Unit 1

Measurement and Errors

1.1 DEFINITION OF PHYSICS

Physics is a branch of science which deals with matter and energy and the relation between them. It perhaps the most comprehensive science, since it examines the behaviour of all kinds of matter from smallest particles to galaxies. It deals with time, temperature and distance of a very wide range.

Physics is the study of nature and its laws. We expect that all these different events in nature take place according to some basic laws and *revealing these laws of nature from the observed* events is physics. For example, the orbiting of the moon around the earth, falling of mango from a tree and tides in a sea on a full moon can all be explained if we know the Newton's law of gravitation and Newton's laws of motion.

Physics is concerned with the basic rules which are applicable to all domains of life. Understanding of physics, therefore, leads to application in many fields including bio and medical sciences.

Physics goes the same way. The nature around us is like a big chess game played by nature. The events in the nature are like the moves of the great game. We are allowed to watch the events of nature and guess at the basic rules according to which the events take place. We may come across new events which do not follow the rules guessed earlier and we may have to declare the old rules inapplicable or wrong and discover new rules.

Since physics is the study of nature, it is real. No one has been given the authority to frame the rules of physics. We only *discover* the rules that are operating in nature.

Newton, Aryabhat, Einstein or Feynman are great physicists because from the observations available at that time, they could guess and frame the laws of physics which explained these observations in a convincing ways. But there can be a new phenomenon any day and if the rules discovered by the great scientists are not able to explain this phenomenon, no one will hesitate to change these rules. For a precise description of any such phenomena, the measurement of quantities is essential.

Matter is anything that has mass and takes up space. It can be in the form of solids, liquids, or gases. When you look at an object, you are able to see many of its properties. Scientists classify matter based on its chemical and physical properties that have been observed and tested. Some physical properties are only known through experimentation, while others are visible to the naked eye.

A **physical property** is a characteristic that can be observed or measured without changing the composition of the sample. It can be used to describe mixtures as well as pure substances. Because these pure substances have uniform and unchanging compositions, they also have consistent and unchanging physical properties.

There are many types of physical properties. Commonly used examples include density, colour, odour, hardness, and volume. Physical properties are further classified based on whether they are extensive or intensive. **Extensive** physical properties are those that are dependent on the amount of the substance preset. **Intensive** physical properties are those that do not depend on the amount of the substance present. This means they will be same whether you have one gram or one thousand kilograms of the substance.

Density is a physical property that is determined by dividing the mass of a given amount of a substances by its volume. It is often reported in units of g/mL, which means 'grams per millilitre. Density is an intensive property because the density of a pure substance will be the same no matter how much of you have. For example, the density of gold is 18.3 g/mL. This means that whether you have .5 grams or 200.5 grams of pure gold, the density will always be 18.3 g/mL when tested. Knowing this standard value enables jewelers to determine whether or not an item is pure gold.

1.1.1 Fundamental Forces in Nature

The following fundamental forces operate in nature:

1.1.1.1 Gravitational Force

Gravitational force is the force between any two masses in the universe and is always attractive in nature. Its range is about 10^{-39} times to nuclear force. Gravitational force is the universal force and interacts to all the particles of the universe whether charged or uncharged, whether atomic or heavenly bodies. Gravitational force can act at a distance without the need of any intervening medium. Thus, it is a non-contact force in nature. It is weakest force in nature but its effects can be strongly realised in heavenly bodies. According to Newton's law of Gravitation, the gravitational force between two bodies is directly proportional to the square of the distance between them. The formation of stars and galaxies, the motion of planets round the sun and the motion artificial and nature satellites are all governed by the gravitational force.

1.1.1.2 Electromagnetic Force

Electromagnetic force is the force which includes both electric and magnetic forces. Although electric and magnetic forces and their effects are considered to be different, they are closely related. Because the state of rest or of motion of a charge is relative. A stationary charge produces an electric field. Any other charge placed in this field experiences and electrical force. A charge in motion gives rise to a current which in turn produces a magnetic field. Also, a charge moving in a magnetic field experiences a force. Thus, the name electromagnetism, the application of magnetic effects of electric current and the phenomenon of electromagnetic induction are governed by these forces.

1.1.1.2.1 Electrostatic Force

Electrostatic force is the force between two static charges. The range of force is infinite and relative strength is about 10^{-2} times of the strong nuclear force. It is an attractive force between two unlike charges while a repulsive force between two like charges *i.e.*, electrostatic force may be attractive or repulsive. Electrostatic force depends on the medium between the two charges. According to Coulomb's law in electrostatics, the force between two stationary charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them. When a comb drawn through the hairs is brought near small pieces of paper, the comb attracts the pieces of paper. The force responsible for this action is the electrostatic force. The process of electrification and various phenomena associated with charged bodies are governed by the electrostatic force.

1.1.1.2.2 Magnetic Force

Magnetic force is the force between the magnetic poles of different magnets. Like electrostatic force magnetic force may be attractive or repulsive. Force is attractive between two unlike poles while repulsive between two like poles. It also depends on the medium in which the poles are situated. According to Coulomb's law in magnetism, the force between two isolated magnetic poles is directly proportional to the product of the pole strengths and inversely proportional to the square of the distance between them. The various effects of earth's magnetic field, the working of a Mariner's compass are governed by magnetic force.

1.1.1.2.3 Nuclear Force

Nuclear force is the force which exists between the nucleons present inside the nucleus of an atom. It is a short range and non-central force. For distances greater than about a fermi (1 fermi *i.e.*, 1 fm = 10^{-15} m) the nuclear force becomes very small and is negligible. It operates between two protons, two neutrons and also between a proton and a neutron. This shows that the nuclear force is charge independent. It is strongly attractive for separation up to about 0.4 fm and for still smaller separations it becomes strongly repulsive. The attractive nuclear force between nucleons which provides stability for the nucleus is called **strong interaction**. The nature of these forces is only partially understood.

Apart from strong interaction which provides stability to a nucleus there is also **weak interaction.** The range of this force is less than that of strong interaction. It plays an important role in certain nuclear processes such a radiative beta decay.

1.2 NEEDS OF MEASUREMENT

If someone ask you to compare between a football and a tennis ball then you will response immediately. But if both the balls are almost same type as in case of a hockey and a cricket ball then you have to measure their size and then can say that which one is bigger or smaller. Hence, after the measurement one can finalized its view.

To know and understand a physical situation completely, one is likely to measure the quantities such as length, speed, time , mass etc. These quantities are called physical quantities.

1.3 PHYSICAL QUANTITIES

All the quantities in terms of which laws of physics are described and whose measurement are essential are called physical quantities.

In almost every physical situation, we realize the necessity of measurement. If someone having fever, the measurement of body temperature is essential. When car runs on road by consuming petrol/diesel which costs us, the measurement of petrol/diesel consumed is must. Thus, we can say that the observation and measurement form the back bone of science and engineering.

The accurate measurements are essential for the development of physics, the perceptions beyond the given data shows are also important.

The process of measurement is basically a comparison process. The measuring process involves the selection of unit of measurement and comparing quantity with the standard unit.

The standard chosen should be of the same nature as that of quantity to be measured. For example, unit of mass measured in terms of mass and unit of length measured in terms of length only and so on.

1.3.1 Fundamental Physical Quantities

Those physical quantities which are independent of each other are called fundamental physical quantities.

Examples: There are seven fundamental quantities namely, length, mass, time, electric current, temperature, amount of substance and luminous intensity.

1.3.2 Derived Physical Quantities

Those physical quantities which can be express in terms of fundamental physical quantities are called derived physical quantities.

Example: Speed, velocity, acceleration, force etc. are derived physical quantities.

1.4 UNITS

A unit of measurement is a definite magnitude of a quantity defined and adopted by convention or by law that is used as a standard for the measurement of same quantity. Any other value of that quantity can be expressed as a simple multiple of the unit of measurement.

For example, length is a physical quantity, metre is the unit of length. The practical use of units of measurement which played a crucial role in human endeavour from early ages of this day. Different use of units used to be very common.

The description is based on the quantitative measurement of the physical quantities. So, length, mass, time, pressure, temperature, current and resistance are physical quantities.

The chosen standard of measurement of a quantity which has essentially the same nature as that of the physical quantity is called the unit.

For example, suppose, we have to measure the line PQ and we choose metre as the unit of measurement. Now, we place the metre scale along PQ and we find that it is contained 2 times in PQ. So, 2 is the numerical value of the length PQ and metre (m) be the unit of measurement. We can write, $PQ = 2$ m.

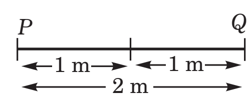


Fig. 1.1

1.4.1 Fundamental and Derive Units

The units chosen for the measurement of fundamental quantities are called **fundamental units.** For example, metre, kilogram, second, kelvin, ampere, candela and mole are fundamental units. The units obtained for derived quantities are called **derived unit.**

For example, unit of speed or velocity is a derived units.

As, $$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\therefore \quad \text{Unit of speed/velocity} = \frac{\text{Unit of distance}}{\text{Unit of time}} = \frac{\text{m}}{\text{s}} = \text{ms}^{-1}$$

Similarly, the derived unit of area is m^2, derived unit of acceleration is ms^{-2} and so on.

Concept of Length

Length of an object may be defined as the distance of separation between any two points at the extreme ends of the object.

The most common unit of length is metre (m). One metre is defined as the distance between two lines marked on a platinum-iridium bar kept at a constant temperature of 273.16 K and at 1 bar pressure.

Concept of Mass

Mass of an object may be defined as the quantity of matter is the object, which can never be zero. The mass of an object is not affected by the presence of other objects.

The most common unit of mass is kilogram (kg). The kilogram (kg) is defined as the mass of one cubic decimetre of water 4°C.

Concept of Time

The concept of time occurred first from the motion of the moon across the sky, the formation of day and night as a result of rotation of the earth around the axis. According the Einstein, time is what clock reads. In fact, the phenomena that repeats itself regularly can serve as a measure of time. Rotation of earth around its axis, revolution of earth around the sun, etc. are examples of repetitive phenomena, which serve as measure of time. The most of common unit of time is second.

One second is defined as the time taken by a simple pendulum of length one metre is going from one extreme position to the other extreme position. One solar day is the time interval between two successive noons.

