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Introduction to Electric Power Systems—Generalised Circuit Constants

Introduction—Transmission System as a Vital Link in Power Systems—Classification of Transmission Lines—Review on Short, Medium and Long Lines—Illustrations—General Circuit Equations—Relations between the Generalized Circuit Constants ABCD—Generalized Constants of Simple Networks—Charts of Transmission Line Constants—Constants of Combined Networks—Ferranti effect—Losses in transmission Lines on open circuit—Tuned Power Lines.

Introduction. Power System Engineering is that special branch of Electrical Engineering which concerns itself with the technology of generation, transmission and distribution of electrical power. The power system growing into a vast and complex system represents one of the most vital systems in every modern nation. The Power System Engineer has an uphill task in designing and operating the Power System, and is confronted with a number of challenging problems.

The special responsibility of a Power System is that, in addition to maintaining the generation of electrical power at adequate level, the power has to be transmitted to the various load centres in response to the changing demands, in proper, form and quality, in accordance with the individual consumers' specifications.

The Power Systems which were hitherto operating in isolation are in the process of being interconnected. The interconnected operation offers many advantages such as improved economy and reliability ; but these benefits can be realised only after the implementation of a number of sophisticated controls both at the local level and at the system's level. Power System Engineer should have a thorough knowledge regarding the operation and control of interconnected Power Systems which will be of immense help, especially when interconnecting the local grids to form an integrated power pool.

In the last few years, there has been a gradual increase in academic interest in the field of Power System Engineering, essentially due to the energy crisis all over the world Numerous problems concerned with the planning and operation of large interconnected Power Systems are being studied both in academic circles as well as by practising engineers. Although the present book is essentially devoted to Analysis, stability and protection of Electric Power Systems, it is worth noting that the courses in this area are being updated with considerable emphasis on the use of computers, control, system theory, & *c*.

The course on hand is a necessary pre-requisite to the study of methods of analysis, stability and protection problems pertaining to the Electric Power Systems.

The current Chapter deals with the principles and performance characteristics of transmission systems and development of generalized circuit constants as applicable to Power System problems.

1.1. Classification of Transmission Lines

The student is expected to have studied the principles of transmission line and its properties, in earlier courses. Essentially, a transmission line possesses resistance, inductance, capacitance and leakance or leakage conductance. While the first two are represented as series constants in the portrayal of a transmission circuit, the last two are displayed as shunt circuit constants. In fact, all the four are distributed parameters and, as such, will have to be taken cognizance of appropriately, depending on the voltage and length of the transmission line in question. The constants may be lumped suitably in the representation on an equivalent circuit depicting the transmission system.

The transmission lines may be classified as short, moderately long (or medium) and long lines. In the case of overhead transmission lines of upto, say, 80 km length, they may be branded as short lines, in which case the shunt constants, *viz.*, capacitive susceptance and leakance may be ignored, with the result that the electrical equivalent circuit may be shown as in Fig. 1.1, by means of a simple series impedance as a lumped constant

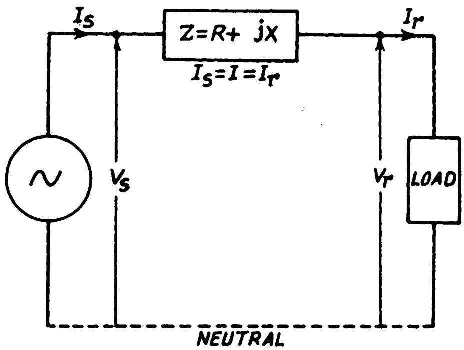


Fig. 1.1. Short transmission line.

$$\dot{Z} = R + jX$$

where R is the total resistance in ohm per phase, X is the total inductive reactance per phase and Z, the series impedance represented in the complex form ;

$$|\dot{Z}| = \sqrt{R^2 + X^2}$$

is the numerical value of the impedance.

The transmission line may be three-phase or single phase. Fig. 1.1 shows an electrical equivalent circuit per phase of the three-phase line. In the case of single-phase line, R and X denote the resistance and reactance of both conductors of the single phase circuit.

The lines of length between 80 km and, say, 240 km may be treated as moderately long (medium) lines, in which case they may be represented by nominal 'T' or 'π' method as shown in Figs. 1.2 and 1.3 respectively. It is to be noted that Z is the total series impedance of the entire line and Y, the total shunt admittance of the line. As in Fig. 1.1, these two figures show the single phase electrical equivalent circuits, irrespective of whether the transmission line is three phase or single phase. Accordingly, the voltages and currents at the sending and receiving ends represent the per-phase values.

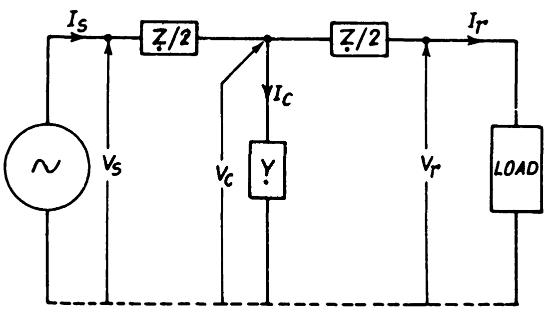


Fig. 1.2. Medium lines : Nominal 'T' circuit.

V_s = sending end voltage per phase

V_r = receiving end voltage per phase

I_s = sending end current per phase

I_r = receiving end current per phase

$\dot{Z} = R + jX$ = series impedance per phase

$\dot{Y} = G + jB$ = shunt admittance per phase

where G = leakance or leakage conductance per phase,

B = capacitive susceptance per phase.

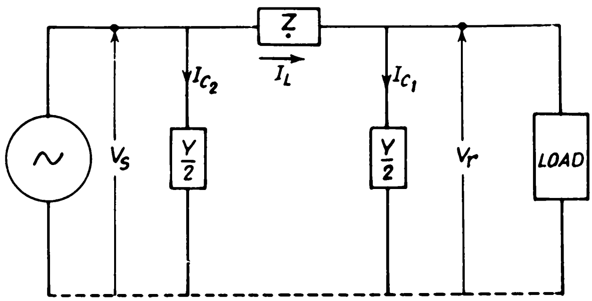


Fig. 1.3. Medium lines : Nominal 'π' circuit.

In case of medium lines, the leakance may generally be ignored, unless otherwise specified, in which case

$$\dot{Y} = jB = j\omega\, C_o$$

where ω = radian frequency of the power supply,

C_o = capacitance to neutral of the transmission line in Farad.

The transmission lines more than 240 km long are considered as long lines, in which case a more rigorous solution is required. The impedance and admittance are treated as uniformly distributed, rather than lumped, unlike the short and medium lines.

With the above introduction, we shall proceed to consider the performance of short, medium and long lines separately, with appropriate examples incorporated in the pertinent articles.

1.2. Short Transmission Lines

As seen from Fig. 1.1, the sending and receiving end currents are equal, $I_s = I_r = I$ (say). The receiving end voltage is equal to the sending end voltage *minus* the voltage drop in the transmission line.

$$\dot{V}_r = \dot{V}_s - \dot{I}\,\dot{Z} \qquad ...(1.1)$$

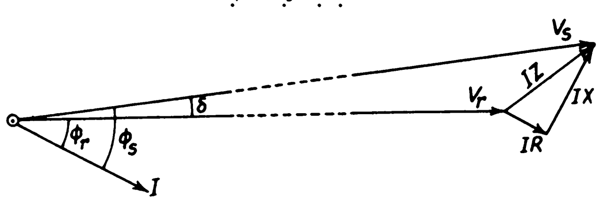


Fig. 1.4. Phasor diagram, for short transmission line.

It should be noted that the Eq. (1.1) is a vector relation between the two end voltages and $\dot{I}\,\dot{Z}$ is the impedance drop in vector form, $\dot{I}$ and $\dot{Z}$ being current and impedance phasors respectively. The impedance drop should be clearly distinguished from the arithmetic difference between the measured values of sending end voltage $\dot{V}_s$ and receiving end voltage V_r. Fig. 1.4 shows the vector relation between the above voltages. It is assumed that the line delivers a load at lagging power factor, ϕ_r being the angle of lag. δ is the angle by which the sending end voltage leads the receiving end voltage and is referred to as "displacement angle".

$$\dot{V}_s = \dot{V}_r + \dot{I}\dot{Z}$$

Sending end voltage is equal to the receiving end voltage *plus* the impedance drop in the line. The above relation is clearly the same as Eq. (1.1). If the load characteristics are specified, *viz.*, V_r, I, and ϕ_r are given, the sending end voltage V_s and power factor cos ϕ_s can be evaluated. ϕ_s is the phase angle between the sending end voltage and current. It is clearly seen that for lagging power factor of load, the sending end power factor is less than the receiving end power factor, as the line adds to the equivalent inductive reactance of the lagging load.

In a practical transmission system, step-up and step-down transformers may be used at the sending and receiving ends of the transmission line, as shown in the single line diagram of Fig. 1.5 with the result that the overall transmission system may be represented by the same model as in Fig. 1.1, by adding the series impedances contributed by the transformers to the series impedance of the transmission line.

Thus $$\dot{Z} = \dot{Z}_{T_1} + \dot{Z}_L + \dot{Z}_{T_2}$$

where $\dot{Z}$ = total transmission system impedance

$\dot{Z}_{T_1}$ = Equivalent series impedance of the transformer T_1, referred to transmission line (H.V. side of the transformer).

$\dot{Z}_{T_2}$ = Equivalent series impedance of the transformer T_2, referred to transmission line (H.V. side of the transformer).

It is assumed that the shunt admittance of each transformer is too small to be taken into consideration. The equivalent single phase circuit of Fig. 1.5 is depicted in Fig. 1.6.

Fig. 1.5. Single line diagram of a transmission system comprising power transformers and transmission line.

The sending end voltage and power factor can be evaluated by adding vectorially the impedance drop to the receiving end voltage to determine the sending end voltage in vector-form. The phase angle at the sending end is determinable from the above. A typical example is given below :

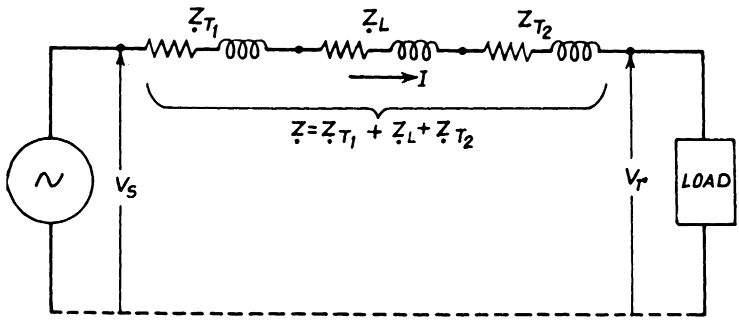


Fig. 1.6. Equivalent single phase circuit of the transmission system in Fig. 1.5.

Example 1.1. *A three-phase load of 5 MVA at 80% power factor lagging is supplied at 11 kV from a step-down transformer having a ratio of 3 : 1 (line values). The primary side of the transformer is connected to a transmission line ; the line constants are : resistance per conductor, 2 ohm, and reactance of line to neutral, 3 ohm. The resistance and reactance per phase of the primary windings of the transformer (star-connected) are 5 ohm and 10 ohm respectively, and the corresponding values for the secondary windings (delta-connected) are 1.5 and 3.0 ohm respectively per phase. Determine the voltage and power factor at the sending end of the transmission line. Neglect shunt admittances of the transformer and line.*

Solution. A single line diagram for the problem is shown in Fig. 1.7. At the receiving end of the transmission line, a transformer is used with a line voltage ratio of 3 : 1, *viz.*, 33 kV to 11 kV. As the primary windings are star-connected and the secondary delta-connected, the phase voltage ratio from primary to secondary would be $\sqrt{3}$: 1.

$$k = \sqrt{3} = \text{primary to secondary turn ratio}$$

The equivalent series impedance of the H.V. winding is $5 + j\,10$ ohm. The equivalent series impedance of the L.V. winding (secondary) referred to H.V. side would be $k^2 \times$ actual impedance.

$$= k^2(1.5 + j\,3.0) = 3\,(1.5 + j\,3.0) \text{ as } k = \sqrt{3}$$
$$= 4.5 + j\,9.0 \text{ ohm.}$$

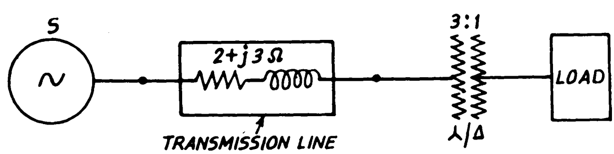


Fig. 1.7. Single line diagram for Example 1.1.

Hence the total equivalent series impedance of the entire transformer referred to H.V. side would be

$$\underset{\cdot}{Z}_T = 5 + j\,10 + 4.5 + j\,9.0$$
$$= 9.5 + j\,19 \text{ ohm.}$$

The single phase equivalent circuit is shown in Fig. 1.8.

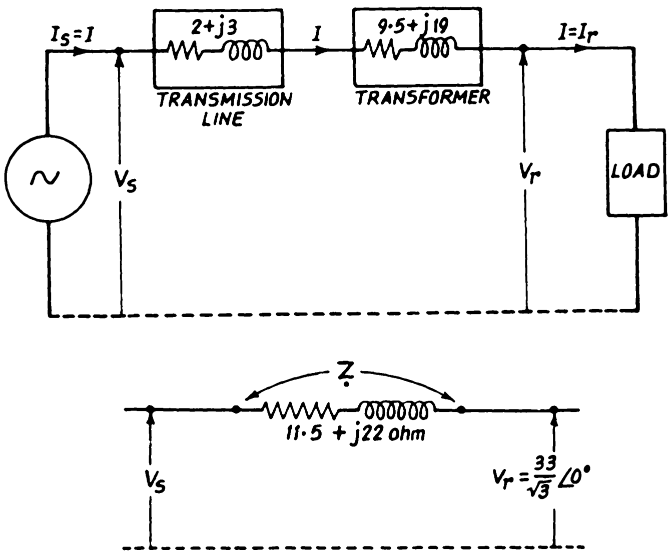


Fig. 1.8. Single phase equivalent circuit for Fig. 1.7.

Taking V_r as the reference, and referring the load data to the H. V. side of the transmission circuit,

$$V_r = \frac{33}{\sqrt{3}} \underline{/\,0^\circ} = 19.05 \text{ kV to neutral } \underline{/\,0^\circ}$$

As the load is 5 MVA at 0.8 p.f. lagging, the load current would be

$$I = \frac{5000}{\sqrt{3} \times 33} = 87.5 \text{ Amp.}$$

$$\underset{\cdot}{I} = 87.5 \,\overline{\big\backslash 36.0^\circ} \text{ Amp.}$$

the angle being $-36.9°$, *viz.* current lagging by 36.9° behind the reference voltage V_r.

$$= 87.5\,(0.8 - j\,0.6)$$
$$= 70 - j\,52.5$$
$$\underset{\cdot}{Z} = 11.5 + j\,22\ \Omega$$
$$\underset{\cdot}{I}\,\underset{\cdot}{Z} = (70 - j\,52.5)\,(11.5 + j\,22)$$
$$= 1960 + j\,936.25$$

Therefore,
$$\dot{V}_s = \dot{V}_r + \dot{I}\dot{Z} = 19050 + j\,0 + 1960 + j\,936.25 \text{ Volt}$$
$$= 21010 + j\,936.25 \text{ Volt}$$
$$= 21031 \text{ volt } \underline{/2.55^\circ}$$
$$= 21.031 \text{ kV } \underline{/2.55^\circ} \text{ (per phase)}$$

Line to line voltage at the sending end
$$V_s\,(L-L) = 36.43 \text{ kV}.$$

In the phasor diagram of Fig. 1.4 as applied to this example,
$$\delta = 2.55^\circ$$

Sending end power factor angle,
$$\phi_s = \phi + \delta = 36.9 + 2.55 = 39.45^\circ.$$

Sending end power factor,
$$\cos\phi_s = \cos 39.45^\circ$$
$$= \mathbf{0.772 \text{ lagging.}}$$

Incidentally, the power at the sending end, which is the power supplied to the transmission system would be
$$\sqrt{3} \times 36.43 \times 87.5 \times 0.772 \text{ kW}$$
$$\mathbf{P_s = 4262.2 \text{ kW}.}$$

Power delivered at the receiving end, *viz.*, to the load
$$P_r = 5000 \times 0.8$$
$$= 4000 \text{ kW}.$$

Efficiency of the transmission system is thus found to be
$$\frac{P_r}{P_s} = \frac{\text{Power delivered to the load}}{\text{Power input to the transmission system}}$$

$$\left.\begin{array}{l}\text{Transmission}\\ \text{Efficiency}\end{array}\right\} = \frac{4000}{4262.2} \times 100\% = \mathbf{93.85\%}.$$

1.2.1. Regulation of Transmission Lines

The voltage regulation of a transmission line (or transmission system) may be defined as the per unit or percentage change in voltage at the receiving end when full load is thrown off, at the prescribed power factor.

If V_r = receiving end voltage at the specified load and power factor, and V_{r_0} = receiving end voltage when the load is thrown off, the regulation ε may be expressed as follows :

$$\varepsilon = \frac{V_{r0} - V_r}{V_r} \text{ p.u.}$$
$$= \frac{V_{r0} - V_r}{V_r} \times 100\% \qquad ...(1.2)$$

If V_r is held constant at the specified load, and V_s adjusted to hold V_r at that level, then $V_{r0} = V_s$ (when the load is thrown off). Consequently, the value of V_{r0} can be determined from the phasor diagram of Fig. 1.4, by resolving V_s into two components, one along V_r-phasor and the other, perpendicularly.

$$V_{r0}^2 = (V_r + IR\cos\phi_r + I \times \sin\phi_r)^2 + (I \times \cos\phi_r - IR\sin\phi_r)^2$$

If the second part on the right hand side of the above expression fades into a negligible quantity, an approximate result will be as follows :

$$V_{r0} = V_s + IR\cos\phi_r + I \times \sin\phi_r \qquad ...(1.3)$$

(ϕ_r is positive for lagging power factor, and negative for leading power factor).

From (1.2) and (1.3), it is seen that the regulation may be approximately expressed as

$$\frac{IR\cos\phi_r + I \times \sin\phi_r}{V_r} \times 100\%$$

As an example, if the full load resistance and reactance drops are given as 3% and 5% of the receiving end voltage on full load, the regulation at any power factor lagging or leading would be ($IR\cos\phi_r + I \times \sin\phi_r$)% if the drops are represented in percentage values.

