ONE

INTRODUCTION

In the study of electric circuits we are interested in the flow of electricity from one device to another. The simplest of electric devices will have a pair of terminals. 'Electricity' enters at one terminal and leaves from the other. The parameters of the device such as resistance etc., are assumed to be lumped or concentrated at one point. This is not quite true as is evident in the case of transmission lines. The resistance and other parameters of a transmission line are distributed throughout its length.

The energy associated with an electric system is measured by quantities like electric charge, potential and current. Historically electric charges were first observed by rubbing certain dry substances together. Atomic physicists now picture the electric charge as one of the building blocks of the universe. The negatively charged electron is one of the constituents of the atom. Its mass is 9.107×10^{-28} gm. As the electron is a very small unit, the practical unit of charge is the coulomb.

$$1 \text{ coulomb} = 6.24 \times 10^{18} \times Q_e$$

where Q_e is the electronic charge. The flow of electric charge (electrons) constitutes a current. The rate of flow of charge is defined as current (i) and its unit is the ampere. If one coulomb of charge is transferred in one second, the current is said to be one ampere.

$$i = \frac{dQ}{dt}.$$

Current has direction depending on the flow of charge.

Potential Difference (e). This is defined with respect to any two points and is measured in volts. Numerically, this is equal to the work done (in joules per coulomb) in moving a coulomb of charge between the two points.

$$1 \text{ volt} = 1 \text{ joule/coulomb.}$$

$$V = \frac{dW}{dQ}$$

Power can be obtained as a product of current and potential difference as

$$p = e \cdot i. \quad \text{...(1)}$$

The extent to which conduction of current (electrons) takes place is determined by the number of electrons which are free to move. In metals, a comparatively large current will be transmitted under a given applied electric pressure, as they have a larger number of free electrons. Silver, Copper and Aluminium are excellent conductors. In contrast rubber and porcelain are extremely poor conductors.

Electric Current

When one coulomb of electric charge continuously passes a given point every second, the electric current is said to be one ampere.

$$I = \frac{Q}{t} \quad \text{...(2)}$$

where I = current in amperes
Q = charge in coulombs
t = time in seconds.

Electromotive force (*EMF*)

This term represents the electric pressure or potential difference between two ends of a conductor that tends to create an electron flow. EMF may be developed in a battery or in a generator.

Electrical Resistance and Resistivity

Resistance in a circuit is the parameter which dissipates energy. Typical examples are the heating elements in stoves, filament bulbs etc. The amount of current passing through a conductor depends not only on the impressed voltage but on the properties of the conductor also. Electrical conductivity varies with different material. All substances may therefore be assumed to have a property which tends to oppose the flow of a current. This is called electrical resistance of a material and is very much like any other property of a material.

The unit of resistance is the ohm. The international ohm is defined to be the resistance at zero degrees centigrade of a column of mercury of uniform cross section, having a length of 106.3 cm and a mass of 14.4521 grams. The resistance of a conductor depends upon :

1. The material of the conductor.
2. Its length.
3. Cross-sectional area, and
4. The temperature.

The resistance of electrical materials are usually given per unit cross-sectional area and per unit length. This is called specific resistance or resistivity of the material represented by the letter ρ. Thus the resistivities of silver, copper and aluminium are 0.0147, 0.0173 and 0.0283 $\mu\ \Omega$-m. If ρ of a material is specified, then the resistance of conductor of that material with length l and area of cross-section a is

$$R = \frac{\rho l}{a} \qquad ...(3)$$

In the MKS system of units, l is in metres a in square metres, and hence the unit of specific resistance is ohm-metre.