The length of a solar day average over a year is called mean solar day.

$$1 \text{ mean solar day} = 24 \times 60 \times 60 = 86400 \text{ seconds}$$

$$\therefore \quad \text{One mean solar second} = \frac{1}{86400} \text{ part of a mean solar day.}$$

1.4.2 Systems of Units

A system of units is the complete set of units, both fundamental and derived for all kinds of physical quantities. The common systems of units used in mechanics are given below:

1.4.2.1 FPS, CGS, MKS and SI Units

1. **FPS System** : In this system, the units of length, mass and time are foot, pound and second respectively. It is the British Engineering System of units.
2. **CGS System.** In this system, the units of length, mass and time are centimetre, gram and second respectively.
3. **MKS System.** In this system, the units of length, mass and time are metre, kilogram and second respectively.
4. **SI System (International system of units).** The 14th General Conference on Weights and Measures in 1971 agreed upon a system of units called the 'International System of Units' abbreviated as SI. The abbreviation is based on its French name '*Le Systeme International d'* units.

The system of units, which is at present internationally accepted for measurement. This system of units is essentially a modification over the MKS system and therefore, called rationalised MKS system.

The SI is based on the following seven fundamental units and two supplementary units are given below in Table 1.1.

Table 1.1. Fundamental and Supplementary Units on SI

S. No.	Fundamental Physical Quantity	Symbol	Fundamental Units	Symbol of Units
1.	Mass	m	kilogram	kg
2.	Length	l	metre	m
3.	Time	t	second	s
4.	Temperature	T	kelvin	K
5.	Electric current	I	ampere	A
6.	Luminous intensity	L	candela	cd
7.	Amount of substance	n	mole	mol

S. No.	Supplementary Physical Quantity	Symbol	Supplementary Units	Symbol of Units
1.	Plane angle	rad	radian	rad
2.	Solid angle	sr	steradian	sr

1.4.2.2 Definition of Fundamental and Supplementary Units in SI

The definitions of fundamental and supplementary units in SI are as follows:

Do you know?

International system of units (SI) applies to all branches of science, whereas MKS system is confined to mechanics only.

1. **The metre.** The metre is the distance of the path travelled by light in vacuum during time interval of $\frac{1}{299,792,458}$ second.

[Taking velocity of light in vacuum, $c = 299,792,458 \text{ ms}^{-1}$.]

2. **The kilogram.** The kilogram is the mass of a cylinder made of platinum-iridium alloy kept at the International Bureau of Weights and Measures at Sevres near Paris (France).

3. **The second.** The second is the time taken by light of specified wavelength emitted by a Cs-133 (cesium-133) atom to execute 9,192,631,770 vibrations.

4. **The ampere.** The ampere is that current which when flowing in each of the two straight parallel conductors of infinite length and negligible cross-section, placed one metre apart in vacuum produces between the conductors, a force of 2×10^{-7} N/m length of each conductor.

5. **The kelvin.** One kelvin is $\frac{1}{273.16}$ of the thermodynamical temperature of the triple point of water.

6. **The mole.** The mole is the amount of substance that contains as many elementary entities (atoms or molecules) as there are atoms is 0.012 kg of pure carbon-12 (C–12). This number is called Avagadro's constant and its value is 6.023×10^{23}.

7. **The candela.** The candela is the luminous intensity of a black body of surface area $\frac{1}{6,00,000}$ sq. m placed at the temperature of freezing platinum (Pt) and at pressure of 101,325 Nm^{-2}, in the perpendicular to its surface.

8. **The radian (rad).** The radian is the angle subtended at the centre of a circle by an arc equal in length to the radius of circle.

$$d\theta = \frac{ds}{r} \text{ radian}$$

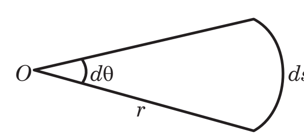


Fig. 1.2

where, $ds \rightarrow$ arc of length and subtends an angle = $d\theta$

9. **The steradian (sr).** The steradian is the solid angle subtended at the centre of the sphere by an area of its surface equal to the square of the radius of the sphere.

$$d\Omega = \frac{dA}{r^2} \text{ steradian}$$

where, $d\Omega$ = solid angle,

dA = area of spherical surface and

r = radius

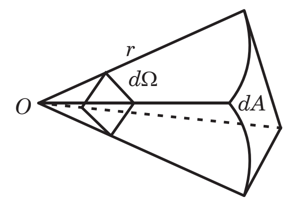


Fig. 1.3

Rules are Formulated for Writing the Symbols of SI Units

The following rules are given below:

(*i*) The symbol of a unit named after a scientist should be written in capital letter. For example, unit temperature namely is kelvin is to written as K. Unit of force namely newton is to be written N.

(*ii*) The symbol of a unit should not be followed by a full stop or any punctuation mark.

(*iii*) The full name of unit, even if it is named after a scientist should be written in small letters. It should not begin with a capital letter. For example, newton, watt, ampere etc.

(*iv*) The plural should not be used for units and their symbols. For example, (*a*) metre (m) and not metres (*b*) second (s) and not seconds etc.

Some Practical Units

(*a*) **Macro-cosm measurement (Very large distances)**

1. **Astronomical Unit (AU):** It is an average distance of the centre of the sun from the centre of the earth.

$$1 \text{ AU} = 1.496 \times 10^{11} \text{ m} \simeq 1.5 \times 10^{11} \text{ m}$$

2. **Light Year (LY):** One light year is the distance travelled by light in vacuum in one year. The velocity of light in vacuum (c) = 3×10^8 ms^{-1}.

and $$1 \text{ year} = 365 \times 24 \times 60 \times 60 \text{ seconds}$$

$\therefore$ $$1 \text{ light year} = 3 \times 10^8 \times 365 \times 24 \times 60 \times 60 \text{ m}$$

$$1 \text{ ly} = 9.46 \times 10^{15} \text{ m}$$

3. **Par sec:** One par sec is the radius of a circle at the centre of which an arc of the circle, 1 AU long subtends an angle 1″.

$$1 \text{ par sec} = 3.1 \times 10^{16} \text{ m}$$

$$1 \text{ par sec} = 3.26 \text{ ly}$$

Some useful units of length are

$$1 \text{ inch} = 0.0254 \text{ m}$$

$$1 \text{ foot} = 0.3048 \text{ m}$$

$$1 \text{ yard} = 0.9144 \text{ m}$$

$$1 \text{ mile} = 1.609 \times 10^3 \text{ m}$$

$$1 \text{ nautical mile} = 1.852 \times 10^3 \text{ m}$$

(*b*) **Micro-cosm measurement (Very small distances)**

(*i*) 1 micron (μ) = 10^{-6} m

(*ii*) 1 nanometre (nm) = 10^{-9} m

(*iii*) 1 angstrom (Å) = 10^{-10} m

(*iv*) 1 fermi (f_m) = 10^{-15} m

(*c*) **Measuring heavy masses**

(*i*) 1 tonne or 1 metric ton = 10^3 kg

(*ii*) 1 quintal = 100 kg

(*iii*) 1 slug = 14.57 kg

*Chandra Shekhar Limit (CSL) is the largest practical unit.

1 CSL = 1.4 times of mass of sun.

(*d*) **Lunar month:** The time taken by the moon to complete one revolution around the earth in its orbit.

1 lunar month = 27.3 days.

(*e*) **Shake:** It is the smallest unit of time.

1 shake = 10^{-8} s.

Multiples and Sub-multiples of Units

The multiples and sub-multiples are the prefixes and suffixes that we frequently use to express or show quantities in scientific terminology. These are presented below for general benefit for students.

Multiples:	Deca =	10
	hecto =	100
	kilo =	1000
	mega =	1000000
	giga =	1000000000
	tera =	1000000000000
Sub-multiples:	deci =	one-tenth
	centi =	one-thousandth
	micro =	one-millionth
	nano =	one-billionth
	pico =	one-thousand billionth

Powers of Ten

In mathematics, a power of 10 is any of the integer powers of the number ten. In other words, ten multiplied by itself a certain number of times (when the power is a positive integer).

By definition, the number one is a power i.e., zeroth power of ten (10°).

SI Prefixes

The physical quantities whose magnitude is either too large or too small can be expressed more compactly by used of certain prefixes.

The prefixes commonly use for powers of 10 are listed below in Table 1.2.

Table 1.2.

S.No.	Power of 10	Prefixes	Symbol
1.	10^{-12}	pico	p
2.	10^{-9}	nano	n
3.	10^{-6}	micro	μ
4.	10^{-3}	milli	m
5.	10^{-2}	centi	c
6.	10^{-1}	deci	d
7.	10^{1}	deca	da
8.	10^{2}	hecto	h
9.	10^{3}	kilo	k
10.	10^{6}	mega	M
11.	10^{9}	giga	G
12.	10^{12}	tera	T

Do you know?

In our country, the responsibility of maintenance of physical standard of length, mass and time etc. have been given to National Physical Laboratory (NPL), New Delhi.

Advantages of SI

The SI system of units has following advantages.

- It is closely related to the CGS system.
- In SI system, only one unit is used for one physical quantity. Therefore, it is a rationalised system of units.
- All derived units can be obtained by suitable manipulation of base and supplementary units and no numerical factors are encountered. Therefore, it is a coherent system of units.
- In metric system, multiples can be expressed as suitable powers of 10.

Table 1.3. Physical quantities and its units

S. No.	Physical quantity	Relation with other quantity	SI units
1.	Area	length × breadth	m^2
2.	Speed/velocity	distance/time	m/s or ms^{-1}
3.	Acceleration	$\frac{\text{change in velocity}}{\text{time taken}}$	ms^{-2}
4.	Force	mass × acceleration	N (newton)
5.	Work	force × distance	J (joule)
6.	Power	$\frac{\text{work}}{\text{time}}$	W (watt)
7.	Density	$\frac{\text{mass}}{\text{volume}}$	kgm^{-3}
8.	Pressure	force/area	Nm^{-2}
9.	Impulse	force × time	Ns
10.	Strain	$\frac{\text{change in dimension}}{\text{original dimension}}$	No units

1.5 DIMENSIONS OF PHYSICAL QUANTITIES

The dimensions of physical quantities are the powers to which the units of base quantities are raised to represent a derived unit of that quantity.

In mechanics, all the physical quantities can be written by the symbols of the dimensions of [L], [M] and [T].

For example, Area = length × breadth

$$= [L] \times [L] = [L^2]$$

The area is said to have two dimensions in length. So, we write, area = $[M^0L^2T^0]$ and say that area has zero dimension in mass and zero dimension is time in addition to 2 dimensions in length.

Similarly, volume = length × breadth × height

$$= [L] \times [L] \times [L] = L^3 = [M^0L^3T^0]$$

Now, $$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{\text{length}}{\text{time}} = \frac{[L]}{[T]} = [LT^{-1}] = [M^0LT^{-1}]$$

So, the dimensions of speed or velocity are: zero in mass, +1 in length and –1 in time.