Here $IR = 3$

$IX = 5$

$\cos\phi_r = 0.8$

$\sin\phi_r = 0.6$

Regulation at 0.8 p.f. lagging

$= (3 \times 0.8 + 5 \times 0.6)\% = 5.4\%.$

Regulation at 0.8 p.f. leading

$= (3 \times 0.8 - 5 \times 0.6)\% = -0.6\%.$

Regulation at unity p.f.

$= 3.0\%.$

It is seen from the above example that at a leading power factor, there is a tendency for the pressure to rise on throwing off the load or an improvement in regulation.

It follows that if a capacitor is connected across a load, for the purpose of improving the overall receiving end power factor, there will be an improvement of regulation due to the reduction in lagging reactive power delivered by the line. To illustrate the effect of the presence or absence of a capacitor, an example is given below.

Example 1.2. *A single phase 50 Hz generating station supplies an inductive load of 6000 KW at a power factor of 0.6 by means of an overhead transmission line 20 km long. The resistance per km of each conductor is 0.015 ohm and the loop inductance is 0.75 mH per km. The voltage at the receiving* *end is maintained constant at 11 kV and a capacitor is connected across the load to raise the power factor to 0.9 lagging. Calculate.*

(a) the capacitance of the capacitor, and

(b) the generating station voltage when the capacitor is (i) in use, (ii) disconnected.

Solution. Resistance of both conductors for 20 km length

$$= 0.015 \times 2 \times 20\ \Omega$$

$$R = 0.6 \text{ ohm.}$$

Loop reactance $X = 2\pi \times 50 \times 0.75 \times 10^{-3} \times 20$

$$X = 4.712 \text{ ohm.}$$

Thus total series impedance of the single-phase transmission lines is

$$Z = 0.6 + j\,4.712 \text{ ohm.}$$

(*a*) Let the original power factor be $\cos \phi_1$ and the improved power factor $\cos \phi_2$ when capacitor forms a part of the receiving end load.

In this example,

$$\cos \phi_1 = 0.6, \qquad \sin \phi_1 = 0.8$$

$$\cos \phi_2 = 0.9, \qquad \sin \phi_2 = 0.436.$$

Fig. 1.9 shows the original and modified KVA, as given by the lengths *OB* and *OC* respectively, the corresponding power factor angles being ϕ_1 and ϕ_2 respectively. The reactive KVA (or KVAR) of the lagging load is given by

$$AB = 6000 \times \frac{0.8}{0.6} = 8000.$$

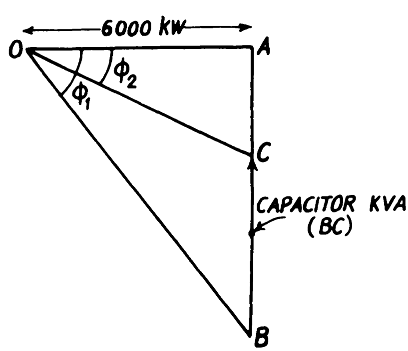


Fig. 1.9. For Example 1.2.

The modified KVAR of the overall receiver load is indicated by the length *AC*, which means that there is a reduction in the reactive power supplied through the transmission line, by virtue of an improved overall receiver power factor. The difference in KVAR, as signified by the length *BC* corresponds to the contribution of the capacitor installed at the receiving end across the load.

Thus, $\quad BC = \text{capacitor KVAR} = AB - AC$

$$= 8000 - 6000 \times \frac{0.436}{0.9}$$

$$= 8000 - 2907 = \mathbf{5093.}$$

Let C be the capacitance of the capacitor used, in Farad.

Then $\quad V_r\,(V_r\omega C) = 5093 \times 1000,$

neglecting capacitor loss.

Substituting $\quad V_r = 11000$ V

$$\omega = 2\pi \times 50$$

$$C = 133.9\ \mu\text{F say, } \mathbf{134\ \mu F.}$$

(*b*) (*i*) *To find the Generating Station voltage, V_s when the capacitor is in service.*

The combination of the load and capacitor will have a lagging power factor of 0.9.

Receiving end current

$$I = \frac{6000}{11 \times 0.6} = 606 \text{ Amp.}$$

Taking current as the reference phasor,

$$I = 606 + j\,0$$

$$V_r = 11000\,(0.9 + j\,0.436) = 9900 + j\,4796 \text{ volt.}$$

$$IZ = 606\,(0.6 + j\,4.712)$$

$$= 363.6 + j\,2855 \text{ volt}$$

$$V_s = V_r + I\,Z = 10263.6 + j\,7651 \text{ V}$$

$$\mathbf{V_s = 12.80\ kV.}$$

(*ii*) *Generating Station voltage when the capacitor is disconnected.*

When the capacitor is disconnected, the transmission line current is exactly the same as the load current, at 0.6 p.f. lagging.

$$I = \frac{6000}{11 \times 0.6} = 909.1 \text{ Amp.}$$

The current will therefore increase by about 50%, resulting in an increase of impedance drop proportionately.

$$V_r = 11000\,(0.6 + j\,0.8)$$

$$= 6600 + 8800 \text{ V.}$$

$$I\,Z = 909.1\,(0.6 + j\,4.712)$$

$$= 545.5 + j\,4283.7 \text{ V}$$

$$V_s = V_r + IZ$$

$$= 7145.5 + j\,13084 \text{ V}$$

$$V_s = \sqrt{7145.5^2 + 13084^2} \text{ V} = 14.91 \text{ kV.}$$

There is thus an increase in voltage required at the sending end, by about 2.1 kV, consequent to disconnection of the capacitor.

1.2.2. Parallel operation of short transmission lines

Duplication or operation in multiple of transmission lines (either overhead or underground) is commonly practised in order to improve the reliability of service by maintaining continuity of supply especially to an important industrial area, so that on the event of one of the lines developing defects or being subjected to faults, the other or others will carry or share the load until the fault is located and rectified. The lines may or may not follow the same transmission-route. The lines may be overhead or underground cables, or both. If all the lines follow the same route and their design features are identical, then the total transmitted load will be shared equally by them. If, however, the routes are different, as also their circuit constants, then the respective share will not be the same and can be computed from the knowledge of the circuit parameters.

Load-division. Consider two transmission lines, in parallel, whose impedances per phase are

$$\dot{Z}_1 = R_1 + jX_1$$

and

$$\dot{Z}_2 = R_2 + jX_2$$

Fig. 1.10 shows an equivalent single phase circuit for two transmission lines in parallel. Let I be the total load current and I_1, I_2 the currents delivered by lines 1, 2 respectively.

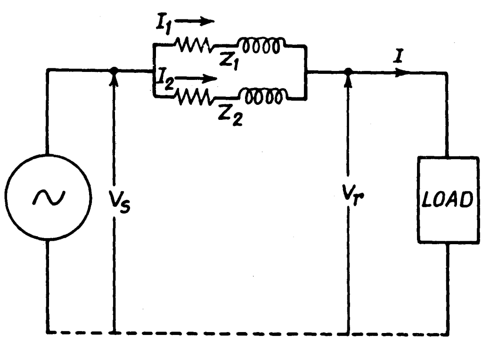


Fig. 1.10. Two transmission lines with impedances Z_1 and Z_2, in parallel.

As the two lines are paralleled at the sending as well as receiving end, they are represented by the two impedances in parallel.

Let Z be the combined impedance of the two lines in parallel.

Then $$\dot{Z} = \frac{\dot{Z}_1 \dot{Z}_2}{\dot{Z}_1 + \dot{Z}_2} \qquad \text{...(1.4)}$$

$$\underset{\cdot}{I} = \underset{\cdot}{I}_1 + \underset{\cdot}{I}_2 \qquad \text{...(1.5)}$$

As Z_1 and Z_2 are in parallel, the voltage drops across them shall be equal. Consequently.

$$\underset{\cdot}{I}_1\underset{\cdot}{Z}_1 = \underset{\cdot}{I}_2\underset{\cdot}{Z}_2 = \underset{\cdot}{I}\,Z \qquad \text{...(1.6)}$$

Hence
$$\underset{\cdot}{I}_1 = \frac{\underset{\cdot}{Z}}{\underset{\cdot}{Z}_1}\underset{\cdot}{I} = \frac{\underset{\cdot}{Z}_2}{\underset{\cdot}{Z}_1 + \underset{\cdot}{Z}_2}\underset{\cdot}{I} \qquad \text{...(1.7)}$$

on substitution from (1.4).

Similarly,
$$\underset{\cdot}{I}_2 = \frac{\underset{\cdot}{Z}}{\underset{\cdot}{Z}_2}\underset{\cdot}{I} + \frac{\underset{\cdot}{Z}_1}{\underset{\cdot}{Z}_1 + \underset{\cdot}{Z}_2}\underset{\cdot}{I} \qquad \text{...(1.8)}$$

It should be noted that all the quantities in the above expressions are phasors, and any difference in the impedance angles should be taken into consideration. The power factor of the transmitted power along the lines need not be the same as that of the total receiver load.

The currents may be replaced by complex powers $\underset{\cdot}{S}_1$, $\underset{\cdot}{S}_2$ and $\underset{\cdot}{S}$ where $\underset{\cdot}{S}_1$ is the KVA or MVA supplied through line 1, represented as a polar vector, the angle associated being the power factor angle. Similarly, S_2 and S are also complex powers.

If the load power factor angle is cos ϕ, and the power factors of the transmitted powers through the lines 1 and 2 are cos ϕ_1 and cos ϕ_2 respectively, the complex powers may be expressed in the polar form as follows :

$$\left.\begin{aligned} \underset{\cdot}{S} &= S \underline{/\phi} \\ \underset{\cdot}{S}_1 &= S_1 \underline{/\phi_1} \\ \underset{\cdot}{S}_2 &= S_2 \underline{/S_2} \end{aligned}\right\} \qquad \text{...(1.9)}$$

Replacing the currents in equations (1.7) and (1.8) by the complex powers,

$$\left.\begin{aligned} \underset{\cdot}{S}_1 &= \frac{\underset{\cdot}{Z}_2}{\underset{\cdot}{Z}_1 + \underset{\cdot}{Z}_2}\underset{\cdot}{S} \\ \underset{\cdot}{S}_2 &= \frac{\underset{\cdot}{Z}_1}{\underset{\cdot}{Z}_1 + \underset{\cdot}{Z}_2}\underset{\cdot}{S} \end{aligned}\right\} \qquad \text{...(1.10)}$$

If S, S_1 and S_2 are expressed in cartesian form,

$$\left.\begin{aligned} \underset{\cdot}{S}_1 &= P_1 + jQ_1 \\ \underset{\cdot}{S}_2 &= P_2 + jQ_2 \\ \underset{\cdot}{S} &= P + jQ \end{aligned}\right\} \qquad \text{...(1.11)}$$

where P and Q represent the total true power and reactive power respectively. If the load is given in MVA, the true power is in MW and the reactive power, in MVAR.

$$\left.\begin{aligned} P &= S\cos\phi\,; & Q &= S\sin\phi \\ P_1 &= S_1\cos\phi_1\,; & Q_1 &= S_1\sin\phi_1 \\ P_2 &= S_2\cos\phi_2\,; & Q_2 &= S_2\sin\phi_2 \end{aligned}\right\} \qquad \text{...(1.12)}$$

The manner in which the parallel lines will share a common load is illustrated by the following examples.

Example 1.3. *Two short three-phase transmission lines operating in parallel supply a 6 MW, 66 kV, balanced load having a power-factor of 0.8 lagging. The resistance and reactance of one line are 4 ohm and 6 ohm respectively and those of the other line, 5 ohm each. Determine the current, power and power factor of the load carried by each line.*

Solution. The impedances of the two lines are expressed in cartesian and polar forms, as follows :

$$Z_1 = 4 + j6 = 7.21 \angle 56.0^\circ \text{ ohm}$$

$$Z_2 = 5 + j5 = 7.07 \angle 45^\circ \text{ ohm}$$

$$Z_1 \,.\, Z_2 = 9 + j11 = 14.21 \angle 50.7^\circ \text{ ohm}$$

$$\text{Load current } I = \frac{6000}{\sqrt{3} \times 66 \times 0.8} = 65.61 \text{ Amp.}$$

Taking the receiving end voltage as reference,

$$V_r = \frac{66}{\sqrt{3}} \angle 0^\circ \text{ kV/phase}$$

$$I = 65.61 \overline{/36.9^\circ} \text{ Amp.}$$

as the p.f. = 0.8 lagging. $$I_1 = \frac{Z_2}{Z_1 + Z_2} I$$

$$= \frac{7.07 \angle 45^\circ}{14.21 \angle 50.7^\circ} \times 65.61 \overline{/36.9^\circ}$$

$$= 32.64 \overline{/42.6^\circ} \text{ Amp.}$$

Power factor of load carried by line 1

$$= \cos 42\ 6^\circ = 0.736 \text{ lagging.}$$

Load carried by line 1

$$= \sqrt{3} \times 66 \times 32.64 \times 0.736 = 2746 \text{ KW}$$

$$\mathbf{P_1 = 2.75\ MW,} \quad \textbf{(say)}$$

$$I_2 = \frac{Z_1}{Z_1 + Z_2} I$$

$$= \frac{7.21 \angle 56.3^\circ}{14.21 \angle 50.7^\circ} \times 65.61 \overline{/36.9^\circ}$$

$$= 33.30 \overline{/31.3^\circ} \text{ Amp.}$$

Power factor of load carried by line 2

$$= \cos 31.3° = 0.854 \text{ lagging}$$

$$P_2 = \sqrt{3} \times 66 \times 33.30 \times 0.854$$

$$= 3251 \text{ KW}$$

$$\mathbf{P_2 = 3.25\ MW\ (say).}$$

If only the MW loading of each line is required, it is not necessary to calculate the current supplied by the line. The expression for S_1 or S_2 as in (1.10) can be directly used to find the complex power, real part of which gives the true power.

Example 1.4. *A 66 kV Generating Station is to supply a load of 15 MW at 63.8 kV and at a power factor of 0.8 lagging. The transmission line is designed such that the efficiency of transmission is 97% :*

(*a*) *What must be the resistance and reactance of the line ?*

(*b*) *If the load carrying capacity of the transmission line has to be raised to 20 MW by erecting a parallel line, all the other conditions remaining unaltered, what must be the constants of the second line ?*

Solution. (*a*) Line current

$$= \frac{15,000}{\sqrt{3} \times 63.8 \times 0.8} = 169.7 \text{ Amp.}$$

As the efficiency of transmission is given as 97%, the total line loss will be 3% of the power supplied at the sending end, *viz.*, $15,000 \times \frac{3}{97}$ KW.

Let R be the resistance of each conductor. Total loss in the three phases

$$= 3I^2 R = 3 \times 169.7^2 \times R$$

$$= \frac{15000 \times 3000}{97} \text{ W}$$

whence $\mathbf{R = 5.37\ ohm.}$

Taking current as the reference phasor,

$$I = 169.7 + j\,0$$

$$V_r = \frac{63.8}{\sqrt{3}} (0.8 + j\,0.6) \text{ kV}$$

$$= (29.47 + j\,22.1) \text{ kV}$$

Impedance drop $\dot{I}\,\dot{Z} = 169.7\,(5.37 + jX)$ volts

$$\dot{V}_r = 29,470 + j22,100 \text{ V}$$

$$\dot{V}_s = \dot{V}_r + \dot{I}\,\dot{Z}$$

$$V_s = \frac{66,000}{\sqrt{3}} \text{ V}$$

$$= 38100 \text{ V/Phase}$$

$$V_s = 29470 + j22100 + 911.3 + j169.7\,X$$

$$(38{,}100)^2 = (30{,}380)^2 + (22{,}100 + 169.7\,X)^2$$

Solving, $\mathbf{X = 5.26\ ohm.}$

Thus, the constants of the line are

$$R_1 = 5.37 \text{ ohm}$$
$$X_1 = 5.26 \text{ ohm}$$
$$Z_1 = 5.37 + j5.26 \text{ ohm.}$$

(*b*) A new parallel line will have to carry the extra load, *viz.*, 20 – 15 = 5 MW, while the original line will continue to carry its design load of 15 MW, under the same conditions, namely, at the same efficiency and the same regulation. As the power factor of the total load is the same and also the receiving end voltage, as in case (*a*), the current transmitted by the new line should be 1/3 of that carried by the first line as the MW loading is 1/3 of that of the first line. As the impedance drops across the two parallel lines should be equal, the new line impedance Z_2 should be thrice that of the first line. As the ratio of line loss to load is the same for both lines, it is also seen that the resistance of the second line should be thrice that of the original line.

Therefore, $\dot{Z}_2 = 3\,\dot{Z}_1 = 3(R_1 + jX_1)$

$$= 3(5.37 + j5.26) = \mathbf{16.11 + j15.78\ ohm.}$$

Example 1.5. *Two single-phase transmission lines are connected in parallel. Their impedances are (2 + j3.5) ohm and (3.5 + j2.5) ohm respectively. The sending end voltage is 11 kV. Deduce and draw a locus diagram for the receiving end voltage for a total current of 600 Amp at various power factors. From the diagram, find the voltage regulation for a power factor at the receiving end of 0.8 lagging. Check the result by calculation. Determine also the KW load transmitted by each line at that power factor.*

Solution. Let Z_1 and Z_2 be the impedances of the two lines.

$$\dot{Z}_1 = 2 + j\,3.5 = 4.03\,\underline{/60.3^\circ}\ \Omega$$

$$\dot{Z}_2 = 3.5 + j\,2.5 = 4.30\,\underline{/35.5^\circ}\ \Omega$$

$$\dot{Z}_1 + \dot{Z}_2 = 5.5 + j\,6.0 = 8.14\,\underline{/47.5^\circ}\ \Omega$$

$$\dot{Z} = \frac{\dot{Z}_1\,\dot{Z}_2}{Z_1 + Z_2}$$

$$= \frac{(4.03 \times 4.30)\,\underline{/95.8^\circ}}{8.14\,\underline{/47.5^\circ}} = 2.13\,\underline{/48.3^\circ}$$

Total current $I = 600\,\underline{/0^\circ}$ Amp.

is chosen as the reference for drawing the locus diagram.

Construction of locus diagram. [See Fig. 1.11].

Draw the reference line OX, which is the line depicting the orientation of the current chosen as the reference vector.

