March 21 and June 22, the sun's declination (δ) will be equal to the colatitude and hence the sun on that day (say *A*) will be circumpolar. From this day onward δ varies and the sun continues to stay above the horizon. From June 22, the sun retraces his path and on another day (say *B)* his declination will be equal to $90° - \theta$. After this day, the sun will again rise and set.

The perpetual day. The period between the days *A* and *B* during which the sun is entirely above the horizon, is called the *perpetual day.* The middle of the perpetual day is June 22.

The perpetual night. The period between the days *A´* and *B´* during which the sun is completely below the horizon is called the *perpetual night.* The middle of the perpetual night is December 22.

4. The observer on the north pole. To an observer on the north pole, the celestial north coincides with the zenith and the celestial equator with the horizon. From March 21, the sun's diurnal paths are small circles above and parallel to the horizon and on June 22, the sun attains maximum altitude. Thereafter it retraces its path and on September 23, it is back on the equator. This period of six months from March 21 to September 23 is the duration of the perpetual day for the observer at the north pole. Similarly, the period of six months from September 23 to March 21, is the perpetual night of the observer on north pole.

The variations in the length of day and night to the observer in the southern hemisphere will be reverse to what has been said for the north hemisphere.

Mathematical relationship between hour angle, declination and latitude of the place. The variations in the length of the day is related as under :

$$\cos h = -\tan \theta \, . \tan \delta \qquad ...(1.64)$$

where h = the hour angle of the sun

θ = the latitude of the place

δ = the declination of the sun (positive or negative as it is north or south of the equator).

1.45. DETERMINATION OF AZIMUTH OF HEAVENLY BODIES

As the azimuth of heavenly bodies is used for the determination of the true bearing of survey lines, the following terms may be clearly understood before making an attempt to know different methods of determination of azimuth.

1. Azimuth. The horizontal angle between the pole and celestial body at the observer's place, is known as *azimuth of the body*. Astronomical azimuths are always reckoned from the north, eastward or west-ward and their values range from 0° to 180°.

2. True Bearing of a line. The horizontal angle between the north meridian and the given line measured clockwisely, is known as *true bearing*. It is reckoned from zero to 360°.

Knowledge of true bearing of survey lines is of prime importance to surveyors and engineers. There are several methods of determining the true meridian. If the azimuth of any celestial body is determined and the horizontal angle between a reference line and the celestial body is also observed, then, the true bearing of the reference line may be easily calculated.

Reference mark (R.M.) or Reference object (R.O). While making astronomical observations, an illuminated object is sighted at the end of the survey line either a triangulation station or any other convenient point. For illumination a hurricane lantern or an electric bulb may be centred on the ground station mark. Such a station or mark is called *reference mark* or *reference object*. R.O. should be placed at a sufficient distance away say 2 km to 5 km, so that focussing of the telescope, is not required to be changed for making observation to the star.

1.46. DETERMINATION OF AZIMUTH BY MAKING OBSERVATIONS ON STARS

The following are the principal methods of determining the azimuth of celestial bodies.

1. By observation on circumpolar star at transit.
2. By observation on a star at equal altitudes.
3. By observation on circumpolar star at elongation.
4. By observation on polaris.
5. By hour angle of stars.
6. By observation on ex-meridian altitude of the star.

1. By observations on circumpolar stars at transit. *Principle of the method.* When a circumpolar star culminates either north of pole or south of pole, its azimuth is zero. Hence, the horizontal angle between the star and the R.O. is the required true bearing of the line measured clockwise from the observer's meridian.

Field Observation. The following steps are involved :

1. Set up the theodolite over the station of observation, centre and level it carefully.
2. Bisect the referring object (R.O.) on face left and note down the reading about 5 to 10 minutes before the calculated time of the transit of the star.
3. Swing right and bring the star in the field of view.
4. About 30 seconds before the exact time of culmination, bisect the star and note down the hroizontal circle reading.
5. Change face and bisect the star again in quick succession and note down the reading again.
6. Swing left and bisect the referring object and note down the horizontal circle reading.
7. Take the means of the readings on both the faces. The difference of the means is the required angle between R.O. and the meridian.
8. The required bearing of the line, is the angle measured clockwise from the meridian.

Specimen Field Book for Observations

Object	*Face*	*Horizontal Angle*		*Mean*	*General*	*Angle*
		A	*B*			
R.O.	L	0°05´30´´	180°05´40´´	0°05´40´´	0°05´35´´	
Star	L	60°25´40´´	240°25´50´´	60°25´45´´		
Star	R	240°25´40´´	60°26´00´´	240°25´50´´	60°25´47´´	60°20´12´´
R.O.	R	180°05´30´´	0°05´40´´	180°05´30´´		

Note. The following points may be noted :

(*i*) When a circumpolar star transits, its movement is only in *azimuth* and not in *vertical* plane.

(*ii*) By observing the star on both faces, few seconds before and few seconds after the exact culmination, we eliminate the error of the line of collimation.

Determination of chronometer time of culmination of a circumpolar star. The time of culmination of a circumpolar star may be determined as follows :

1. Ascertain the exact longitude of the place of observation by noting down the Greenwich wireless signals.
2. Obtain the sidereal time of Greenwich Mean Mid-Night on that day from a Nautical Almanac.
3. Convert the longitude to solar mean time.
4. Convert the interval of longitude in solar mean time to sidereal time interval.
5. Calculate local sidereal time (L.S.T.) as under : L.S.T. at L.M.M. = G.S.T. at G.M.M. ± difference in solar time interval expressed in sidereal time. The +ve sign to be accepted if the place is west of Greenwich and negative if it is east of Greenwich.
6. Obtain the right ascension (R.A.) of the star on that day from the Nautical Almanac.
7. Calculate the interval in S.T. between local mean mid night and the culmination, *i.e.* difference of the steps (6) and (5). If right ascension (R.A.) is less than L.S.T. at L.M.M., add 24 hours to R.A.
8. Convert the sidereal interval obtained in step (7) to the interval of solar time by subtracting @ 9.8295 sec per hour.
9. Calculate the L.M.T. of the chronometer. If the interval of solar time is more than 12h, subtract 12h.

Example 1.42. *Calculate the local mean time (L.M.T.) of the upper culmination of the polaris* (α *Ursae Minors) at Delhi (Longi-tude 76°15´E) on 30th Dec. 1980. Given :*

(i) G.S.T. at G.M.M. : 6h 29m 32.23s.

(ii) R.A. of the polaris : 1h 32m 27.40s.

Solution.

Longitude of Delhi = 76°15´E (given)

Longitude in time = 5h 5m

As the place is east of Greenwich, sidereal time at Delhi will be less than the sidereal time at G.M.M. @9.8565s per hour of the solar time.

Difference for 5h = 49.2825s

Difference for 5m = 0.8214s

Total = 50.1039 sec.

∴ L.S.T. at L.M.M. = 6h 29m 32.23s 50.10s

= 6h 28m 42.13s

Now, R.A. of the polaris = 1h 32m 27.40s (given)

and L.S.T. at L.M.M. = 6h 28m 42.13s

∴ Interval in sidereal time between L.M.M. and culmination of star

= R.A. – L.S.T. at L.M.M.

= (1h 32m 27.40s + 24h) – 6h 28m 42.13s

= 19h 03m 45.27s.

Convert the sidereal time interval to solar time by deducting @9.8265 sec per sidereal hour.

19h S.T. = 186.76s

3m S.T. = 0.49s

45.27s = 0.12s

Total = 187.37s

= 03m 07.37s

The interval of solar time = 19h 03m 45.27s. – 3m 7.37s

= 19h 00 m 37.90

∴ Chronometer time of culmination of polaris

= 7h 00 m 37.90s P.M. **Ans.**

2. By observation to Stars at Equal Altitudes

To over-come the difficulty of bisecting the circumpolar star exactly at culmination, it is advisable to observe the same star at equal altitudes, once before culmination and again after culmination. In this method, neither the latitude nor the local time, is required and also no calculations are involved.

Principle of the Method. The principle of the method is based on the following fact:

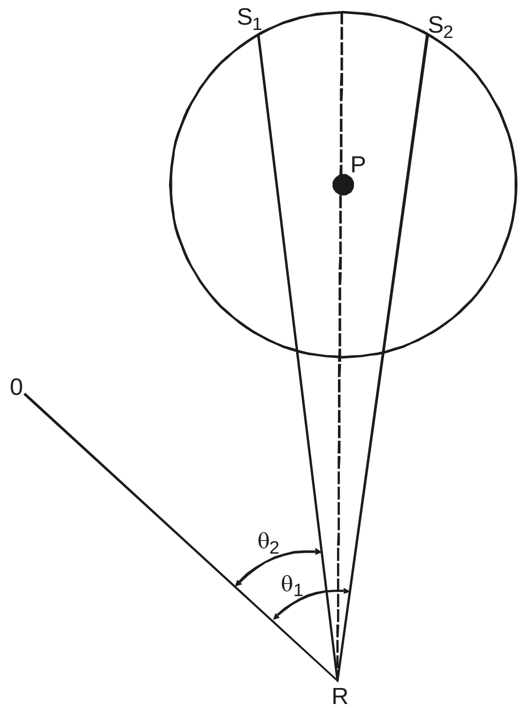


Fig. 1.54.

"The angle between the referring object and meridian of the place is equal to the half algebraic sum of two horizontal angles at'equal altitudes".

Let S_1 and S_2 be the positions of a star when its altitude is equal to θ. Let θ_1 and θ_2 be the respective horizontal angles between R.O. and the star in two positions S_1 and S_2. As the star moves in a circular path round the meridian of the observer, the horizontal distance between the meridian and star at same altitude will be equal. (Fig. 1.54).

i.e. $\angle S_1RP = \angle S_2RP$

But $\angle S_2RS_1 = \theta_1 - \theta_2$

or $\angle ORP = \theta_2 + \dfrac{\theta_1 - \theta_2}{2}$

$= \dfrac{\theta_1 + \theta_2}{2}$

Hence, the azimuth of the line is equal to half the sum of two angles, if the positions of the star are to the same side of the survey line. In case, the line lies such that the star attains the same altitude once on its right and again on its left, the azimuth will be equal to half the difference of the observed angles, which may be easily proved by the reader himself.

Field observations : The following steps-are involved :

1. Set up the instrument at R, the station of observation and level it accurately.
2. Sight the referring object and make the horizontal circle reading approximately zero degree and few minutes.
3. Unclamp the upper plate and swing the telescope clockwise to bisect the star at position S_1 and clamp both the horizontal and vertical circles.
4. Read the horizontal circle reading as well as vertical circle reading (α).
5. Unclamp the upper plate, swing the telescope and follow the star till it is again seen through the telescope.
6. When the star attains the same altitutde (α), clamp both the clamps.
7. Read the horizontal circle reading as well as vertical circle reading (α).

Calculations :

1. Find the horizontal angle (θ_1) between R.O. and the star at position (S_1).
2. Find the horizontal angle (θ_2) between R.O. and the star at position S_2.
3. The angle between R.O. and meridian is equal to half the algebrak sum of θ_1 and θ_2.

Note : It should be ensured that the theodolite is set up on firm ground and levelling of the instrument is done with the help of altitude bubble so that altitude bubble remains central in all positions of the telescope.

Field observations with an imperfect instrument. The procedure stated above is only suitable if the instrument is in perfect adjustment. If it is not, proceed as under :

1. Set up the instrument at R and bisect the referring object O and clamp its both plates.
2. Read the horizontal circle.

3. Loosen the upper plate, swing the telescope in azimuth and bisect the star at S_1.
4. Note down the horizontal and vertical circle readings.
5. Obtain the horizontal angle θ_1 by subtracting the second reading from the first and let the vertical angle be α_1.
6. Change face and bisect O again and clamp both the plates. During this duration, the star moves upward and westward to position S_2.
7. Unclamp upper clamp, and swing the telescope in azimuth to sight the star at S_2. Clamp the vertical circle. Read horizontal and vertical circles again.
8. Obtain the horizontal angle θ_2 by subtracting the seventh reading from the sixth and let the yertical angle be α_2.
9. With the vertical circle clamped at α_2, swing the telescope in azimuth and bisect thie star at S_3 when it again attains the altitude α_2. Read the horizontal circle reading.
10. Swing the telescope and bisect the referring object again. Read the horizontal circle reading.
11. Obtain the horizontal angle θ_3 between the star at S_3 and R.O.
12. Change the face, bisect R.O. and read horizontal circle. Set the vertical circle at α_1.
13. Unclamp the upper plate, swing the telescope in azimuth and bisect the star when it attains its altitude α_1 again.
14. Clamp both the plates and read the horizontal reading.
15. Obtain the value of horizontal angle between R.O. and the star at position S_4.

Calculations :

Let θ_1, θ_2, θ_3 and θ_4 be the horizontal angles and α_1, α_2, α_2, α_1, be the vertical angles respectively for the four positions of the star S_1, S_2, S_3 and S_4.

Apparently mean horizontal angle for the position S_1 and S_2 is $\frac{\theta_1 + \theta_2}{2}$ and its vertical angle is $\frac{\alpha_1 + \alpha_2}{2}$. Again, when the star crosses the meridian, the mean horizontal angle for the positions S_3 and S_4 is $\frac{\theta_3 + \theta_4}{2}$ and its vertical angle is $\frac{\alpha_2 + \alpha_1}{2}$. In other words, the mean altitude is the same for both mean horizontal angles.

If both average positions of the star at the time of observa-tion are on the same side of the line, the azimuth.

$$A = \frac{1}{2}\left[\frac{\theta_1 + \theta_2}{2} + \frac{\theta_3 + \theta_4}{2}\right]$$

$$A = \frac{(\theta_1 + \theta_2) + (\theta_3 + \theta_4)}{2} \qquad ...(1.65)$$

If both average positions of the star at the time of observation are on either side of the line, the azimuth

$$A = \frac{1}{2}\left[\frac{\theta_1 + \theta_2}{2} + \frac{\theta_3 + \theta_4}{2}\right]$$

$$A = \frac{(\theta_1 + \theta_2) - (\theta_3 + \theta_4)}{2} \qquad ...(1.66)$$

3. By observations on a circumpolar star at elongation.

When a circumpolar star is at elongation, it is at its greatest distance east or west of the meridian. The paralactic angle *ZSP* apparently becomes 90´´. (Fig. 1.55)

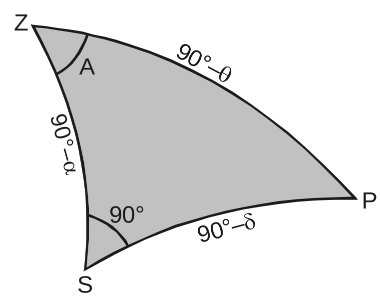


Fig. 1.55.

Let α be the altitude of the star.

δ be the delination of the star.

θ be the latitude of the place.

Applying the sine formula to the right angle spherical triangle ZSP at S, we get

$$\frac{\sin A}{(\sin 90° - \delta)} = \frac{\sin 90}{\sin(90° - \theta)}$$

or

$$\frac{\sin A}{\cos \delta} = \frac{\sin 90°}{\cos \theta}$$

or

$$\sin A = \cos \delta \sec \theta \qquad ...(1.67)$$

Field Observations. The following steps are involved :

1. Calculate the exact time of elongation as discussed earlier.
2. Set up the theodolite on the station of observation about 20 minutes before the time of elongation and level it accurately.