1.5.1 Dimensional Formula and Dimensional Equations

The expression for a physical quantity obtained in terms of fundamental quantities shows the powers or indices to which the fundamental quantities are raised. These indices are the dimensions of the given physical quantities, and the expression obtained is called the **dimensional formula.**

For example, speed = $[M^0LT^{-1}]$ signifies that the dimensional formula for the speed is $[M^0LT^{-1}]$.

The relation between the physical quantity and the fundamental quantities expressed in the form of equation is called dimensional equation.

For example, if speed/velocity represented by (v).

Then, $[v] = [M^0LT^{-1}]$ is called dimensional equation of velocity.

Table 1.4, gives the dimensional formulae and units of various derived physical quantities.

Table 1.4. Physical quantities and its dimensions formulae.

S.No.	Physical quantity	Relation	Dimensional other quantity
1.	Volume	length × breadth × height	$[L] \times [L] \times [L] = L^3 = [M^0\ L^3\ T^0]$
2.	Force	mass × acc.	$M \times LT^{-2} = [M^1\ L^1\ T^{-2}]$
3.	Acceleration	$\frac{\text{change in velocity}}{\text{time taken}}$	$\left[\frac{L/T}{T}\right] = [LT^{-2}] = [M^0\ L^1\ T^{-2}]$
4.	Acceleration due to gravity (g)	$\frac{\text{change in velocity}}{\text{time taken}}$	$\frac{L/T}{T} = LT^{-2} = [M^0\ L^1\ T^{-2}]$
5.	Density	$\frac{\text{mass}}{\text{volume}}$	$\frac{[M]}{[L^3]} = [M^1\ L^{-3}\ T^0]$
6.	Pressure	force/area	$\frac{MLT^{-2}}{L^2} = [M^1\ L^{-1}\ T^{-2}]$

7.	Impulse	force × time	$MLT^{-2} \times T = [M^1 L^1 T^{-1}]$
8.	Specific gravity	$\dfrac{\text{density of body}}{\text{density of water at } 4°\text{C}}$	$\dfrac{[M/L^3]}{[M/L^3]} = 1 = [M^0 L^0 T^0]$ (Dimensionless)
9.	Work	force × distance	$MLT^{-2} \times L = [M^1 L^2 T^{-2}]$
10.	Universal constant of gravitation (G)	From Newton's law of gravitation. $F = \dfrac{Gm_1m_2}{r^2}$ or $G = \dfrac{F\,r^2}{m_1m_2}$ where, F is force between masses m_1, m_2 at a distance r	$G = \dfrac{(MLT^{-2})\,L^2}{MM} = [M^{-1}L^3T^{-2}]$
11.	Moment of force	force × distance	$MLT^{-2} \times L = [M^1 L^2 T^{-2}]$
12.	Power	$\dfrac{\text{work}}{\text{time}}$	$\dfrac{ML^2\,T^{-2}}{T} = [M^1 L^2 T^{-3}]$
13.	Surface tension	$\dfrac{\text{force}}{\text{length}}$	$\dfrac{MLT^{-2}}{L} = [M^1 L^0 T^{-2}]$
14.	Surface energy	Energy of free surface	$[M^1 L^2 T^{-2}]$
15.	Thrust/Tension	force	$[M^1 L^1 T^{-2}]$
16.	Strain	$\dfrac{\text{change in dimension}}{\text{original dimension}}$	$\dfrac{L}{L} = 1 = [M^0 L^0 T^0]$ (Dimensionless)
17.	Coefficient of elasticity	$\dfrac{\text{stress}}{\text{strain}}$	$\dfrac{M^1\,L^{-1}\,T^{-2}}{1} = [M^1 L^{-1} T^{-2}]$
18.	Radius of gyration (K)	distance	$L = [M^0 L^1 T^0]$
19.	Moment of inertia (I)	mass(radius of gyration)^2	$ML^2 = [M^1 L^2 T^0]$
20.	Angle (θ)	length(l)/radius (r)	$\dfrac{L}{L} = 1 = [M^0 L^0 T^0]$ (Dimensionless)
21.	Angular velocity (ω)	$\dfrac{\text{angle }(\theta)}{\text{time }(t)}$	$\dfrac{1}{T} = T^{-1} = [M^0 L^0 T^{-1}]$
22.	Angular momentum	I ω	$(ML^2)\,(T^{-1}) = [M^1 L^2 T^{-1}]$
23.	Torque	I α	$(ML^2)\,(T^{-2}) = [M^1 L^2 T^{-2}]$
24.	Frequency (ν)	number of vibrations/sec.	$1/T = T^{-1} = [M^0 L^0 T^{-1}]$

25.	Angular frequency (ω)	2π × frequency	$T^{-1} = [M^0 L^0 T^{-1}]$
26.	Planck's constant (h)	$\dfrac{\text{energy } (E)}{\text{frequency } (\nu)}$	$\dfrac{ML^2 T^{-2}}{T^{-1}} = [M^1 L^2 T^{-1}]$
27.	Heat (Q)	Energy	$[M^1 L^2 T^{-2}]$
28.	Temperature (θ)	kelvin	$[M^\circ L^\circ T^\circ K^1]$
29.	Latent heat (L)	$\dfrac{\text{heat } (Q)}{\text{mass } (m)}$	$\dfrac{M L^2 T^{-2}}{M} = [M^\circ L^2 T^{-2}]$
30.	Electric current (I)	ampere (A) is fundamental unit of current	$[M^\circ L^\circ T^\circ A^1]$

1.5.2 Principle of Homogeneity of Dimensions

According to principle of homogeneity, the dimensions of all the terms in the physical equation must be same. This principle basically signifies that only similar physical quantities can be added, subtracted or equated.

1.5.2.1 To Check the Correctness of an Equation

To check the correctness of the given equation, we shall write the dimensions of the quantities of both sides of the equation. For the correct equation, the principle of homogeneity of dimensions is obeyed.

Example. *A car starting with a velocity (u) and moving with a constant acceleration (a) travels a distance (s) in time (t). To check the correctness of a given equation related these quantities is* $s = ut + 1/2\, at^2$.

Solution. Given equation is $s = ut + \dfrac{1}{2}\, at^2$

The correctness of the given equation is checked using the dimensions as follows:

$$[s] = [L],\ [ut] = [\text{velocity} \times \text{time}]$$

$$[LT^{-1}]\,[T] = [M^0\, LT^0] \qquad ...(i)$$

$$[at^2] = [\text{acceleration} \times \text{time} \times \text{time}]$$

$$= [M^0\, LT^{-2}]\,[T^2] = [M^0\, LT^0] \qquad ...(ii)$$

Equating eqns. (i) and (ii), we get,

$$[M^0\, LT^0] = [MLT^0] \quad \Rightarrow \quad \text{LHS} = \text{RHS}$$

Hence, the equation is correct dimensionally.

Example. *Velocity of a particle is given by the equation,* $v = a + \dfrac{b}{c+t}$*, where t stands time. Find the dimensions of constants a, b,and c.*

Solution. Applying principle of homogeneity of equation,

1. Dimensions on both sides of equation remain same
2. Same dimension (physical quantities) are added or subtracted.

Since, v is velocity and having dimension $[LT^{-1}]$, all the other factors on RHS should have the same dimension.

Hence, $\quad a \Rightarrow [LT^{-1}]$

Now see the denominator of second factor, here c is added to t (time), hence it is also time and the dimension of c should be as of t, i.e., [T].

again $$\frac{b}{[T]} = LT^{-1}$$

or $$b = LT^{-1} \times [T] = [L]$$

1.5.2.2 To Convert Unit of One System into Another

The conversion of unit based on the fact that magnitude of a physical quantity remains the same, whatever be the system of its measurement.

$$Q = n_1 u_1 = n_2 u_2 \qquad ...(i)$$

where u_1 and u_2 are two units of measurement of the quantity Q and n_1 and n_2 are their numerical values.

Let M_1, L_1, T_1 are the fundamental units of mass, length and time in one system and M_2, L_2, T_2 are the fundamental units of mass, length and time in other system. Let a, b and c are the respective dimensions of mass, length and time of the both the systems.

Now, the units of measurement u_1 and u_2 of the quantity of the two systems would be

$$u_1 = [M_1^a \, L_1^b \, T_1^c]$$

and $$u_2 = [M_2^a \, L_2^b \, T_2^c] \qquad ...(ii)$$

From eq. (*i*), we get $$n_2 = \frac{n_1 u_1}{u_2} \qquad ...(iii)$$

Putting the values of u_1 and u_2 in eq. (*iii*), we get

$$n_2 = n_1 \frac{[M_1^a \, L_1^b \, T_1^c]}{[M_2^a \, L_2^b \, T_2^c]}$$

$\Rightarrow$ $$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c \qquad ...(iv)$$

Example. *Convert 1 newton into dyne.*

Solution. One newton is the unit of force in SI unit and one dyne is unit of force in CGS system. The dimensional formula of force, $[F] = [MLT^{-2}]$. Now, the dimensions of force in mass, length and time are $a = 1$, $b = 1$, $c = -2$.

So, we convert SI system into CGS system,

$$M_1 = 1 \text{ kg}, \quad L_1 = 1 \text{ m} \quad \text{and} \quad T_1 = 1 \text{ s}$$

and $$M_2 = 1 \text{ g}, \quad L_2 = 1 \text{ cm} \quad \text{and} \quad T_2 = 1 \text{ s}$$

$$n_1 = 1 \text{ newton and } n_2 = ?$$

We know that, $$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$= \left(\frac{1 \text{ kg}}{1 \text{ g}}\right)^1 \left(\frac{1 \text{ m}}{1 \text{ cm}}\right)^1 \left(\frac{1 \text{ s}}{1 \text{ s}}\right)^{-2}$$

$$= \left(\frac{10^3 \text{ g}}{1 \text{ g}}\right)\left(\frac{10^2 \text{ cm}}{1 \text{ cm}}\right) \times 1$$

$$= 10^3 \times 10^2 = 10^5$$

$\therefore$ $$1 \text{ newton} = 10^5 \text{ dyne.}$$

1.5.2.3 To Deduce a Relation Connecting the Physical Quantities

Using the principle of homogeneity, equate the powers of M, L and T on the both sides of the dimensional equation. The equation solved to obtained the values of three unknown dimensions. Putting these values in the equation, we obtain the preliminary form of the formula.

Example. Deduce $F = \frac{mv^2}{r}$, where F is force, which may depend upon, mass (m), velocity (v) and radius (r).

Solution. Let $F \propto m^a v^b r^c$

where a, b and c are the dimensions.

or $$F = k\, m^a v^b r^c \quad ...(i)$$

where k is the dimensionless constant.