Voltage drop in the line

$$= \underset{\cdot}{I}\,\underset{\cdot}{Z} = 600 \times 2.13 \underline{/48.3^\circ}$$

$$= \mathbf{1278 \underline{/48.3^\circ} \text{ volts.}}$$

The impedance drop phasor makes an angle 48.3° with the reference axis. Draw OA equal to 1278 volt (to an appropriate scale), making an angle of 48.3° with the horizontal, as shown in Fig. 1.11. Draw the horizontal AB

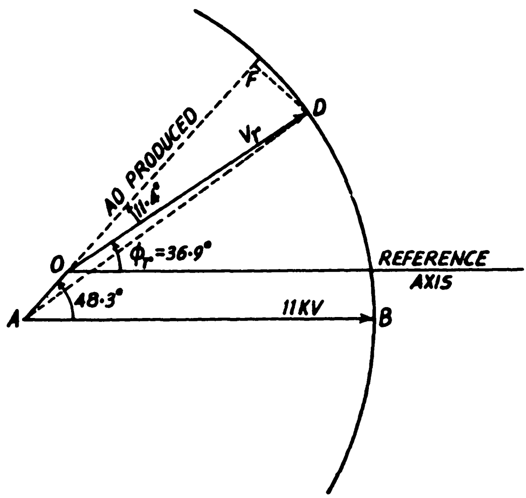


Fig. 1.11. Locus diagram for receiving end voltage V_r.

equal to 11000 V (the sending end voltage). With A as centre and AB as radius, draw a circle which is the locus of the tip of the V_r phasor. The magnitude and phase of the receiving end voltage V_r is readily obtained by drawing a straight line such as OD making an angle equal to that of load power factor with the horizontal [Above the horizontal for lagging power factor, and below for leading power factor].

By measurement, for 0.8 p.f. lagging

$$OD = 9900 \text{ volts}$$

$$\mathbf{V_r = 9.9 \text{ kV.}}$$

Calculation. For a p.f. of 0.8 lagging,

$$\phi_r = 36.9^\circ$$

Consider ΔOAD ; draw a perpendicular DF to AO produced.

$$AD^2 = CA^2 + OD^2 + 2 \,.\, OA \times OF$$

$$OD^2 = AD^2 - OA^2 - 2 \,.\, OA \times OF$$

$$= 11000^2 - 1278^2 - 2 \times 1278 \times OD \times \cos 11.4^\circ$$

Putting $OD = V_r$ and rearranging, we obtain a quadratic equation in V_r.

$$V_r^2 + 2500\, V_r - 122.63 \times 10^6 = 0$$

Solving, $V_r = 9900$ V

$= 9.9$ kV, discarding the negative value as the root.

This agrees with the above result.

1.3. Medium Transmission Lines

In dealing with the short transmission lines, the line capacitance was ignored. In fact, the effect of capacitance of transmission lines designed upto 20 kV fades into insignificance but as the length of the line and the operating voltage increase, the shunting effect of the capacitance becomes pronounced and will, therefore, have to be given due recognition in determining the characteristics of the transmission lines. For voltages upto, say, 100 kV, line calculations may be based on nominal T and π modes of representation of the transmission circuit. [Vide Figs. (1.2) and (1.3)].

In both nominal T and π-methods of approach, the sending end voltage and current are expressed in terms of receiving end voltage and current in the following general forms :

$$V_s = A\,V_r + BI_r \qquad ...(1.13)$$

$$I_s = C\,V_r + DI_r \qquad ...(1.14)$$

The subscripts s and r denote the sending and receiving end parameters respectively, and A, B, C, D are referred to as generalised transmission line constants, determinable from the values of series impedance and shunt admittance of the line.

Next, let us proceed to express these constants as referred to T-and π-networks separately.

1.3.1. Nominal-T method

Fig. 1.2 shows a T-network in which Z = series impedance per phase and Y = shunt admittance per phase, which is entirely concentrated at the middle of the line. It may, therefore, be referred to as Middle-Capacitor method.

Now we have to express V_s and I_s, each in terms of receiving end voltage V_r and current I_r. Let V_c = voltage across the shunt admittance Y.

Then $$V_c = V_r + I_r\,(Z/2)$$

$$I_c = V_c Y = (V_r + \tfrac{1}{2}\,I_r Z)\,Y$$

Hence, $$I_s = I_r + I_c = I_r + (V_r + \tfrac{1}{2} I_r Z)\,Y$$

$$I_s = V_r Y + I_r\,(1 + \tfrac{1}{2}\,Z Y) \qquad ...(1.15)$$

$$V_s = V_r + \tfrac{1}{2}\,(I_s + I_r)\,Z$$

$$= V_r(1 + \tfrac{1}{2} Z Y) + I_r Z\,(1 + \tfrac{1}{4} ZY) \qquad ...(1.16)$$

By comparison of Eq. (1.16) with (1.13), and (1.15) with (1.14), the generalized constants *A, B, C, D,* are readily obtained in terms of line parameters, as follows :

$$\left.\begin{aligned} \dot{A} &= 1 + \tfrac{1}{2}\dot{Z}\dot{Y} \\ \dot{B} &= \dot{Z}(1 + \tfrac{1}{4}\dot{Z}\dot{Y}) \\ \dot{C} &= \dot{Y} \\ \dot{D} &= 1 + \tfrac{1}{2}\dot{Z}\dot{Y} \end{aligned}\right\} \qquad ...(1.17)$$

It should be noted that $\dot{Z}$ and $\dot{Y}$ are phasors and the constants *A, B, C, D* are consequently in complex form.

From (1.17), it is seen that

$$\dot{A} = \dot{D} \qquad ...(1.18)$$

This is true in case of symmetrical networks, as is the *T*-net work with *Z*/2 on either side of *Y*. If the two impedances are not equal, $A \neq D$.

If may also be proved that

$$AD - BC = 1 \qquad ...(1.19)$$

Proof. $AD - BC = (1 + \frac{1}{2}ZY)^2 - YZ(1 + \frac{1}{4}ZY)$

$$= 1 + YZ + \tfrac{1}{4}(ZY)^2 - YZ - \tfrac{1}{4}(ZY)^2 = 1.$$

1.3.2. Nominal π method

Fig. 1.3 shows a π-network in which *Z* and *Y* are the same as in the case of *T*-network. The essential difference is that in the π-network, the entire *Z* is lumped as a series impedance and the admittance *Y* is divided into equal halves, each half being localized at the sending as well as the receiving end. This method of representation of medium lines is also referred to as 'Split-Capacitor method' or 'localized, capacitor method'.

In Fig. 1.3, I_L represents the current through the impedance *Z*. I_{c_1} and I_{c_2} are charging currents at the receiving and sending end respectively, each corresponding to one-half of the total capacitance of the line.

Thus, $\dot{I}_{c_1} = \frac{1}{2}\dot{V}_r\dot{Y}$

$$\dot{I}_L = \dot{I}_r + \dot{I}_{c_1} = \dot{I}_r + \tfrac{1}{2}\dot{V}_r\dot{Y}$$

$$\dot{V}_s = \dot{V}_r + \dot{I}_L\dot{Z}$$

$$= \dot{I}_r + (\dot{I}_r + \tfrac{1}{2}\dot{V}_r\dot{Y})\dot{Z}$$

$$\dot{V}_s = \dot{V}_r(1 + \tfrac{1}{2}\dot{Z}\dot{Y} + \dot{I}_r(\dot{Z}) \qquad ...(1.20)$$

$$\dot{I}_{c_2} = \tfrac{1}{2}\dot{V}_s\dot{Y}$$

$$= \tfrac{1}{2}\dot{Y}[\dot{V}_r(1 + \tfrac{1}{2}\dot{Z}\dot{Y}) + \dot{I}_r\dot{Z}]$$

Hence
$$I_s = I_L + I_{c_2}$$
$$= I_r + \tfrac{1}{2} V_r Y + \tfrac{1}{2} Y[V_r(1 + \tfrac{1}{2} ZY) + I_r Z]$$
$$I_s = V_r Y(1 + \tfrac{1}{4} ZY) + I_r (1 + \tfrac{1}{2} ZY) \qquad ...(1.21)$$

By comparison of Eqs. (1.20) and (1.21) with (1.13) and (1.14), respectively, it is seen that

$$\left.\begin{aligned} A &= 1 + \tfrac{1}{2} ZY = D \\ B &= Z \\ C &= Y(1 + \tfrac{1}{4} ZY) \end{aligned}\right\} \qquad ...(1.22)$$

It may also be proved, as in the case of nominal T-approach, that

$$AD - BC = 1 \qquad ...(1.23)$$

The relations (1.18) and (1.19) hold good for all symmetrical networks, while the latter is true for asymmetrical networks as well. This will be proved later in the case of general transmission networks.

1.3.3. Phasor diagrams for T and π circuits

Phasor diagram for nominal T method. Phasor diagram for voltages and currents may be drawn by taking either the receiving end voltage or current as reference phasor. Fig. 1.12 shows the phasor diagram, by taking the

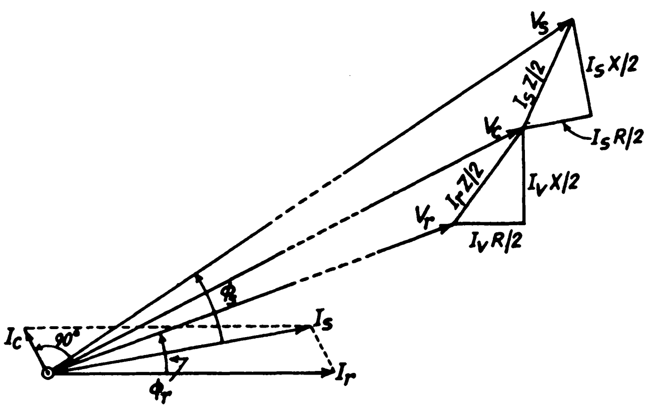


Fig. 1.12. Phasor diagram for voltages and currents : Nominal T method.

receiving end current I_r as reference. The power factors at the receiving end and sending end are cos ϕ_r and cos ϕ_s respectively. The receiving end power factor is assumed lagging, as is usually the case. V_r phasor leads I_r by an angle ϕ_r. V_c, the voltage across the middle-capacitor ($Y = j\omega C_0$) is obtained by adding victorially the impedance drop vector $\dot{I}_r \dot{Z}/2$ to $\dot{V}_r$. The charging current phasor $\dot{I}_c$ leads $\dot{V}_c$ by 90°, as shown. Adding $\dot{I}_c$ to $\dot{I}_r$ gives $\dot{I}_s$. The impedance drop due to I_s, added to V_c, yields the sending end voltage phasors V_s. The phase angle between V_s and I_s gives the power factor angle of the power input to the transmission line (at the sending end).

Phasor diagram for Nominal π method. In the nominal T-method, I_r was taken as the reference. Just for a change, we shall take V_r as the reference and draw the phasor diagram for the π-model network (*Vide* Fig. 1.13). I_{C_1} and I_{C_2} are the charging currents at the receiving and sending end respectively and lead the corresponding voltages V_r and V_s respectively by 90°. The line current I_L is obtained by vector addition of I_{C_1} to I_r ; and I_s is obtained by adding I_{C_2} vectorially to I_L. The phase angle between the sending end voltage and current gives the power factor angle of the power input at the sending end.

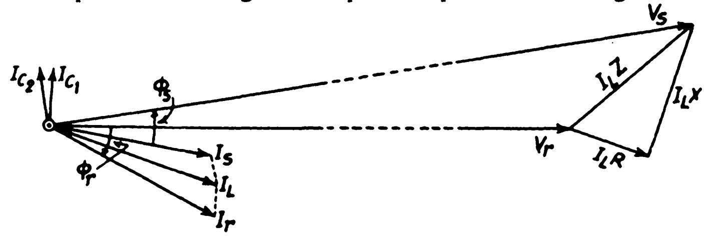


Fig. 1.13. Phasor diagram for voltages and currents.
Nominal π-method.

Now we are ready to take up some examples on the evaluation of characteristics of medium lines by nominal T-and π methods of approach. The problems may be solved either by using the expressions for V_s and I_s, each in terms of V_r and I_r, together with the $ABCD$ constants, or by proceeding systematically from the receiving end toward the sending end. The latter approach is adopted in the following examples.

Example 1.6. *A three-phase transmission line has the following constants (line to neutral) : R = 12 ohm, inductive reactance = 20 ohm, capacitive susceptance = 5 × 10⁻⁴ mho. Using the middle-capacitor (Nominal T) method, calculate the sending end voltage, current and power factor and also the cfficiency of transmission when supplying a balanced three-phase load of 12 MW at 66 kV, at a power factor of 0.8 lagging.*

Solution. Fig. 1.14 shows the nominal-T circuit of representation of the transmission network. (equivalent single phase)

$$6 + j\,10 = \frac{Z}{2} = 6 + j\,10$$

3 φ load ; 12,000 KW at 0.8 p.f. lagging

$$I_r = \frac{12{,}000}{\sqrt{3} \times 66 \times 0.8} = 131.2 \text{ Amp.}$$

$$\dot{Z}/2 = 6 + j10 = 11.66 \underline{/59°}\ \Omega$$

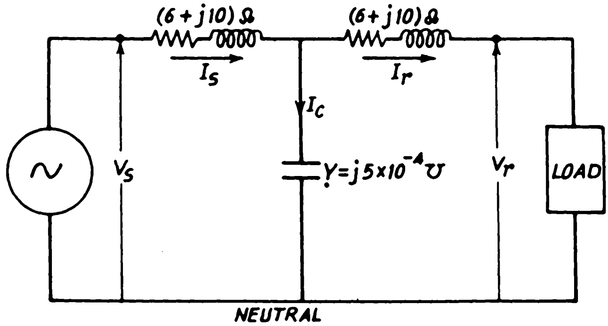


Fig. 1.14. For Example 1.6.

Taking the receiving end current (load) as the reference phasor,

$$I_r = 131.2 + j0 \text{ Amp.}$$

$$\dot{V}_r = \frac{66}{\sqrt{3}}(0.8 + j0.6) \text{ kV/phase}$$

$$= (30.485 + j22.864) \text{ kV/phase}$$

$$\dot{I}_r \dot{Z}/2 = 131.2\,(6 + j10)$$

$$= 787.2 + j\,1312 \text{ V/phase}$$

$$\dot{V}_c = \dot{V}_r + \dot{I}_r \dot{Z}/2$$

$$= 31.272 + j24.176 \text{ kV/phase}$$

$$\dot{I}_c = \dot{V}_c \dot{Y}$$

$$= (31.272 + j24.176) \times j5 \times 10^{-4}$$

$$= -12.1 + j15.64 \text{ Amp.}$$

$$\dot{I}_s = \dot{I}_r + \dot{I}_c$$

$$= 119.1 + j15.64$$

$$= \mathbf{120.1 \underline{/7.5°}\ Amp.}$$

$$\dot{I}_s \dot{Z}/2 = (120.1 \underline{/7.5°}\,(11.66 \underline{/59°})$$

$$= 1400 \underline{/66.5°} \text{ volt.}$$

$$= 1400\,(0.3987 + j0.917)$$

$$= 558 + j1284.$$

$$\dot{V}_s = \dot{V}_c + \dot{I}_s \dot{Z}/2$$

$$= 40.76 \underline{/38.7°} \text{ kV/phase}$$

$$|\dot{V}_s|\,(L - L) = \sqrt{3} \times 40.76$$

Line voltage at the sending end

$$= \mathbf{70.6\ kV}$$

Power factor angle at the sending end

$$= (38.7 - 7.5)^\circ = 31.2^\circ$$

Sending end power factor

$$\mathbf{\cos \phi_s = 0.855 \text{ lagging.}}$$

Phasor diagram in Fig. 1.15 depicts the relative positions of voltage and current phasors at the sending and receiving ends.

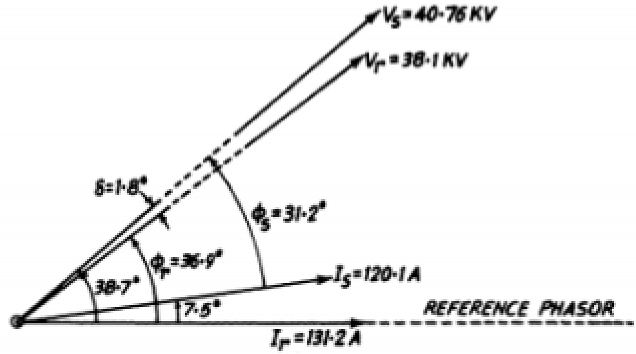


Fig. 1.15. Phasor diagram for the Nominal-T circuit of Fig. 1.14.

To find the efficiency of transmission. Power input to the transmission line at the sending end

$$= \sqrt{3} \times 70.6 \times 120.1 \times 0.855 \text{ KW}$$
$$= 12{,}560 \text{ KW}$$

Power delivered at the receiving end

$$= 12{,}000 \text{ KW}$$

Hence efficiency of transmission

$$= \frac{12{,}000}{12{,}560} \times 100\% = 95.5\%.$$

Alternatively, line loss may be evaluated and added to the power delivered at the load-end to determine the power at the sending end of the transmission line.

Total line loss in all the three phases

$$= 3\,[(131.2)^2 + (120.1)^2] \times 6 \text{ W} = 569 \text{ KW}$$

Therefore, $P_s = 12{,}569$ KW

$$\eta = \frac{P_r}{P_s} = \frac{12{,}000}{12{,}569} \times 100\% = 95.5\%$$

This closely agrees with the earlier result.

Example. 1.7. *Find the characteristics of the load at the sending end and the efficiency of transmission of a three phase transmission line 160 km* *long, consisting of hard-drawn copper conductors spaced in 3 metre delta arrangement, when the receiving end delivers 15 MVA load at 110 kV, 50 Hz and power factor 0.9 lagging. The resistance of the conductors is 0.25 ohm per km and the effective conductor-diameter is 8.75 mm. Neglect leakance and use the Nominal π (Split-capacitor) method for your calculations.*

Solution. Resistance of each line conductor

$$160 \text{ km long} = 0.25 \times 160 = 40 \text{ ohm}$$

$$\mathbf{R = 40\ ohm.}$$

The student will recapitulate from his earlier course on *Field Theory and Transmission Lines* and derive the formulae for inductance and capacitance per phase of a three phase transmission line with the conductors equilaterally spaced D (metre) apart and with radius r (in metre).