3. About five minutes before the time of elongation, bisect R.O. on face left and read the horizontal circle.
4. Loosen the upper plate, swing the telescope in azimuth and bisect the star on the vertical wire.
5. Note down the horizontal circle at every half a minute interval.
6. Near the elongation, change the face of the theodolite and intersect the star on vertical wire.
7. Note down the horizontal circle at every half a minute interval. Equal number of observations should be taken on either face.
8. Finally intersect R.O. on the face right and note down the horizontal circle reading.

Calculations:

At elongation, the star moves on the vertical wire and hence there is no change in azimuth. By inspection of the field observations, select an equal number of face left and face right readings, when the horizontal angle readings remain constant.

The mean of face left and face right readings, gives the read-ing of the star at elongation.

$\therefore$ The angle between R.O. and the star at elongation

= Reading of star – Reading of R.O.

Azimuth of the line = Reading to star at elongation – Reading to R.O. $\pm A$ according as star is at east or west elongation.

Note. The following points may be noted :

1. *When the star is at eastern elongation, the star appears to move vertically downward and at western elongation, it appear to move vertically upward.*
2. *At both elongations the altitude of star is greater than the elevation of the pole.*
3. *The observations should be limited within five minutes before and after elongation.*
4. *For observations extending over a period exceeding 5 minutes, a correction = 1.96 tan A* $\sin^2 \delta (t_E - t)^2$ *(in seconds) need be applied where* $(t_E - t)$ *is the sidereal interval in minutes between the time of elongation and time of observation.*
5. *Observations 30 minutes before or after the time of elon-gation, should never be accepted.*

Effect of an Error in latitude on the azimuth. We know that the azimuth of a circumpolar star at elongation is calculated from the following formula :

$$\sin A = \frac{\cos\delta}{\cos\theta} \qquad ...(1.68)$$

The accurate value of the declination may be obtained from the star almanac. Accurate value of the latitude needs be obtained. Let dA be the error in azimuth due to an error $d\theta$ in lati-tude.

Differentiating the equation (1.65) w.r.t. θ we get

$$\cos A . dA = \frac{\cos\delta(-\sin\theta)}{\cos^2\theta} . d\theta$$

$$dA = \frac{-\cos\delta\sin\theta}{\cos A\cos^2\theta} . d\theta$$

Substituting $\frac{\cos\delta}{\cos\theta} = \sin A$ from (Eqn. 1.64), we get,

$$dA = \frac{-\sin A\cos\theta}{\cos A\cos\theta} . d\theta$$

or $$dA = - \tan A . \tan q . d\theta \qquad ...(1.69)$$

From equation (1.66) it is clear that the error in azimuth, is directly proportional to the azimuth itself and also the latitude, *i.e.*

1. The closer the circumpolar star to the pole, the smaller it its azimuth and consequently, the smaller is the error dA.
2. The error dA will be more the higher latitudes and less for lower latitudes.

Example 1.43. *The Polaris elongates on the west of meridian at a place having latitude 30°22´15´´, at 7h 50m 00.2s local sidereal time. If the horizontal angle between the R.O. and the star at elongation is 47°55´38´´, calculate the true bearing of the line given that the declination of the star is 89°03´45´´.*

Solution. (Fig. 1.47)

Given : θ = 30° 22´ 15´´; δ = 89° 03´ 45´´

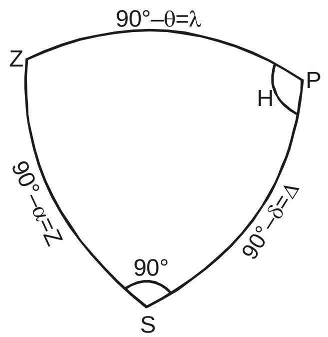
Applying the Napier's formula, we get

Sine of middle part = Product of consine of opposite parts

$$\sin\Delta = \cos(90° - A) \times \cos(90° - \lambda)$$

$$\sin\Delta = \sin A . \sin\lambda$$

or $$\sin A = \frac{\sin\Delta}{\sin\lambda} \qquad ...(i)$$



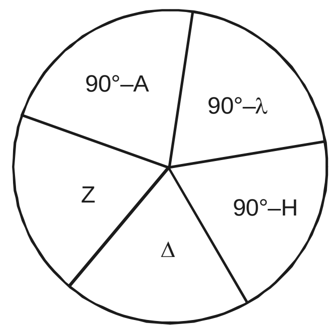


Fig. 1.56.

Here $\Delta = [90° - 89°03'45''] = 0°56'15''$

$\lambda = [90° - 30°22'15''] = 59°37'45''$

$$\sin A = \frac{\sin \Delta}{\sin \lambda} = \frac{\sin 0°56'15''}{\sin 59°37'45''}$$

or $\sin A = 0.018964244$

$A = 1°05'12''$

Azimuth of R.O. = Horizontal angle between R.O. and the star + Azimuth of the star

$= 47°55'38'' + 1°05'12'' = 49°00'50''$

∴ True bearing of R.O. $= 360° - 49°00'50'' = 310°59'10''$ **Ans.**

By observations to two circumpolar stars at elongation. To eliminate the latitude altogether from the formula, observe two circumpolar stars at elongation within a short time.

Let δ_1 and δ_2 be the declinations of two stars which elongate within a short time at the place of observation, having θ latitude.

$$\sin A_1 = \frac{\cos \delta_1}{\cos \theta} \quad ...(i)$$

and

$$\sin A_2 = \frac{\cos \delta_2}{\cos \theta} \quad ...(ii)$$

Dividing eq. (*i*) by eqn. (*ii*), we get

$$\frac{\sin A_1}{\sin A_2} = \frac{\cos \delta_1}{\cos \delta_2} = k, \text{ a constant}$$

Two cases may arise according to the elongatijon of the stars.

(1) **Elongation of the stars on the same side of the pole** (Fig. 1.57)

Let the difference in azimuth $(A_2 - A_1)$ of the stars at eastern elongation be a,

$$A_1 = A_2 - a$$

or $$\sin A_1 = \sin (A_2 - a)$$

$$= \sin A_2 \cos a - \cos A_2 \sin a$$

But, $$\sin A_1 = k \,.\, \sin A_2$$

$\therefore$ $$k \sin A_2 = \sin A_2 \cos a - \cos A_2 \sin a$$

or $$k = \cos a - \cot A_2 \sin a$$

or $$\cot A_2 = \frac{\cos a - k}{\sin a} \qquad \text{...(1.70)}$$

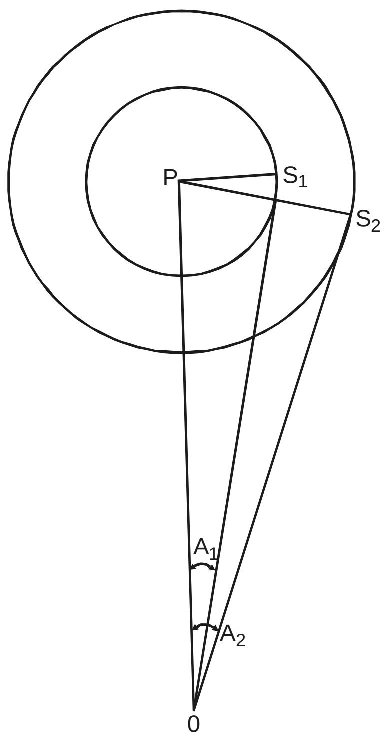


Fig. 1.57.

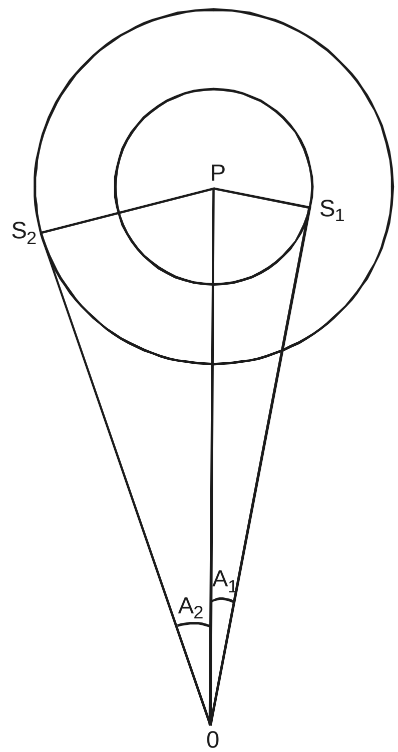


Fig. 1.58.

(2) Elongation of the stars on either side of the pole (Fig. 1.58).

In this case $A_1 + A_2 = a$

$$A_1 = a - A_2$$

$$\sin A_1 = \sin (a - A_2) = \sin a \cos A_2 - \cos a \sin A_2$$

But, $\sin A_1 = k \sin A_2$

$\therefore$ $\sin a \cos A_2 - \cos a \sin A_2 = k \sin A_2$

or $\sin a \cot A_2 - \cos a = k$

or
$$\cot A_2 = \frac{k + \cos a}{\sin a} \qquad ...(1.71)$$

Equations (1.67) and (1.68) may be combined into one equation.

i.e.,
$$\cot A_2 = \frac{\cos a \pm k}{\sin a} \qquad ...(1.69)$$

Using +ve sign for the elongations in opposite side and –ve sign for the same side elongations of stars.

Example 1.44. A *star of declination 82°06´45´´ N was observed at E elongation when the clockwise angle from a reference object was 110°24´50´´. Immediately afterwards another star of declination 75°42´20´´ N was observed at E elongation, and clock-wise horizontal angle observed was 125°42´40´´. Determine the azimuth of R.O.*

Solution. (Fig. 1.48)

Given : $\delta_1 = 82°06´45´´$; $\delta_2 + 75°42´20´´$

$$K = \frac{\cos \delta_1}{\cos \delta_2} = \frac{\cos 82°06´45´´}{\cos 75°42´20´´}$$

$$= \frac{0.1372284}{0.246905} = 0.55579676$$

Now $a = 125°42´40´´ - 110°24´50´´ = 15°17´50´´$

From eqn. (1.67) we get

$$\cot A_2 = \frac{\cos 15°17´50´´ - 0.55579676}{\sin 15°17´50´´}$$

$$= \frac{0.9645702 - 0.55579676}{0.268262}$$

$$= \frac{0.4087759}{0.2638262} = 1.949439$$

or $A_2 = 32°50´18´´$

$\therefore$ Azimuth of R.O. $= 125°42´40´´ - 32°50´18´´$

$= 92°52´22´´$ East of the meridian. **Ans.**

4. By observations to Polaris (Fig. 1.59).

Polaris. The pole star, Polaris or a Ursa Minor is the star on which observations for latitude and azimuth, are generally made in latitudes between 20°to 40°North. Its distance from the pole is roughly 1°. The maximum change in its declination is less than half a second. Its location on the celestial sphere can be easily determined with the help of its neighbouring constellations ojf Ursa major and Cassoipeia. α and β stars of the constellation of Ursa major, are called the *pointers* because the line joining them passes very nearly to the celestial north pole. The constellation of cassiopeia always remains on the same side of the pole as Polaris. '

If cassiopia is in the north, polaris is at its upper culmination.

If cassiopia is in the south, polaris is near its lower culmination.

If cassiopia is in the east, polaris is near its east elongation.

If cassiopia is in the west, polaris is near its west elongation.

The line passing through τ ursa major and δ cassiopia passes nearly through the polaris and north pole. If the line is nearly horizontal, the polaris is at its either elongation. If it is nearly vertical, the polaris is on its either culmination.

Observations to determine the azimuth, are usually made when polaris is at elongation when it appears to move vertically. Observations to determine the latitudes, are usually made when polaris is at culmination, when the star appears to move horizontally.

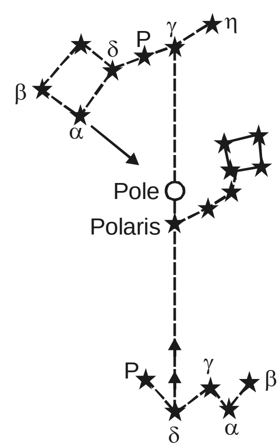


Fig. 1.59. Polaris.

The data regarding culminations and elongations of the pole star are published in Star Almanac annually for the use of astronomers and field surveyors.

We know that the polaris at its elongation moves very rapidly in altitude whereas its azimuth remains constant. Error in altitude if any, will not affect the azimuth. Moreover, when the hour angle of the polaris is 90°, error in the assumed value of the latitude of the place will be least.

To utilise these favourable conditions, usually observations are taken when the polaris is within three hours of its elongation.

Field Observations. The following steps are involved :

1. Set up the instrument over the station mark, centre and level it accurately,
2. Bisect the referring object (R.O.) on face left and note down the horizontal circle reading.
3. Swing the telescope to the Polaris and bisect it at the cross of the hair.
4. Note down the horizontal circle reading and observe vertical angle, ensuring that altitude bubble is central.
5. Change the face quickly and repeat the observations. This constitutes one set.
6. Take a minimum of three sets and finally bisect R.O. on face right to eliminate the collimation error, if any.
7. Note down the temperature and barometric pressure at the beginning and at the end of observations.

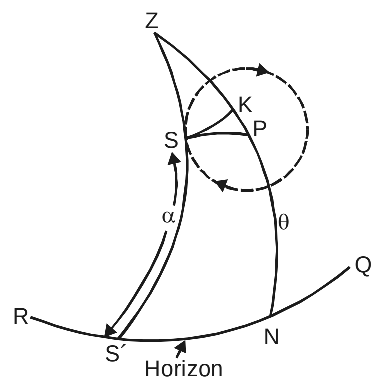


Fig. 1.60.

Computations.

Let P be the pole

Z be the zenith

RQ be the horizon

S be the position of polaris

α be the altitude of the polaris

θ be the latitude of the place.

Construction : Draw SK parallel to $S'N$ where S' is the projection of S on the horizon RQ. (Fig. 1.60)

Here $$KP = KN - PN$$
$$= SS' - PN$$
$$= \alpha - \theta = a$$

The right angled triangle PKS being small, may be treated as a plane triangle without introducing any appreciable error.

$$\cos H = \frac{KP}{SP} = \frac{a}{\Delta}$$

where Δ is north polar distance

or $$H = \cos^{-1}\frac{a}{\Delta}$$

and $$\sin H = \frac{SK}{SP} = \frac{SK}{\Delta}$$

∴ $$SK = \Delta \sin H \quad ...(1.73)$$

and Azimuth $$A = NS' = SK \sec \alpha \quad ...(1.74)$$

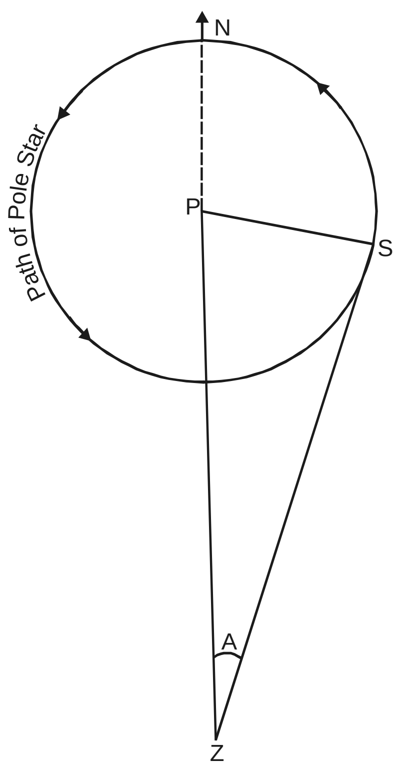


Fig. 1.61.