Example 1.1. *Find the resistance of a copper wire 1 km long and 0.5 cm dia given the specific resistance of copper as 1.7×10^{-8} Ω-m*

Solution.
$$R = \frac{\rho l}{a}$$
$$l = \text{length in metres} = 1000$$
$$R = \frac{1.7 \times 10^{-8} \times 1000}{\frac{\pi}{4} \times \left(\frac{0.5}{100}\right)^2}$$
$$= \mathbf{0.866\ ohm}$$

Example 1.2. *A copper wire whose diameter is 0.162 cm has a resistance of 0.4 ohm. If the wire is drawn through a series of dies until its diameter is reduced to 0.032 cm, what is the resistance of the lengthened conductor ? Assume ρ remains constant.*

Solution. Let l_1 and a_1 be the original length and area of cross section of the conductor and let l_2 and a_2 be the corresponding values after passing through the dies.

$$0.4 = \frac{\rho l_1}{a_1}$$

where $$a_1 = \frac{\pi}{4}\left(\frac{0.162}{100}\right)^2$$

New Resistance $$R = \frac{\rho l_2}{a_2}$$

where $$a_2 = \frac{\pi}{4}\left(\frac{0.032}{100}\right)^2$$

As the volume of copper remains unchanged in the process,

$$a_1 l_1 = a_2 l_2$$

or

$$l_2 = \frac{a_1}{a_2} l_1$$

$$\therefore \quad \frac{R}{0.4} = \frac{l_2}{l_1} \frac{a_1}{a_2} = \left(\frac{a_1}{a_2}\right)^2 = \left(\frac{0.162}{0.032}\right)^4$$

i.e.

$$R = 0.4 \left(\frac{0.162}{0.032}\right)^4 = \mathbf{262.7\ ohms.}$$

Example 1.3. *A bread toaster has a Nichrome unit ($\rho = 112 \times 10^{-8}$ Ω-m) of resistance 32 ohms. What is the resistance of a copper conductor of equal area of cross section but 10 times as long as the Nichrome wire ?*

Solution.

$$\rho_{Cu} = 1.7 \times 10^{-8} \text{ ohm-m}$$

$$\rho_{Ni} = 112 \times 10^{-8} \text{ ohm-m}$$

$$R_{Ni} = 32 = 112 \times 10^{-8} \times \frac{l_{Ni}}{a_{Ni}}$$

$$R_{Cu} = \rho_{Cu} \frac{l_{Cu}}{a_{Cu}}$$

$$\frac{R_{Cu}}{32} = \frac{\rho_{Cu}}{\rho_{Ni}} \cdot \frac{l_{Cu}}{l_{Ni}} \cdot \frac{a_{Ni}}{a_{Cu}}$$

$$= \frac{1.7 \times 10^{-8}}{112 \times 10^{-8}} \times 10 = \frac{17}{112}$$

$$\therefore \quad R_{Cu} = \frac{32 \times 17}{112} = \mathbf{4.857\ ohms.}$$

Example 1.4. *An overhead busbar of copper is a tube with 3 cm internal diameter and 0.5 cm thickness. Find the resistance between the two ends of the busbar, if its length is 10 metres.*

Solution.

$$R = \frac{\rho l}{a}$$

$$\rho = 1.7 \times 10^{-8} \text{ ohm-m for copper}$$

$$l = 10 \text{ m}$$

$$a = \frac{\pi}{4}\left(\frac{4^2 - 3^2}{100 \times 100}\right) \text{sq. m}$$

$$\therefore \quad R = \frac{1.7 \times 10^{-8} \times 10 \times 4 \times 10000}{\pi \times 7}$$

$$= \mathbf{3.09 \times 10^{-4}\ ohm.}$$

Example 1.5. *A 240 V generator supplies a motor through a pair of aluminium wires each 400 metres long. The terminal* *voltage at the motor is 230 V and the current supplied is 20 A. Find the area of cross-section of the wires required (specific resistance of aluminium 2.83×10^{-8} ohm-m).*

Solution. Voltage drop in the wires

$$= 240 - 230 = 10 \text{ V}$$

Drop in each conductor $= 5$ V

Current flowing $= 20$ A

$\therefore$ Resistance of each wire $= \frac{5}{20} = 0.25\ \Omega$

$$= \frac{\rho l}{a}$$

$$\therefore \quad a = \frac{2.83 \times 10^{-8} \times 400}{0.25} \text{ sq. m}$$

$$= \mathbf{0.4528 \text{ sq. cm.}}$$

Conductance and Conductivity

The reciprocal of resistance is called conductance, it is represented by the letter G and its unit is the mho (Ω) siemens.

$$G = \frac{1}{R}$$

The reciprocal of resistivity is called conductivity and is represented by the symbol ν

$$\nu = \frac{1}{\rho} \text{ mho/metre.}$$

Temperature-Resistance Effect

It has been experimentally found that the resistance of most of the metals increases as the temperature is raised. This change is approximately proportional to the temperature change, except for very low and very high temperatures. Therefore within the usual operating range, the resistance may be said to vary linearly with temperature changes.