If L_0 and C_0 are the inductance and capacitance to neutral respectively per km length, it may be proved that

$$L_0 = 0.050 + 0.4606 \log_{10}\left(\frac{D}{r}\right) \text{ mH/km}$$

$$C_0 = \frac{0.0241}{\log_{10}(D/r)} \text{ ηF/km}$$

Inductance per km length

$$= 0.05 + 0.4606 \log_{10}\left(\frac{3000}{4.375}\right) \text{ mH}$$

$$= 1.356 \text{ mH/km}$$

$$L_0 = 1.356 \times 160 \times 10^{-3} \text{ H}$$

$$= 0.217 \text{ H/phase}$$

Inductive reactance per phase

$$X = 2\pi f L_0 = 2\pi \times 50 \times 0.217 = \mathbf{68.2\ ohm.}$$

Capacitance to neutral

$$C_0 = \frac{0.0241}{\log_{10}\left(\frac{3000}{4.375}\right)} \times 160 \text{ μF} = 1.36 \text{ μF}.$$

Capacitive susceptance would be

$$\omega C_0 = 2\pi \times 50 \times 1.36 \times 10^{-6}$$

$$\frac{Y}{2} = j\frac{\omega C_0}{2} = j0.214 \times 10^{-3} \text{ ℧}$$

$$\mathbf{Z = 40 + j68.2\ ohm.}$$

The π model of the transmission network per phase is shown in Fig. 1.16, with the constants marked thereon.

Load current $I_r = \dfrac{15000}{\sqrt{3} \times 110} = 78.73$ A.

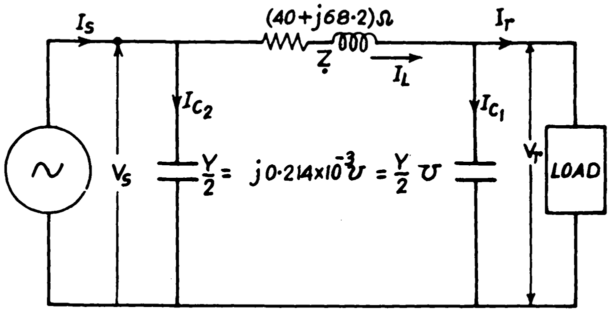


Fig. 1.16. π-network for Example 1.7.

Taking I_r as the reference phasor,

$$\dot{I}_r = 78.73 \angle 0° \text{ A}$$

$$\dot{V}_r = \frac{110}{\sqrt{3}} (0.9 + j\,0.436) \text{ kV/phase}$$

$$= (57,160 + j\,27,690) \text{ volts/phase}$$

charging current at the receiving end

$$\dot{I}_{c_1} = j0.214 \times 10^{-3} (57160 + j27690)$$

$$= -5.93 + j12.29 \text{ A}$$

$$\dot{I}_r = 78/73 + j0$$

$$\dot{I}_L = \dot{I}_r + \dot{I}_{c_1} = 72.8 + j12.29 \text{ A}$$

$$\dot{I}_L \dot{Z} = (72.8 + j12.29)(40 + j68.2)$$

$$= 2074 + j5457$$

$$\dot{V}_r = 57160 + j27690 \text{ V}$$

$$\dot{V}_s = \dot{V}_r + \dot{I}_L \dot{Z}$$

$$= 59.234 + j33.147 \text{ kV/phase}$$

$$= 67.9 \angle 29.2° \text{ kV/phase.}$$

Line voltage at the sending end

$$V_s (L - L) = 67.9 \sqrt{3}$$

$$= \mathbf{117.6\ kV.}$$

Charging current at the sending end

$$\dot{I}_{c_2} = j0.214 \times 10^{-3} (59234 + j33147) \text{ A}$$

$$= (-7.09 + j12.68) \text{ A}$$

$$\dot{I}_L = 72.80 + j12.29$$

$$\dot{I}_s = 65.71 + j24.97$$

Sending end current = **70.3 ∠ 20.8° Amp.**

Power factor at the sending end

$$= \cos [29.2 - 20.8]° = \cos 8.4° = 0.9893 \text{ lagging.}$$

Power at the sending end

$$P_s = \sqrt{3} \times 117.6 \times 70.3 \times 0.9893$$
$$= 14166 \text{ KW.}$$

Power delivered at the receiving end

$$P_r = 15000 \times 0.9 = 13500 \text{ KW.}$$

Efficiency of transmission

$$\eta = \frac{P_r}{P_s} = \frac{13{,}500}{14{,}166} \times 100\% = 95.3\%.$$

The student is advised to draw phasor diagram for voltages and currents as in Example 1.6.

1.3.4. Dr. Steinmetz' split-capacitor method for medium transmission lines

The above two methods *viz.* T and π modes of network-representation are subject to error as the length increases. A more accurate method has been suggested by Dr. Steinmetz, in which the line capacitance is assumed to be divided into three parts, as shown in Fig. 1.17. Out of the total shunt admittance Y, 2/3 of it is assumed to be concentrated at the centre of the transmission line, and $(1/6)Y$ is assumed to be localized at either end of the line.

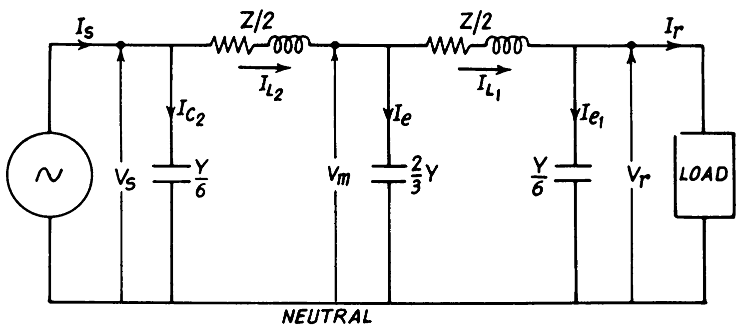


Fig. 1.17. Steinmetz' split-capactor method.

The calculations are done by step-by-step approach as for nominal T and π modes.

An example is taken up for illustration of the method of solving the characteristics of the transmission lines.

Example 1.8. *A three phase, 50 Hz line, 170 km long, has the following constants per km :*

Resistance	*= 0.08 ohm*
Inductive reactance to neutral	*= 0.50 ohm.*

Capacitive susceptance to neutral = 3.16×10^{-6} ℧
Leakance, negligible.

The full load receiver input is 60 MW at 80% lagging power factor. The receiving end voltage is maintained at 132 kV. For full-load receiver input, calculate by Dr. Steinmetz' split-capacitor method, the following sending end quantities : (a) voltage (b) current, (c) power factor. What is the percentage line loss at full-load ? Using the receiver voltage as the reference phasor, draw a phasor diagram showing the various component voltages and currents that combine to give the sending end voltage and current.

Solution. Total resistance of the line per phase

$$R = 170 \times 0.08 = 13.6 \text{ ohm}$$

$$\mathbf{R/2 = 6.8 \text{ ohm.}}$$

Total reactance of the line per phase

$$X = 170 \times 0.5 = 85 \text{ ohm}$$

$$\mathbf{X/2 = 42.5 \text{ ohm.}}$$

Total admittance per phase

$$\dot{Y} = j\,3.16 \times 10^{-6} \times 170 = \mathbf{j5.37 \times 10^{-4}} \text{ ℧}$$

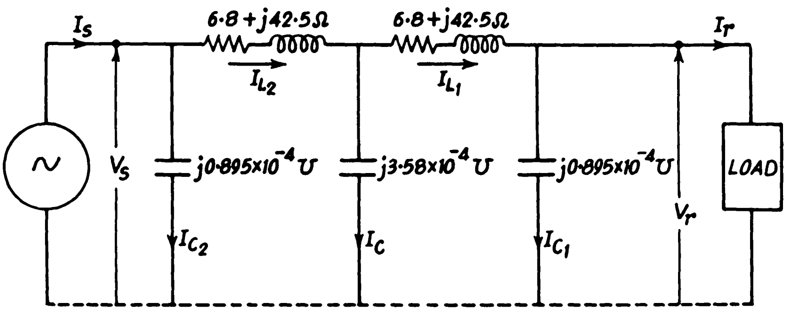


Fig. 1.18. Steinmetz' split-capacitor network for Example 1.8.

The equivalent Steinmetz' circuit per phase, with all parameters marked thereon, is shown in Fig. 1.18 :

$$\tfrac{1}{6}Y = j\,0.895 \times 10^{-4} \text{ ℧}$$

$$\tfrac{2}{3}Y = j\,3.58 \times 10^{-4} \text{ ℧}$$

Load supplied = 60,000 KW

Receiving end voltage V_r

$$= \frac{132}{\sqrt{3}} \text{ kV/phase.}$$

Taking V_r as reference

$$V_r = \frac{132}{\sqrt{3}} + j0 \text{ kV} = 76212 \underline{/0^\circ} \text{ V/phase.}$$

Load current $$I_r = \frac{60{,}000}{\sqrt{3} \times 132 \times 0.8} = 328 \text{ Amp.}$$

$$\dot{I}_r = 328\,(0.8 - j\,0.6) = (262.4 - j197)\text{ A}$$

$$\dot{I}_{c_1} = \dot{V}_r \times j0.895 \times 10^{-4}$$

$$= 76{,}212 \times j0.895 \times 10^{-4} = j6.82 \text{ Amp.}$$

$$\dot{I}_{L_1} = \dot{I}_r + \dot{I}_{c_1} = 262.4 - j190.2 \text{ A}$$

$$\dot{I}_{L_1} = \mathbf{324 \text{ Amp.}}$$

$$\dot{V}_m = \dot{V}_r + \dot{I}_{L_1}(\dot{Z}/2)$$

$$= 76{,}212 \times (262.4 - j190.2)(6.80 + j42.5) \text{ volt.}$$

$$= (86.08 + j9.86) \text{ kV}$$

$$\dot{I}_c = j3.58 \times 10^{-4} \times V_m = (-3.53 + j30.8) \text{ A}$$

$$\dot{I}_{L_2} = \dot{I}_c + \dot{I}_{L_1}$$

$$= 258.9 - j159.4 \text{ A}$$

$$\dot{I}_{L_2} = \mathbf{304 \text{ Amp.}}$$

$$\dot{I}_{L_2}(\dot{Z}/2) = 8535 + j9919 \text{ V}$$

$$\dot{V}_s = \dot{V}_m + \dot{I}_{L_2}(\dot{Z}/2)$$

$$= 94.615 + j19.78 \text{ kV/Phase}$$

$$V_s = \mathbf{96.7 \underline{/\,11.8^\circ} \text{ kV/Phase}}$$

Line voltage at the sending end

$$V_s\,(L - L) = \sqrt{3} \times 96.6 = \mathbf{167.5 \text{ kV}}$$

current at the sending end

$$\dot{I}_s = \dot{I}_{L_2} + \dot{I}_{c_1}$$

$$\dot{I}_{c_1} = j\,0.895 \times 10^{-4}\,(94{,}615 + 19{,}780)$$

$$= -1.77 + j\,8.47 \text{ Amp.}$$

$$\dot{I}_s = 257.13 - j\,150.93 \text{ Amp} = \mathbf{298 \underline{/\,30.4^\circ} \text{ Amp.}}$$

Phase angle between V_s and I_s phasors,

$$\phi_s = 11.8 + 30.4 = 42.2^\circ.$$

Hence, power factor at the sending end

$$= \cos\phi_s = 0.74 \text{ lagging.}$$

Line loss per phase = $(324^2 + 304^2)$ 6.8 W = 1342.3 KW.

Line loss for all phases = 1342.3 × 3 = 4027 KW.

Power delivered at the receiving end

= 60,000 KW

% Line loss $$= \frac{4027}{60{,}000} \times 100$$

Total line loss = **6.71% of full load.**

Taking V_r as the reference-phasor, a phasor diagram is drawn in Fig. 1.19 for the above example.

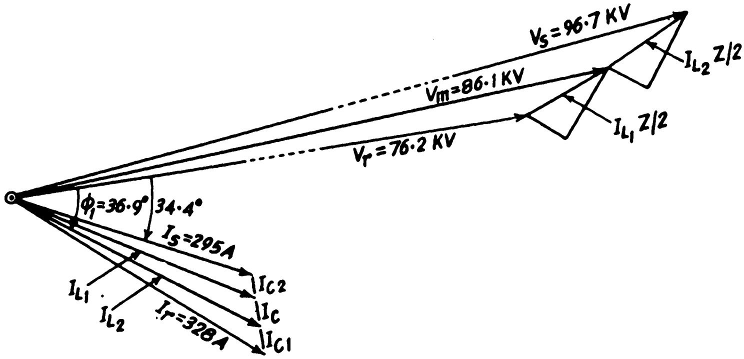


Fig. 1.19. Phasor diagram for voltages and currents, Dr. Steinmetz' split capacitor method, for example 1.8.

1.4. Long Transmission Lines

In the preceding articles, short and medium transmission lines were dealt with, approximating their representation through lumped parameters (impedance and admittance) without seriously affecting the accuracy of results obtainable for assessing their performance under load conditions. However, in the case of long transmission lines, an accurate solution can be achieved only by considering the distributive features of the constants, and approximations are not justifiable, unlike the short and medium lines. The resistance, inductive reactance and shunt admittance are all assumed to be uniformly distributed over the entire length of the line.

The following nomenclature is used :

r = resistance in ohm per km length

x = inductive reactance in ohm per km length

$z = r + j\,x$, the series impedance in ohm per km length

g = leakance (leakage conductance in mho per km length

b = capacitive susceptances in mho per km length

$y = g + j\,b$, the shunt admittance in mho per km length.

The distributive nature of the transmission line constants is depicted in Fig. 1.20.

Now we shall establish relations between the sending end voltage and current and the receiving end voltage and current. Consider an elemental length δl of the line, in Fig. 1.21, depicting a part of the long line (represented per phase).

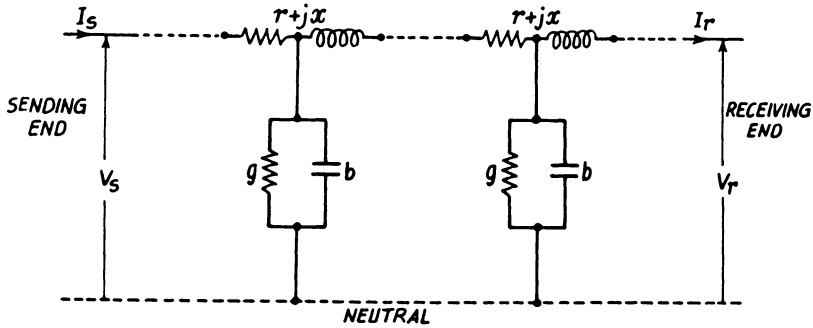


Fig. 1.20. Distributive feature of long transmission line parameters.

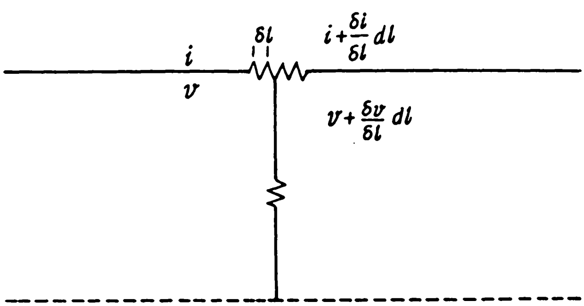


Fig. 1.21. For solution of long line problems.

Let i be the current on the left hand side boundary plane of the element δl. As the current varies from point to point along the line due to the shunt parameter, let us assume that the current changes at the rate $\delta i/\delta l$, with the result that the change for a length δl will be $(\delta i/\delta l)\, dl$. Hence the current on the right hand side boundary plane of the element becomes $i + (\delta i/\delta l) \,.\, dl$. Let v be the potential difference between line and neutral on the left hand side boundary of the element δl, and that on the right hand side $\left(v + \frac{\delta v}{\delta l} \,.\, dl\right)$. The current required for the shunt admittance of length δl can be written as $v(g + jb)\, \delta l$.

Hence the change in current may be equated with $v(g + jb)\, \delta l$.

Thus, $$-\frac{\delta i}{\delta l} \,.\, dl = v(g + jb)\, dl$$

$$-\frac{\delta i}{\delta l} = v(g + jb) \qquad \text{...(1.23)}$$

Next, considering the impedance drop over a length δl.

$$-\frac{\delta v}{\delta l}\, dl = i(r + jx)\, dl$$

$$AD = 1 + (Z_1 + Z_2)Y + Z_1 Z_2 Y^2$$

$$BC = (Z_1 + Z_2)Y + Z_1 Z_2 Y^2$$

Clearly, $$AD - BC = 1 \quad \text{....(1.65)}$$

The relation $AD - BC = 1$ holds good, irrespective of whether the network is symmetrical or not.

If the above procedure is repeated for the unsymmetrical network of Fig. 1.27 (*b*), the following relations may be proved. (This may be taken as an exercise).

$$\left.\begin{aligned} V_s &= (1 + ZY_2)\, V_r + (Z)\, I_r \\ &= A V_r + B I_r \end{aligned}\right\} \quad \text{...(1.66)}$$

$$\left.\begin{aligned} I_s &= (Y_1 + Y_2 + ZY_1Y_2)\, V_r + (1 + ZY_1)\, I_r \\ &= C V_r + D I_r \end{aligned}\right\} \quad \text{...(1.67)}$$

For π network,

$$\left.\begin{aligned} &A = 1 + ZY_2\, ; B = Z \\ &C = Y_1 + Y_2 + ZY_1Y_2\, ; D + 1 = ZY_1 \end{aligned}\right\} \quad \text{...(1.68)}$$

Again, the above relations show that A and D are not equal unless $Y_1 = Y_2$ (symmetrical). However, it can be proved by substitution for A, B, C, D that

$$AD - BC = 1 \quad \text{...(1.69)}$$

[as in Eq. (1.65)]

The above relation may be proved for the most general network as shown in Fig. 1.28, as follows,

If, as at (*a*), receiving end is short-circuited, with $V_s = E$

$$V_s = A V_r + B I_r$$

$$E = 0 + B I_r \quad \text{...(1.70)}$$

Now with the sending end terminal pair short-circuited, the same voltage is applied at the receiving end, *viz.* $V_r = E$.

By reciprocity theorem, the ratio of excitation to response involves no distinction between the terminal pairs, *viz.* the ratio of applied voltage at one terminal pair to the current through the short-circuit at the other terminal pair is the same.