Substituting the value of SK from eqn. (1.73), we get

$$A = A \sin H \sec \alpha$$

Most suitable position of the polaris for observations

We know that the polaris attains maximum azimuth at elongation and zero at its culmination. At elongation, the variables which affect the azimuth, are the paralactic angle and the altitude of the star. Let us discuss the effect of an error in altitude (Fig. 1.61).

Let P be the pole

S be the position of Polaris

Z be the zenith of observer

A be the azimuth of the polaris

α be the altitude of the polaris

δ be the declination of the polaris

θ be the latitude of the place of observation.

Solving the spherical triangle *ZPS,* we get

$$\cos A = \frac{\cos PS - \cos ZS \,.\, \cos ZP}{\sin ZS \,.\, \sin ZP}$$

or
$$\cos A = \frac{\sin\delta - \sin\alpha \,.\, \sin\theta}{\cos\alpha \,.\, \cos\theta} \qquad ...(1.75)$$

Differentiating eqn. (1.75) with respect to α, we get, – sin*A*.*dA*

$$= \frac{\cos\alpha.\cos\theta(-\sin\theta\cos\alpha) - (\sin\delta - \sin\alpha\sin\theta)(-\cos\theta\sin\alpha)\delta d}{\cos^2\alpha\cos^2\theta}$$

$$= \frac{-\sin\theta\cos^2\alpha + \sin\delta\sin\alpha - \sin\theta\sin^2\alpha}{\cos\theta\cos^2\alpha} d\alpha$$

$$= \frac{\sin\delta\sin\alpha - \sin\theta}{\cos\theta\cos^2\alpha} d\alpha$$

or $$dA = \frac{\sin\theta - \sin\delta\sin\alpha}{\sin A \,.\, \cos\theta\cos^2\alpha} d\alpha$$

Substituting the value of $\sin\theta - \sin\delta\sin\alpha = \cos\delta\cos\alpha\cos S$ we get

$$dA = \frac{\cos\delta\cos\alpha \,.\, \cos S}{\sin A \,.\, \cos\theta\cos^2\alpha} dx \qquad ...(1.76)$$

Again, applying the sine formula to Δ*ZPS* we get

$$\frac{\sin A}{\sin PS} = \frac{\sin S}{\sin PZ}$$

or
$$\sin A \sin PZ = \sin S \sin PS$$

or
$$\sin A \cos\theta = \sin S \cos\delta \qquad ...(1.77)$$

Substituting the value of sin *A* cos θ from (Eqn. 1.77) in Eqn. (1.76) we get

∴ $$dA = \frac{\cos\delta\cos\alpha\cos S}{\sin S \,.\, \cos\delta\cos^2\alpha} d\alpha$$

$$= \cot S \sec\alpha \, d\alpha \qquad ...(1.78)$$

i.e. error in azimuth (dA) increases as paralactic angle S decreases or as altitude α increases. The maximum value of altitude is attained when the star is farthest away from the meridian.

Note. The following points may be noted :

(*i*) If the vertical angles increase, the polaris is east of the meridian.

(*ii*) If vertical angles decrease, the polaris is west of the meridian.

Example 1.45. *The polaris (declination 89°03´54´´.9N) was observed west of the meridian when the anticlockwise angle from a reference object was 28°49´38´´. The angle of elevation corrected for refraction was 30°50´19´´ and the latitude of the place of observation was 30°19´25´´. Calculate the azimuth of the R.O.*

Solution.

Here $\Delta = 90° - \delta = (90° - 89°03'54.9'')$

$\therefore$ $56'5.1'' = 3365''.1$

Altitude of the polaris = 30°50´19´´

Latitude of the place = 30°19´25´´

$\therefore$ True value of α = 0°30´54´´ = 1854´´

$$\cos H = \frac{\alpha}{\Delta} = \frac{1854}{3365.1} = 0.5509494$$

$$H = 56°34'04''$$

Substituting the values of H and α in eqn. (1.71) we get

$$A = \Delta \sin H \cdot \sec \alpha$$

$$= 3365.1 \times 0.8345381 \times 1.1646663$$

$$= 3270''.5$$

$\therefore$ $A = 0°54'30''.5$

As the polaris is west of the meridian,

Azimuth of R.O. = Angle between Polaris and R.O. – Azimuth of the polaris

$$= 28°49'38'' - 0°54'30.5''$$

$$= 27°55'7.5''$$ **Ans.**

Example 1.46. *The polaris (declination 89°03´48´´ ; R.A. 1h 52m 16.8s.) was observed west of meridian at a place of latitude 30°22´15´´. The L.S.T. of observation was 8h 37m 02.3s. Calculate the bearing of the polaris.*

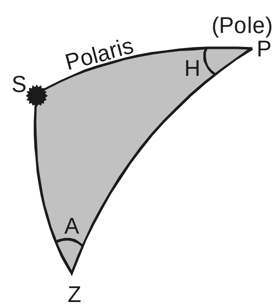


Fig. 1.62.

Solution. (Fig. 1.62.)

Let P be the pole of celestial sphere
S be the Polaris
Z be the zenith of observer.
A be the azimuth

Given : Declination of polaris

$= 89°03'48''$

R.A. of polaris $= 1°52'16''.8$

Latitude of the place $= 30°22'15''$

In spherical triangle PSZ we get

$$PS = 90° - 89°03'48'' = 0°56'12''$$

$$PZ = 90° - 30°22'15'' = 59°37'45''$$

Hour angle $H = \text{L.S.T.} - \text{R.A.}$

$= 8\text{h } 37\text{m } 02.3\text{s} - 1\text{h } 52'\text{m } 16.8\text{s}$

$= 6\text{h } 44\text{m } 45.5\text{s}$

H in arc $= 101°11'22.4''$

Applying cosine formula to ΔPSZ we get

$$\cos ZS = \cos 0°56'12'' \cos 59°37'45'' + \sin 0°56'12'' \sin 59°37'45'' \times \cos 101°11'22''.4$$

$$= 0.999866 \times 0.505595 + 0.0163472 \times 0.862771 \times (-0.194055)$$

$$= 0.50552725 - 0.0027369303 = 0.50279027$$

$$ZS = 59°48'54''.7$$

Applying sine formula to ΔPZS we get

$$\sin A = \frac{\sin 0°56'12'' \times \sin 101°11'22.4''}{\sin 59°48'54.7''}$$

$$= \frac{0.0163472 \times 0.980991}{0.864408}$$

$$\sin A = \frac{0.016036456}{0.864408}$$

$$= 0.018551952$$

$\therefore$ $A = 1°03'46\text{-}8''$

Bearing of Polaris $= 360° - 1°03'46''.8$

$= 358°56'13''.2$ **Ans.**

Example 1.47. *The following observations were made to Polaris to calculate the bearing of a survey line :*

Latitude of the place $= 30°22'15''$

Declination of Polaris $= 89°03'49''\ N$

Field Observations

Object	Face	Horizontal angles			Vertical Angles	
		Reading	General Mean	Angles	Reading	Vertical Angles
R.O.	*L*	*301°02´48´´*	*301°01´45´´*			
	R	*121°00´52´´*	*301°01´50´´*			
Polaris	*L*	*222°03´27´´*	*222°02´08´´*	*78°59´37´´*	*30°25´16´´*	*30°24´38´´*
	R	*42°00´49´´*			*149°36´00´´*	
Polaris	*R*	*42°00´47´´*	*222°02´07´´*	*78°59´38´´*	*140°36´55´´*	*30°22 ’31´´*
	L	*222°03´26´´*			*30°21´57´´*	
R.O.	*L*	*301°02’ 37´´*	*301°01´40´´*			
	R	*121°00´43´´*				

Solution. (Fig. 1.63)

As the vertical angle of the polaris decreases, the star is on the west of the meridian.

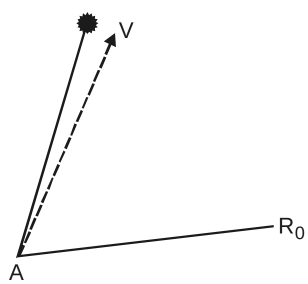


Fig. 1.63.

Let P = celestial pole,

Z = zenith of observer

S = the polaris

Observed altitude of polaris

= 30°24´38´´

Refraction correction 57 cot α

= (–) 1´39´´ = 30°22', 59´´

In astronomical triangle PZS (Fig. 1.64) we get

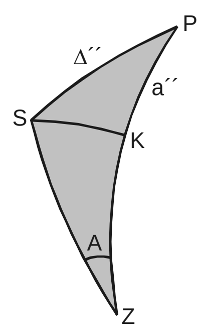


Fig. 1.64.

PZ = 90°– 30°22´15´´= 59°37´45´´

PS = 90°– 89°03´49´´= 0°56’11´´

ZS = 90° – 30°22´59´´= 59°37´01´´

Let A be the azimuth of the polaris

$\cos PS = \cos PZ . \cos SZ + \sin PZ \sin SZ . \cos A.$

$$\cos A = \frac{\cos PS - \cos PZ \;.\; \cos Z}{\sin PZ \;.\; \sin SZ}$$

$$= \frac{\cos 0°56'11'' - \cos 59°37'45'' \cos 59°37'01''}{\sin 59°37'45'' \sin 59°37'01''}$$

$$= \frac{0.9998660 - 0.25571933}{0.862771 \times 0.862663}$$

$$= \frac{0.74414670}{0.74428061}$$

$$= 0.99982009$$

$$A = 1°05'12''.5$$ **Ans.**

∴ Azimuth of the Polaris = 1°05′12″.5 W.

By approximate formula

When the poiaris is at its elongation, the astronomical triangle *SZP* may be treated as a plane triangle.

Let *H* be the hour angle

$$\therefore \quad \cos H = \frac{a}{\Delta} \quad ...(i)$$

Here a = (observed altitude – latitude) = 30°22″ 59″ – 30°22′15″

Δ = north polar distance in seconds Substituting the values in eqn. (i) we get

$$\cos H = \frac{44}{3371} = 0.013052506$$

$$\therefore \quad H = 89°15'7''$$

Now, substituting the values in eqn. (1.70) we get

$$A = \Delta \sin H \sec \alpha$$

$$= \frac{3371 \sin 89°15'7''}{\cos 30°22'59''} = \frac{3371 \times 0.999915}{0.862663}$$

$$= 3907''.33$$

or $$A = 1°05'07.33''$$ **Ans.**

Now clockwise angle between polaris and R.O.

= 78°59′37″

∴ Bearing of R.O. = 78°59′37″ –1°05′12″.5

= 77°54′24″.5 **Ans.**

5. By Hour Angle of the Star

Principle of the method. If we know the hour angle of the star at the time of observation the azimuth of the star may easily be calculated without any knowledge of the altitude of star.

Suitability of the method.; As the altitude of the star is not involved atmospheric refraction does not affect the accuracy of the result.

Field Observations. Following steps are involved :

1. Set up the theodolite over the station of observation and level it accurately.
2. Select a suitable star preferably near the prime-vertical.
3. Bisect the R.O. on the face left and read the horizontal circle.
4. Unclamp the upper plate, swing the telescope in azimuth and bring the star in the field of view.
5. When the star is exactly at the intersection of the cross hair, note down the chronometer time accurately.
6. Note down the horizontal circle reading.
7. Change the face of the transit and bring the star in the field of view.
8. When the star is exactly at the intersection of the cross hair, note down the chronometer time accurately.
9. Note down the horizontal circle reading.
10. Swing the telescope in azimuth, bisect the R.O. and read the horizontal circle.

Calculations.

1. The mean of the chronometer time is the required time of the observation and the difference of the means of horizontal circle readings taken on both faces to R.O. and star, is the required angle between the star and the R.O.
2. Convert the observed chronometer mean time, duly corrected for chronometer error, to the local sidereal time.
3. Calculate the hour angle of the star from the relation, *i.e.* L.S.T. = R.A. ± Hour Angle.

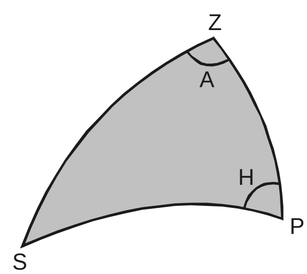


Fig. 1.65.

4. Solve the spherical astronomical triangle *PZS* where colatitude *PZ,* co-declination PS and the hour angle *H,* are known to get the azimuth of the star (Fig. 1.65).

Let

$$PZ = (90 - \theta) = \lambda = \text{co-latitude}$$

$$PS = (90 - \delta) = \Delta = \text{co-declination}$$

$$\angle ZPS = H = \text{the hour angle}$$

then, $\tan A = \tan H \,.\, \cos B \,.\, \text{cosec}\,(B - \theta)$

where $B = \tan^{-1} (\tan \delta \,.\, \sec H)$

Disadvantages of the method. As separate observations for time are necessary, this method is generally not preferred to.

Note. The following points may be noted :

(*i*) *The error in time has very little effect on the azimuth if star is observed on the prime vertical.*

(*ii*) *The motion of the star in between two observations of time is not linear but circular. Hence, a correction for the curvature of the path is applied to the face left 'and face right observations.*

(*iii*) *The correction (ΔA in seconds) to be applied to the azimuth may be calculated by the formula.*

$$\Delta A'' = \frac{1}{8} \sin A \cos \theta \sec^2 a \,(\cos a \sin \delta - 2 \cos A \cos \theta)$$

$$\Delta t^2 \times \sin 1'' \qquad ...(1.76)$$

where Δt = the difference of time between the face right and face left observations.

Example 1.48. *At a place (latitude 34°30', longitude 82°30′E), the following observations were taken on an eastern star.*

Observed clockwise angle between R.O. and the star = 125°36′15″

R.A. of the star	:	*12h 17m 13.74s*
Declination of the star	:	*20°06′48″.4*
G.M.T. of observation	:	*16h 32m 26.5s*
G.M.T. of G.M.M.	:	*11h 32m 34.2s*

Calculate the true bearing of the reference object.

Solution.

In this case no altitude of the star is observed, hence azimuth of the star may be calculated from the hour angle of the star at the time of observation.

Calculation of the hour angle

G.S.T. of G.M.M. = 11h 32m 34.2s (Given)

Since the place of observation is east of Greenwich, a retardation @9.8565 seconds per hour for the longitude in time, is applied to G.S.T. of G.M.M. for calculating the L.S.T. of L.M.M.

Longitude = 82°30′*E* = 5h 30m *E*

5h × 9.8565 = 49.28 seconds

30m × 3.1642 = 4.93 seconds

Total = 54.21 seconds

∴ L.S.T. of L.M.M. = G.S.T. of G.M.M. – Retardation

= 11h 32m 34.2s – 54.2s

= 11h 31m 40s

But, L.M.M. of observation = G.M.M. of observation + Longitude in time

= 16h 32m 26.5s + 5h 30m

∴ L.M.T. of observation = 22h 02m 26.5s

To convert M.T. interval to S.I. an acceleration @ 9.8565 seconds per mean time interval is added.

22h × 9.8565 = 216.8430 seconds

2m × 0.1642 = 0.3284 seconds

26.5 × 0.0027 = 0.0716 seconds

Total = 217.2430 seconds = 3m 37.24s

S.I. = Mean time interval + Acceleration

= 22h 2m 26.5s + 3m 37.24s

= 22h 06m 03.74s

∴ L.S.T. of observation = L.S.T. of L.M.M.+ S.T.