Now, write the dimensions in terms of M, L and T on the both sides of eqn. (i), we get

$$[MLT^{-2}] = M^a [LT^{-1}]^b [L]^c$$

$$\Rightarrow \quad [M^1L^1T^{-2}] = [M^a L^{b+c} T^{-b}]$$

Applying the principle of homogeneity of dimensions, we get

$$a = 1, \quad b + c = 1 \quad \text{and} \quad b = 2 \quad ...(ii)$$

we have, $$b + c = 1$$

$$\Rightarrow \quad 2 + c = 1 \quad \Rightarrow \quad c = -1$$

Putting the values of a, b and c in eqn. (i), we get

$$F = k\, m^1 v^2 r^{-1}, \text{ where } k \text{ is dimensionless quantity.}$$

$$\Rightarrow \quad F = \frac{mv^2}{r}$$

1.5.2.4 Limitations of Dimensions

There are no limitations of the method of dimensions. The method of dimensions is subject to the following limitations in its applications.

1. The constant of proportionally cannot be determined.
2. The method of dimensions gives us no information about the dimensionless constants in the formula. For example, 1, 2, 3, ... π, etc.
3. We cannot derive the formulae containing trigonometrical functions, large functions, exponential functions etc. which have no dimensions.
4. The relation connecting a set of physical quantities can be deduced only if the relation is in the form of a product.
5. The method of dimensions cannot be used to derive an exact form of relation, when it consists of more than one part on any side.

   For example, the exact form of the formula $v^2 = u^2 + 2as$ cannot be obtained.
6. When the dimensions are given, the physical quantity may not be unique as many physical quantities have the same dimensions.

1.6 ERRORS IN MEASUREMENT

The difference between the true value and the measured value of a quantity is called error in measurement. The true value is the average of the infinite number of measurements and the measured value is the precise value.

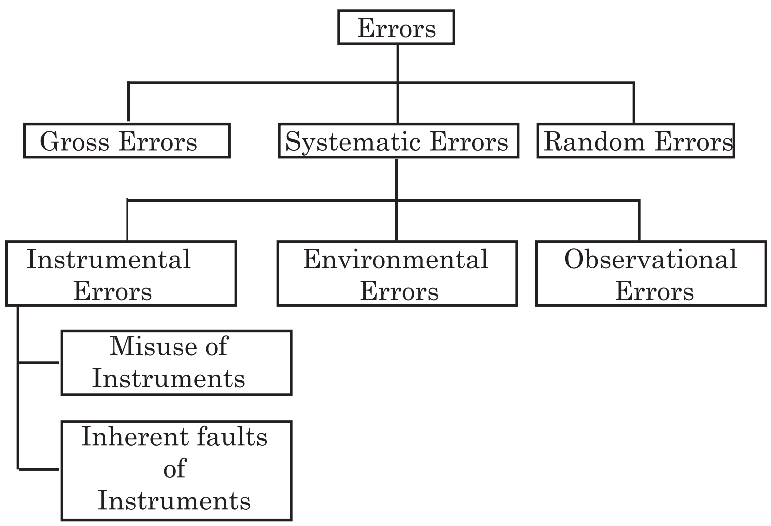
Errors may be due to faulty equipment, carelessness of the experimenter or random causes. The first two types of errors can be removed after detecting their cause but the random errors still remain. No specific cause can be assigned to such errors.

When an experiment is repeated many times, the random errors are sometimes positive and sometimes negative. Hence, the average of a large number of the results of repeated experiments is close to the true value.

1.6.1 Types of Errors

There are mainly three types of errors:

1. Random errors
2. Systematic errors
3. Gross errors



1.6.1.1 Random Errors

Random errors are due to unknown cause. Sometimes, these errors are called as *chance errors*. They depend on the individual measuring person. Even the same person repeating an observation may get different readings every time. For example, while measuring diameter of a wire with a screw gauge, one may get different readings in different observations due to non-uniform area of cross-section of wire at different places, the screw might have been tightened unevenly in observations, etc. In order to minimise random errors, measurements are repeated many times and arithmetic mean of all measurements is taken as the true value of the measured quantity. If a number of observation is made n times, the random errors reduced to $\left(\frac{1}{n}\right)$ times.

For example, when random errors in the arithmatic mean (AM) of 50 observations in m, then random error is the AM of 200 observations will be $\left(\frac{1}{200}\right)$.

1.6.1.2 Systematic Errors

Those errors that tend to be in one direction either positive or negative are called systematic errors. The systematic errors can be minimised by improving experimental techniques, removing personal errors and selecting better instruments as far as possible. For a given experimental set up, the systematic errors can be estimated to a certain extent.

Systematic errors are of various types:

(*i*) **Instrumental errors:** These errors are introduced due to improper designing and manufacturing defects of instrument. Often there may be zero error. For example, a meter scale may be worn off at the end of zero mark.

(*ii*) **Errors due to external factors:** These errors are due to fluctuation in atmospheric conditions like temperature, pressure and humidity.

(*iii*) **Errors due to imperfection:** These are introduced due to negligence of facts. For example, error in weighing of an object arising out of buoyancy, *i.e.*, usually ignored.

(*iv*) **Personal errors:** These errors are introduced due to lack of proper care of the observer. For example, lack of proper setting of the apparatus, recording the reading without applying proper precautions.

1.6.1.3 Gross Errors

The gross error occurs because of the human mistakes. For examples, consider the person using the instruments takes the wrong reading, or they can record the incorrect data. Such type of error comes under the gross error. The gross error can only be avoided by taking the reading carefully.

For example: The experimenter reads the 41.5°C reading while the actual reading is 31.5°C. This happens because of the oversights. The experimenter takes the wrong reading and because of which the error occurs in the measurement.

Such type of error is very common in the measurement. The complete elimination of such type of error is not possible. Some of the gross error easily detected by the experimenter but some of them are difficult to find.

Two methods can remove the gross error. These methods are:

(*i*) The reading should be taken very carefully.

(*ii*) Two or more reading should be taken of the measurement quantity. The readings are taken by the different experimenter and at a different point for removing the error.

Estimation of Errors, Absolute, Average, Percentage and Relative Errors

Estimation of Errors: Any statistical process which seeks to approximate an unknown value, such as a distribution is the estimation of errors.

Absolute Error: Suppose a physical quantity measured n times. Let the measured values are $a_1, a_2, a_3, ..., a_n$.

$$\therefore \quad \text{Arithmetic mean} = \frac{a_1 + a_2 + a_3 \ldots a_n}{n} \quad \ldots(i)$$

or

$$a_m = \frac{1}{n}\sum_{i=1}^{i=n} a_i \quad \ldots(ii)$$

Thus, the arithmetic mean a_m is taken as the true value of the quantity.

According to definition, absolute errors is the individual measured values of the quantity are

$$\Delta a_1 = a_m - a_1$$
$$\Delta a_2 = a_m - a_2$$
$$\ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ldots\ldots$$

$$\Delta a_n = a_m - a_n$$

The absolute errors can be positive in certain cases and negative in certain other cases.

Average absolute error: The average absolute error is the arithmetic mean of the magnitudes of absolute errors in all the measurements of the quantity. It is denoted by Δa_{av}. So,

$$\Delta a_{av.} = \frac{|\Delta a_1| + |\Delta a_2| + \ldots |\Delta a_n|}{n}$$

$$\Rightarrow \qquad \Delta a_{av} = \frac{1}{n}\sum_{i=1}^{i=n} |a_i|$$

So, we can write the final value.

$$a = a_m \pm \Delta a_{av}$$

Thus, the measurement of the quantity is likely to lie between $(a_m + \Delta a_{av})$ and $(a_m - \Delta a_{av})$.

Percentage Error: The relative error is expressed is percentage, is called percentage error.

So, percentage error, $\delta a = \dfrac{\Delta a_{\text{mean}}}{a_m} \times 100\%$

Relative Error: The ratio of mean absolute error to the mean value of the quantity measured is called relative error. It is denoted by δa. So,

$$\text{Relative error, } \delta a = \frac{\text{mean absolute error}}{\text{mean value}} = \frac{\Delta a_{\text{mean}}}{a_m}$$

Example. Actual length of a field is 100 meter. A measuring instrument shows the length to be 98 meter. Find,

(*a*) Absolute error in the measured length of field.
(*b*) Relative error in the measured length of the field.
(*c*) Percentage error in the measured length of the field.

Solution. (*a*) Absolute error = | 98 – 100 | = 2 meters.

(*b*) Relative error = $\dfrac{|98-100|}{100} = 0.02$

(c) Percentage error = relative error × 100 = 2.0%

1.6.2 Propagation of Errors

We know that the result of an experiment is calculated by performing mathematical operations (addition, subtraction, multiplication, division, raising to some power etc.) on several measurements, which have different degrees of accuracy. To find the net error in the result, we should study how errors propagate in different mathematical operations. The propagation of errors in the five different mathematical operations are given below:

1.6.2.1 Error in a Sum

Let $$x = a + b \qquad \ldots(i)$$

Suppose Δa = absolute error in measurement of a, Δb = absolute error in measurement of b and Δx = absolute error in calculation of x

i.e., sum of a and b

From equ. (*i*), we get

$$x \pm \Delta x = (a \pm \Delta a) + (b \pm \Delta b)$$

$$= (a + b) \pm \Delta a \pm \Delta b$$
$$= x \pm \Delta a \pm \Delta b$$
$$\therefore \qquad \pm \Delta x = \pm \Delta a \pm \Delta b.$$

The four possible values of Δx are $(+ \Delta a + \Delta b)$; $(+ \Delta a - \Delta b)$; $(- \Delta a + \Delta b)$; $(-\Delta a - \Delta b)$

Hence, the maximum absolute error in x is given as $\Delta x = \pm (\Delta a + \Delta b)$

Thus, the maximum absolute error in sum of the two quantities is equal to sum of the absolute errors in the individual quantities.

1.6.2.2 Error in a Difference

Let $x = a - b$ ...(*ii*)

Suppose Δa = absolute error in the measurement of a, Δb = absolute error in the measurement of b, Δx = absolute error in calculation of x *i.e.*, the difference of a and b.

From equ. (*ii*), we get,

$$x \pm \Delta x = (a \pm \Delta a) - (b \pm \Delta b)$$
$$= a \pm \Delta a - b \mp \Delta b$$
$$= (a - b) \pm \Delta a \mp \Delta b$$
$$x \pm \Delta x = x \pm \Delta a \mp \Delta b$$
$$\therefore \qquad \Delta x = \pm \Delta a \mp \Delta b$$

The four possible values of Δx are $(+ \Delta a - \Delta b)$; $(+\Delta a + \Delta b)$, $(- \Delta a - \Delta b)$ and $(- \Delta a + \Delta b)$.

Hence, the maximum absolute error in x is

$$\Delta x = \pm (\Delta a + \Delta b)$$

Thus, the maximum absolute error in difference of two quantities is equal to sum of the absolute errors in the individual quantities.