This ratio is $E/I_r = B$ [from (1.70)]

With the voltage applied at the receiving end,

$$\left.\begin{aligned} 0 &= AE + BI_r \\ I_s &= -\frac{E}{B} = CE + DI_r \end{aligned}\right\} \quad \text{...(1.71)}$$

and current, with a view to ultimately express the sending end voltage and current.

Putting $l = 0$, $v = V_r$ and $i = I_r$ in the Eqs. (1.27) and (1.28), we get

$$\left.\begin{aligned} V_r &= A' + B' \\ \text{and} \quad nI_r &= B' - A' \end{aligned}\right\} \quad ...(1.29)$$

$$\left.\begin{aligned} \text{whence} \quad A' &= \frac{V_r - nI_r}{2} \\ \text{and} \quad B' &= \frac{V_r + nI_r}{2} \end{aligned}\right\} \quad ...(1.30)$$

To find V_s and I_s, substitute $l = -L$ in the Eqs. (1.27) and (1.28), replacing the constants A' and B' by the terms from (1.30).

Then,
$$V_s = \frac{V_r - nI_r}{2} \cdot e^{-mL} + \frac{V_r + nI_r}{2} \cdot e^{mL}$$

$$= V_r \frac{e^{mL} + e^{-mL}}{2} + nI_r \frac{e^{mL} - e^{-mL}}{2}$$

$$V_s = V_r \cosh mL + nI_r \sinh mL$$

$$mL = \sqrt{zy} L = \sqrt{zL\, yL}$$

$$= \sqrt{Z \cdot Y}$$

where $Z = zL$ = total series impedance per phase over the whole length of the line.

and $Y = yL$ = total shunt admittance per phase over the whole length of the line.

$$n = \sqrt{\frac{z}{y}} = \sqrt{\frac{zL}{yL}} = \sqrt{\frac{Z}{Y}}$$

Replacing m and n by expressions containing Z and Y, Eq. (1.31) may be rewritten as

$$V_s = V_r \cosh \sqrt{ZY} + I_r \sqrt{\frac{Z}{Y}} \sinh \sqrt{ZY} \quad ...(1.32)$$

Similarly, as for V_s,

$$nI_s = B' e^{mL} - A' e^{-mL}$$

$$= V_r \frac{e^{mL} - e^{-mL}}{2} + nI_r \frac{e^{mL} + e^{-mL}}{2}$$

$$I_s = V_r/n \frac{e^{mL} - e^{-mL}}{2} + I_r \frac{e^{mL} + e^{-mL}}{2}$$

$$I_s = V_r \sqrt{\frac{Y}{Z}} \sinh \sqrt{ZY} + I_r \cosh \sqrt{ZY} \quad ...(1.33)$$

As expressed in the general form of Eqs. (1.13) and (1.14), the expressions for V_s and I_s may be reproduced as

$$V_s = AV_r + BI_r \qquad ...(1.13)$$

$$I_s = CV_r + DI_r \qquad ...(1.14)$$

which, by comparison with the pertinent relations in Eqs. (1.32) and (1.33) yield A, B, C, D constants as follows, in terms of line constants Z and Y:

$$\begin{aligned} A &= D = \cosh \sqrt{ZY} \\ B &= \sqrt{Z/Y} \sinh \sqrt{ZY} \\ C &= \sqrt{Y/Z} \sinh \sqrt{ZY} \end{aligned} \qquad ...(1.34)$$

Clearly, $AD = \cosh^2 \sqrt{ZY}$

$BC = \sinh^2 \sqrt{ZY}$

Thus $AD - BC = 1 \qquad ...(1.35)$

as for medium lines.

If a constant γ is introduced (same as m) being the square root of the product of z and y, *viz.* $\gamma = \sqrt{zy}$, it is seen to be a dimensionless quantity. γ is designated as 'Propagation Constant' and is a complex quantity expressible in the form $\gamma = \alpha + j\beta$ where α is referred to as 'Attenuation constant' and β, as 'Phase constant'. The following units are used in practice. Attenuation constant is expressed in Neper per km length and phase constant in radian per km length. The expression $\sqrt{Z/Y}$ has the dimension of an impedance and is referred to as characteristic or surge impedance of the line.

$$Z_c = \sqrt{Z/Y}$$

is the characteristic of natural impedance of the line. Introducing Z_c and γ into the expression for sending end voltage and current, Eqs. (1.32) and (1.33) may be rewritten as follows :

$$V_s = V_r \cosh \gamma L + I_r Z_c \sinh \gamma L \qquad ...(1.36)$$

$$I_s = I_r \cosh \gamma L + (V_r/Z_c) \sinh \gamma L \qquad ...(1.37)$$

Dividing (1.36) by (1.37), we get an expression for the input impedance of the line of length L.

$$\begin{aligned} Z_s = \frac{V_s}{I_s} &= \frac{V_r \cosh \gamma L + I_r Z_c \sinh \gamma L}{I_r \cosh \gamma L - (V_r/Z_c) \sinh \gamma L} \\ &= Z_c \frac{Z_r \cosh \gamma L + Z_c \sinh \gamma L}{Z_c \cosh \gamma L + Z_r \sinh \gamma L} \end{aligned} \qquad ...(1.38)$$

(as $V_r = Z_r I_r$)

where Z_r is the equivalent impedance of load.

When the line is terminated in its characteristic impedance, *i.e.*, $Z_r = Z_c$ the equation (1.38) gets reduced to

$$\boxed{Z_s = Z_c} \qquad ...(1.39)$$

In the case of a very long line, hypothetically of infinite length, the input impedance, Z_s may be found by letting L approach infinity in Eq. (1.38).

The result is $\quad \dot{Z}_s = \dot{Z}_c$ ...(1.40)

It is thus seen that the result in Eqs. (1.39) and (1.40) are the same, leading us to the conclusion that "a line of finite length terminated in a load whose equivalent impedance is equal to the characteristic impedance of the line, appears to the sending end generator as an infinite line". In other words, a finite line terminated in its characteristic impedance and an infinite line are indistinguishable by measurements at the sending end.

Evaluation of A, B, C, D constants by expansion of the hyperbolic series functions :

In general,
$$\cosh\theta = \frac{e^{\theta}+e^{-\theta}}{2}$$
$$= 1+\frac{\theta^2}{2\,!}+\frac{\theta^4}{4\,!}+\frac{\theta^6}{6\,!}+\ldots\ldots$$

neglecting the higher powers

$$\sinh\theta = \frac{e^{\theta}-e^{-\theta}}{2}$$
$$= \theta+\frac{\theta^3}{3\,!}+\frac{\theta^5}{5\,!}+\frac{\theta^7}{7\,!}+\ldots\ldots$$

Putting $\theta = \sqrt{ZY}$, we obtain the following results :

$$A = \cosh\sqrt{ZY}$$
$$= 1+\frac{ZY}{2}+\frac{Z^2Y^2}{24}+\frac{Z^3Y^3}{720}+\ldots\ldots$$
$$B = \sqrt{Z/Y}\,\sinh\sqrt{ZY}$$
$$= Z\left(1+\frac{ZY}{6}+\frac{Z^2Y^2}{120}+\frac{Z^3Y^3}{5040}+\ldots\ldots\right)$$
$$C = \sqrt{Y/Z}\,\sinh\sqrt{ZY}$$
$$= Y\left(1+\frac{ZY}{6}+\frac{Z^2Y^2}{120}+\frac{Z^3Y^3}{5040}+\ldots\ldots\right)$$

Alternatively, the following expressions may be used for evaluating the hyperbolic functions :

$$\cosh\sqrt{ZY} = \cosh(a+jb)$$
$$= \cosh a\cos b + j\sinh a\sin b$$
$$\sinh\sqrt{ZY} = \sinh(a+jb)$$
$$= \sinh a\cos b + j\cosh a\sin b$$

The values of hyperbolic functions may be read off from the standard tables.

Example 1.9. *A long transmission line delivers a load of 60 MVA at 154 kV, 50 Hz, at 0.8 power factor lagging. The line constants (to neutral) are as follows :*

R = 25.3 ohm (total), total inductive reactance X = 66.5 ohm and admittance due to capacitance, $j\,0.442 \times 10^{-3}$ mho.

(*a*) *Evaluate the constants A, B, C, D.*

(*b*) *Find the sending end voltage and current and also the power factor at the sending end.*

(*c*) *Determine the efficiency of transmission at that load.*

(*d*) *Estimate the receiving end regulation.*

Solution. (*a*) $Z = 25.3 + j\,66.5$ ohm

$$Y = j\,0.442 \times 10^{-3} \text{ mho}$$

$$\sqrt{ZY} = \sqrt{(25.3 + j\,66.3)\,j\,0.442 \times 10^{-3}}$$
$$= 0.0327 + j\,0.174$$

$$\sqrt{\frac{Z}{Y}} = \sqrt{\frac{25.3 + j\,66.5}{j\,0.442 \times 10^{-3}}}$$
$$= 393 - j\,72.3$$

$$A = \cosh\sqrt{ZY} = \cosh(0.0327 + j\,0.174)$$
$$= 0.9855 + j\,0.00553$$
$$A = O = 0.986\,\underline{/\,0.32^\circ}$$

$$\sinh\sqrt{ZY} = \sinh(0.0327 + j\,0.174)$$
$$= 0.0315 + j\,0.1728$$

$$B = \sqrt{\frac{Z}{Y}}\,\sinh\sqrt{ZY}$$
$$= (393 - j\,72.3)\,(0.0315 + j\,0.1728)$$
$$= 25 + j\,65.6$$
$$B = 70.3\,\underline{/\,69.2^\circ}$$

$$C = \frac{\sin\sqrt{ZY}}{\sqrt{Z/Y}} = \frac{0.315 + j\,0.1728}{393 - j\,72.3}$$
$$= 0.00813 + j\,4.44)\,10^{-4}$$
$$\approx j\,4.44 \times 10^{-4}$$
$$C = 4.44 \times 10^{-4}\,\underline{/\,90^\circ}$$

(*b*) *Load.* 60 MVA at 154 kV ($L - L$)

Load current $\quad I_r = \dfrac{60{,}000}{\sqrt{3} \times 154} = 225$ Amp

Taking I_r as the reference phasor,

$$I_r = 225\,\underline{/\,0^\circ} \text{ Amp}$$

$$V_r = \frac{154}{\sqrt{3}} \angle 36.9^\circ \text{ kV/phase}$$

$$= 88.8 \angle 36.9^\circ \text{ kV/phase}$$

$$V_s = AV_r + BI_r$$

$$= (0.986 \angle 0.32^\circ)(88.8 \angle 36.9^\circ) + \frac{(70.3 \angle 69.2^\circ) \times 225}{1000}$$

$$= 75.32 + j67.58 \text{ kV} = 101.2 \angle 41.9^\circ \text{ kV}$$

Line to line voltage, at the sending end its

$$V_s\,(L-L) = \sqrt{3} \times 101.2 \text{ kV}$$

$$= 175.5 \text{ kV}$$

Sending end current is given by

$$I_s = CV_r + DI_r$$

$$= (0.44 \times 10^{-4} \angle 90^\circ)(88{,}800 \angle 36.9^\circ) + (0.986 \angle 0.32^\circ)(225 \angle 0^\circ)$$

$$= 219.66 + j\,4.34 = 219.7 \angle 1.1^\circ \text{ Amp.}$$

Power factor angle at the sending end

$$= 41.9 - 1.1 = 40.8^\circ$$

Power factor $= \cos \phi_s = \cos 40.8^\circ$

$$= 0.757 \text{ lagging}$$

(*c*) Power input to the transmission line

$$P_s = \sqrt{3} \times 175.5 \times 219.7 \times 0.757 = 50{,}600 \text{ kW}$$

$$P_r = 60{,}000 \times 0.8 = 48{,}000 \text{ kW}$$

Efficiency of transmission

$$\eta = \frac{48{,}000}{50{,}600} \times 100 = 94.8\%$$

(*d*) Voltage regulation at the receiving end is given by the increase in voltage at the receiving end when the load is thrown off. When there is no load, $I_r = 0$ Hence $V_s = AV_{ro}$, where V_{ro} is the no load voltage, assuming V_s is held at the value adjusted for rated voltage at the receiving end *viz.*

$$V_r = 154 \text{ kV } (L-L)$$

Receiving end voltage at no load will therefore be given by

$$V_{ro}\,(L-L) = \frac{V_s\,(L-L)}{A} = \frac{175.5}{0.986} = 178 \text{ kV}$$

Hence voltage regulation

$$= \frac{V_{ro} - V_r}{V_r} = \frac{178 - 154}{154} = \mathbf{15.6\%.}$$

Example 1.10. *The following data apply to a long, three-phase transmission line. The resistance per phase is 63.5 ohm, reactance per phase, 167 ohm. Capacitive susceptance to neutral, 1.1 × 10^{-3} mho. Determine the ABCD constants of the line.*

Solution. First of all, express the total series impedance $\dot{Z}$ and shunt admittance $\dot{Y}$ in complex from and then use the expressions for *A, B, C, D* in terms of *ZY* ; [*vide* Eq. (1.41)].

$$\dot{Z} = 63.5 + j\,1.67 = 178.7 \underline{/\,69.2^\circ} \text{ ohm}$$

$$\dot{Y} = j\,1.1 \times 10^{-3} = 1.1 \times 10^{-3} \underline{/\,90^\circ} \text{ mho}$$

$$\dot{Z}\dot{Y} = 178.7 \times 1.1 \times 10^{-3} \underline{/\,159.2^\circ}$$

$$= 0.1966 \underline{/\,159.2^\circ}$$

$$(\dot{Z}\,\dot{Y})^2 = 0.0387 \overline{/\,41.6^\circ}$$

$$= 3.87 \times 10^{-2} \overline{/\,41.6^\circ}$$

$$(\dot{Z}\,\dot{Y})^3 = (1.966)^3 \times 10^{-3} \underline{/\,117.6^\circ}$$

$$= 7.6 \times 10^{-3} \underline{/\,117.6^\circ}$$

$$A = D = 1 + \frac{ZY}{2} + \frac{(ZY)^2}{24} + \frac{(ZY)^3}{720} + \ldots..$$

$$= 0.9092 + j\,0.0338$$

$$= 0.9098 \underline{/\,2.13^\circ}$$

$$A = D \simeq 0.91 \underline{/\,2.13^\circ}$$

In fact, the term $\frac{(ZY)^3}{720}$ becomes insignificant and so all the terms of higher powers starting from this are negligible.

$$B = Z\left[1 + \frac{ZY}{6} + \frac{(ZY)^2}{120} + \frac{(ZY)^3}{5040} + \ldots\ldots\right]$$

$$= 178.7 \underline{/\,69.2^\circ} \,[0.970 \underline{/\,0.67^\circ}]$$

$$B = 173.3 \underline{/\,69.9^\circ}$$

$$C = Y\left[1 + \frac{ZY}{6} + \frac{(ZY)^2}{120} + \frac{(ZY)^3}{5040} + \ldots\ldots\right]$$

$$= (1.1 \times 10^{-3} \underline{/\,90^\circ} \times (0.974 \underline{/\,0.67^\circ})$$

$$C = 1.067 \times 10^{-3} \underline{/\,90.7^\circ}$$

1.4.1 Phase-Modifier for voltage control of transmission system.

It is seen from the examples on performace-evaluation of transmission lines that the line parameters have a significant impact on the voltage regulation of the system. One method of keeping down the disparity between sending and receiving end voltages, as a means of voltage regulation, is to control the current in transmission line in magnitude and phase, by installing a synchronous phase Modifier at the receiving end. By controlling its excitation, the MVAR contribution of the modifier would render the reactive power flow through the transmission line feasible.

The student may review the theory on 'Machinery' and 'Transmission Lines', for better grasp of the problem on hand.

An example is given below to illustrate the method of estimating the capacity of a phase modifier designed for the above purpose.

Example 1.11. *The sending end voltage per phase of a long transmission line is given by the expression :*

$$\dot{V}_s = (0.986 \underline{/0.32°})\, V_r + (70.3 \underline{/69.2°})\, I_r$$

Determine the capacity of a Phase Modifier to be installed at the receiving end so that, when a Load of 50 MVA is delivered at 132 kV and power factor 0.707 lagging, the sending and voltage can also be 132 kV.

Solution. $\dot{V}_s = \dot{A}\, \dot{V}_s + \dot{B}\, \dot{I}_r$

where $\dot{A} = 0.986 \underline{/0.32°}$

$$\dot{A} = 0.986 + j\, 0.0055$$

and $\dot{B} = 70.3 \underline{/69.2°}$

$$\dot{B} = 24.96 + j\, 65.72$$

Load current $= \dfrac{50{,}000}{\sqrt{3} \times 132} = 218.7$ Amp

Taking the receiving end voltage as reference phasor,

$$\dot{V}_r = \frac{132}{\sqrt{3}} \underline{/0°} = 76.2 \underline{/0°} \text{ kV/phase}$$

Load current $\dot{I}_r = 218.7 \overline{\backslash 45°}$ Amp. (as p.f. is 0.707 lagging)

$$= 154.6 - j\, 154.6 \text{ A}$$

The problem is to determine the capacity of a Phase Modifier to be installed at the receiving end. Assuming that the modifier losses are negligible and that the current is $\dot{I}_m = j\, I_m$, the overall receiving end current would then be

$$\dot{I}_r' = \dot{I}_r + \dot{I}_m$$

$$= (154.6 - j\, 154.6) + j\, I_m$$

$$\dot{I}_r' = 154.6 - j\, (154.6 - I_m)$$

The sending end voltage expression would then the

$$\dot{V}_s = \dot{A}\, \dot{V}_r + \dot{B}\dot{I}_r'$$

$$= (0.986 + j\,0.0056)\,76{,}200$$
$$+ (24.96 + j\,65.72)(154.6 - j\,154.6 + jI_m)$$
$$\dot{V}_s = (89152.33 - 65.72\,I_m) + j\,(6738.21 + 24.96\,I_m)$$

Given. $V_s = V_r = 76{,}200$ V/phase

Hence, $(76{,}200)^2 = (89152.33 - 67.72\,I_m)^2 + (6728.21 + 24.96\,I_m)^2$

Simplifying the above results into the following simple quadratic equation :

$$I_m^2 - 2304\,I_m + 442.6 \times 10^3 = 0$$

and solving, $I_m = 2093$ or **212 Amp.**

Although both values of I_m seem to be valid, they are only mathematical results. In practice, the higher value is invalid as it lies outside the region of stable operation, with the result that the lower value, *viz.* $I_m = 212$ A is accepted. Hence the phase modifier capacity to meet the specifications would be

$$\frac{\sqrt{3} \times 132 \times 212}{1000} \text{ MVAR}$$
$$= \mathbf{48.47\ MVAR.}$$

The phase modifier may be rated at **50 MVAR**, to enable the sending end voltage to be adjusted to the same voltage, *viz.* 132 kV $(L - L)$ when the load delivered at the receiving end is 50 MVA at 0.707 lagging power factor.