= 11h 31m 40.0s + 22h 6m 03.74s

= 33h 37m 43.74s

Subtract R.A. of the star = 12h 17m 13.74s

Hour angle of the star = 21h 20m 30s

= 320°07´30´´ (westernly)

∴ Eastern hour angle of the star *(i.e.* smallest arc of the hour angle)

= 360°- 320°07´30´´

or H = 39°52´30´´

∴ The value of the hour angle *(H)* = 39°52´30´´

Calculation of the azimuth of the star

We know that

$$\tan A = \tan H \,.\, \cos B \,.\, \text{cosec}\,(B - \theta)$$

where $\tan B = \tan \theta \,.\, \sec H.$

= tan 20°06´48´´.4 sec 39°52´30´´

= 0.3662141 × 1.3030249

or $\tan B = 0.477186$

$B = 25°30'35''$

and $B - \theta = 25°30'35'' - 34°30'00'' = -8°59'35''$

$\tan A = \tan 39°52'30'' \times \cos 25°30'35'' \times \operatorname{cosec}(-8°59'25'')$

$= 0.8353890 \times 0.9025120 \times 6.3993037$

$= 4.8247459$

or $A = 78°17'25''$

Azimuth of the R.0. $= 125°36'15'' - 78°17'25'' = 47°18'50''$

∴ True bearing of R.O. $= 360° - 47°18'50''$

$= 312°41'10''$. **Ans.**

6. By observation on ex-meridian altitude of a star. *Suitability of the method.* In lower latitudes, polaris attains lower altitude and as such the refraction becomes uncertain. This is why polaris is not observed at lower latitude. To compensate the effect of refraction both east and west stars are observed.

Position of stars. As the refraction correction is almost uncertain for stars very near the horizon, the stars are observed only when these attain altitude at least 30°. Again, a star when observed should move more in altitude and less in azimuth. Such conditions are ahieved only when the star is on the *prime vertical.*

Selection of pair of stars. We know that when a star is on the prime vertical, its azimuth is 90°. Hence, by solving the right angled triangle at zenith, we get $\sin \delta = \sin \theta \sin \alpha$. For any place latitude is constant and for a particular day, declination of the star is also constant. To avoid uncertain refraction, altitude should not be less than 30°. Hence, the selected star should be such that its declination is equal to $\sin^{-1} (\sin \theta \sin 30°)$.

Field observations. The following steps are involved.

1. Set up the transit over the ground station.
2. Clamping both the plates to zero, sight R.O. on face left.
3. Swing the telescope and bisect the star.
4. Note down horizontal and vertical angles.
5. Change face and bisect the star again.
6. Swing the telescope and bisect R.O.
7. Note down the horizontal and vertical angles to R.O.
8. Take a number of sets in the same manner with different zeros.

Computation. Let *Z, P* and *S* represent the zenith, the pole and the star at an altitude (α). (Fig. 1.66).

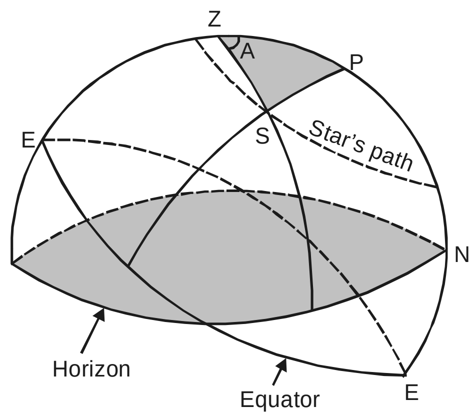


Fig. 1.66.

In the spherical triangle *ZPS*

$$ZP = \text{co-latitude} = 90° - \theta = \lambda$$

$$PS = \text{co-declination} = 90° - \delta = \Delta$$

$$ZS = \text{co-altitude} = 90° - \alpha = Z$$

The azimuth *A* of the star may be calculated by one of the following formulae.

$$\sin \tfrac{1}{2}A = \sqrt{\frac{\sin(S-Z)\sin(S-\lambda)}{\sin Z \,.\, \sin\lambda}}$$

$$\cos \tfrac{1}{2}A = \sqrt{\frac{\sin S.\sin(S-\Delta)}{\sin Z \,.\, \sin\lambda}}$$

$$\tan \tfrac{1}{2}A = \sqrt{\frac{\sin(S-Z)\sin(S-\lambda)}{\sin S \,.\, \sin(S-\Delta)}}$$

where $2S = (\lambda + \Delta + Z)$

Effect of an error in altitude on the azimuth.

We know that

$$\cos Z = \frac{\cos PS - \cos PZ \cos ZS}{\sin PZ \sin ZS}$$

$$= \frac{\cos(90° - \delta) - \cos(90° - \theta)\cos(90° - \alpha)}{\sin(90° - \theta)\sin(90° - \alpha)}$$

or $$\cos Z = \frac{\cos\delta - \sin\theta \,.\, \sin\alpha}{\cos\theta \,.\, \cos\alpha} \qquad \text{...(1.80)}$$

Differentiating eqn. (1.80) with respect to α, we get

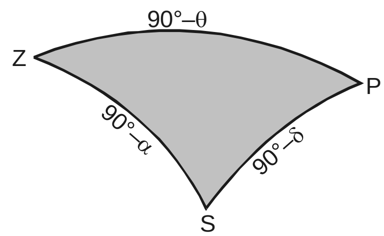


Fig. 1.67.

$$-\sin Z dZ = \left(\frac{\sin\delta\sin\alpha}{\cos\theta\cos^2\alpha} - \frac{\sin\theta}{\cos\theta} \times \frac{1}{\cos^2\alpha}\right) d\alpha$$

$$= \left(\frac{\sin\delta\sin\alpha - \sin\theta}{\cos\alpha\cos^2\alpha}\right) d\alpha$$

or $$dZ = \left(\frac{\sin\theta - \sin\delta\sin\alpha}{\cos\theta\cos^2\alpha\sin Z}\right) d\alpha \qquad \text{...(1.81)}$$

Again, applying cosine formula to Δ*PZS* we get

$$\cos = \frac{\cos PZ - \cos PS\cos ZS}{\sin PS\sin ZS}$$

$$= \frac{\cos(90° - \theta) - \cos(90° - \delta)\cos(90° - \alpha)}{\sin(90° - \delta)\sin(90° - \alpha)}$$

$$= \frac{\sin\theta - \sin\delta\sin\alpha}{\cos\delta\cos\alpha}$$

or $$\cos S \cos\delta \cos\alpha = \sin\theta - \sin\delta\sin\alpha \qquad \text{...(1.82)}$$

Substituting the value of sin θ – sin δ sin α from eqn. (1.82) in eqn. (1.81), we get

$$dZ = \frac{\cos S\cos\delta\cos\alpha}{\cos\theta\cos^2\alpha\sin Z} \,.\, d\alpha.$$

Applying sine rule to Δ*PZS* we get

$$\frac{\sin Z}{\sin(90° - \delta)} = \frac{\sin S}{\sin(90° - \theta)}$$

or $$\sin Z = \frac{\sin S\cos\delta}{\cos\theta}$$

$$\therefore \qquad dZ = \frac{\cos S \,.\, \cos\delta\cos\alpha \,.\, \cos\theta}{\cos\theta\cos^2\alpha\sin S\cos S}\,d\alpha$$

or
$$dZ = \cot S \sec\alpha\, d\alpha \qquad ...(1.83)$$

i.e. For minimum error in azimuth, the altitude should be barely minimum and S should be nearly 90°or star should be near elongation.

Effect of an error in latitude on azimuth.

We know that

$$\cos Z = \frac{\cos PS - \cos SZ\cos PZ}{\sin SZ\sin PZ}$$

$$= \frac{\cos(90° - \delta) - \cos(90° - \alpha)\cos(90° - \theta)}{\sin(90° - \alpha)\sin(90° - \theta)}$$

or
$$\cos Z = \frac{\sin\delta - \sin\alpha\sin\theta}{\cos\alpha\cos\theta} \qquad ...(1.84)$$

Differentiating eqn. (1.81) w.r.t. θ we get

$$-\sin Z dZ = \left(\frac{\sin\delta}{\cos\alpha}.\frac{\sin\theta}{\cos^2\theta} - \frac{\sin\alpha}{\cos\alpha\cos^2\theta}\right)d\theta$$

or
$$dZ = \frac{\sin\alpha - \sin\delta\sin\theta}{\sin Z\cos\alpha\cos^2\theta}\,d\theta \qquad ...(1.85)$$

Again, applying cosine formula to ΔPZS we get

$$\cos P = \frac{\cos ZS - \cos PS\cos PZ}{\sin PS\sin PZ}$$

$$= \frac{\cos(90° - \alpha) - \cos(90° - \delta)\cos(90° - \theta)}{\sin(90° - \delta)\sin(90° - \theta)}$$

or
$$\cos P = \frac{\sin\alpha - \sin\delta\sin\theta}{\cos\delta\cos\theta}$$

$$\therefore \quad \sin\alpha - \sin\delta\sin\theta = \cos P\cos\delta\cos\theta$$

Substituting the value in eqn. (1.85) we get

$$dZ = \frac{\cos P\cos\delta\cos\theta}{\sin Z\cos\alpha\cos\theta}\,d\theta \qquad ...(1.86)$$

But
$$\frac{\sin Z}{\sin P} = \frac{\sin PS}{\sin ZS} = \frac{\sin(90° - \delta)}{\sin(90° - \alpha)} = \frac{\cos\delta}{\cos\alpha}$$

$$\therefore \qquad \sin Z = \frac{\sin P\cos\delta}{\cos\alpha}$$

Substituting the value of sin Z in eqn. (1.85) we get

$$dZ = \frac{\cos P \cos \delta \cos \theta \cos^2 \alpha}{\sin P \cos \delta \cos \alpha \cos \theta}\ d\theta$$

or $$dZ = \cot P \sec \theta \,.\, d\theta. \qquad ...(1.87)$$

i.e. the error in azimuth will be minimum if the hour angle of the celestial body is 90°or 6 hours. Also, the error in azimuth will be more in higher latitude.

Effect of an error in declination on the azimuth.

We know that

$$\cos Z = \frac{\cos PS \cos PZ \cos ZS}{\sin PZ \sin ZS}$$

$$= \frac{\cos(90° - \delta) - \cos(90° - \theta)\cos(90° - \alpha)}{\sin(90° - \theta)\sin(90° - \alpha)}$$

$\therefore$ $$\cos Z = \frac{\sin \delta - \sin \theta \sin \alpha}{\cos \theta \cos \alpha} \qquad ...(1.88)$$

Differentiating eqn. (1.88) with respect to δ we get

$$-\sin Z dZ = \frac{\cos \delta}{\cos \theta \cos \alpha}\ d\delta$$

or $$dZ = \frac{-\cos \delta\ d\delta}{\sin Z \cos \theta \cos \alpha} d\delta$$

$$\sin Z = \frac{\sin S \cos \delta}{\cos \theta}$$

But $$Z = \frac{\cos S \cos \theta}{\sin S \cos \delta \cos \theta \cos \alpha} d\delta$$

or $$dZ = -\text{cosec}\ S \sec \alpha \qquad ...(1.89)$$

i.e. the error in azimuth will be minimum if δ is 90°and also it will be more for higher altitudes.

1.47. DETERMINATION OF AZIMUTH BY MAKING OBSERVATIONS ON SUN

The following are some of the principal methods used for determining the azimuth from the sun.

1. By observations on the sun at equal altitudes.
2. By hour angle of the sun.
3. By ex-meridian observations on the sun.

1. By observations on sun at equal altitudes. The principle of the method and sequence of the observations are the same as that of a star at equal altitudes. As the sun's centre cannot be bisected, observations are made on either right hand limb or left hand limb of the sun with the telescope normal and inverted, before meridian and after meridian respectively. During this interval of observations, the sun's declination changes considerably. Hence, the mean of the horizontal angles needs be corrected to determine the azimuth of the lines accurately by the following formula :

$$C = \tfrac{1}{2}(\delta E - \delta m) \sec \theta \,.\, \text{cosec}\, t \qquad ...(1.90)$$

where C = Angular correction to the mean of the horizontal angles.

δE = Sun's average declination for after-noon observations.

δm = Sun's average declination for before-noon observations.

θ = Latitude of the place of observation.

t = Half of the time interval between two observations.

For detailed procedure refer to article 1.41.

2. By hour angle of the sun. The principle of the method and the sequence of the observations are the same as in the case of stars.

For detailed procedure, refer to article 1.41.

3. By ex-meridian observations of the sun. As the declination of the sun changes rapidly, an exact time of observation is required. To the observed altitudes of the sun, refraction correction and also parallax correction are applied to get accurate altitude.

To get the required altitude and the horizontal angle to the sun's centre, the cross hairs are set tangential to the two limbs simultaneously. The opposite limbs are then observed by changing the face as shown in Fig. 1.68.

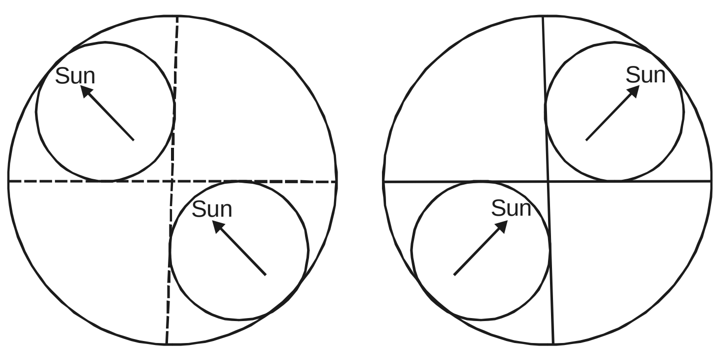
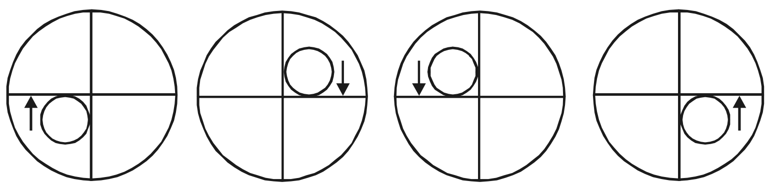


Fig. 1.68.

Field procedure for afternoon observations

Following steps are involved :

1. Set the theodolite over the ground station mark and level it accurately.
2. Unclamp the lower plate, swing the telescope and bisect the R.O. using the lower tangent screw. Note down the reading.
3. Swing the telescope and bring the sun into the lower left quadrant of the object glass. The sun is moving upward [Fig. 1.69(*a*)]. Clamp both the plates.

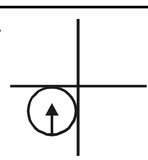


(*a*) (*b*) (*c*) (*d*)

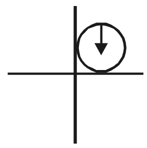
Fig. 1.69.

4. Keep the vertical wire on the apparent right limb of the sun by using the tangent screw of the upper plate.
5. When the upper limb touches the horizontal wire, remove hand from the tangent screw and note down the horizontal and vertical readings.
6. Change face and intersect the sun on the upper right quadrant Fig. 1.69(*b*).
7. Keep the vertical wire on the apparent left limb of the sun by moving the tangent screw of the upper plate.
8. When the apparent lower limb touches the horizontal wire, remove hand from the tangent screw and note down the vetical and horizontal readings.
9. Swing the telescope to the R.M. on face right and bisect it accurately. Note down the reading.

One set of observations will therefore be as under:

Point sighted	*Position of the Sun*	*Readings*
R.M.		Horizontal reading to R.M. on face left
Sun 	Apparent right and upper limbs of the sun touch vertical and horizontal wires	Horizontal and vertical readings to the sun on face left.

Change face and swing the transit to get the sun in the field of view.