1.6.2.3 Error in a Product

Let, $x = a \times b$ ...(*iii*)

Suppose Δa = absolute error in the measurement of a, Δb = absolute error in the measurement of b, Δx = absolute error in calculating of x *i.e.*, the product of a and b

From equ. (*iii*), we get

$$\therefore \qquad (x \pm \Delta x) = (a \pm \Delta a) \times (b \pm \Delta b)$$
$$x\left(1 \pm \frac{\Delta x}{x}\right) = a\left(1 \pm \frac{\Delta a}{a}\right) b\left(1 \pm \frac{\Delta b}{b}\right)$$
$$= ab\left(1 \pm \frac{\Delta a}{a} \pm \frac{\Delta b}{b} \pm \frac{\Delta a}{a}\frac{\Delta b}{b}\right)$$
$$\Rightarrow \qquad x\left(1 \pm \frac{\Delta x}{x}\right) = x\left(1 \pm \frac{\Delta a}{a} \pm \frac{\Delta b}{b} \pm \frac{\Delta a}{a}\frac{\Delta b}{b}\right)$$
$$\Rightarrow \qquad 1 \pm \frac{\Delta x}{x} = 1 \pm \frac{\Delta a}{a} \pm \frac{\Delta b}{b} \pm \frac{\Delta a}{a}\frac{\Delta b}{b}$$
$$\Rightarrow \qquad \pm \frac{\Delta x}{x} = \pm \frac{\Delta a}{a} \pm \frac{\Delta b}{b} \pm \frac{\Delta a}{a}\frac{\Delta b}{b}$$

As $\left(\frac{\Delta a}{a}\right)$ and $\left(\frac{\Delta b}{b}\right)$ both are small, their product is still smaller and can be neglected.

$\therefore$ $$\pm\frac{\Delta x}{x} = \pm\frac{\Delta a}{a} \pm \frac{\Delta b}{b}$$

Four possible values of $\frac{\Delta x}{x}$ are given as $\left(+\frac{\Delta a}{a}+\frac{\Delta b}{b}\right);\left(+\frac{\Delta a}{a}-\frac{\Delta b}{b}\right);$ $\left(-\frac{\Delta a}{a}+\frac{\Delta b}{b}\right)$ and $\left(-\frac{\Delta a}{a}-\frac{\Delta b}{b}\right)$.

$\therefore$ The maximum possible value of

$$\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a}+\frac{\Delta b}{b}\right).$$

Thus, maximum fractional error or relative error in product of quantities is equal to sum of the frictional or relative errors in the individual quantities.

1.6.2.4 Error in a Quotient

Let, $$x = \frac{a}{b} \qquad \ldots(iv)$$

Let Δa = absolute error in the measurement of a, Δb = absolute error in the measurement of b and Δx = absolute error in the division of a and b

From equ. (iv), we get

$$x \pm \Delta x = \frac{a \pm \Delta a}{b \pm \Delta b}$$

$\Rightarrow$ $$x\left(1 \pm \frac{\Delta x}{x}\right) = \frac{a(1 \pm \Delta a / a)}{b(1 \pm \Delta b / b)}$$

$\Rightarrow$ $$x\left(1 \pm \frac{\Delta x}{x}\right) = \frac{a}{b}\left(1 \pm \frac{\Delta a}{a}\right)\left(1 \pm \frac{\Delta b}{b}\right)^{-1}$$

As, $\frac{\Delta b}{b} << 1$, so expanding by

Binomially, we get

$$x\left(1 \pm \frac{\Delta x}{x}\right) = x\left(1 \pm \frac{\Delta a}{a}\right)\left(1 \mp \frac{\Delta b}{b}\right)$$

$\Rightarrow$ $$1 \pm \frac{\Delta x}{x} = 1 \pm \frac{\Delta a}{a} \mp \frac{\Delta b}{b} \pm \frac{\Delta a}{a}\frac{\Delta b}{b} \qquad \ldots(v)$$

As $\left(\frac{\Delta a}{a}\right)$ and $\left(\frac{\Delta b}{b}\right)$ both are small, then their product $\left(\frac{\Delta a}{a}\right)\left(\frac{\Delta b}{b}\right)$ may be taken as negligibly small.

From equ. (v), we get $\pm\frac{\Delta x}{x} = \pm\frac{\Delta a}{a} \mp \frac{\Delta b}{b}$

$\therefore$ The four possible values of $\frac{\Delta x}{x}$ are $\left(+\frac{\Delta a}{a}-\frac{\Delta b}{b}\right);\left(+\frac{\Delta a}{a}+\frac{\Delta b}{b}\right),$ $\left(-\frac{\Delta a}{a}-\frac{\Delta b}{b}\right)$ and $\left(-\frac{\Delta a}{a}+\frac{\Delta b}{b}\right)$

Hence, the maximum value of $\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a}+\frac{\Delta b}{b}\right)$.

Thus, the maximum value of fractional or relative error in division of quantities is equal to sum of the fractional/relative errors in the individual quantities.

1.6.2.5 Error in a Measured Quantity Raised to a Power

Let $$x = \frac{a^n}{b^m} \quad \ldots(vi)$$

Let Δa = absolute error in the measurement of a, Δb = absolute error in the measurement of b and Δx = absolute error in calculation of x,

Taking log on both sides of equ, (vi), we get

$$\log x = \log a^n - \log b^m$$

$$\Rightarrow \quad \log x = n \log a - m \log b$$

Differentiating both sides, we get

$$\frac{dx}{x} = n\frac{da}{a} - m\frac{db}{b}$$

In terms of fractional error, we may rewrite the equation as

$$\pm\frac{\Delta x}{x} = \pm n\frac{\Delta a}{a} \mp m\frac{\Delta b}{b}$$

Hence, the maximum value of

$$\frac{\Delta x}{x} = \pm\left(n\frac{\Delta a}{a} \mp m\frac{\Delta b}{b}\right) \quad \ldots(vii)$$

From equ. (vii), we see that the fractional error or relative error in a quantity raised to power (n) is n times the fractional/relative error in the individual quantity.

Thus, the quantity with the maximum power should be measured with the highest degree of accuracy *i.e.*, with the least error.

Generally, let $X = \dfrac{a^l b^m}{c^n}$

$$\therefore \quad \frac{\Delta X}{X} = l\left(\frac{\Delta a}{a}\right) + m\left(\frac{\Delta b}{b}\right) + n\left(\frac{\Delta c}{c}\right).$$

Hence, the maximum relative error or percentage error in X = (l times the relative error or percentage error in a) + (m times the relative error or percentage error in b) + (n times the relative error or percentage error in c).

1.6.3 Significant Figures

The digits that are known reliably plus the first uncertain digit are called as significant figures.

The numerical value is read generally from some calibrated scale. To measure the length of an object we can put a metre scale in contact with the object. One end of the object may be made to coincide with the zero of the metre scale and the reading just in front of the other end is noted from the scale. If an electric current is measured by using an ammeter reading of the pointer on the graduation of the ammeter is noted. The value noted down includes all the digits that can be read directly from the scale and one doubtful digit at the end. The doubtful digit corresponds to the estimation of eye within the smallest sub-division is called the least count of the instrument. In a metre scale, the major graduations are at an interval of 1 centimetre and 10 sub-divisions are made between two consecutive major graduations. Hence, the smallest sub-division measures in millimetre. When one end of the object coincides with the zero mark of the metre scale, the other end may fall between 9.4 cm and 9.5 cm mark of the scale (Fig. 1.4). We estimate the distance between the 9.4 cm mark and the edge of the body as follows.

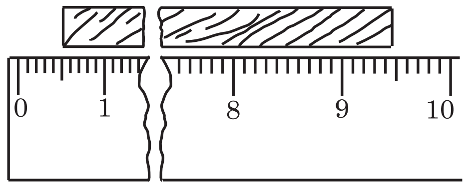


Fig. 1.4

We may note down the reading as 9.46 cm. The digits 9 and 4 are certain but 6 is doubtful. All these digits are significant figures/digits. We observe that the length is measured up to three significant figures/digits. The rightmost or the doubtful digit is called the **least significant figure/digit** and the leftmost digit is called the **most significant figure/digit**.

The significant figures express the accuracy with which a physical quantity may be expressed. The digits whose values are accurately known in a measurement are called its significant figures. In making a measurement it is necessary to read the instrument to its smallest scale division; least count. Lesser the least count of the instrument more will be the significant figures.

1.6.3.1 Significant Figures (Digits) in Calculations

If two or more numbers are added, subtracted, multiplied or divided, how to decide about the number of significant figure in the answer? For example, let the mass of an object X is measured to be 15.0 kg and of another object Y to be 7.0 kg. Find the ratio of the mass of X to the mass of Y. In arithmetic, ratio it is given as

$$\frac{15.0}{7.0} = 2.14285 \ldots$$

Here, all the digits of this answer cannot be significant. The zero of 15.0 is a doubtful digit and the zero of 7.0 is also doubtful. The quotient cannot have so many reliable digits. The rules for deciding the number of significant figures in an arithmetic calculation are listed below.

In a multiplication or division of two or more quantities, the number of significant figures in the answer is equal to the number of significant figures in the quantity which has the minimum number of significant figures. Hence, $\frac{15.0}{7.0}$ will have two significant figure only.

The insignificant figures are dropped from the result, when they appear after the decimal point. They are replaced by zeros, when they appear to the left of the decimal point. The least significant figure is rounded according to the rules given below.

When the digit next to the one rounded more than 5, the digit to the rounded is increased by 1. If the digit next to the one rounded is less than 5, the digit to be rounded is left unchanged. If the digit next to the one rounded is 5, then the digit to be rounded is increased by 1 if it is odd and is left unchanged if it is even.

For addition or subtraction when the numbers one below the other with all the decimal points in one line. Now locate the first column time has a doubtful digit. All digits right is the column are dropped from all the numbers and sounding is done to this column. The addition or subtraction is now performed to get the answer.

For examples, rounding off the following numbers of significant figures (*i*) 18381, (*ii*) 15.835, (*iii*) 15.850 (*iv*) 12.750×10^8

Here, (*i*) The 3rd significant figure is 3. This digit is rounded up. The next digit is 8 > 5. The 3rd digit must be increased by 1.

(*i*) Hence, 18381 becomes 18400, on rounding off three significant figures.

(*ii*) The 3rd significant figure in 15.835 is 8. The number next to it is less than 5 *i.e.*, 3 < 5. Hence, 15.835 becomes 15.8 on rounding of three significant figures.

(*iii*) 15.850 will becomes 15.8, the digit to be rounding off is odd and the digit next to it is 5.

(*iv*) 12.750×10^8 will become 12.8×10^8 because the digit to be rounding off is even and the digit next to it 5.