Fig. 1.22 shows a phasor diagram of voltages and currents, to demonstrate how the sending and receiving end voltages are equalized in magnitude, but displaced in phase, when a phase modifier is installed at the receiving end across the load. It will be visualized that without the modifier

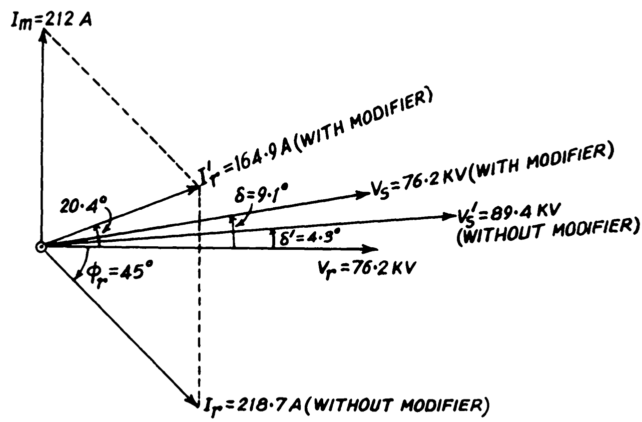


Fig. 1.22. Phasor diagram to demonstrate the voltages and currents at both ends of transmission line with and without Phase Modifier.

the transmission line has to carry a current of 218.7 Amp at 0.707 lagging, whereas the installation of a phase Modifier with appropriate excitation control results in a reduction of the magnitude of the current to 164.7 A (about 75% of the original) leading the receiving end voltage by 20.4°. This boosts the overall receiving end power factor from 0.707 lagging to 0.937 leading, as

$$I_r' = 154.6 - j\,(154.6 - 212)$$

$$= 164.9 \angle 20.4^\circ \text{ Amp.}$$

Had the phase modifier not been used, the sending end voltage should have been adjusted to

$$\dot{V}_s' = \dot{A}\dot{V}_r + \dot{B}\dot{I}_r$$

$$= 0.986 \angle 0.32^\circ (76.2 \angle 0^\circ) + \frac{70.3 \angle 69.2^\circ \times 218.7 \angle -45^\circ}{1000}$$

$$= \mathbf{89.4 \angle 4.3^\circ \text{ kV}}$$

about 17% more than the receiving end voltage, and displacement angle

$$\delta' = 4.3^\circ$$

With phase modifier used, the sending and voltage could be made equal to 76.2 kV.

$$V_s = (89152.33 - 65.72 \times 212) + j\,(6728.21 + 24.96 \times 212) \text{ V/Phase}$$

$$= \mathbf{76.2 \angle 9.1^\circ \text{ kV/Phase}}$$

The corresponding displacement angle is given by

$$\delta = 9.1^\circ.$$

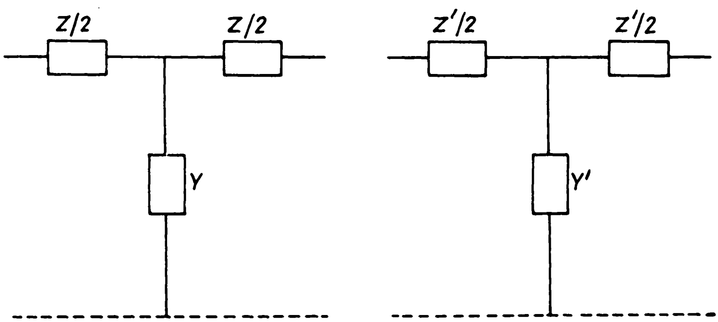
1.4.2. Equivalent circuits for long lines

It may be recalled that in dealing with problems on Medium lines, an approximate representation of the transmission lines is resorted to either by Nominal – T or Nominal – π mode. These two methods will suffice, though not exact, for lines of moderate length, say upto 200—240 km. These localized capacitance methods of solving problems are fairly accurate, the percentage error involved in calculation of sending end voltage from the receiving end data are well within 1% for medium lines. Nominal T approach overcompensates (giving slightly lower value for V_s) whereas Nominal π method results in under-compensation. This statement may be verified by solving a common problem by both methods. Of course, the three capacitor method of Dr. Steinmetz offers a better degree of accuracy.

In fact, the localized or concentrated capacitance methods are chiefly of class-room interest in so far as long lines are concerned, as an accurate method of representing the equivalent circuit is possible, taking cognizance of the distributive nature of the line constants, and without involving too much of work in calculations.

The long lines can be represented by equivalent T and equivalent π-circuits which are exactly the models obtainable from the rigorous solution of such lines.

Equivalent T-network. The parts (*a*) and (*b*) of Fig. 1.23 indicate Nominal T and Equivalent T-circuits respectively ; while the former is an approximate circuit, the latter affords an accurate model of the transmission line. Z and Y are the total series impedance per phase of the long line and the total shunt admittance of the line, respectively. In the equivalent T-circuit, appropriate correction has to be applied to obtain the equivalent impedance $Z'/2$ on either side of the shunt admittance Y' which is also the corrected value of Y.



(*a*) Nominal T-circuit (*b*) Equivalent T-circuit

Fig. 1.23

Now we have to evaluate Z' and Y' in terms of Z and Y. In the case of Nominal T-circuit, it is seen from Eq. (1.17) that

$$A = 1 + \tfrac{1}{2} ZY$$

and $$C = Y$$

Similarly, we may express the constants for the equivalent T-circuit by replacing Z and Y by Z' and Y' respectively.

Consequently, $$\left.\begin{aligned} A &= 1 + \tfrac{1}{2} Z'Y' \\ C &= Y' \end{aligned}\right\} \qquad \text{...(1.42)}$$

For long lines, as seen from the Eq. (1.34),

$$\left.\begin{aligned} A &= \cosh \sqrt{ZY} \\ C &= \sqrt{Y/Z} \sinh \sqrt{ZY} \end{aligned}\right\} \qquad \text{...(1.34)}$$

By comparison of the above two sets of relations,

$$\left.\begin{aligned} \frac{Z'Y'}{2} &= \cosh \sqrt{ZY} - 1 \\ \text{and} \quad Y' &= \sqrt{Y/Z} \sinh \sqrt{ZY} \end{aligned}\right\} \qquad \text{...(1.43)}$$

Simplification of the above relations yields the following results, expressing $Z'/2$ in terms of $Z/2$, with a correction factor k_z and Y' in terms of Y, with a correction factor k_y.

$$\left.\begin{aligned}\frac{Z'}{2} &= \frac{Z}{2} \cdot \frac{\tanh \sqrt{ZY/2}}{\sqrt{ZY/2}} \\ &= (k_z)\frac{Z}{2}, \\ \text{where} \quad k_z &= \frac{\tanh \sqrt{ZY}/2}{\sqrt{ZY}/2}\end{aligned}\right\} \quad \text{...(1.44)}$$

$$\left.\begin{aligned}Y' &= Y\frac{\sinh \sqrt{ZY}}{\sqrt{ZY}} \\ &= (k_y)\,Y \\ \text{where} \quad k_y &= \frac{\sinh \sqrt{ZY}}{\sqrt{ZY}}\end{aligned}\right\} \quad \text{...(1.45)}$$

Fig. 1.24 shows the equivalent T-circuit with the series and shunt arms in terms of Z and Y.

Equivalent π-network. The parts (*a*) and (*b*) of Fig. 1.25 indicate Nominal π and Equivalent π-circuits respectively ; while the former is an approximate model, the latter depicts an exact equivalent model of the transmission line. Z and Y are the total series impedance per phase and the total shunt admittance per phase respectively of the long line.

In the exact equivalent π-circuit, the equivalent impedance and admittance are shown as Z'' and Y'' respectively with $Y''/2$ concentrated at each end ot the transmission line.

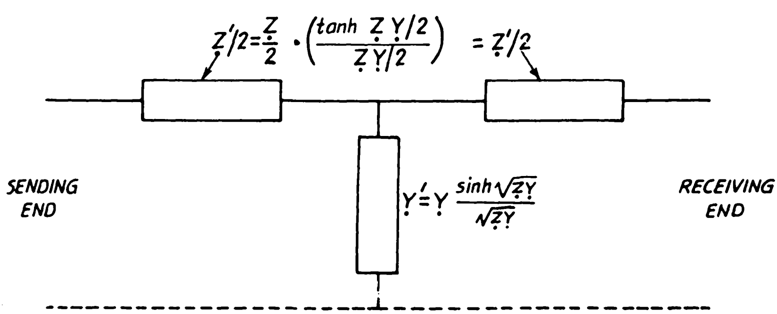
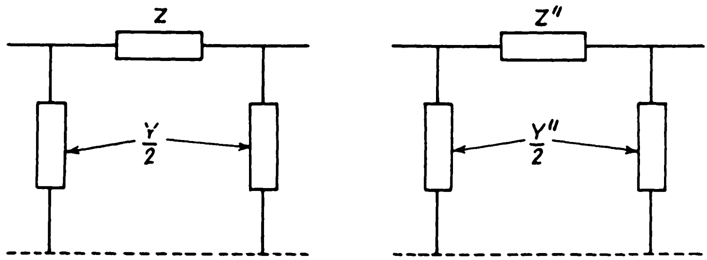


Fig. 1.24. Equivalent T-circuit for a long line.

Z'' and $Y''/2$ are the corrected values of Z and $Y/2$, and it is now necessary to evaluate the correction factors k_z' and k_y' for Z and $Y/2$ to obtain Z'' and $Y''/2$ respectively. In the case of Nominal – π circuit, it is seen from Eq. (1.22), that

$$A = 1 + \frac{1}{2} ZY$$
$$B = Z$$



(*a*) Nominal circuit (*b*) Equivalent circuit

Fig. 1.25

Based on the above, the constants for the equivalent π-circuit may be expressed by replacing Z and Y by Z'' and Y'' respectively.

Consequently,
$$\left.\begin{aligned} A &= 1 + \tfrac{1}{2} Z''Y'' \\ B &= Z'' \end{aligned}\right\} \qquad ...(1.46)$$

For long lines, as seen from the Eq. (1.34),
$$\left.\begin{aligned} A &= \cosh \sqrt{ZY} \\ B &= \sqrt{Z/Y} \sin \sqrt{ZY} \end{aligned}\right\} \qquad ...(1.34)$$

By comparison of the above two sets of relations,
$$\left.\begin{aligned} \frac{Z''Y''}{2} &= \cosh \sqrt{ZY} - 1 \\ \text{and} \quad Z'' &= \sqrt{Z/Y} \sinh \sqrt{ZY} \end{aligned}\right\} \qquad ...(1.47)$$

These relations are similar to those obtained for equivalent T-circuit, *vide* Eq. (1.43).

Simplification and repetition of the process as for equivalent T will lead to
$$\left.\begin{aligned} Z'' &= Z \,.\left(\frac{\sinh \sqrt{ZY}}{\sqrt{ZY}} \right) \\ &= (k_z') \,.\, Z, \\ \text{where} \quad k_z' &= \frac{\sinh \sqrt{ZY}}{\sqrt{ZY}} \end{aligned}\right\} \qquad ...(1.48)$$

and
$$\frac{Y''}{2} = \frac{Y}{2} \cdot \left(\frac{\tanh \sqrt{ZY}/2}{\sqrt{ZY}/2} \right)$$
$$= (k_y') \,.\, Y/2 \qquad ...(1.49)$$

where $$k_y' = \frac{\tanh \sqrt{ZY}/2}{\sqrt{ZY}/2}$$

It is interesting to note that the correction factors k_y' of equivalent π and k_z of equivalent T circuit are the same; similarly, the correction factors k_z' of equivalent π and k_y of equivalent T circuit are the same.

Fig. 1.26 depicts the equivalent π circuit with the series and shunt arms in terms of Z and Y. It should be noted that whereas the nominal T and nominal

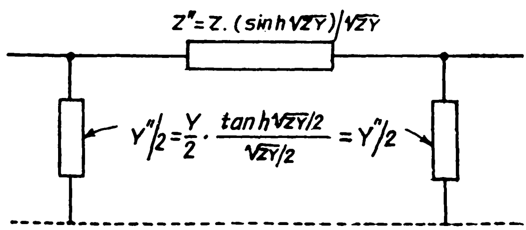


Fig. 1.26. Equivalent π-circuit for a long transmission line.

π being each an approximate circuit model are not equivalent to each other, the equivalent T and π-circuits being exact are equivalent to each other and may be interchangeably used, without any difference resulting in the calculations for any given problem.

1.4.3. Charts for transmission lines

We have already seen that the *ABCD* constants of long lines are hyperbolic functions of ZY or γl. Consequently, the evaluation of these constants from the line parameters is an uphill task unless ready-made charts are available for simplification of the computational work. Povejsil and Johnson have published a set of charts from which the components of the complex constants *ABCD* may be read off.

The principle involved in reading off the chart is explained below

$$\dot{Z} = \dot{z}l = (r + jx)l \qquad \text{...(1.50)}$$

where l is the length in km.

$$\left.\begin{aligned} \dot{Y} &= \dot{y}l = (g + jb)l \\ &= (g + j\,2\pi f C_0)l \end{aligned}\right\} \qquad \text{...(1.51)}$$

where C_0 = shunt capacitance in farad per km.

In the usual overhead transmission lines during normal operation, there is no corona discharge, and leakance across insulators may also be negligible. Consequently, $g = 0$ and the Eq. (1.51) gets reduced to

$$\dot{Y} = j(2\pi f C_0)l \qquad \text{...(1.52)}$$

In the construction of transmission line charts, the product L_0C_0 (the product of inductance in Henry per km and capacitance in farad per km) is assumed constant.

From the velocity of light

$$v = \frac{1}{\sqrt{L_0C_0}} = 3 \times 10^5 \text{ km per second}$$

$$\sqrt{L_0C_0} = \frac{1}{3 \times 10^5}, \text{ leading to}$$

$$L_0C_0 = \frac{10^{-10}}{9} \qquad ...(1.53)$$

This may be treated as constant for standard copper or ACSR conductor transmission lines

$$\sqrt{ZY} = \gamma l = \sqrt{zy}\, l = \sqrt{(r + jx)\, j\omega C_0}\, .\, l$$

$$= \sqrt{(r + j\omega L_0) j\omega C_0}\, .\, l$$

$$= \sqrt{j\omega L_0 \left(1 + \frac{r}{j\omega L_0}\right) j\omega C_0}\, .\, l$$

$$\sqrt{ZY} = \gamma . l = j2\pi f .\, l \sqrt{L_0C_0}\, .\, \sqrt{1 - jr/x}$$

$$= j\,\frac{2\pi f\, l}{3 \times 10^5} \sqrt{1 - jr/x} \qquad ...(1.54)$$

as $\sqrt{L_0C_0} = 1/(3 \times 10^5)$

Thus it is seen that $\sqrt{ZY}$ and hyperbolic functions of $\sqrt{ZY}$ are functions of : (*i*) *fl*, the product of frequency in Hz and length of line in km ; (*ii*) r/x, the ratio of resistance to reactance ; (*iii*) $\sqrt{L_0C_0}$, which may be treated as constant, as indicated earlier.

The *ABCD* constants in charts are represented in terms of the magnitude of the characteristic impedance of the line, *i.e.* in per unit of the characteristic impedances, $|\dot{Z}_c|$.

If $\dot{Z}_c = |Z_c| \underline{/\xi}$

$$\left.\begin{aligned} \dot{A} &= \cosh \dot{\gamma} l = \dot{D} \\ \dot{B} &= \sqrt{\dot{Z}/\dot{Y}} \sin \gamma l \\ &= Z_c (\sinh \gamma l) \underline{/\xi} \text{ ohm} \\ C &= \frac{\sinh \gamma l}{Z_c} \overline{/\xi} \text{ ohm} \end{aligned}\right\} \qquad ...(1.55)$$

Clearly, *A* and *D* are dimensionless, and are independent of Z_c. When expressed in per unit,

$$\left.\begin{aligned} A &= \cosh \gamma l = D \\ B &= \frac{B \text{ in ohm}}{Z_c} = (\sinh \gamma l)\,\underline{/\xi} \\ C &= Z_c \times (C \text{ in mho}) = (\sinh \gamma l)\,\overline{/\xi} \end{aligned}\right\} \quad ...(1.56)$$

Substituting from Eq. (1.54) for γl in Eq. (1.56), and letting

$$\begin{aligned} \frac{2\pi f}{3 \times 10^5} &= \frac{2\pi \times 50}{3 \times 10^5} \\ &= 104.8 \times 10^{-5} \quad (\text{for } f = 50 \text{ Hz}) \\ &= 1.048 \times 10^{-3} \end{aligned}$$

we obtain the constants in p.u. in the following form

$$\left.\begin{aligned} A &= \cosh\left(j\,1.048 \times 10^{-3}\, l\sqrt{1 - j\frac{r}{x}}\right) = D \\ B &= \sinh\left(j\,1.048 \times 10^{-3}\, l\sqrt{1 - j\frac{r}{x}}\right)\underline{/\xi} \\ C &= \sinh\left(j\,1.048 \times 10^{-3}\, l\sqrt{1 - j\frac{r}{x}}\right)\overline{/\xi} \end{aligned}\right\} \quad ...(1.57)$$

The angle ξ associated with the characteristic impedance is also a function of r/x.

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{r + j\omega L_0}{j\omega C_0}} \quad ...(1.58)$$

$$= Z_c \,\underline{\bigg/ \frac{\tan^{-1}(\omega L_0/r) - 90^\circ}{2}} \quad ...(1.59)$$

Thus $\quad \xi = -\frac{1}{2}(\tan^{-1} r/x) \quad ...(1.60)$

Since the constants are complex functions, the real and quadrature components are plotted separately in the charts and expressed in per unit in terms of r/x and l

$$\left.\begin{aligned} A &= A_1 + jA_2 \\ B &= B_1 + jB_2 \\ C &= C_1 + jC_2 \end{aligned}\right\} \quad ...(1.61)$$

The charts come in very handy as a ready reckoner in preparation for problems on long transmission lines.