Sun		Apparent left and lower limbs of the sun touch vertical and horizontal wires	Horizontal and vertical readings to the sun on face right
R.M.			Horizontal reading to R.M. on face right.

Example 1.49. *At a place (latitude 30°22′15′′ N, Longitude 77°50′00′′ E), the following observations were taken on the sun at 03.10 P.M. on 18th March, 1980.*

Observed angle between the R.M. and the sun

= 175°14′15′′ (clockwise)

Declination of the sun at 03 10 P.M. = 0°53′06′′S

Observed corrected altitude of the sun = 40°11′38′′

Calculate the true bearing of the R.M.

Solution. We know that in a spherical triangle

$$\tan A/2 = \sqrt{\frac{\sin(S-\lambda)\sin(S-z)}{\sin S \,.\, \sin(S-\Delta)}}$$

Here

$$z = (90° - \alpha) = 49°48′22′′$$

$$\lambda = (90° - \theta) = 59°37′45′′$$

$$\Delta = (90° - \delta) = 90°53′06′′$$

$$\text{Sum} = 2S = 200°19′13′′$$

$$S = 100°09′37′′$$

$$S - A = 9°16′31′′$$

$$S - \lambda = 40°31′52′′$$

$$S - z = 50°21′15′′$$

Substituting the values in eqn. (*i*) we get

$$\tan A/2 = \sqrt{\frac{\sin 40°31′52′′ \sin 50°21′15′′}{\sin 100°09′37′′ \sin 9°16′31′′}}$$

$$A/2 = 0.24943575$$

$$A/2 = 60°37′02′′$$

$$A = 121°14′04′′$$

Bearing of the sun $= 360° - 121°14′04′′$

$= 238°45′56′′$ sun being in west

Bearing of the R.M. = Bearing of the Sun – angle between R.M. and sun

= 238°45′56″ – 175°14′15″

= 63°31′41″ **Ans.**

Example 1.50. *Find the bearing of the line ML from the following ex-meridian observations to the sun.*

S.N.	*Object*	*Face*	*Horizontal Circle Verniers*		*Vertical Circle Verniers*	
			A	*B*	*C*	*D*
1	*R.O.*	*L*	*30°33′19″*	*33′17″*		
		R	*210°33′04″*	*33′12″*		
2	*Sun*	*R*	*25°52′15″*	*52′10″*	*40°42′12″*	*42′22″*
		L	*205°42′52″*	*43′15″*	*140′17′15″*	*17′12″*
3	*Sun*	*L*	*206°23′38″*	*23′58″*	*140°51′00″*	*50′42″*
		R	*27°41′50″*	*41′30″*	*39°08′00″*	*08′12″*
4	*R.O.*	*R*	*210°33′28″*	*33′32″*		
		L	*30°33′40″*	*33′40″*		

Latitude of station M

= 30°22′15″N

Longitude of the station M

= 77°50′00″ E

Declination of the Sun at G.M.N.

= 00°50′46″ S

(decreasing 1′per hour on 18th March 1976) L.M.T. of two observations= 3h 10m 0s P.M.

= 3h 16m 0s P.M.

P.M. Correction for horizontal parallax

= 10.8″

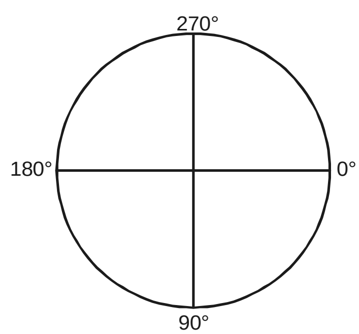


Fig. 1.70.

Correction for refraction = 51.5″ cot (apparent altitude)

Mode of graduations of vertical circle is as shown in Fig. 1.69.

Solution.

Mean horizontal reading to R.O.

$= \frac{1}{2}(30°33'13'' + 30°33'35'') = 30°33'24''$

Horizontal angle between R.O. and the Sun at first position

$= \frac{1}{2}(205°52'13'' + 205°43'04'') - 30°33'24''$

$= 205°47'39''-30°33'24'' = 175°14'15''$

Horizontal angle between R.O. and the Sun at second position

$= \frac{1}{2}(206°23'48'' + 207°41'40'') - 30°33'24''$

$= 207°02'44'' - 30°33'24'' = 176°29'20''$

Vertical angle of the Sun at first position

$= \frac{1}{2}[40°42'17'' + 180° - 140°17'13''] = 40°12'32''$

∴ Apparent zenith distance V_1

$= 90° - 40°12'32'' = 49°47'28''$

Vertical angle of the sun at second position

$= \frac{1}{2}[(180° - (140°50'51'') + 39°08'06'']$

$= 39°08'38''$

∴ Apparent zenith distance V_1

$= 90° - 39°08'38'' = 50°51'22''$

Refraction correction for V_2

$= -51.5'' \tan 49°47'28'' = 0°01'01''$

Refraction correction for V_2

$= -51.5'' \tan 50°51'22'' = 0°01'03''$

Parallax correction for V_2

$= 10.8'' \cos 40°12'32'' = 0°00'08''$

Parallax correction for V_2

$= 10.8'' \cos 39°00'38'' = 0°00'08''$

∴ Corrected zenith distance

1st position		Second position
= + 49°47′28″	+	50°51′22″
+ 0°01′01″	+	0°01′03″
– 0°00′08″	–	0°00′08″
= 49°48′21″	=	50°52′17″

Calculation of declination

	1st position	2nd position
Local Standard Time	= 15h 10m 0s	15h 16m 0s
Deduct East Longitude	= 5h 30m 0s	5h 30m 0s
∴ G.S.T. of observation	= 9h 40m 0s	9h 46m 0s
Sun's declination G.S.T.	= 0°50´46´´	0°50´46´´
Variation @ 60´´ per hour for interval of G.S.T. and G.N.T.	= + 0°02´20´´	0°02´14´´
Declination at the time of observation	= 0°53'06´´S	0°53'00´´ S
Zenith distance Z	= 49°48´21´´	50°52´17´´
Co-latitude λ	= 59°37´45´´	59°37´45´´
Co-declination Δ	= 90°53´06´´	90°53´00´´
	$2S$ = 200°19'12´´	201°23'02´´
	S= 100°09´36´´	100°41'31´´
or	$S–\Delta$ = 9°16´30´´	9°48´31´´
	S–λ = 40°31'51´´	41°03'46´´
	$S–Z$ = 50°21´15´´	49°49´15´´

Substituting these values in the formula

$$\tan A/2 = \sqrt{\frac{\sin(S-Z)\sin(S-\lambda)}{\sin S \sin(S-\Delta)}}\text{, we get}$$

$$\tan A/2 = \sqrt{\frac{\sin 50°21'15'' \sin 40°31'51''}{\sin 100°09'36'' \sin 9°16'30''}}$$

$$= \sqrt{\frac{0.7700031 \times 0.6498608}{0.9843181 \times 0.16111779}}$$

or $A/2 = 60°37'03''$

$A = 121°14'06''$ West

$360° - A = 238°45'54''$

Bearing of the line = Bearing of the Sun – Horizontal angle

$= 238°45'54'' - 175°14'15'' = 63°31'39''$

Similarly, from second observation, we get

$$\tan A/2 = \sqrt{\frac{\sin 49°49'15'' \sin 40°03'46''}{\sin 100°41'31'' \sin 9°48'31''}}$$

$$= \sqrt{\frac{0.7640244 \times 0.6568855}{0.98433181 \times 0.1705582}}$$

$A/2 = 59°59'32''$

$A = 119°59'04''$ West

$\therefore \quad 360° - A = 240°00'56''$

Bearing of line $= 240°00'56'' - 176°29'20''$

$= 63°31'36''$.

Mean bearing of the line $= \frac{1}{2}\,(63°31'39'' + 63°31'36'')$

$= 63°31'37\text{-}5''$ **Ans.**

Example 1.51. *The following notes are recorded at 4 P.M. on Jan. 14, while determining the azimuth of a reference point P from a station A of a triangulation survey. The opposite faces of the theodolite were used in observing the upper and lower limbs of the sun :*

Latitude of the station A	:	*41°40´40´´ N*
True altitude of the Sun	:	*34°32'50´´*
Declination of the Sun at 4 P.M.	:	*23°17'18´´*

The mean observed horizontal angle of the sun, right of the reference point was 202°26´43´´. Find the azimuth of the reference point.

Solution. (Fig. 1.71)

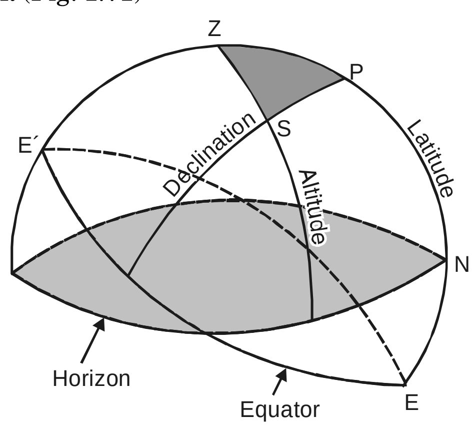


Fig. 1.71.

Given :

Latitude of station A,

$\theta = 41°40'40''$ N

Altitude of the sun, $\alpha = 34°32'50''$

Declination of the Sun at the time of observation, *i.e.* 4 *P.M.*

$$\delta = 23°17'18''$$

In the spherical triangle *PZS,* we have

Co-latitude $\lambda = 90° - 0 = 48°19'20''$

Co-altitude $Z = 90° - \alpha = 55°27'10''$

Co-declination $\Delta = 90° - 5 = 66°42'42''$

$$2S = 170°29'12''$$

$$S = 85°14'36''$$

and $S - \lambda = 36°55'16''$

$$S - Z = 29°47'26''$$

$$S - \Delta = 18°31'54''$$

Substituting the values of $(S - \lambda)$, $(S - Z)$, $(S - \Delta)$ and S in the following equation.

$$\tan A/2 = \sqrt{\frac{\sin(S - Z)\sin(S - \lambda)}{\sin S \,.\, \sin(S - \Delta)}}$$

$$\tan A/2 = \sqrt{\frac{\sin 29°47'26'' \sin 36°55'16''}{\sin 85°14'36'' \sin 18°31'54''}}$$

$$= \sqrt{\frac{0.4168309 \times 0.6007148}{0.9965559 \times 0.3178287}}$$

$$= \sqrt{0.9707102}$$

$$\tan A/2 = 0.9707102$$

$$A/2 = 44°08'55''$$

or $A = 88°17'50''$.

Now, Azimuth of the Sun measured anticlockwise

$$= 88°17'50''$$

Angle between R.O. and Sun

$$= 202°26'43''$$

$\therefore$ Azimuth of the R.O. measured clockwise

$$= 290°44'33''$$

$\therefore$ Azimuth of the R.O. measured clockwise

$$= 360° - 290°44'33''$$

$$= 69°15'27''$$ **Ans.**

Example 1.48. *At a place A (latitude 52°30´20´´), the sun was observed on the western sky. The following data were available :*

Mean corrected altitude = 33°35´10´´

Declination of the sun at the time of observation = + 22°05´36´´

What was the azimuth of the sun ?

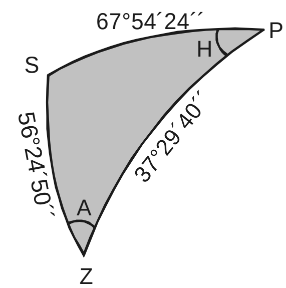


Fig. 1.72.

Solution. (Fig. 1.72).

Let Z be the zenith of observer
P be the celestial pole
S be the Sun's position in western sky.

Given :

Altitude = 33°35´10´´

Declination = + 22°05´36´´

Latitude = 52°30´20´´

ZS = coaltitude = 90° – 33°35´10´´
= 56°24´50´´

PS = Codeclination
= 90° – 22°05´36´´ = 67°54´24´´

PZ = Colatitude = 90° – 52°30´20´´
= 37°29´40´´

Applying cosine rule to the astronomical triangle PSZ, we get

$$\cos A = \frac{\cos PS - \cos ZP \cos ZS}{\sin ZP \sin ZS} \qquad ...(i)$$

Substituting the values in eqn. (*i*), we get

$$\cos A = \frac{\cos 67°54'24'' - \cos 37°29'40'' \cos 56°24'50''}{\sin 37°29'40'' \cos 56°24'50''}$$

$$= \frac{0.376177 - 0.793412 \times 0.553190}{0.608685 \times 0.8333055}$$

$$= \frac{0.376177 - 0.43890758}{0.50706808}$$

or $\cos A = -0.12383068$

As value sine of A is negative, the angle A lies between 90° and 180°.

$\therefore$ $\cos(180° - A) = 0.12383068$

or $180° - A = 82°53'12''.48$

$\therefore$ Azimuth of the sun $= 180° - 82°53'12''.5$

$= 97°06'47''.5$ **Ans.**

Example 1.53. *The greatest azimuth attained by a circumpola star is 45°. If the latitude of the observer's place is 45°N, prove that star's declination is 60°.*

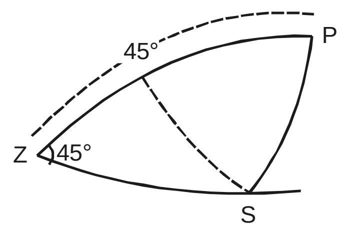


Fig. 1.73.

Solution. (Fig. 1.73).

ZP = Colatitude $= 90° - 45° = 45°$

$\therefore$ $\angle ZSP$ = right angle, star being at elongation

$\angle PZS = 45°$ azimuth (given)

Applying sine rule to the spherical triangle PZS we get

$$\frac{\sin PS}{\sin 45°} = \frac{\sin PZ}{\sin 90°}$$

or $$\sin PS = \frac{\sin 45° \times \sin 45°}{\sin 90°} = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$$

or $PS = 30°$ = co-declination

$\therefore$ Declination $= 90° - 30° = 60°$.

Example 1.54. *In determining the azimuth of a line in Calcutta observation was made to the sun and the following data were available.*

Mean horizontal angle of the sun

Clockwise angle from the reference line $= 47°07'30''$

Mean observed altitude of the sun $= 10°03'00''$

Declination of the sun at the time of observation

$= 180°30'00''$ *South*

Horizontal parallax $= 8''.9$

Refraction correction $= 57'' \cot \alpha$

Latitude of Calcutta $= 22°30'$ *North*

The sun was observed on the west sky

Calculate the azimuth of the line.

Solution. (Fig. 1.74.)

Let P represents the celestial pole

Z represents the zenith

S represents the sun's position

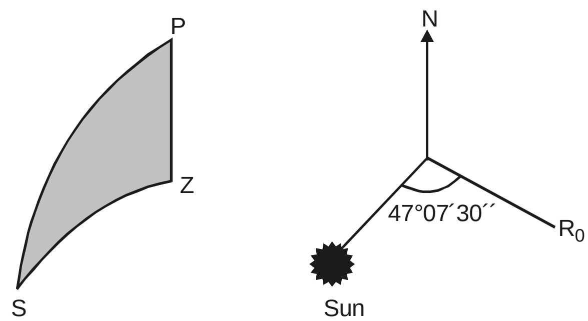


Fig. 1.74.