2. When a measured distance is to be 576.8 m, it has 4 significant figures 5, 7, 6 and 8. The digits 5, 7 and 6 are certain and reliable digits, while 8 is uncertain digit. Similarly, the radius measured by screw gauge is 3.24 cm, it has three significant figures 3, 2 and 4. The digits 3 and 2 are certain and reliable, while digit 4 is uncertain.

Clearly, the larger the number of significant figures is a measurement, greater is the precision of measurement and vice-versa.

Rules and identification of significant figures

Following rules are important for determining the number of significant figures:

Rule 1. All non-zero digits are significant. 91 has two significant digits 9 and 1 similarly 92.5 has three significant figures, 9, 2 and 5.

Rule 2. All zeros occurring between two non-zero digits are significant. 200.8 Kg has 4 significant figures, 2, 0, 0 and 8.

Rule 3. All zeros to the right of a decimal point and to the left of a non-zero digit are not significant. 0.0047 has two significant figures, 4 and 7.

Rule 4. All zeros to the right of the last non-zero digit are not significant. On the other hand, all zeros to the right of the last non-zero digit are significant provided they come from a measurement.

During calculations, the following rule is obeyed for identifications of the number of significant figures.

Do not retain a greater number of decimal places in a result computed from addition or subtraction or multiplication or division than in the observation which has the lowest decimal places.

SUMMARY

- Physics is a branch of science which deals with matter and energy and the relation between them.
- All the quantities in terms of which laws of physics are described and whose measurement are essential are called physical quantities.
- Those physical quantities which are independent of each other are called fundamental quantities. All other physical quantities which can be express in terms of fundamental quantities are called derived quantities.
- The chosen standard of measurement of a quantity which has essentially the same nature as that of the physical quantity is called the unit.
- The units chosen for the measurement of fundamental quantities are called fundamental units. The units obtained for derived quantities are called derived unit.
- In FPS system, the units of length, mass and time are foot, pound and second respectively. It is the British Engineering System of units.
- In CGS system, the units of length, mass and time are centimetre, gram and second respectively.

- In MKS system, the units of length, mass and time are metre, kilogram and second respectively.
- In SI System (International system of units), the 14th General Conference on Weights and Measures in 1971 agreed upon a system of units called the 'International System of Units' abbreviated as SI.
- The dimensions of a physical quantity are the powers to which the units of base quantities are raised to represent a derived unit of that quantity.
- The difference between the true value and the measured value of a quantity is called error in measurement.
- Random errors are due to unknown cause. Sometimes, these errors are called as *chance errors*.
- The systematic errors that tend to be in one direction either positive or negative.
- The gross error occurs because of the human mistakes. It can only be avoided by taking the reading carefully.
- Propagation of errors are the result of an experiment is calculated by performing mathematical operations (addition, subtraction, multiplication, division, raising to some power etc.) on several measurements, which have different degrees of accuracy.
- The digits that are known reliably plus the first uncertain digit are called as significant figures.
- For addition/ subtraction the result should have as many decimal places as the measured number with the smallest number of decimal places.(ex. 100.0 + 1.111 = 101.1)

SOLVED EXAMPLES

Example 1. *Check the correctness of the equation* $t = 2\pi\sqrt{\frac{l}{g}}$ *, where l is the length, t is the time period of pendulum and g is acceleration due to gravity.*

Solution. We have, $$t = 2\pi\sqrt{\frac{l}{g}}$$

We write the dimensions of the both sides of equation.

$$\therefore \quad \text{LHS} = [t] = [T] = [M^0L^0T] \qquad ...(i)$$

and $$\text{RHS} = 2\pi\sqrt{\frac{l}{g}} = \sqrt{\frac{L}{LT^{-2}}} = \sqrt{T^2} = T = [M^0L^0T] \qquad ...(ii)$$

Since eqn (*i*) and (*ii*) are equal.

$$\therefore \quad \text{LHS} = \text{RHS}$$

Thus, the given equation is correct.

Note:

(*i*) Numbers and dimensionless.

(*ii*) Mathematical constant like, π, e, etc. are dimensionless.

Example 2. *Convert 1 joule into erg.*

Solution. 1 joule is the unit of work in MKS system and 1 dyne is the unit of force in CGS system. The dimensional formula of work $[W] = [ML^2T^{-2}]$. Then, the dimensions of work in mass, length and time are $a = 1$, $b = 2$ and $c = -2$.

So, we convert MKS system into CGS system.

$$M_1 = 1 \text{ kg}, \quad L_1 = 1 \text{ m} \quad \text{and} \quad T_1 = 1 \text{ s}$$

and $$M_2 = 1 \text{ g}, \quad L_2 = 1 \text{ cm} \quad \text{and} \quad T_2 = 1 \text{ s}$$

Since, $$n_1 = 1 \text{ joule} \quad \text{and} \quad n_2 = ?$$

As, $$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c = 1\left(\frac{1 \text{ kg}}{1 \text{ g}}\right)^1 \left(\frac{1 \text{ m}}{1 \text{ cm}}\right)^2 \left(\frac{1 \text{ s}}{1 \text{ s}}\right)^{-2}$$

$$= \left(\frac{10^3 \text{ g}}{1 \text{ g}}\right)^1 \left(\frac{10^2 \text{ cm}}{1 \text{ cm}}\right)^2 \times 1$$

$$= 10^3 \times 10^2 = 10^7$$

$\therefore$ $$1 \text{ joule} = 10^7 \text{ erg.}$$

Example 3. *Consider a simple pendulum having a bob attached to a string that oscillates under the action of a force of gravity. Suppose that the period of oscillation of the simple pendulum depends on the length (l), mass of the bob (m) and acceleration due to gravity (g). Derive the expression for its time period using method of dimensions.*

Solution. Suppose $T \propto l^a g^b m^c$

where a, b and c are the dimensions.

or $$T = k\, l^a g^b m^c \quad \ldots(i)$$

where k is dimensionless constant. Writing the dimensions on both sides, we get

$$[M^{\circ} L^{\circ} T^1] = [L]^a [LT^{-2}]^b [M]^c$$

$$= [M^c L^{a+b} T^{-2b}]$$

Equating the dimensions of M, L and T on both sides, we get

$$a = 0, \quad a + b = 0, \quad -2b = 1 \quad \Rightarrow \quad b = -1/2 \text{ and } c = 0$$

$$a + b = 0$$

$\therefore$ $$a - \frac{1}{2} = 0 \quad \Rightarrow \quad a = 1/2$$

Putting the values of a, b and c in eqn. (i), we get

$$T = k\, l^{1/2} g^{-1/2} m^0$$

$\Rightarrow$ $$T = k\sqrt{\frac{l}{g}}$$

$\Rightarrow$ $$T = 2\pi\sqrt{l/g} \quad \text{where,} \quad k = 2\pi$$

Example 4. *Show that the expression $h = kT/(r\,\rho\, g)$ for ascent h of a liquid of surface tension T and density ρ in a capillary tube of radius r is dimensionally correct.*

Solution. First, we write the dimensions of the terms used,

$$h = [L], r = [L],$$

$$T = \text{surface tension} = [MT^{-2}],$$

$$\rho = [ML^{-3}] \text{ and } g = [LT^{-2}]$$

Now, taking the dimensions of both sides, we get

$$[L] = \frac{[M\,T^{-2}]}{[L]\,[M\,L^{-3}]\,[L\,T^{-2}]} = [L] \qquad (\because \ k \text{ has dimensionless})$$

As the dimensions of both sides of the equation are same. Thus, the relation is correct.

Example 5. *Show dimensionally that the frequency n of the transverse waves in a string of length l and mass per unit length m under a tension T is given by*

$$n = \frac{k}{l}\sqrt{\left(\frac{T}{m}\right)}$$

Solution. Given, $n = l^a T^b m^c$

or $n = k\, l^a T^b m^c$

Taking the dimensions on both sides, we get

$$[T^{-1}] = [L]^a [M L T^{-2}]^b [M L^{-1}]^c$$

or $T^{-1} = L^{a+b-c} M^{b+c} T^{-2b}$

Now, comparing the powers of L, M and T on both sides, we get

$$a + b - c = 0,\ b + c = 0 \text{ and } -2b = -1$$

Solving, we get $a = -1$, $b = (1/2)$ and $c = (1/2)$

$\therefore$ $n = k\, l^{-1} T^{1/2} m^{-1/2}$

or $$n = \frac{k}{l}\sqrt{\left(\frac{T}{m}\right)}$$

This is the required relation.

Example 6. *The focal length of convex mirror obtained by a student in repeated experiments are given below. What is the average value of focal length with uncertainty in ± σ limit?*

No. of observation	Focal length (in cm)
1	15.4
2.	15.2
3.	15.5
4.	15.1
5.	15.3
6.	15.2
7.	15.3
8.	15.4

Solution. We have, $\bar{f} = \frac{1}{8}\sum_{i=1}^{8} f_i \approx 15.3$

The value of σ is given in the table below:

i	f_i (cm)	$f_i - \bar{f}$ (cm)	$(f_i - \bar{f})^2$(cm^2)	$\Sigma(f_i - \bar{f})^2$ cm^2
1	15.4	0.1	0.01	
2	15.2	–0.1	0.01	
3	15.5	0.2	0.04	
4	15.1	–0.2	0.04	
5	15.3	–0.0	0.00	0.12
6	15.2	–0.1	0.01	
7	15.3	0.0	0.00	
8	15.4	0.1	0.01	

$$\sigma = \sqrt{\frac{1}{8}\Sigma_i(f_i - \overline{f})^2} = \sqrt{0.0312\ \text{cm}^2} = 0.12\ \text{cm} = 0.1\ \text{cm}.$$

Hence, the focal length is $f = (15.3 \pm 0.1)$ cm

Example 7. *In an experiment, two capacities measured are (1.2 ± 0.1)* μ *F and (2.3 ± 0.4)* μ*F. Find the total capacity in parallel with percentage error.*

Solution. Given, $C_1 = (1.2 \pm 0.1)\ \mu$ F and $C_2 = (2.3 \pm 0.4)\ \mu$ F.