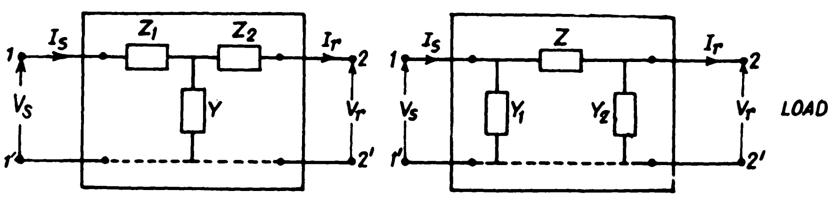
1.5. Generalized Circuit Constants

1.5.1. General circuit equations

In the preceding articles exclusively devoted to transmission lines, represented on a single-phase circuit, the generalized circuit constants $ABCD$ were introduced, in terms of which the sending end voltage and current were expressed as functions of receiving end voltage and current.

In practice the transmission line may have terminal apparatus, such as power transformers, both at the sending and receiving ends. If the Power Transformers are not identical, the transmission network comprising transformers and lines will cease to be symmetrical, and the T or π-networks will be unsymmetrical circuits such as in Fig. 1.27, in which the network is portrayed as a four-terminal (or two-terminal-pair) network. The terminals are marked 1, 1′ at one end, and 2, 2′ at the other end.

Proceeding systematically from the terminal pair 2, 2′ (receiving end) to the pair 1, 1′ (sending end), the student will prove the following relations for the T-network.



(*a*) Unsymmetrical T (*b*) Unsymmetrical π

Fig. 1.27. General two-terminal-pair network.

$$V_s = (1 + Z_1Y)\,V_r + (Z_1 + Z_2 + Z_1Z_2Y)\,I_1$$

$$= AV_r + BI_r \qquad ...(1.62)$$

$$I_s = YV_r + (1 + Z_2Y)\,I_r \qquad ...(1.63)$$

$$= CV_r + DI_r$$

It is thus seen that

$$\left.\begin{array}{l} A = 1 + Z_1Y\,;\; B = Z_1 + Z_2 + Z_1Z_2Y \\ C = Y\,;\; D = 1 + Z_2Y \end{array}\right\} \qquad ...(1.64)$$

Eqns. (1.64) show that

A and D are not necessarily equal, as Z_1 and Z_2 may differ. In case of symmety,

$$Z_1 = Z_2$$

and $$A = D$$

$$\dot{I}_s = (\dot{C} + \dot{D}\dot{Y}_T)\dot{V}_L + [\dot{C}\dot{Z}_T + \dot{D}(1 + \dot{Z}_T\dot{Y}_T)]\dot{I}_L \quad ...(1.120)$$

$$= C'\, V_L + D' I_L$$

whence

$$\left.\begin{aligned} \dot{C}' &= \dot{C} + \dot{D}\dot{Y}_T \\ \dot{D}' &= \dot{C}\dot{Z}_T + \dot{D}(1 + \dot{Z}_T\dot{Y}_T) \end{aligned}\right\} \quad ...(1.121)$$

The Eqs. (1.119) and (1.121) give the expressions for the new constants A', B', C', D' which can now be evaluated as all the constants except C are known. C may be evaluated by using the popular relations $AB - BC = 1$.

$$\dot{A}' = \dot{A} + \dot{B}\dot{Y}_T$$

$$= 0.94\,\underline{/1.5^\circ} + (150\,\underline{/67.2^\circ}) \times (2.5 \times 10^{-4}\,\overline{/7.5^\circ})$$

$$A' = \mathbf{0.977\,\underline{/1.1^\circ}}$$

$$\dot{B}' = \dot{A}\dot{Z}_T + \dot{B}(1 + \dot{Z}_T\dot{Y}_Y)$$

$$= (0.94\,\underline{/1.5^\circ}) \times (100\,\underline{/7.0^\circ}) + (150\,\underline{/67.2^\circ})$$

$$\times (1 + 100 \times 2.5 \times 10^{-4}\,\underline{/70^\circ - 75^\circ})$$

Simplifying, $B' = \mathbf{247.6\,\underline{/68.7^\circ}}$ **ohm**

Before calculating C', we have to find the value of C.

As $\quad AD - BC = 1,$

$$\dot{C} = \frac{\dot{A}\dot{D} - 1}{B}$$

$$= \frac{(0.94\,\underline{/1.5^\circ})^2 - 1}{150\,\underline{/67.2^\circ}}$$

$$\dot{C} = \mathbf{8.7 \times 10^{-4}\,\underline{/91.3^\circ}}\ \textbf{mho}$$

$$\dot{C}' = \dot{C} + \dot{D}\dot{Y}_T$$

$$= 8.7 \times 10^{-4} \times \underline{/91.3^\circ} + (0.94\,\underline{/1.5^\circ})$$

$$\times (2.5 \times 10^{-4}\,\overline{/7.5^\circ})$$

$$C' = \mathbf{6.42 \times 10^{-4}\,\underline{/85.8^\circ}}$$

(on simplification of the above)

$$\dot{D}' = \dot{C}\dot{Z}_T + \dot{D}(1 + \dot{Z}_T\dot{Y}_T)$$

$$= (8.7 \times 10^{-4} \times 100)\,\underline{/161.3^\circ} + (0.94\,\underline{/1.5^\circ})$$

$$\times (1 + 100 \times 2.5 \times 10^{-4}\,\underline{/70^\circ - 75^\circ})$$

$$D' = \mathbf{0.882\,\underline{/3.3^\circ}}.$$

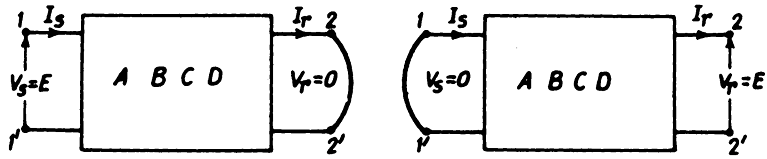
As $$I_r = -\frac{A}{B}E, \text{ we get}$$

$$-\frac{E}{B} = CE + D\,[-(A/B)E]$$

$$-\frac{1}{B} = C - \frac{AD}{B}$$

$$\left(\frac{AD}{B}\right) - C = \frac{1}{B},$$

whence $$AD - BC = 1.$$



(*a*) the receiving end (*b*) the sending end

Fig. 1.28. Two-terminal pair network with a short-circuit.

1.5.2. Transmission line with transformers at both ends

Fig. 1.29 shows an equivalent single phase network corresponding to a transmission system comprising two transformers, one at each end of a transmission line. The transformer T_1 at the sending end has an equivalent series impedance Z_{T_1} and negligible shunt admittance. The transmission line has the general constants *A, B, C, D*. The transformer T_2 at the receiving end has an equivalent series impedance Z_{T_2} and negligible shunt admittance.

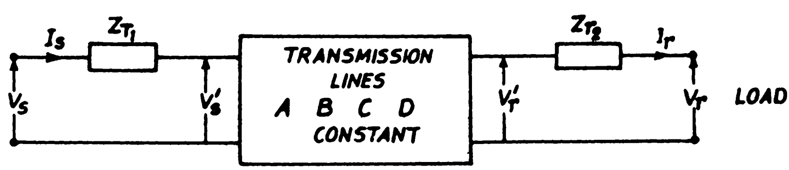


Fig. 1.29. Transmission system network for transmission line with terminal transformers.

The problem is to evaluate the constants A_0, B_0, C_0, D_0 for the overall transmission system. As the transformers are portrayed by merely the series impedances Z_{T_1} and Z_{T_2}, the supply current I_s is the same as the input current to the transmission line, and similarly, the output current of the line, I_r is the same as the load current. However, the voltage V_s (input to the sending end transformer) is different from the input voltage V_s' to the transmission line. Similarly the output voltage of the line V_r' is different from the voltage V_r

across the load at the receiving end. The voltage and current relations are as follows

$$V_s' = A V_r' + B I_r \quad ...(1.72)$$

where $$V_r' = V_r + I_r Z_{T_1} \quad ...(1.73)$$

Hence $$V_s' = A(V_r + I_r Z_{T_2}) + B I_r \quad ...(1.74)$$

$$V_s = V_s' + I_s Z_{T_1} \quad ...(1.75)$$

$$I_s = C V_r' + D I_r$$

$$= C(V_r + I_r Z_{T_2}) + D I_r$$

$$= C V_r + (C Z_{T_2} + D) I_r \quad ...(1.76)$$

Combining (1.74) and (1.75), substituting for I_s from (1.76) and rearranging the terms containing V_r and I_r, we get

$$V_s = (A + C Z_{T_1}) V_r + [B + A(Z_{T_1} + Z_{T_2}) + C Z_{T_1} Z_{T_2}] I_r \quad ...(1.77)$$

Eq. (1.77) is of the form

$$V_s = A_0 V_r + B_0 I_r$$

where $$A_0 = A + C Z_{T_1}$$

and $$B_0 = B + A(Z_{T_1} + Z_{T_2}) + C Z_{T_1} Z_{T_2} \quad ...(1.78)$$

Eq. (1.76) is of the form

$$I_s = C_0 V_r + D_0 I_r$$

where $$C_0 = C$$

and $$D_0 = C Z_{T_2} + D$$

$$= C Z_2 + A \quad ...(1.79)$$

as $A = D$ for transmission line.

1.5.3. Transmission line with series impedance at the receiving end

Fig. 1.30 depicts a transmission line with $ABCD$ constants, at the receiving end of which is connected a series impedance Z_r between the line and load.

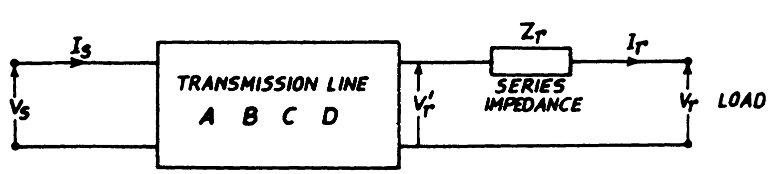


Fig. 1.30. Transmission line with series impedance Z_r.

It is required to find the A_0, B_0, C_0, D_0 of the overall transmission system.

Letting the transmission line output voltage be V_r' and the load voltage V_r, we can write

$$V_r' = V_r + I_r Z_r \qquad ...(1.80)$$

$$\begin{aligned} V_s &= A V_r' + B I_r \\ &= A(V_r + I_r Z_r) + B I_r \\ &= A V_r + (A Z_r + B) I_r \end{aligned} \qquad ...(1.81)$$

$$\begin{aligned} I_s &= C V_r' + D I_r \\ &= C V_r + D + C Z_r) I_r \end{aligned} \qquad ...(1.82)$$

For the transmission system as a whole

$$\left.\begin{aligned} V_s &= A_0 V_r + B_0 I_r \\ I_s &= C_0 V_r + D_0 I_r \end{aligned}\right\} \qquad ...(1.83)$$

By comparison of the above with Eqs. (1.81) and (1.82), we obtain

$$\left.\begin{aligned} A_0 &= A \\ B_0 &= A Z_r + B \\ C_0 &= C \\ D_0 &= D + C Z_r \end{aligned}\right\} \qquad ...(1.84)$$

1.5.4. Transmission line with series impedance at the sending end

Fig. 1.31 shows a transmission line with *ABCD* constants, at the sending end of which is connected a series impedance Z_s between the supply and the line.

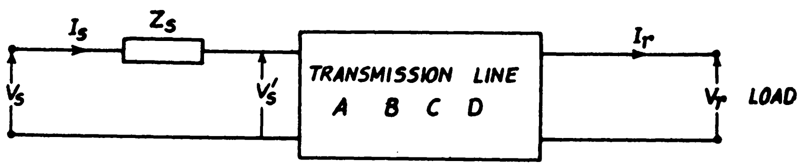


Fig. 1.31. Transmission line with series impedance Z_s.

Proceeding as in the preceding example with Z_r in series with the line, the student will prove the following relations, as an exercise :

$$\left.\begin{aligned} A_0 &= A + C Z_s \\ B_0 &= B + D Z_s = B + A Z_s \\ C_0 &= C \\ D_0 &= D \end{aligned}\right\} \qquad ...(1.85)$$

1.5.5. Generalized constants of combined networks

(*a*) **Two networks in tandem (series).** Two transmission networks, 1 and 2, are connected in series, such that the output of the network 1 is the input to the network 2. The input to network 1 and output of network 2 are the overall input and output, respectively of the combined system.

Fig. 1.32 shows the two networks connected in tandem.

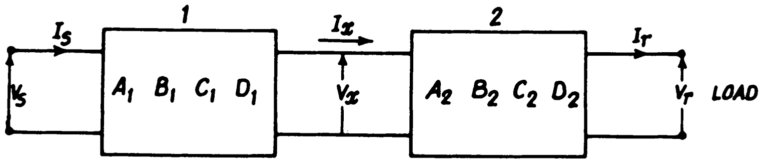


Fig. 1.32. Series-connection of two transmission networks.

The voltage and current relations for the network 1 are (in matrix form)

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_x \\ I_x \end{bmatrix} \qquad ...(1.86)$$

Similarly, for the network 2, we have

$$\begin{bmatrix} V_x \\ I_x \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix} \qquad ...(1.87)$$

For the combined networks,

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix} \qquad ...(1.88)$$

where *ABCD* are the constants for the overall system, combining the two networks. From Eqs. (1.86), (1.87) and (1.88), we get the *ABCD* constants by matrix multiplication, as follows :

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \times \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \qquad ...(1.89)$$

The expressions for *A, B, C, D* are readily obtained from Eq. (1.89)

$$\left.\begin{aligned} A &= A_1 A_2 + B_1 C_2 \\ B &= A_1 B_2 + B_1 D_2 \\ C &= C_1 A_2 + D_1 C_2 \\ D &= C_1 B_2 + D_1 D_2 \end{aligned}\right\} \qquad ...(1.90)$$

(*b*) **Two networks in parallel.** If the two networks 1 and 2 are connected in parallel, at both ends, voltages at the sending and receiving ends will be common for them.

Fig. 1.33 shows the parallel-connected networks with constants A_1 B_1 C_1 D_1 and A_2 B_2 C_2 D_2 respectively.

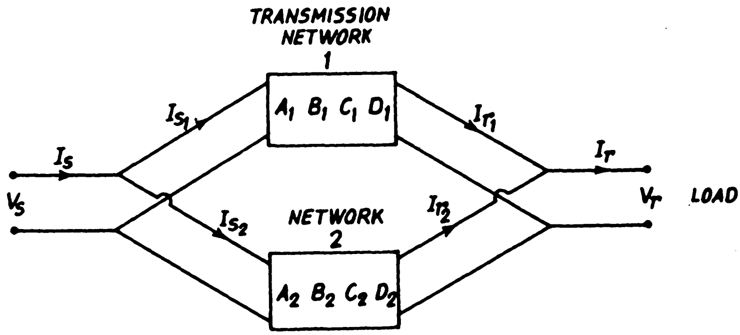


Fig. 1.33. Transmission networks in parallel.

The total current at the sending end I_s is divided into I_{s_1} and I_{s_2} ; similarly, at the receiving end, the currents are I_{r_1} and I_{r_2} when the total load current is I_r.

$$I_s = I_{s_1} + I_{s_2}$$
$$I_r = I_{r_1} + I_{r_2}$$

It the combined system has the constants A, B, C, D, then

$$\left.\begin{aligned} V_s &= A V_r + B I_r \\ I_s &= C V_r + D I_r \end{aligned}\right\} \qquad \text{...(1.91)}$$

where A, B, C, D have to be evaluated in terms of the constants of the parallel connected networks.

For network 1,

$$\left.\begin{aligned} V_s &= A_1 V_r + B_1 I_{r_1} \\ I_{s_1} &= C_1 V_r + D_1 I_{r_1} \end{aligned}\right\} \qquad \text{...(1.92)}$$

and for 2,

$$\left.\begin{aligned} V_s &= A_2 V_r + B_2 I_{r_2} \\ I_{s_2} &= C_2 V_r + D_2 I_{r_2} \end{aligned}\right\} \qquad \text{...(1.93)}$$

Equating the expressions for V_s in Eqs. (1.92) and (1.93),

$$A_1 V_r + B_1 I_{r_1} = A_2 V_r + B_2 I_{r_2} \qquad \text{...(1.94)}$$

As $\quad I_{r_2} = I_r - I_{r_1}$

I_{r_2} may be eliminated from Eq. (1.94),

whence $\quad I_{r_2} = \dfrac{B_2 I_r - V_r (A_1 - A_2)}{B_1 + B_2}$

substituting which in the expression for V_s in Eq. (1.92), we get

$$V_s = \left(\frac{A_1 B_2 + A_2 B_1}{B_1 + B_2} \right) V_r + \left(\frac{B_1 B_2}{B_1 + B_2} \right) I_r \qquad ...(1.95)$$

Adding, I_{s_1} and I_{s_2} from Eqs. (1.92) and (1.93) and replacing I_{r_2} by the expression for I_{r_1}

$$I_s = I_{s_1} + I_{s_2}$$

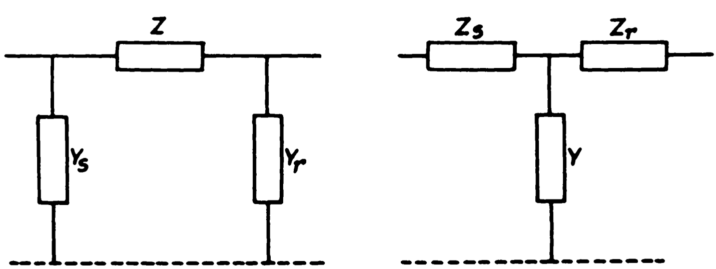
$$= \left[C_1 + C_2 + \frac{(A_1 - A_2)(D_2 - D_1)}{B_1 + B_2} \right] V_r + \left[\frac{D_2 B_1 + B_2 D_1}{B_1 + B_2} \right] I_r' \qquad ...(1.96)$$

Comparison of relations in Eqs. (1.95) and (1.96) with (1.91) yields the constants $A\ B\ C\ D$, as follows :

$$\left.\begin{aligned} A &= \frac{A_1 B_2 + A_2 B_1}{B_1 + B_2} \\ B &= \frac{B_1 B_2}{B_1 + B_2} \\ C &= \frac{(A_1 - A_2)(D_2 - D_1)}{B_1 + B_2} + C_1 + C_2 \\ D &= \frac{D_2 B_1 + B_2 D_1}{B_1 + B_2} \end{aligned}\right\} \qquad ...(1.97)$$

Example 1.12. *Determine the series impedances and shunt admittances of the unsymmetrical π-and T-networks in terms of the ABCD constants.*

Solution. Fig. 1.34 shows the two networks in parts (*a*) and (*b*) respectively.