Here observed altitude	α =	10°03´00´´
Horizontal parallax correction	=	+ 8´´.9
Corrected angle	=	10°03´08´´.9
Refraction correction (–) (57´´ cot 10°03')	=	05´21´´.6
Corrected angle	=	09°57´47´´-3

In spherical triangle *PZS,* we get

PS = Co-declination = 90°– (– 18°30')
= 108°30´

ZS = Co-altitude = 90°– 09°57´47´´.3
= 80°02´12´´.7

PZ = Co-latitude = 90°– 22°30´
= 67°30´

Let *A* be the azimuth of the sun

Applying cosine formula, we get

$$\cos A = \frac{\cos PS - \cos SZ \cos PZ}{\sin SZ \sin PZ}$$

$$= \frac{\cos 108°30´ - \cos 80°02´12´´.7 \cos 67°30´}{\sin 80°02´12´´.7 \sin 67°30´}$$

$$= \frac{-0.17305 - 0.173015 \times 0.382684}{0.984919 \times 0.923879}$$

$$= \frac{0.317305 - 0.06621}{0.90994598}$$

$$\cos A = \frac{0.383515}{0.90994598} = -0.42147011$$

$$A = 144°55'37''.2 \text{ westernly}$$

The azimuth of the line

$$= 114°55'37''.2 + 47°07'30''$$
$$= 162°03'07''.2 \text{ westernly.}$$ **Ans.**

Example 1.55. A *star was observed at western elongation at a station A in latitude 54°30′N and longitude 52°30′W. The declina-tion of the star was 62°12′21′′ N. The mean observed horizontal angle between the referring object P and the star was 65°18′42′′. Find :*

(i) Hour angle of the star ;

(ii) The altitude of the star at elongation,

(iii) The azimuth of the line AP.

Solution. (Fig. 1.75)

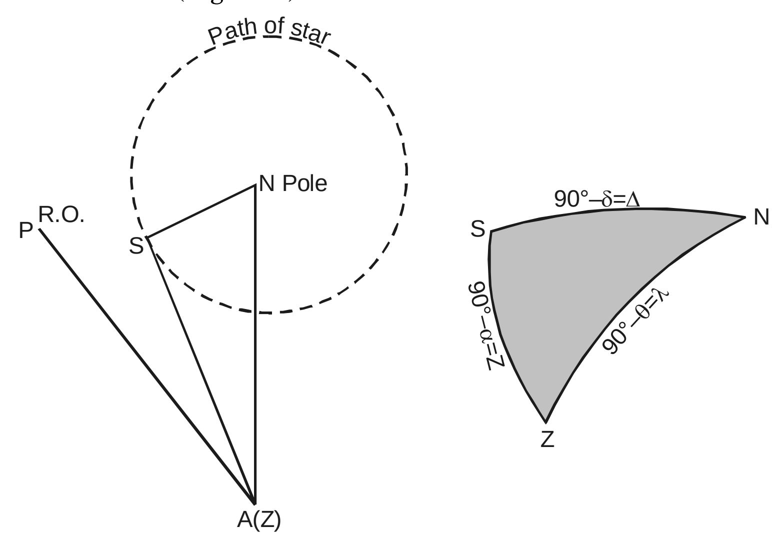


Fig. 1.75

Let, S be the star's position

N be the North pole ; Z be the zenith of A.

The star being at western elongation ΔNSZ is a right angled triangle.

From Napier's rule for circular parts, we get

$$\sin \alpha = \frac{\sin \theta}{\sin \delta}$$

$$= \frac{\sin 54°30'}{\sin 62°12'21''} = \frac{0.814116}{0.884628}$$

$= 0.92029191$

or $\alpha = 66°58'7''.68$

∴ The altitude of star at elongation

$= 66°58'7''.68$ **Ans.**

Again, $$\sin A = \frac{\cos\delta}{\cos\theta} = \frac{\cos 62°12'21''}{\cos 54°30'}$$

$$= \frac{0.466297}{0.580703} = 0.80298706$$

or Azimuth of sun $A = 53°24'58''.68\ W$

∴ Azimuth of line AP = Azimuth of the star + Horizontal angle between the line and the star.

$= 53°24'58''.68 + 65°18'42''$

$= 118°43'40''.68$

∴ Azimuth of line AP $= 118°43'40''.68\ W$ **Ans.**

Again, $$\cos H = \frac{\tan\theta}{\tan\delta} = \frac{\tan 54°30'}{\tan 62°12'21''}$$

$$= \frac{1.40195}{1.89714} = 0.73898078$$

or Hour angle $H = 42°21'19''.08$ **Ans.**

Example 1.56. *At a place of 39°N, the declination and hour angle of a star were 19° and 42° respectively. Find the altitude and azimuth of the star.*

Solution.

In the astronomical triangle PZS, we have

Colatitude $PZ = 90° - 39° = 51°$

Codeclination $PS = 90° - 19° = 71°$

Hour angle $H = 42°$

Applying cosine formula (1.5) to ΔPZS, we get

$$\cos SZ = \cos PS \,.\, \cos PZ + \sin PS \,.\, \sin PZ.\ \cos H.$$

or $$\cos(90° - \alpha) = \cos 71° \,.\, \cos 51° + \sin 71° \sin 51° \cos 42°$$

$= 0.325568 \times 0.629321 + 0.9455180 \times 0.777146 \times 0.743145$

$= 0.20488677 + 0.54606705$

$$= 0.75095382$$

$$90° - \alpha = 41°19'36''.8$$

$$\text{Altitude } (a) = 90° - 41°19'36''.8$$

$$= 48°40'23''.2. \quad \textbf{Ans.}$$

Again, applying sine rule to triangle *PZS,* we have

$$\sin A = \frac{\sin PS \,.\, \sin H}{\sin SZ} = \frac{\sin 71° \times \sin 42°}{\sin 41°12'36''.8}$$

$$= \frac{0.945518 \times 0.669131}{0.660354}$$

$$\sin A = 0.9580852$$

or Azimuth $(A) = 73°21'09''.$ **Ans.**

1.48. DETERMINATION OF LATITUDE

Knowledge of the latitude at different places on the surface of the earth, is very necessary for the land surveyors and civil engineers. The most practical and generally accepted methods for determining the latitude of any place are as under :

1. By meridian altitude of a star.
2. By equal meridian altitudes of two stars on either side of zenith.
3. By meridian altitude of a circumpolar star at its upper or lower culminations.
4. By ex-meridian observations of a star.
5. By altitude of the star on the prime vertical.

1. **Latitude by meridian altitude of a star**

Principle of the method. This method is based on the fact that the latitude of any place is equal to the altitude of the pole star at that place.

Field Procedure. Following steps are involved :

1. Determine the meridian of the place and fix two pegs at considerable distance apart to define it.
2. Set up the theodolite on the north peg if the star is in south direction and on the south peg if the star is in north direction.
3. Bisect the distant peg and clamp both the plates.
4. Rotate the telescope in the vertical plane till the star is bisected on the horizontal wire.

5. Read both the verniers of vertical scale and take the mean to get apparent meridian altitude of the star.
6. Change the face and repeat the steps (1) to (5) to get apparent meridian altitude on face right.
7. The mean of the two altitude observations of both faces, is the required altitude.

The observed altitude should now be corrected for the refraction as discussed earlier.

Calculations :

Given data :

(*i*) declination of the star.

(*ii*) meridian altitude of the star.

Depending upon the position of the star in the celestial sphere four cases may arise.

Let S_1, S_2, S_3 and S_4 be the positions of the star (Fig. 1.76).

NS is the north south direction and Z and *P* are the zenith and pole respectively

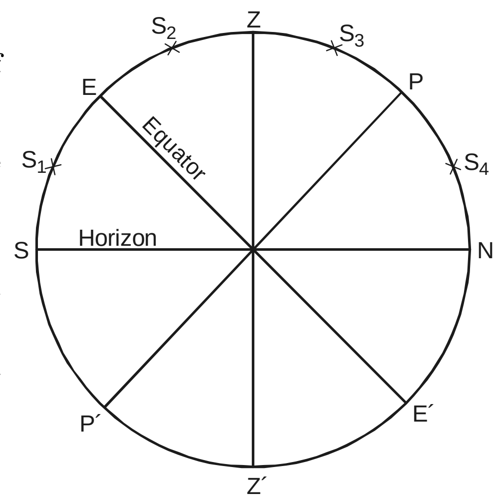


Fig. 1.76.

Case I. Star (S_1) between the horizon and the equator.

The latitude $\theta = NP$

$\therefore$ Co-latitude $(90° - \theta) = ZP$

But $EZ = 90° - ZP$

$EZ = \theta$ = latitude

Angle, $SS_1 = \alpha_1$ = meridian-altitude of the star S_1

$ZS_1 = 90° - \alpha_1 = z$

= zenith distance of the star

Angle, $ES_1 = \delta_1$ = declination of the star (south)

$EZ = ZS_1 . ES_1$

$EZ = (90° - \alpha_1) - \delta_1$

or $$\theta = z_1 - \delta_1$$

$\therefore$ Latitude = Zenith distance – Declination.

Case II. Star (S_2) between the equator and the zenith.

Here $SS_2 = \alpha_2$ meridian altitude of the star

$ZS_2 = (90° - \alpha_2) = z_2$ = zenith distance

$ES_2 = \delta_2$ = declination of the star (North)

Now $EZ = ZS_2 + ES_2$

$$\theta = (90° - \alpha_1) + \delta_2$$

or $$\theta = z_2 + \delta_2$$

$\therefore$ Latitude = Zenith distance + Declination.

Case III. Star (S_3) between the zenith and the pole.

Here $NS_3 = \alpha_3$ = altitude of the star

$ZS_3 = (90° - \alpha_3) = z_3$ = zenith distance

$ES_3 = \delta_3$ = declination of the star (North)

Now, $EZ = ES_3 - ZS_3$

$$= \delta_3 - (90° - \alpha_3)$$

or $$EZQ = \delta_3 - z_3$$

$\therefore$ Latitude = Declination – Zenith distance.

Case IV. Star (S_4) between the horizon and the pole

Here $NS_4 = \alpha_4$ = altitude of the star.

$ZS_4 = (90° - \alpha_4) = z_4$ = zenith distance.

$E'S_4 = \delta_4$ = declination of the star.

Now, PN = altitude of the pole

= latitude of the place θ

$= NS_4 + PS_4 = \alpha_4 + (PE' - E'S_4)$

$= \alpha_4 + (90° - \delta_4) = (90° - Z_4) + (90° - 5_4)$

$= 180° - (24 + \delta_4)$

$\therefore$ Latitude = 180° – (Zenith distance + Declination)

Disadvantages of the method. The following are the disadvantages of the method :

(*i*) During the interval of changing face, the star moves out of the meridian.

(*ii*) The direction of the meridian of the place needs be determined before actual observations are made.

2. Latitude by equal meridian altitudes of two stars on either side of zenith (Tal Cott Method)

Principle of the method. The error of observations, refraction and instrument may be reduced by making observations upon two stars which culminate on the opposite sides of the observer's zenith, and having zenith distances approximately equal.

Field Procedure. Following steps are involved :

1. Select two stars which culminate within an interval of 10 to 30 minutes, such that difference of right ascensions of the two stars is equal to the interval of times between their culminations.
2. Observe the meridian altitude of the star which culminates first, accurately.
3. Swing the telescope and observe the meridian altitude of the second star.

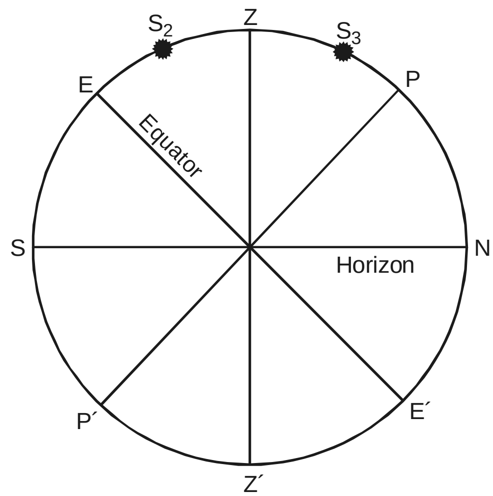


Fig. 1.77.

Calculations : Proceed as under.

Let S_1 and S_2 be the two stars which culminate within an interval of 20 minutes. S_1 culminates south of zenith and S_2 culminates north of the zenith.

Let α_1 and α_2 be the meridian altitudes, and δ_1 and δ_2 be the declinations.

Apparently for S_1, latitude

$$\theta = EZ = (90° - \alpha_1) + \delta_1$$

For S_2 latitude $\theta = EZ = \delta_2 - (90° - \alpha_2)$

$\therefore$ Average latitude $= \frac{1}{2}\,[90° - \alpha_1) + \delta_1 + \delta_2 - (90° - \alpha_2)]$

or

$$\theta = \frac{\alpha_2 - \alpha_1}{2} + \frac{\delta_1 + \delta_2}{2} \quad ...(1.91)$$

Note. The following points may be noted :

(*i*) *The correction for refraction gets cancelled as the difference in the altitudes of stars is not involved and these are approximately equal.*

(*ii*) *The instrumental errors also get cancelled as the observations to both stars are made under identical conditions.*

(*iii*) *The face of the theodolite remains the same.*

3. By meridian altitudes of a circumpolar star at its upper and lower culminations

Principle of the method. The north polar distance of a circumpolar star at lower culmination is equal to the north polar distance at its upper culmination *i.e., the mean altitude of the circumpolar star at upper and lower culminations is equal to the altitude of the pole and hence equals the latitude of the place.*

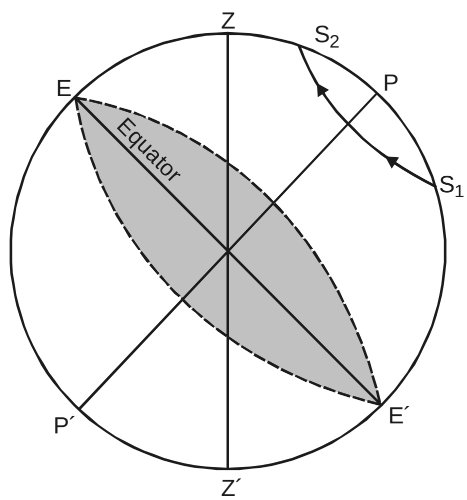


Fig. 1.78.

Proof.

Let S_1 and S_2 be the two positions of the circumpolar star at its lower and upper culminations. The path of the star is denoted by arrows (Fig. 1.78).

Let α_1 be the altitude of the star at lower culmination

α_2 be the altitude of the star at upper culmination.

But, latitude of the place = altitude of the pole

i.e. $\theta = NP$

$$NP = NS_1 + PS_1 = \alpha_1 + PS_1 \quad ...(i)$$

$$NP = NS_2 - PS_2 = \alpha_2 - PS_2 \quad ...(ii)$$

Adding equations (*i*) and (*ii*), we get

$$2\,PN = \alpha_1 + \alpha_2 + (PS_1 - PS_2)$$

But $PS_1 = PS_2$ = Co-declination of the star

$\therefore$ $2NP = \alpha_1 + \alpha_2$

or $$NP = \frac{\alpha_1 + \alpha_2}{2} \quad ...(1.92)$$

i.e. the latitude of the place is equal to half the sum of the altitudes of the circumpolar star at its upper and lower culminations.

Field procedure. Following steps are involved :

1. Determine the meridian of the place.
2. Select a suitable circumpolar star whose both culminations occur within night.
3. Observe the meridian altitude at its upper and lower culminations.

Calculations : Proceed as under.