In parallel,
$$C_p = C_1 + C_2$$
$$= 1.2 + 2.3 = 3.5\ \mu\text{F}$$
$$\Delta C_P = \pm(\Delta C_1 + \Delta C_2)$$
$$= \pm(0.1 + 0.4) = \pm 0.5$$

∴ $$\text{Percentage error} = \pm\frac{0.5}{3.5}\times 100 = \pm 8.6\%$$

Thus,
$$C_p = (3.5 \pm 0.5)\ \mu F$$
$$= 3.5\ \mu\text{F} \pm 8.6\%$$

Example 8. *The lengths of two rods are measured to be l_1 = (3.51 ± 0.02) cm and l_2 = 3.43 + 0.05 cm. Find difference in lengths with error limits.*

Solution. Given,
$$l_1 = (3.51 \pm 0.02)\ \text{cm},$$
$$l_2 = (2.43 \pm 0.05)\ \text{cm}$$

∴ $$l' = l_1 - l_2 = 3.51 - 2.43 = 1.08\ \text{cm}.$$

Now,
$$\Delta l' = \pm(\Delta l_1 + \Delta l_2)$$
$$= \pm(0.02 + 0.05) = \pm 0.07\ \text{cm}$$

∴ $$\text{Percentage error} = \pm\ \frac{0.07}{1.08} \times 100 = 6.48\%$$

Thus, difference in lengths
$$= (1.08 \pm 0.07)\ \text{cm} = 1.08\ \text{cm} \pm 6.48\%.$$

Example 9. *The length and breadth of a rectangular park are measured to be (3.1 ± 0.3) cm and (1.4 ± 0.2) cm. Find the area of the rectangular park with error limit.*

Solution. Given, length, (l) = (3.1 ± 0.3) cm and breadth (b) = (1.4 ± 0.2) cm

As,
$$\text{Area } (A) = \text{length } (l) \times \text{breadth } (b)$$

or
$$A = l \times b = 3.1 \times 1.4 = 4.34\ \text{cm}^2$$

Now,
$$\frac{\Delta A}{A} = \pm\left(\frac{\Delta l}{l} + \frac{\Delta b}{b}\right)$$
$$= \pm\left(\frac{0.3}{3.1} + \frac{0.2}{1.4}\right) = \pm\frac{0.52}{2.17}$$
$$\Delta A = \pm\frac{0.52}{2.17}\times A = \pm\frac{0.52}{2.17}\times 4.34 = \pm 1.04$$

∴ $$A = (4.34 \pm 1.04)\ \text{cm}^2.$$

Example 10. *The determination of $g = 4\pi^2 l/t^2$, when l and t are measured with ± 3% and ± 4% errors respectively. Find the percentage error.*

Solution. Given, $g = 4\pi^2 l/t^2$

$$\therefore \quad \frac{\Delta g}{g}\times 100 = \pm\left(\frac{\Delta l}{l}\times 100 + 2\frac{\Delta t}{t}\times 100\right)$$
$$= \pm(3 \pm 3 \times 4) = \pm 15\%.$$

Example 11. *Find 32.27 + 0.0813 + 378.2.*

Solution: We have,

32.27
0.0813
378.2

All the figures right to the column should be dropped after proper rounding. Then, we get

32.3
0.1
378.2
―――
410.6

Thus, required sum is 410.6.

Example 12. *Find* $\frac{43.1\times 1564}{23.4}$. *All the figures in this expression are significant.*

Solution. We have, $\frac{43.1\times 1564}{23.4} = 2880.70085.$

Here, the numbers 43.1, 23.4 have 3 significant figures and 1564 has four.

∴ Now, rounding 2880.70085... to three significant figures, it becomes 2881.

Hence, $\frac{43.1\times 1564}{23.4} = 2881.$

Example 13. *A student measure the period of oscillation of a simple pendulum. In successive measurements, the readings turn out to be 2.56 s, 2.63 s, 2.71 s, 2.42 s and 2.80 s. Find the absolute error, relative error and percentage error.*

Solution. Given, the observed values of time period are:

$$t_1 = 2.56\ s,\quad t_2 = 2.63\ s,\quad t_3 = 2.71\ s,\quad t_4 = 2.42\ s \quad \text{and} \quad t_5 = 2.80\ s.$$

∴ The mean period of oscillation of a simple pendulum,

$$t_m = \frac{2.56 + 2.63 + 2.71 + 2.42 + 2.80}{5} = \frac{13.12}{5} = 2.624\ s$$

∴ The mean time period to the second decimal place, $t_m = 2.62\ s$.

Now, the absolute errors in the measurement are:

$$t_m - t_1 = 2.62 - 2.56 = 0.06\ s$$
$$t_m - t_2 = 2.62 - 2.63 = -0.01\ s$$
$$t_m - t_3 = 2.62 - 2.71 = -0.09\ s$$
$$t_m - t_4 = 2.62 - 2.42 = 0.20\ s$$
$$t_m - t_5 = 2.62 - 2.80 = -0.18\ s$$

∴ Mean absolute error,

$$\Delta t_{\text{mean}} = \frac{0.06 + 0.01 - 0.20 + 0.18}{5}\ s = \frac{0.54}{5}\ s = 0.11\ s$$

$\therefore$ Period of simple pendulum, $t = (2.62 \pm 0.11)s$

So, t lies between $(2.62 + 0.11)$ s and $(2.62 - 0.11)$ s as 2.73 s and 2.51 s.

$$\text{The relative error} = \delta a = \pm \frac{0.11}{2.62} = \pm 0.041$$

$\therefore$ The percentage error,

$$\delta a \times 100 = \pm 0.41 \times 100 = \pm 4.1\%.$$

Example 14. *Find the area enclosed by a circle of diameter 1.06 m to correct number of significant figures.*

Solution. Given, $r = \dfrac{1.06}{2} = 0.53$ m

As, area $= \pi r^2$

$= 3.14 \times (0.53)^2 = 0.882026\ m^2 = 0.882\ m^2$

(Rounded to three significant figures).

TRUE/FALSE STATEMENTS

1. Newton is the unit of force in CGS system. (*F*, MKS system)
2. Work and energy have same dimensions. (*T*)
3. Dimension of light year is [L]. (*T*)
4. Sometimes dimensional analysis fails to determine the value of a constant. (*T*)
5. 1 erg is equal to 10^7 J. (*F*, 10^{-7} J)
6. Force is a fundamental physical quantity. (*F*, derived)
7. A dimensionless quantity may have a unit. (*T*)
8. Some constants have dimensions. (*T*, Plank's and Gravitational constant)
9. Time is derived quantity. (*F*, fundamental)
10. The round off value of 25.87 in the three significant figures is 25.8. (*F*, 25.9)
11. Systematic errors, unlike random errors, shift the results in one dimension (*T*)

FILL IN THE BLANKS

1. Physical quantities are used to ………and …..the physical situations.
2. Mass, ….. and ….. are the fundamental quantities.
3. SI stands for………. .
4. Dimension formula for kinetic energy is …….
5. The numbers of significant figures in 251.0 is…..
6. 234.6 + 17.12 = ……. as according to significant figures.
7. If the length of rod is 10 m and the error in measurement is 2 m, the measured value of rod will be ….. or 12 m.
8. The base quantity among the speed, area, length and weight is …………

Answers

1. express measure,	2. Length time,
3. System International	4. ML^2T^{-2}
5. 4	6. 251.7
7. 8 m	8. length

EXERCISE

1. What are fundamental forces of nature?
2. What do you know about fundamental and derived units?
3. Define astronomical unit, light year and par sec. How are they related?
4. The density of wood is 0.5 g/cc. Find its value in SI. [**Ans.** 500 kg/m^3]
5. Express the average distance of earth from the sun is
(*i*) light year (*ii*) par sec [**Ans.** (*i*) 1.58×10^{-5} ly (*ii*) 4.86×10^{-6} par sec]
6. Name three physical quantities which have same dimensions.
7. Explain the principle of homogeneity of dimensions. Find its uses.
8. Find the dimensions of universal gravitational constant. If its value in SI units is 6.6×10^{-11}, find its values in CGS system.
9. Does a quantity have different dimensions in different system of units.
10. What is meant by significant figures? Give rules for counting significant figures.
11. What are the limitations of dimensional analysis?
12. Convert an acceleration of 1 km/h^2 into cm/s^2.
13. Define dimensions of a physical quantity. What is dimensional formula?
14. Distinguish between fundamental and derive units.
15. Give the SI prefixes and their symbols for 10^{-15} and 10^{18}.
16. If velocity, time and force are choosen as base quantities find the dimensions of mass.
17. Find the relative error in z, if $z = \dfrac{A^4 B^{1/3}}{CD^{1/3}}$.
18. What is the propagation of errors. Give its detail operations.
19. If $f = a^2$, relative error in f would be how many times the relative error in a?
[**Ans.** two times]
20. Name the quantities represented by the dimensional formula [$M^1L^2T^{-1}$], [$M^1L^2T^{-2}$], [$M^1L^3T^0$.]
21. Energy and Young's modulus have the same dimension, comment.
22. Write the dimensions of a and b in the formula, $v = a + bt$, where v is the velocity and t is time. [**Ans.** LT^{-1}, LT^{-2}]
23. Name the three physical quantities which have same dimensions.

PROBLEMS

1. Check the correctness of the relation $t = 2\pi, \sqrt{\dfrac{l}{g}}$, where l is length, t is time period of a simple pendulum and g is acceleration due to gravity.
2. The frequency of vibration (ν) of a string may depend upon length (l) of the string, tension (T) in the string and mass per unit length (m) of the string. Using the method of dimensions, derive the formula for ν.
3. Check by the method of dimensions, the formula $v = \dfrac{1}{\lambda}\sqrt{\dfrac{K}{d}}$, where v is the velocity of longitudinal waves, λ is wavelength of wave, K is coefficient of elasticity and d is density of the medium. [**Ans.** Wrong]
4. The wavelength (λ) of matter waves may depend upon Planck's constant (h), mass (m) and velocity (v) of the particle. Use the method of dimensions to derive the formula.