The change in resistance per ohm per degree temperature variation is called temperature co-efficient of resistance and is represented by the symbol α.

α itself varies with temperature. Thus α for copper is 0.00402 per °C at 20°C.

If R_1 is the resistance of a conductor at t_1 °C and

R_2 is its resistance at t_2 °C, then

$$R_2 = R_1\, [1 + \alpha_1(t_2 - t_1)] \qquad \text{...(4)}$$

The value of α_1 is the temperature co-efficient of resistance at the temperature t_1 °C.

Example 1.6. *The resistance of a coil of wire increases from 50 ohms at 25°C to 62 ohms at 75°C. Calculate the temperature coefficient of the material of the wire at 0°C.*

Solution. The resistance at 25°C, $R_{25} = 50 = R_0\ [1 + \alpha_0\ (25 - 0)]$ and at 75°C, $R_{75} = 62 = R_0\ [1 + \alpha_0\ (75 - 0)]$

Dividing one by the other,

$$\frac{62}{50} = \frac{1 + 75\ \alpha_0}{1 + 25\ \alpha_0}$$

$$1.24\ (1 + 25\ \alpha_0) = 1 + 75\ \alpha_0$$

$$0.24 = \alpha_0\ (75 - 31) = 44\ \alpha_0$$

or

$$\alpha_0 = 0.24/44 = 0.00545.$$

Example 1.7. *An incandescent lamp takes 2.5 Amps at 240 V at the instant of switching it on. At the normal operating temperature the current drops to 0.2 Amp. Find the temperature of the heated filament. Take room temperature as 20°C and the corresponding temperature co-efficient of the filament as 0.005 per °C.*

Solution. Let t be the normal operating temperature.

$$R_{20} = \frac{240}{2.5} = 96 \text{ ohms.}$$

$$R_t = \frac{250}{0.2} = 1200 \text{ ohms}$$

$$R_t = R_{20}\ [1 + \alpha_{20}\ (t - 20)]$$

$$1200 = 96\ [1 + 0.005\ (t - 20)]$$

or

$$t = \frac{1113.6}{0.48} = \mathbf{2320°C.}$$

Example 1.8. *Two conductors, one of copper and the other of iron, are connected in parallel. At 20°C, they carry equal currents. If the temperature is now raised to 150°C, what proportion of the total current will each conductor carry ?*

$\alpha_{Cu} = 0.0043$; $\alpha_{Fe} = 0.0063$ per°C at 20°C.

Solution. At 20°C, the currents carried by the 2 conductors are equal. Hence their resistances are equal at 20°C say R.

Let R_{Cu} be the resistance of the copper conductor at 150°C, and R_{Fe} the corresponding resistance of the iron conductor at 150°C.

$$R_{Cu} = R\ [1 + 0.0043\ (150 - 20)] = 1.559\ R$$

$$R_{Ee} = R\ [1 + 0.0063\ (150 - 20)] = 1.819\ R$$

The currents will be in the inverse proportion of the resistances.

Hence current in the copper conductor : Current in the iron conductor

i.e. 1.819 : 1.559

∴ Current in the copper conductor

$$= \frac{1.819}{1.819 + 1.559} = \frac{1.819}{3.378}$$

$= \mathbf{0.539}$ **of the total current.**

Example 1.9. *The resistance temperature co-efficient for a certain copper wire is 0.004 and for carbon filament it is 0.0003. How many ohms of carbon filament in series with a 75 ohm copper wire will make the total combined resistance invariant with temperature.*

Solution. Let the required carbon filament resistance be R ohms. The resistance of the copper wire increases with temperature and is given by

$$R_{Cu} = 75\,(1 + 0.004\,t)$$

where t is the difference of temperature.