Let α_1 and α_2 be the altitudes of the star at lower and upper culminations

$\therefore$ Latitude $\qquad \theta = \dfrac{\alpha_1 + \alpha_2}{2}$

Note. The following points may be noted :

(*i*) *This method is not preferred to as 12 sidereal hours elapse between two observations.*

(*ii*) *If the duration of night is less than 12 hours, one of the culminations of the star will be in day.*

(*iii*) *The declination of the star is not involved in the computation.*

(*iv*) *The error of refraction for both altitudes is not same.*

Example 1.57. *In northern hemisphere in longitude 76°30′E on December 20th 1978, observations for latitude were made on a circumpolar star whose altitude at lower and upper culminations were 25°37′15″ and 35°24′40″ respectively. Calculate the latitude of the place.*

Assume refraction in seconds

= 58″ × Tangent of apparent zenith distance

= 58″ × Cotangent of apparent altitude.

Solution.

Apparent altitude at lower culmination	= 25°37′15″
Refraction correction = 58″ × 2.0852143	= – 02′01″
∴ Correct altitude at lower culmination	= 25°35′14″
Apparent altitude at upper culmination	= 35°24′40″
Refraction correction = 58″ × 1.4065587	= – 1′22″
∴ Correct altitude at upper culmination	35°23′18″

∴ Latitude of the place θ = ½(25°35′14″ + 35°23′18″)

= 30°29′16″ *N.*

Ans.

Example 1.58. *Find the latitude of a place in the northern hemisphere at which a star of declination N 6°18′34′′ will have an altitude at upper transit of 45°03′04′′.*

Solution. (Fig. 1.79)

As the declination of star is 6°18′34′′ north, it lies between zenith and equator.

∴ Latitude of the place = declination + zenith distance.

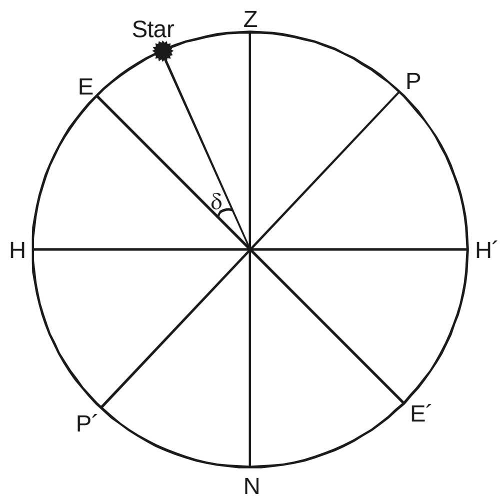


Fig. 1.79.

= 6°18′34′′ + (90° – 45°03′04′′)

= 51°15′30′′ *N*. **Ans.**

Example 1.59. *Find the latitude of a place at which the meridian zenith distances of a circumpolar star were observed to be 75°29′30′′ and 47°31′24′′. Take coefficient of refraction = 58′′.*

Solution.

Refraction correction for lower transit

= 58′′ tan 75°2′30′′ = 224′′ .13

Refraction correction for upper transit

= 58′′ tan 47°31′24′′ = 63′′.35

∴ Corrected co-altitude of star at lower transit

= 75°29′30′′ + 0°03′44′′.13

z_1 = 75°33′14′′.13

Corrected co-altitude of star at upper transit

= 47°31′24′′ + 0°01′03′′.35

z_2 = 47°32′27′′.35

∴ Co-latitude of the place

= ½ $(z_1 + z_2)$

= ½ [75°33′14′′13 + 47°32′27′′.35]

$= \frac{1}{2}\,(123°05'41''.48) = 61°32'50''.74$

$\therefore$ Latitude of the place = 90° – co-latitude

$= 28°27'09''.26\ N.$ **Ans.**

4. By ex-meridian observations of star or Sun. This method involves the following elements of the astronomical triangle.

1. The altitude of the star or sun.
2. The exact time of observation.
3. The right ascension of the star or sun.

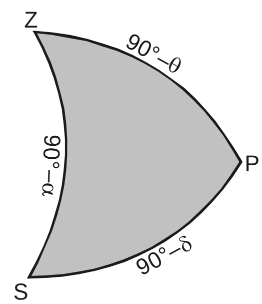


Fig. 1.80.

Field observations. The following steps are followed.

1. Observe the altitude of the celestial body.
2. Note down the correct time of observation.

Calculations : Proceed as under.

We know L.S.T. = R.A. of the star + H.A. of the star.

In the astronomical triangle *SZP* (Fig. 1.80).

$$SZ = 90° - \alpha = Z$$
$$SP = 90° - \delta = \Delta$$
$$\angle ZPS = H$$

From the cosine formula, we get

$$\cos(90° - \alpha) = \cos(90° - \theta)\cos(90° - \delta) + \sin(90° - \theta)\sin(90° - \delta)\cos H$$

or $$\sin\alpha = \sin\theta\sin\delta + \cos\theta\cos\delta\cos H \quad ...(1.93)$$

By substituting the values in equation (1.90) the value of θ can be evaluated.

Alternatively

Applying the sine formula to the astronomical triangle *SZP* we get

$$\sin SZP = \frac{\sin PS \,.\, \sin SPZ}{\sin ZS}$$

i.e. the azimuth of the star can be computed

Again, $$\tan\frac{PZ}{2} = \frac{\sin\frac{1}{2}(A+H)}{\sin\frac{1}{2}(A-H)}\tan\ \tfrac{1}{2}(SP - SZ)$$

$$= \frac{\sin\frac{1}{2}(A+H)}{\sin\frac{1}{2}(A-H)} \tan\ \tfrac{1}{2}(90° - \delta - 90° + \alpha)$$

$$\tan \frac{\rho_2}{2} = \frac{\sin\frac{1}{2}(A+H)}{\sin\frac{1}{2}(A-H)} \tan\ \tfrac{1}{2}(d - \delta) \qquad ...(1.94)$$

Note : The solution of the equation (1.93) may be done as follows.

$$\sin \alpha = \sin \theta \sin \delta + \cos \theta \cos \delta \cos H$$

Let $\sin \delta = x \sin y$ ...(1.95)

and $\cos \delta \cos H = x \cos y$ ...(1.96)

Dividing equation (1.94) by equation (1.95) we get

$$\frac{\sin \delta}{\cos \delta \cos H} = \tan y$$

$$\tan \delta \sec H = \tan y \qquad ...(1.97)$$

Substituting the values of equations (1.95) and (1.96) in equation (1.90)

$$\sin \alpha = \sin \theta \,.\, x \sin y + \cos \theta \,.\, x \cos y$$

$$\sin \alpha = x (\sin \theta \sin y + \cos \theta \cos y)$$

$$\sin \alpha = x \cos (\theta - y)$$

$$x = \sin \alpha \sec (\theta - y) \qquad ...(1.98)$$

Substituting the value of x in equation (1.95) we get

$$\sin \delta = \sin \alpha \,.\, \sec (\theta - y) \sin y$$

or $\cos(\theta - y) = \sin \alpha \,.\, \sin y \operatorname{cosec} \delta$ ...(1.99)

Substituting the value of y from equation (1.97) in equation (1.99) we may compute the value of θ.

5. By altitude of a star on prime vertical. (Fig. 1.81)

Let P be the celestial pole

Z be the zenith

ZSE be the prime vertical

α be the altitude

δ be the declination of star

θ be the latitude of place.

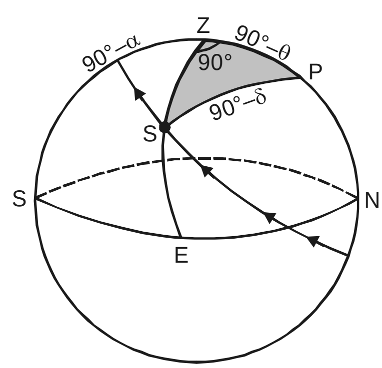


Fig. 1.81.

When the star is on the prime vertical, its azimuth is 90°. *i.e.* Spherical triangle *PZS* is a right angled triangle.

$$\therefore \quad \cos PS = \cos ZS \,.\, \cos ZP.$$

or $$\cos (90° - \delta) = \cos (90° - \alpha) \cos (90° - \theta)$$

or $$\sin \delta = \sin \alpha \sin \theta$$

$$\sin \theta = \sin \delta \operatorname{cosec} \alpha$$

$$= \sin \delta \sec z \qquad ...(1.100)$$

where z is zenith distance

Knowing the values of δ and z we can compute the value of the latitude.

Example 1.60. *The altitude of a star when on the prime vertical is seen to be 30°and its meridian altitude is 45°. Calculate the latitude of the place.*

Solution.

Given : $$\alpha_1 = \text{altitude on prime vertical} = 30°$$

$$z = 60°$$

$$\alpha_2 = \text{altitude on meridian} = 49°$$

or $$z = 45°$$

Substituting the values in eqn. (1.97) we get

$$\sin \theta = \sin \theta \sec 60° \qquad ...(i)$$

Again, we know that

$$\theta_1 = z + \delta$$

$$\sin (z + \delta) = \sin \delta \sec 60° \qquad ...(ii)$$

$$\sin (45° + \delta) = \sin \delta \sec 60°$$

$$\sin 45° \cos \delta + \cos 45° \sin \delta = 2 \sin \delta$$

or $$\frac{1}{\sqrt{2}} \cot \delta + \frac{1}{\sqrt{2}} = 2$$

$$\frac{1}{\sqrt{2}} \cot \delta = 2 - \frac{1}{\sqrt{2}}$$

$$\cot \delta = 2\sqrt{2} - 1$$

$$\cot \delta = 1.828427$$

$$\delta = 28°40'30''$$

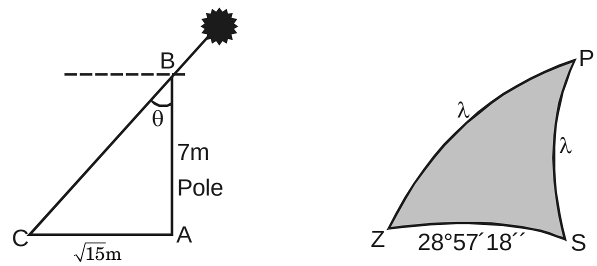
Substituting the value of δ in equation (*ii*) we get

$$\theta = 45° + 28°40'30''$$

$\therefore$ Latitude $\theta = 73°40'30''$ **Ans.**

Example 1.61. *The length of the shadow of a7m high pole at 10 O'clock in the morning is* $\sqrt{15}$ *m. If the pole does not cast any shadow at noon, calculate the latitude of the place.*

Solution. (Fig. 1.82)



(*a*) (*b*)

Fig. 1.82

Let *AC* be the shadow of the pole *AB* at 10 o'clock.

Apparently, zenith angle

$$= \angle CBA = 0$$

$$\tan\theta = \frac{\sqrt{15}}{7} = 0.55328332$$

or $$\theta = 28.955 = 28°57'18''$$

At noon, the pole does not cast any shadow *i.e.* sun is in zenith.

Co-latitude of the observer = Co-declination of sun = λ

At 10 O'clock in morning hour angle = 15° × 2 = 30°

Applying cosine formula to spherical triangle *PZS* we get

$$\cos^2 x + \sin^2 x \cos 30° = \cos 28°57'18''$$

$$(1 - \sin^2 x) + \sin^2 x \times 0.866026 = 0.875$$

$$1 - \sin^2 x + 0.866026 \sin^2 x = 0.875$$

or $$\sin^2 x\,(1 - 0.866026) = 1 - 0.875$$

$$\sin^2 x = \frac{0.125}{0.133974} = 0.93301685$$

$$\sin x = 0.96592797$$

$$x = 75°$$

∴ Latitude of the place = 90° – 75°

= 15°*N*. **Ans.**

Example 1.62. *An observation for latitude was made at a place in longitude 7°20´15´´ W. The meridian altitude of the sun's lower limb was observed to be 44°12´30´´, the sun being to the south of the zenith. Sun's declination at G.A.N. at 6.82 seconds per hour and semi-diameter of sun 15´45´´.86. Find the latitude of the place of observation.*

Solution.

Observed meridian altitude of the sun	= 44°12´30´´.00
Correction for refraction (– 57´´ cot 44°12´30)´´	(–) 59´´.60
Semi-diameter correction (+ve)	15´45´´.86
Correction for parallax (8´´.78 cos 44°12´30´´)	+ ve 6´´.30
.-. Corrected meridian altitude	= 44°27´22´´.56
Zenith distance z	= 45°32´37´´.44
L.A.T. of observation	= 0h 0m 0s
Time interval between Greenwich and observer's meridian 7°20´15´´ *W*	= 0h 29m 21s
G.A.T. of observation	= 0h 29m 21s west
Declination of sun at G.A.T.	= 22°18´12´´.80
Increase in sun's declination in Oh 29m 21s @ 6.82 sec per hour	= + 3´´.34
Sun's declination at L.A.N.	= 22°18´16´´.14

From the established relationship we get

$$\theta = z + \delta$$

∴ Latitude = 45°32´37´´.44 + 22°18´16´´.14

= 67°50´53´´.58 **Ans.**

Example 1.63. *Observations were taken on a star at some place in northern hemisphere and the following data were obtained.*

True altitude of the star = 41°00´15´´

Declination of the star =16°31´45´´

Hour angle of the star = 50°35´20´´

Calculate the latitude of the place of observation.

Solution.

We know from Eqn. (1.93), that

$$\tan y = \tan \delta \,.\, \sec H \qquad ...(i)$$

Substituting the values in Eqn. (*i*)

$$\tan y = \tan 16°31'45'' \times \sec 50°35'20''$$
$$= 0.2967672 - 1.57510 = 0.467438$$

or $$y = 25°03'11''.5$$

Substituting the values in Eqn. (1.95)

$$\cos(\theta - y) = \sin\alpha \,.\, \sin y \,.\, \operatorname{cosec}\delta$$
$$= \sin 41°00'15'' \times \sin 25°03'11'' \times \operatorname{cosec} 16°31'45''$$
$$= 0.656114 \times 0.423457 \times 3.5149014$$

or $$= 0.97656637$$
$$\theta - y = 12°25'41''.5$$

or $$\theta = 12°25'41''.5 + 25°03'11''.5$$

Latitude of the place $= 37°28'53''$ North. **Ans.**

Example 1.64. *The observed altitude of the sun's lower limb when crossing the meridian of a station in the northern hemisphere was 42°16′46″. The G.M.T. of observation was 12h 52m 45s. Calculate the latitude and longitude of the station.*

Given : Sun's declination = *1°33′55″ S increasing 58″.475 per hour,*

Equation of time = + *8m 55.05[3] increasing 20.1s/day.*

Semi-diameter of the sun = 15′58″-55

Correction for refraction = *57* cot α

Correction for parallax = *8″.8 cos α*

Solution.

Equation of time at midnight at Greenwich

= 8m 55.05s

Change in equation of time during

12h 52m 45s = 10.79s

∴ Equation of time at the time of observation

= 9m 05.84s

∴ Equation of time at the time of observation

= G.M.T. + E.T.

= 12h 52m 45s + 9m 05.84s

= 13h 01m 50.84s

Local apparent solar time at noon = 12h

∴ Longitude of the place = Apparent solar time at Greenwich–local apparent solar time

= 13h 01m 50.84s – 12h

= 1h 01m 50.84s

Because, the apparent solar time is ahead of the local apparent solar time, the longitude is west.

= 15°27′42″.59 *W*. **Ans.**

Calculation of the latitude

Observed meridian altitude of sun = 42°16′46″

(*i*) Correction for semidiameter (+ve) = 15′58″.55

Altitude of the sun corrected for semi-diameter

= 42°32′44″.55

(*ii*) Correction for refraction (–ve) = (–) 0°01′02″.69

57 cot 42°16′46″ = 62.69s.