$$\left[\textbf{Ans. } \lambda = \frac{kh}{mv}\right]$$

5. If the fundamental quantities are velocity (v), mass (m), time (t), what will be the dimensions of η in the equation, $V = \frac{\pi \rho r^4}{8 l \eta}$, where the symbols have their usual meanings [**Ans.** $M^1 L^{-1} T^{-2}$]
6. A physical quantity x is calculated from the relation, $x = \frac{a^2 b^3}{c\sqrt{d}}$. If the percentage error in a, b, c and d are 2%, 1%, 3% and 4% respectively, find the percentage error is x. [**Ans.** ± 12%]
7. In an experiment, the refractive index of glass was observed to be 1.44, 1.56, 1.45, 1.53, 1.54 and 1.54. Find (*i*) mean value of refractive index (*ii*) mean absolute error (*iii*) relative error (*iv*) percentage error.
   Express the result in terms of absolute error and percentage error.
   [**Ans.** (*i*) 1.51 (*ii*) ± 0.04 (*iii*) ± 0.03 (*iv*) ± 3%; μ = 1.51 ± 0.04, μ = 1.51 ± 3%]
8. The velocity of an object which has fallen freely under gravity varies $g^p h^q$, where g is acceleration due to gravity at a place and h is the height through which the object has fallen. Find the values of p and q. $\left[\textbf{Ans. } p = q = \frac{1}{2}\right]$
9. To deduce $P = \rho gh$, pressure exerted by a column of liquid depends upon (*i*) height of liquid column (*ii*) density ρ of the liquid (*iii*) acceleration due to gravity g at the place. [**Ans.** $P = k \rho gh$]
10. The volume of sphere is 2.78 m. Find its volume with due to regard to significant figures. [**Ans.** 11.3 m^3]
11. An object travels uniformly a distance of (13.8 ± 0.2) m is a time (4.0 ± 0.3)*s*. Find its velocity with error limits. Find the percentage error in velocity. [**Ans.** ± 9%]
12. A liquid of coefficient of viscosity η is flowing steadily through a capillary tube of radius r and length l. If V is the volume of liquid flowing per second. Find the pressure difference P at the end of the tube. $\left[\textbf{Ans. } P = \frac{8\eta \rho V}{\pi r^4}\right]$
13. In an experiment to measure the height of a bridge by dropping stone into water underneath, if the error in measurement of time is 0.1 s at the end of 2s, find the error in estimation of height of bridge. [**Ans.** 1.96 m]
14. If the energy, $E = G^p h^q c^r$, where G is the universal gravitational constant, h is the Planck's constant and c is the velocity of light. Find the values of p, q and r respectively. $\left[\textbf{Ans. } p = -\frac{1}{2}, q = \frac{1}{2} \text{ and } r = \frac{5}{2}\right]$
15. 5.74 g of a substance occupies 1.2 cm^3. Express its density by keeping the significant figures. [**Ans.** ρ = 4.8 g cm^{-3}]
16. Round off to four significant figures.
    (*i*) 37.873 (*ii*) 2.0096 [**Ans.** (*i*) 37.87, (*ii*) 2.009]
17. Write the dimensions of a/b in the relation $F = a\sqrt{x} + bt^2$, where F is force, x is distance and t is true. [**Ans.** $L^{-1/2}T^2$]

18. Determine the value of 60 J/min, on a system which has 100g, 100 cm and 1 min, as the fundamental units. [**Ans.** 2.16×10^6 new units]
19. The centripetal force (F) acting on a body may depend upon mass of the body (m), radius of circle (r) and frequency of revolution (ν). Derive the formula dimensionally. [**Ans.** $F = kmr\,\nu^2$]
20. The length and breadth of a rectangular hall are (5.7 ± 0.1) m and (3.4 ± 0.2) m. Find the area of the rectangular hall with error limits. [**Ans.** (19 ± 1.5) m^2]
21. Two cylinders having lengths measured as (1.8 ± 0.2)m and (2.3 ± 0.1) m. Find their combined length with error limits. [**Ans.** (4.1 ± 0.3)m]
22. The original length of a rope is (153.7 ± 0.6) cm. It is stretched to (155.3 ± 0.2) cm. Find the elongation in the wire with error limits. [Ans. (1.6 ± 0.8) cm]
23. To measure radius of curvature of a convex mirror using a spherometer, it was found that $l = (4.4 \pm 0.1)$ cm and $h = (0.085 \pm 0.001)$ cm. Find the maximum possible error in the radius of curvature. [**Ans.** 2.5 cm]
24. Each side of a cube is measured to be 7.203 m. Find the total surface area and the volume of the cube to appropriate significant figure. [**Ans.** 311.299 m^2, 373.7 m^3]

MULTIPLE CHOICE QUESTIONS

1. Which of the following have no dimension?
   (*a*) angle (*b*) work
   (*c*) force (*d*) speed
2. The length of a rod is (11.05 ± 0.05) cm. What is the total length of 2 such rods?
   (*a*) (22.10 ± 0.15) cm (*b*) (22.10 ± 0.10) cm
   (*c*) (22.10 ± 0.05) cm (*d*) (22.15 ± 0.10) cm
3. The dimensions of gravitational constant are
   (*a*) $[ML^2\,T^{-2}]$ (*b*) $[M^{-1}\,L^{-2}\,T^2]$
   (*c*) $[M^{-1}\,L^3\,T^{-2}]$ (*d*) $[M\,L^{-3}\,T^{-2}]$
4. Which one of the following has the dimensions of pressure?
   (*a*) $[ML^{-1}\,T^{-2}]$ (*b*) $[ML^{-1}\,T^{-1}]$
   (*c*) $[ML^{-2}\,T^{-2}]$ (*d*) $[MLT^{-2}]$
5. The number of significant figure in 3400 is
   (*a*) 1 (*b*) 2
   (*c*) 3 (*d*) 4
6. Length cannot be measured in
   (*a*) light year (*b*) micron
   (*c*) fermi (*d*) debye
7. The dimensions of energy are
   (*a*) $ML^2\,T^{-2}$ (*b*) $M^2\,L^2\,T^{-2}$
   (*c*) $ML\,T^{-2}$ (*d*) $ML\,T^{-1}$
8. The correct number of significant figures in 0.0003056 is
   (*a*) six (*b*) four
   (*c*) seven (*d*) eight
9. The unit of electric current is
   (*a*) ampere (*b*) coulomb
   (*c*) faraday (*d*) newton

10. 1 angstrom (Å) is equal to
   (*a*) 10^{-5} m (*b*) 10^{-6} m
   (*c*) 10^{-8} m (*d*) 10^{-10} m
11. If the error in the measurement of radius of a sphere is 2%, then the error in the determination of volume the sphere will be
   (*a*) 6% (*b*) 4%
   (*c*) 8% (*d*) 10%
12. Par sec is the unit of
   (*a*) time (*b*) distance
   (*c*) frequency (*d*) acceleration
13. What is percentage error in volume of a sphere, when error in measuring its radius is 2%?
   (*a*) ± 2% (*b*) ± 3%
   (*c*) ± 4% (*d*) ± 6%
14. Joule × sec is the unit of
   (*a*) energy (*b*) momentum
   (*c*) angular momentum (*d*) power
15. Which of the following can be expressed as dynes/cm^2?
   (*a*) Pressure (*b*) Longitudinal stress
   (*c*) Longitudinal strain (*d*) Young's modulus of elasticity
16. The velocity of a particle is given by
   $v = a\,t^2 + b\,t + c$
   If v is measured in ms^{-1} and t is measured in s, the unit of
   (*a*) a is ms^{-1} (*b*) b is ms^{-1}
   (*c*) c is ms^{-1} (*d*) a and b is same but that of c is different
   [**Hint:** unit of a = ms^{-3}, unit of b = ms^{-2} and unit of c = ms^{-1}]
17. The dimensional formula for force is
   (*a*) $M^0 L^2 T^{-2}$ (*b*) $M L T^{-2}$
   (*c*) $M L^2 T^{-2}$ (*d*) $M L^2 T^{-1}$
18. The dimensional formula for Planck's constant (h) is
   (*a*) $M L^{-2} T^{-3}$ (*b*) $M L^2 T^{-2}$
   (*c*) $M L^2 T^{-1}$ (*d*) $M L^{-2} T^{-2}$
19. The dimensions of velocity is
   (*a*) $M^0 L^2 T^{-2}$ (*b*) $M^0 L^2 T^{-2}$
   (*c*) $M L T^{-2}$ (*d*) $M^0 L T^{-1}$
20. The dimensional formula for bulk modulus of elasticity is
   (*a*) $M^1 L^{-2} T^{-2}$ (*b*) $M^1 L^{-3} T^{-2}$
   (*c*) $M^1 L^2 T^{-2}$ (*d*) $M^1 L^{-1} T^{-2}$

$$\left[\textbf{Hint: } k = \frac{\text{Volume stress}}{\text{Volume strain}} = \frac{\text{F/A}}{\Delta/\text{V}} = \frac{MLT^{-2}}{L^2} = ML^{-1}\,T^{-2}\right]$$

21. Out of the following the only pair that does not have identical dimensions is
   (*a*) angular momentum and Planck's constant
   (*b*) moment of inertia and moment of a force
   (*c*) work and torque
   (*d*) impulse and momentum
   [**Hint:** Moment of inertia = $M L^2$ and Moment of force = $M L^2 T^{-2}$]

22. Which of the following errors that tend directions either positive or negative?
    (*a*) random errors (*b*) systematic errors
    (*c*) gross errors (*d*) propagation errors
23. If the unit of length, mass and time each be doubled, the unit of work is increased
    (*a*) two times (*b*) four times
    (*c*) six times (*d*) no change
    [**Hint:** $W = Fs = M\,L\,T^{-2} \times L = M\,L^2\,T^{-2} = M\,(L^2/T^2)$
    When units are doubled, then new unit of work will be
    $$= 2M\,\frac{(2L)^2}{(2T)^2} = 2M(L^2/T^2)$$
    $\therefore$ Unit becomes two times.]
24. The dimensional formula for impulse is
    (*a*) $M\,L\,T^{-2}$ (*b*) $M\,L\,T^{-1}$
    (*c*) $M\,L^2\,T^{-1}$ (*d*) $M^2\,L\,T^{-1}$
25. Rounding off the significant digits of 14.650×10^{12} is
    (*a*) 14.7×10^{12} (*b*) 1.47×10^{12}
    (*c*) 14.7×10^{14} (*d*) 14.7×10^{15}
26. The dimensional formula for angular momentum is
    (*a*) $M\,L^2\,\mathrm{T}^{-2}$ (*b*) $M\,L^2\,\mathrm{T}^{-1}$
    (*c*) $M\,L\,T^{-1}$ (*d*) $M^0\,L^2\,T^{-2}$
27. 1 newton = ... dyne
    (*a*) 10^5 (*b*) 10^6
    (*c*) 10^7 (*d*) 10^8
28. The dimensions of power are
    (*a*) $M^1\,L^2\,T^{-3}$ (*b*) $M^2\,L^1\,T^{-2}$
    (*c*) $M^1\,L^2\,T^{-1}$ (*d*) $M^1\,L^1\,T^{-2}$

ANSWERS

1. (*a*)	**2.** (*b*)	**3.** (*c*)	**4.** (*a*)	**5.** (*b*)	**6.** (*d*)
7. (*a*)	**8.** (*b*)	**9.** (*a*)	**10.** (*d*)	**11.** (*a*)	**12.** (*b*)
13. (*d*)	**14.** (*c*)	**15.** (*a, b, d*)	**16.** (*c*)	**17.** (*b*)	**18.** (*c*)
19. (*a*)	**20.** (*d*)	**21.** (*b*)	**22.** (*b*)	**23.** (*a*)	**24.** (*b*)
25. (*a*)	**26.** (*b*)	**27.** (*a*)	**28.** (*a*)		