The increase in copper wire resistance is

$$75\,(1 + 0.004\,t) - 75 = 0.3\,t.$$

The resistance of the carbon filament decreases with temperature

$$R_c = R\,(1 - 0.0003\,t)$$

The decrease $= R - R\,(1 - 0.0003\,t) = 0.0003\,Rt$.

If the total resistance of copper wire and the carbon filament is to remain constant, the increase of resistance of the copper wire with temperature must be compensated by the decrease of resistance of the carbon filament.

i.e., $0.3\,t = 0.0003\,Rt$

$$\therefore \quad R = \frac{0.3}{0.0003} = \mathbf{1000\ ohms.}$$

Example 1.10. *Show that* α_1, *the resistance temperature co-efficient at* t_1 *°C is given by*

$$\frac{1}{\alpha_1} = \frac{1}{\alpha_0} + t,$$

where α_0 *is the resistance temperature co-efficient at 0°C.*

Solution. Let R_0, R_1 and R_2 be the resistances at temperatures 0°, t_1° and t_2°C respectively.

Then $R_1 = R_0 (1 + \alpha_0 t_1)$

$R_2 = R_0 (1 + \alpha_0 t_2)$

Also $R_2 = R_1 [1 + \alpha_1 (t_2 - t_1)]$

Substituting for R_1 and R_2.

$$R_0 (1 + \alpha_0 t_2) = R_0 (1 + \alpha_0 t_1) [1 + \alpha_1 (t_2 - t_1)]$$

$$1 + \alpha_0 t_2 = (1 + \alpha_0 t_1) [1 + \alpha_1 (t_2 - t_1)]$$

or $$0 = \alpha_1 (t_2 - t_1) - \alpha_0 (t_2 - t_1) + \alpha_0 \alpha_1 (t_2 - t_1) t_1$$

i.e., $$\alpha_0 \alpha_1 t_1 + \alpha_1 = \alpha_0$$

or $$\frac{1}{\alpha_1} = \frac{1}{\alpha_0} + t_1.$$

Similarly it can be shown that α_2, the temperature co-efficient at t_2°C is given by

$$\frac{1}{\alpha_2} = \frac{1}{\alpha_1} + (t_2 - t_1)$$

Insulation Resistance

Current carrying cable conductors are provided with one or more layers of insulating material over them. Although the normal current is along the length of the conductor, small cross wire currents called leakage currents, exist and flow radially outward through the insulation. This is not a desirable condition and is as a result of the insulating material not being perfect. The resistance offered to the flow of the leakage current is called insulation resistance.

To compute the insulation resistance of a cable of length 'l' metres, conductor radius r_1 metres, outer cable radius r_2m, consider an annular strip at a distance of r metres from the centre and of thickness dr metres, as shown in Fig. 1.1.

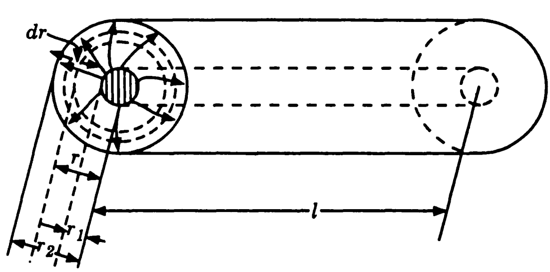

Fig. 1.1

The length of leakage current flow in the strip = *dr* metres (radially out)

area of cross-section = $2\pi rl$

Resistance of the annular strip

$$= \frac{\rho dr}{2\pi rl} \text{ ohms.}$$

where ρ is the specific resistance of the insulating material used. The total resistance offered by the insulating material is

$$R_{in} = \int_{r_1}^{r_2} \frac{\rho dr}{2\pi rl} = \frac{\rho}{2\pi l} \ln\left(\frac{r_2}{r_1}\right) \text{ohms} \qquad ...(5)$$

The insulation resistance decreases with increase of length of cable.