Correction for parallax

8.8 cos 42°16′46″ = = + 0°00′6″.51

∴ Altitude of the sun corrected for semi-diameter

= 42°32′44″.55

(–) 0°01′02″.69

(+) 0°00′ 6″.51

∴ Corrected altitude of the sun = 42°31′48″.37

Corrected zenith distance = 47°28′11″.63

Corrected declination of sun

δ = 1°33′55″ + 12′41″98

= 1°46′36″.98

Latitude of the sun $\theta = z - \delta$

∴ Latitude of the place = 47°28′11″.63

– (– 1°46′36″.98)

θ = 49°14′48″.61 *N*. **Ans.**

Example 1.65. *If the apparent altitude of a star of declination 52°39′ 30″ S at upper transit is 24°20′20″ s, what is the observer's latitude* ?

Solution.

Refraction correction $= 58'' \cot \alpha$

$= 58'' \cot 24°20'20'' = 127''.93$

$= 2'7''.93$

$\therefore$ Correct altitude of star

$= 24°20'20'' - 2'7''.93$

$= 24°18'12''.07$

Zenith distance of star $= 90° - 24°18'2''.07$

$= 65°41'47''.93$

As the star is between horizon and equator, we get

Latitude θ = Zenith distance of star – declination of star

$= 66°41'47''.93 - 52°39'30''$

$= 13°02'17''.93$

$\therefore$ Latitude of the place $= 13°02'17''.93$ **Ans.**

1.49. DETERMINATION OF LONGITUDE

The longitude of any place with respect to another is the angular measure between the meridians of two places measured along the equator. Longitudes are reckoned east or west of the fixed reference meridian up to 180°. The fixed meridian or the standard meridian universally chosen is that of Greenwich, a small town west of London.

We know that the difference of longitudes of two places is connected with the difference of times taken at two places at the same instant. Therefore, we may infer that the difference in local times of two places is equal to the difference in their longitudes.

The various methods of determining longitudes are as under :

1. By transportation of chronometers.
2. By listening to radio signals.
3. By observing the stars which culminate at the same time.

Example 1.66. *At the station A, altitude of Sun was observed in the morning of a certain date in the month of May, 80 and the following data was recorded.*

Corrected altitude of Sun = 43°38′00″

Declination of sun = + 18°45′50″ N

E.T. to be subtracted from apparent time = 3m 43s

Latitude of station = *42°20´N*

G.M.T. of observation = *16h 22m 55s*

Find the longitude of station A.

Solution. (Fig. 1.83)

In the spherical triangle *ZPS* we have

$$ZS = 90° - \alpha = 90° - 41°38'00'' = 46°22'00''$$

$$PS = 90° - \delta = 90° - 18°45'50'' = 71°14'10''$$

$$PZ = 90° - \theta = 90° - 42°20'00'' = 47°40'00''$$

$$2S = 165°16'10''$$

$$S = 82°38'05''$$

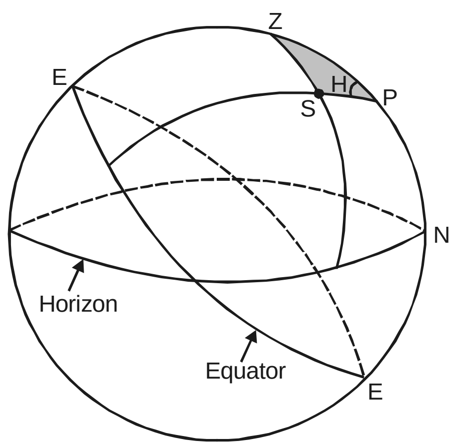


Fig. 1.83.

Solving the triangle *ZPS* for the hour angle (H) we get

$$\tan\frac{H}{2} = \sqrt{\frac{\sin(S-PZ)\sin(S-PS)}{\sin S \,.\, \sin(S-ZS)}} \quad ...(i)$$

Substituting the values in equation (i) we get

$$\tan\frac{H}{2} = \sqrt{\frac{\sin 35°28'05'' \sin 11°53'55''}{\sin 83°08'05'' \sin 36°46'05''}}$$

$$= \sqrt{\frac{0.5802489 \times 0.2061804}{0.9928299 \times 0.598577}}$$

$$= \sqrt{0.2013106} = 0.4486764$$

$$\therefore \quad \frac{H}{2} = 24.164648$$

$$\frac{H}{2} = 24°09'52''$$

or $H = 48°19'44''$

As the observation to Sun was made in the morning.

L.A.T. of observation $= 12h - \dfrac{48°19'44''}{15}$

$= 12h - 3h\ 13m\ 19s$

∴ L.A.T. of observation = 8h 46m 41s

Subtract E.T. from L.A.T. = 3m 43s

Local mean time = 8h 42m 58s

Difference of G.M.T. of observation and L.M.T. of observation

= 16h 22m 55s

= 08h 42m 58s

∴ Longitude in time = 7h 39m 57s

Longitude of the place = 7h × 15 = 105°00′00″

39m × 15 = 9°45′00″

57s × 15 = 14′15″

114°59′15″

As the L.M.T. of observation is behind the G.M.T., the place is west of Greenwich.

∴ Longitude of the place = 114°59′15″ West. **Ans.**

1.50. CONSTELLATIONS

The fixed stars are at varying distances from the earth, but they only appear to lie upon the surface of a sphere known as *celestial sphere*. For the purpose of classification, the relatively fixed stars, have been arranged into groups known as *constellations*. These groups of stars bear the names of animals, birds and other familiar objects, they resemble.

According to Bayer, the various stars of the same constel-lation are designated in order of their brightness by the name of the constellation preceded by the small Greek letters. In case the constellation contains more stars than 24 Greek letters, the 25th and onward stars are designated by Roman letters. For example the stars of the constellation Taurus, are designated as under. The brightest star is a Tauri. the next bright star in β Tauras and so on.

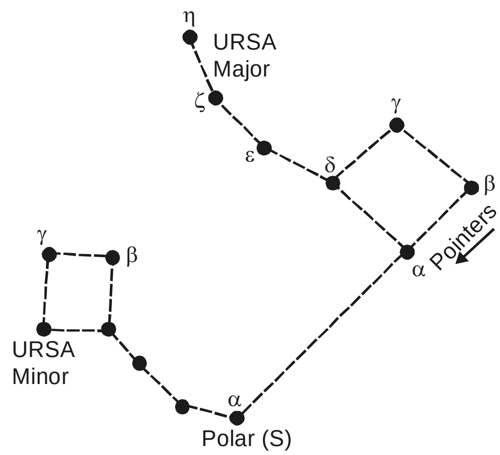


Fig. 1.84

Some brightest stars of the sky bear individual names and designated by their names. For an example, the brightest stars a of the constellation, 'The Little Bear' is popularly known as *polaris* or the *pole star*. It may be easily located in the north sky by a line through the stars β and α (known as pointers) of Ursa Majors and prolonging the same to pass through the polaris (Fig. 1.84).

Sirius, Canopua, Capella, Arcturus, Aldebran, Vega, etc. are other stars which are identified by their names.

According to Flamsteed (1729), the telescopic stars have been numbered consecutively from west to east across the constellation in the order of their Right Ascension (R.A.).

Zodiacal constellations. The imaginary belt between two small circles parallel to the ecliptic at a distance of 8°, on either side, is called the *zodiac*. The motions of all the planets and the moon are within the zodiac. The zodiac is further divided into 12 equal signs, each sign being of 30°. Each sign contains a constellation of stars, which is named after its resemblance with the animals or objects. These tweleve constellations which are called *zodiacal constellations* are named as under :

1. Aeries (Rama)	2. Taurus (Bull)
3. Gemini (Twins)	4. Cancer (Crab)
5. Leo (Lion)	6. Virgo (Virgin)
7. Libra (Balance)	8. Scorpio (Scorpion)
9. Sagittarius (Archer)	10. Capricornus (Goat)

11. Aquarius (water-carrier) 12. Pisces (Fish).

1.51. STAR ALMANACS AND STAR CHARTS

The celestial coordinates (right ascension and declination) of a selection of 650 stars for various dates corresponding with latitudes and longitudes on the earth are given in the Star Almanac for land surveyors published annually by Her Majesty's Nautical Almanac office, London. The position of a celestial body at any time can be obtained by interpolation. The publication most widely used by astronomers in India is "the Star Almanac for Land Surveyors". This is published annually in advance. The declination and right ascension of the listed stars in the star almanacs, are determined by making observations at, fixed observatories by the astronomers. The location of Stars are published in Star Chart as shown in Fig. 1.85.

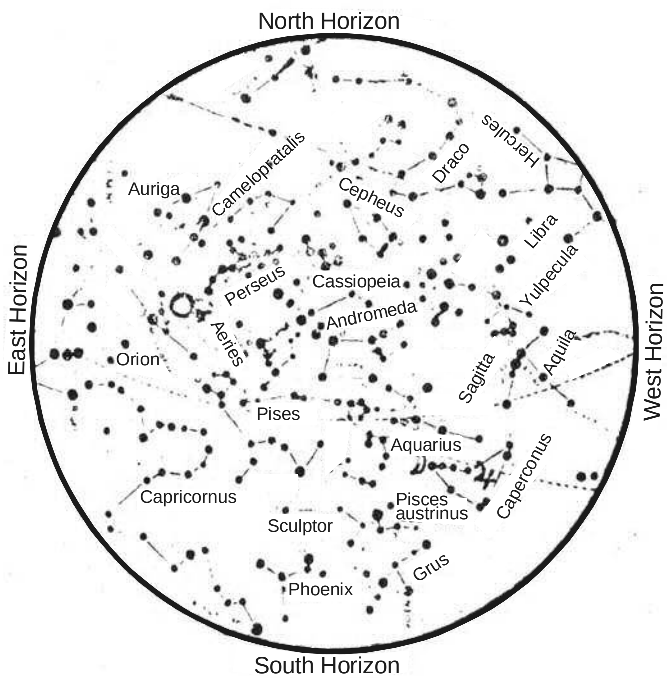


Fig. 1.85. Stars at New Delhi in November month

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* The magnitude of a star is a number which indicates its bright-ness. Magnitude increases as the brightness decreases. The magnitude of the sun, the brightest star is -26.7, for full moon -12.5 and Aldebran is 1.06.

Exercise 1

1. If the latitude of a place is 33°, find the zenith distance of the north pole. **[Ans. 57°]**
2. If the latitude of New Delhi is 28°32´45´´ N, calculate the declination of the star which may culminate at the New Delhi. **[Ans. 28°32´45´´ N]**
3. If the latitude of a place is 28°45´30´´ N and the declination of a star is 25°30´45´´ N, ascertain whether the star culminates north or south of zenith. **[North of zenith]**
4. If the declination of a star is 15°25´20´´ S, calculate its zenith distance when it is on the meridian of a place having latitude 25°32´00´´ N. **[Ans. 40°57´20´´]**
5. The declination of polaris is 89°02´40´´ N. Calculate the altitude of Polaris at its upper and lower culminations at a place of latitude 5°32´40´´. **[Ans. 6°33´00´´, 4°35´20´´]**
6. Find the altitude of the sun at upper transit at place in latitude 30°30´20´´ given that the declination is 12°29´40´´s. **[Ans. 47° in the South]**
7. Find the altitude at upper and lower transits of a star (declination N 62°32´30´´) at a place in latitude 28°24´30´´ N. **[Ans. 55°52´, 0´57´´]**
8. Find L.S.T. at L.M.N. (1200 hrs) in longitude (*i*) 90°E (*ii*) 90°W if G.S.T. at G.M.N. is 2h 02m 22.4s.

   [Ans. 2h 01m 23-3s, 2h-03m 21.5s]
9. Find L.S.T. at 0800 A.M. (Local Mean Time) in longitude 75°W if G.S.T. at G.M.M. is 14h 48m 08.7s.

   [Ans. 22h 50m 16-9s]
10. Determine L.A.T. of an observation at a place in longitude 15°W if L.M.T. is 15h 30m 20s ; the equation of time at G.M.N. is 5 min. 58.7 sec additive to apparent time and increasing at 0.22 sec/hr. **[Ans. 15°24´20´´.31s]**
11. Find the azimuth of a reference mark from the following obser-vations taken on a star at its eastern elongation.

    Declination of the star = 75°24´30´´

    Latitude of the place = 45°20´N.

    Clockwise angle from the sun to the reference mark

    = 45°42´29´´ **[Ans. 66°43´30´´]**

12. A star of declination 82°04′30″ N was observed at East Elongation when the anti-clockwise angle from a reference mark was 110°25′50″. Immediately afterwards a star of declination 63°45′35″ was observed at East elongation and the anti-clockwise angle observed was 75°20′20″. Determine the azimuth of R.M.

[Ans. 123°57′49″ clockwise from North]

13. Following observations were taken on the sun to determine the azimuth of a reference mark.

The mean observed altitude	= 22°34′30″
Readings of the altitude level	L. 4.5 E 3.5
bubble	R. 5.5 E 2.5
One division of altitude	= 15″
The Mean time of observation	= 3h 12m 57s.
The declination of the star	= 3°25′06″
Latitude of the place	= 53°29′19″ N
The mean horizontal angle between	
R.M. and the sun	= 120°07′14″

[Ans. Azimuth of R.M. = 108°clockwise from North]

14. Find the latitude of a place in the northern hemisphere at which a star of declination *N* 16°18′30″ will have an altitude at upper transit of 45°45′45″. **(Ans. 60°32′45″)**

15. A star was observed to reach an altitude of 50°20′30″ at 9hr 46 min. 16 sec G.S.T. and to return to the same altitude at 10 hr 58 min 20 sec G.S.T. The R.A. of the star was 10 hrs 6 min 6 sec. Determine the longitude of the observer. **(Ans. 4°5′W)**

16. The meridian altitude of a star was observed to be 74°26′20″ on 5th April 1980, the star lying between the zenith and the equator. The declination of the star was 40°18′56-50″ N. Find the latitude of the place of observation. **[Ans. 65°52′52-37″]**

17. To determine the azimuth of a line *AB,* a star was observed at its eastern elongation and the following data was observed :

Latitude of the place	= 45°30’20″ S
Longitude of the place	= 82°30′00″ E

Declination of the star = 75°25´32-5´´ S

Clockwise horizontal angle from the line AB to the star 120°25´30´´

Calculate the azimuth of the line *AB*.

[Ans. 38°31´59´´.5 clockwise from North]

18. Find the Indian standard time of the western elongation of Polaris for a place in latitude 27°12´35´´ N and longitude 31°30´30´´ E on 30th November 1980. The standard meridian is 82°30´E. Find also the azimuth at elongation.

[Ans. 6 h 38 m 07 1 s ; 1°04´10´´]

19. Polaris was observed at its western elongation on Aug. 30, 1980 and the horizontal angle between the star and the referring mark which is west of star is 45°30´20´´.

Declination of Polaris is 88°52´22´´.73 N. Latitude of the place of observer is 50°N. Calculate the azimuth of the referring mark.

[Ans. 312°44´27´´4 clockwise from North]

20. Observations were taken to the sun at place in longitude 80°30´45´´ E, the observed meridian altitude of the lower limb being 38°30´40´´, Sun's declination at G N A, on that data was 22°17´41´´ S, decreasing at the rate of 8´´.2/hour. Find the latitude of the place, the sun being on the south of the observer's zenith. Semi-diameter of the sun = 16´17´´.

[Ans. 28°55´39´´.1 N]

21. Explain the following terms in brief :
    (*i*) Ecliptic (*ii*) Sensible horizon
    (*iii*) Equation of time (*iv*) local meantime.

(UPSC Engg. Exam, 1999)