Example 1.11. *An insulated cable has an insulation resistance of 600 Mega ohms-per km. The internal and external diameters of the insulation are 1.5 and 3 cms. Calculate the specific resistance of the insulation material used.*

If the insulation resistance is to be increased by 50% by using another layer of insulating material of resistivity 6×10^{12} Ω-m, find the additional thickness of insulation required.

Solution. $l = 1000$ metres

$r_2 = 1.5$ cm $\quad r_1 = 0.75$ cm

$$R = 600 \times 10^6 = \frac{\rho}{2\pi\ 1000} \log_e \frac{1.5}{0.75}$$

$$\rho = \frac{2\pi \times 1000 \times 600 \times 10^6}{\log_e 2}$$

$$= 5.44 \times 10^{12} \text{ ohm-m.}$$

Extra insulation resistance

$$= 300 \times 10^6$$

$$= \frac{\rho}{2\pi \times 1000} \log_e \frac{D/2}{3/2}$$

$$300 \times 10^6 = \frac{6 \times 10^{12}}{2\pi \times 1000} \log_e \frac{D}{3}$$

$$\log_e \frac{D}{3} = \frac{\pi}{10} = 0.314$$

Solving $\quad D = 4.12$ cm.

thickness of insulation required

$$= \frac{4.12 - 3}{2} = \mathbf{0.56\ cm.}$$

EXERCISES

1.1. A coil of copper wire ($\rho = 1.7 \times 10^{-8}$ ohm-m) has a length of 600 metres. What is the length of an aluminium conductor ($\rho = 2.83 \times 10^{-8}$ ohm-m) if its area of cross-section and the resistance are the same as those of the copper wire. [**Ans.** 360.4 m]

1.2. A copper wire of a certain length and diameter having a resistance of 4 ohms is drawn successively through dies till its diameter is halved. What is the resistance of the lengthened conductor. Assume that the resistivity remains unchanged. [**Ans.** 64 ohms]

1.3. A copper wire of unknown length has a diameter of 0.25 cm and a resistance of 0.28 ohm. By passing through several successive dies the diameter is reduced to 0.05 cm. Assume ρ to remain unchanged. Find the resistance of the reduced size wire. [**Ans.** 175 ohms]

1.4. The copper field winding of an electric machine has a resistance of 46 ohms at 22°C. What will be its resistance at 75 °C ($\alpha = 0.0043$ per °C)

[**Ans.** 56.5 Ω]

1.5. An incandescent lamp has a tungsten filament whose resistance is 1000 ohms at the operating temperature of 2800°C. Calculate the resistance of the filament just before switching on the lamp at room temperature of 20°C. $\alpha_{20} = 0.005$ per °C. [**Ans.** 67.1 Ω]

1.6. The resistance of a given electric device is 40 ohms at 25°C. If the temperature co-efficient of resistance of the material is 0.00454 at 20°C, determine the temperature at which the resistance of the device is 80 ohms.

[**Ans.** 250.3°C]

1.7. A platinum resistance thermometer has a resistance of 20 ohms at 20°C. When placed in a furnace, the resistance increases to 134 ohms. Find the temperature of the furnace assuming temperature co-efficient of resistance of platinum at 20°C as 0.0032 per °C. [**Ans.** 1801.25°C]

1.8. Repeat Problem 1.7 if α is given as 0.00342 per°C. [**Ans.** 1800.67°C]

1.9. The conductor and insulation resistances of a cable 100 meter long are 0.001 Ω and 2000 Mega Ω respectively. What will these resistances be for a similar cable 100 km long. [**Ans.** 1 Ω, 2 M Ω]

1.10. Find the conductor and insulation resistances of a cable 15 km long given the following : conductor diameter = 1.5 cm thickness of insulation 0.5 cm, resistivity of conductor material = 1.7×10^{-8} Ω-m ; resistivity of insulation material = 6×10^{12} Ω-m. If the cable is cut into two equal bits what are the values of the conductor and insulation resistances of each bit ?

[**Ans.** 1.443 Ω, 32.5 MΩ, 0.7215Ω, 65 MΩ]

1.11. If three lengths of cable having insulation resistances of 330, 660 and 990 Mega ohms respectively are joined together to form a single cable, what is its insulation resistance. [**Ans.** 180 MΩ]