(*a*) (*b*)

Fig. 1.34. Unsymmetrical π- and *T*-networks for Example 1.12.

For unsymmetrical π-circuit,

$$\left.\begin{aligned} A &= 1 + ZY_r \\ B &= Z \\ C &= Y_s + Y_r + Z\,Y_s\,Y_r \\ D &= 1 + Z\,Y_1 \end{aligned}\right\} \quad \text{... From Eq. (1.68)}$$

Substituting for Z in the expression for A,

$$A = 1 + BY_r$$

whence

$$\boxed{\begin{aligned} Y_r &= \frac{A-1}{B} \\ Y_s &= \frac{D-1}{Z} = \frac{D-1}{B} \\ Z &= B \end{aligned}} \qquad \text{...(1.98)}$$

For unsymmetrical T-circuit, it is seen from Eq. (1.64), that

$$\begin{aligned} A &= 1 + Z_sY \\ B &= Z_s + Z_r + Z_sZ_rY \\ C &= Y \\ D &= 1 + Z_rY \end{aligned}$$

Solving the above, as for π-circuit,

$$\boxed{\begin{aligned} & Z_s = \frac{A-1}{C}\,; \quad Z_r = \frac{D-1}{C} \\ & Y = C \end{aligned}} \qquad \text{...(1.99)}$$

1.6. Interpretation of ABCD Constants for Power System Studies

We have seen that the sending and receiving end data are mutually expressible in terms of *ABCD* constants

$$V_s = A\,V_r + B\,I_r$$

from which

$$I_r = \frac{V_s - A\,V_r}{B}$$

$$\boxed{I_r = \frac{V_s}{B} - \frac{A}{B}\,V_r} \qquad \text{...(1.100)}$$

Thus the receiving end current is expressed as a function of V_s and V_r. The first part may be interpreted as the receiving end current due to voltage applied at the sending end, with receiving end short-circuited *viz.*, $V_r = 0$.

$$I_r = \frac{V_s}{D} \text{ with } V_r = 0$$

$$I_r = \frac{V_s}{Z_t}, \quad \text{where } Z_t = B$$

Z_t is referred to as the transfer impedance of the system.

$$\boxed{Z_t = B} \qquad \text{...(1.101)}$$

The sending end voltage and current may be expressed in matrix form

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix} \qquad \text{...(1.102)}$$

whence, V_r and I_r may be expressed in terms of V_s and I_s by matrix inversion

$$\begin{bmatrix} V_r \\ I_r \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} V_s \\ I_s \end{bmatrix}$$

As $AD - BC = 1$, we obtain

$$\begin{bmatrix} V_r \\ I_r \end{bmatrix} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{bmatrix} V_s \\ I_s \end{bmatrix} \qquad \text{...(1.103)}$$

$$\left.\begin{aligned} V_r &= D V_s - B I_s \\ I_r &= -C V_s + A I_s \end{aligned}\right\} \qquad \text{...(1.103 } a\text{)}$$

With a voltage V_r applied at the receiving end, and sending end terminal-pair short-circuited, $V_s = 0$.

$$\frac{V_r}{-I_r} = \frac{B}{A} = Z_r$$

Z_r may be referred to as the driving point impedance at the receiving end.

Eq. (1.100) may be written as

$$\boxed{I_r = \frac{V_s}{Z_t} - \frac{V_r}{Z_r}} \qquad \text{...(1.104)}$$

Receiving end driving point impedance is given by

$$Z_r = \frac{B}{A}$$

$$= \frac{\dot{B}\,\angle\beta}{\dot{A}\,\angle\alpha}, \quad \text{expressed in polar form,}$$

$$\dot{Z}_r = \frac{\mathbf{B}}{\mathbf{A}}\angle\beta - \alpha \qquad \text{...(1.105)}$$

Using the relation in Eq. (1.103 *a*),

$$\dot{V}_r = \dot{D}\dot{V}_s - \dot{B}\,\dot{I}_s$$

If V_s is applied at the sending end, with receiving end short-circuited, *viz.* $V_r = 0$, we get

$$\dot{Z}_s = \frac{\dot{V}_s}{\dot{I}_s} = \frac{\dot{B}}{\dot{D}}$$

$$= \frac{\dot{B}\,\angle\beta}{\dot{A}\,\angle\Delta} = \frac{B}{D}\angle\beta - \Delta$$

This impedance is the sending end driving point impedance

$$\dot{Z}_s = \frac{\mathbf{B}}{\mathbf{D}}\angle\beta - \Delta \qquad \text{...(1.106)}$$

As $$\dot{V}_r = \dot{D}\dot{V}_s - \dot{B}\,\dot{I}_s$$

$$\dot{I}_s = \frac{\dot{D}\dot{V}_s}{\dot{B}} - \frac{\dot{V}_r}{\dot{B}}$$

$$\boxed{\dot{I}_s = \frac{V_s}{\dot{Z}_s} - \frac{\dot{V}_r}{\dot{Z}_t}} \qquad \text{...(1.107)}$$

The Eqs. (1.104) and (1.107) give the receiving end current and sending current respectively, each in terms of the transfer impedance and the respective driving point impedance.

Further analysis on the use of the above impedance will be taken up in developing expressions for power under 'Power Flow' in Chapter 4.

The impedances Z_r and Z_s are driving point impedances, with sending and receiving ends, respectively, short-circuited. With the receiving end open circuited, if measurement of impedance were done at the sending end, the measured impedance would be

$$Z_{so} = \left.\frac{V_s}{I_s}\right]_{(I_r = 0)}$$

Similarly the measured impedance at the receiving end, with sending end open circuited, would be

$$Z_{ro} = \left.\frac{V_r}{I_r}\right]_{(I_s = 0)}$$

Recalling, $\dot{V}_s = \dot{A}\dot{V}_r + \dot{B}\dot{I}_r$; $\dot{V}_r = \dot{D}\dot{V}_s - \dot{B}\dot{I}_s$

$\dot{I}_s = \dot{C}\dot{V}_r + \dot{D}\dot{I}_r$; $\dot{I}_r = -\dot{C}\dot{V}_s + \dot{A}\dot{I}_s$

$$\left.\begin{aligned} \dot{Z}_{so} &= \left.\frac{\dot{V}_s}{\dot{I}_s}\right]_{(I_r = 0)} = \frac{\dot{A}}{\dot{C}} = \frac{A\,\angle\alpha}{C\,\angle\gamma} \\ &= \frac{A}{C}\angle\alpha-\gamma \end{aligned}\right\} \quad \text{....(1.108)}$$

$$\dot{Z}_{ro} = \left.\frac{\dot{V}_r}{-\dot{I}_r}\right]_{(I_s = 0)} = \frac{\dot{D}}{\dot{C}}$$

$$= \frac{D\,\angle\Delta}{C\,\angle\gamma} = \frac{D}{C}\angle\Delta-\gamma \quad \text{...(1.109)}$$

$$\dot{Z}_{ro} - \dot{Z}_r = \frac{\dot{D}}{\dot{C}} - \frac{\dot{B}}{\dot{A}} = \frac{AD - BC}{\dot{A}\dot{C}} = \frac{1}{\dot{A}\dot{C}} \quad \text{...(1.110)}$$

$$\dot{Z}_{so} = \frac{\dot{A}}{\dot{C}} \quad \text{...(1.111)}$$

From Eqs. (1.110) and (1.111),

$$\frac{\dot{Z}_{so}}{\dot{Z}_{ro} - \dot{Z}_r} = \frac{\dot{A}}{\dot{C}} \times \dot{A}\,\dot{C} = (\dot{A})^2$$

Therefore $$\boxed{\dot{A} = \sqrt{\frac{\dot{Z}_{so}}{\dot{Z}_{ro} - \dot{Z}_r}}} \quad \text{...(1.112)}$$

As $\dot{Z}_{so} = \dfrac{\dot{A}}{\dot{C}}$,

$$\dot{C} = \frac{\dot{A}}{\dot{Z}_{so}} = \frac{1}{\dot{Z}_{so}}\sqrt{\frac{\dot{Z}_{so}}{\dot{Z}_{so} - \dot{Z}_r}}$$

$$\boxed{\dot{C} = \sqrt{\frac{1}{\dot{Z}_{so}(\dot{Z}_{ro} - \dot{Z}_r)}}} \quad \text{...(1.113)}$$

As $\dot{Z}_{ro} = \dfrac{\dot{D}}{\dot{C}}$,

$$\boxed{\dot{D} = \dot{C}\dot{Z}_{ro} = \dot{Z}_{ro}/\sqrt{\dot{Z}_{so}(\dot{Z}_{ro} - \dot{Z}_r)}} \quad \text{...(1.114)}$$

It is seen from Eqs. (1.112) and (1.114), that A and D are generally not equal. However, if $Z_{ro} = Z_{so}$ by virtue of symmetry, then $A = D$, *viz.*, for symmetrical networks.

$$Z_r = \frac{B}{A}$$

Hence $$B = AZ_r$$

$$\boxed{B = Z_r \sqrt{\left(\frac{Z_{so}}{Z_{ro} - Z_r}\right)}} \qquad \text{...(1.115)}$$

The Eqs. (1.112) to (1.115) give the values of the $ABCD$ constants each in terms of complex impedances.

A few typical examples are given below, followed by those under Exercises to afford practice to the student in Power System problems on generalized circuit constants as well as transmission line performance.

Example 1.13. *A three phase transmission line has the generalised circuit constants given by*

$$A = D = 0.98 \angle 1.5^\circ\,;$$
$$B = 75.5 \angle 80^\circ\,;$$
$$C = 0.0004 \angle 91^\circ.$$

If an impedance 2.64 + j 42.3 ohm is connected in series with the line at the sending end, determine the modified values of the above constants for the entire transmissions system. (*Madurai University, Nov. 1969*)

Solution. Referring to Article 1.5.4, we find that, in Fig. 1.31, the transmission system constants would be as given by Eq. (1.85) reproduced below

$$A_0 = A + CZ_s$$
$$B_0 = B + DZ_s$$
$$C_0 = C$$
$$D_0 = D$$

In this example, $Z_r = 2.64 + j\,42.3$ ohm
$= 42.4 \angle 86.4^\circ$ ohm

Hence $A_0 = 0.98 \angle 1.5^\circ = (0.0004 \angle 91^\circ) \times (42.4 \angle 86.4^\circ)$

Simplifying, we get

$$\boxed{A_0 = 0.963 \angle 1.53^\circ}$$

$$B_0 = B + D\,Z$$

$$= 75.5 \underline{/80^\circ} + (0.98 \underline{/1.5^\circ})(42.4 \underline{/86.4^\circ})$$

Simplifying,

$$\boxed{B_0 = 116.8 \underline{/82.8^\circ}}$$

$$C_0 = C = 0.0004 \underline{/91^\circ}$$

$$\mathbf{D_0 = D = 0.98 \underline{/1.5^\circ}}$$

The constants C_0 and D_0 are not affected by the introduction of a series impedance at the sending end. One should not conclude that the current given by the expression $I_s = CV_r + DI_r$ is not affected by Z_s. It should be noted that if the load impedance is constant, I_r is constant for a given value of V_r, in which case I_s does not change. However, for a given V_s, the values of V_r and I_r will be affected by the presence of Z_s for a given equivalent load impedance. If V_r is restricted, then V_s will be affected, and has to be regulated.

Example 1.14. *(a) A three phase transmission line delivers 20 MW at a power factor of 0.8 lagging at 32 kV. The transmission line constants for the line, considered as a π-network are as follows :*

$$A = D = 1$$

$$B = (1 + j\,3) \text{ ohm}$$

$$C = j\,2 \times 10^{-4} \text{ mho}$$

Determine the sending end current and voltage of the line and approximate value for the reactive KVA taken by the line.

(b) Describe the operation of apparatus that could be installed at the receiving end of the line in order to maintain the receiving end voltage constant for a given variation in load. (*Madurai University Nov. 1979*)

Solution. (*a*) The problem is to evaluate the sending end voltage and current for the specified receiving end conditions and also the reactive power absorbed by the line.

Taking the receiving end voltage as the reference phasor,

$$V_r = \frac{32}{\sqrt{3}} \underline{/0^\circ} = 18.5 \underline{/0^\circ} \text{ kV/Phase}$$

$$I_r = \frac{20{,}000}{\sqrt{3} \times 32 \times 0.8} = 451.1 \text{ Amp.}$$

$$I_r = 451.1 \overline{\big/36.9^\circ} \text{ Amp.} = 451.1\,(0.8 - j\,0.6)$$

$$V_s = A\,V_r + B\,I_r$$

$$= 1 \times 18500 + (1 + j\,3)\,451.1(0.8 - j\,0.6) \text{ volt}$$

$$= 19.7 \underline{/2.4^\circ} \text{ kV/Phase}$$

Hence line to line voltage at the sending end,

$$V_s\,(L-L) = \sqrt{3} \times 19.7$$
$$= \mathbf{34.1\ kV}$$
$$\underset{\cdot}{I_s} = \underset{\cdot}{C}\underset{\cdot}{V_r} + \underset{\cdot}{D}\,\underset{\cdot}{I_r}$$
$$= j\,2 \times 10^{-4} \times 18{,}500 + 1 \times 451.1\,(0.8 - j\,0.6)$$
$$= 361 - j\,267.3$$

$$\mathbf{I_s = 450\,\angle 36.5^\circ\ Amp.}$$

Fig 1.35 depicts the phase relations of voltages and currents.

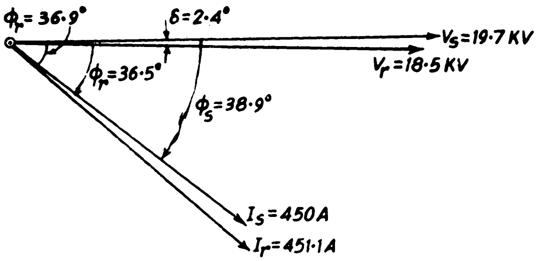


Fig. 1.35. Phasor diagram for Example 1.14.

Reactive power at the sending end of transmission line

$$= 3V_sI_s \times \sin \phi_s$$
$$= 3 \times 19.7 \times 450 \times 0.627 \text{ KVAR}$$
$$= \mathbf{16.7\ MVAR\ (lagging).}$$

Reactive power absorbed by the load at the receiving end

$$= \frac{20}{0.8} \times 0.6 = \mathbf{15\ MVAR\ (lagging)}$$

Hence the reactive power taken by the line

$$= 16.7 - 15 = \mathbf{1.7\ MVAR.}$$

Example 1.15. *A transmission line consists of two circuits 1 and 2 connected in series, circuit 1 being at the serding end of the line. The circuits have the following auxiliary constants :*

Circuit 1	*Circuit 2*
$\underset{\cdot}{A_1} = 0.982\,\angle 1.2^\circ$	$\underset{\cdot}{A_2} = 0.808\,\angle 2.0^\circ$
$\underset{\cdot}{B_1} = 77.3\,\angle 80.0^\circ$	$\underset{\cdot}{B_2} = 30.0\,\angle 45.0^\circ$
$\underset{\cdot}{C_1} = 0.000452\,\angle 91.0^\circ$	$\underset{\cdot}{C_2} = 0.001\,\angle 92.0^\circ$
$\underset{\cdot}{D_1} = 0.982\,\angle 1.2^\circ$	$\underset{\cdot}{D_2} = 0.808\,\angle 2.0^\circ$

Develop expressions for the constants A, B, C, D of the entire line and calculate the numerical value of the constant A.

Solution. The student will refer to the Article, 1.5.5, part (*a*), for the complete derivation of the expressions for the *ABCD* constants. It is seen that

$$\dot{A} = \dot{A}_1 \dot{A}_2 + \dot{B}_1 \dot{C}_2$$

Substituting for the constants

A_1, A_2, B_1 and C_2,

$$A = (0.982 \angle 1.2^\circ) \times (0.808 \angle 2.0^\circ) + (77.3 \angle 80.0^\circ) \times (0.001 \angle 92.0^\circ)$$

Simplifying $\quad \dot{A} = 0.7151 + j\,0.0550 = 0.717 \angle 4.4^\circ$

Numerical value of $\dot{A}$ would be

$$A = \mathbf{0.717.}$$

Example 1.16. *Determine the driving point and transfer impedances of a 350 km long, 132 kV transmission line which has the network constants as follows :*

$$\dot{A} = D = 0.943 \angle 1.3^\circ$$

$$\dot{B} = 140.1 \angle 69.8^\circ$$

$$\dot{C} = 0.865 \times 10^{-3} \angle 90^\circ.$$

Solution. For theoretical background, refer to Article 1.6.

The transfer impedance is given by the constant *B* itself.

$$\dot{Z}_t = \mathbf{140.1 \angle 69.8^\circ \text{ ohm}}$$

The driving point impedances are

$$\dot{Z}_s = \frac{B}{\dot{D}} \text{ (at the sending end)}$$

and
$$\dot{Z}_r = \frac{B}{\dot{A}} \text{ (at the receiving end)}$$

[vide Eqs. (1.106) and (1.105) respectively.]

As $A = D$, for the symmetrical network (from the data), clearly, the sending and receiving end driving point impedances are equal.

$$\dot{Z}_s = \dot{Z}_r = \frac{140.1 \angle 69.8^\circ}{0.943 \angle 1.3^\circ}$$

$$= \mathbf{148.6 \angle 68.5^\circ \text{ ohm}.}$$

Example 1.17. *The sending and receiving end voltages and currents of a long transmission line are given by the expressions*

$$V_s = AV_r + BI_r$$

and
$$I_s = CV_r + DI_r$$

The line has generalized constants given by $A = D = 0.94 \underline{/1.5^\circ}$ *and* $B = 150 \underline{/67.2^\circ}$ *ohm and has at the load end a transformer equivalent to a shunt admittance of* $Y_T = 2.5 \times 10^{-4} \overline{/75^\circ}$ *mho across the receiving end of the line and a series impedance of* $Z_T = 100 \underline{/70^\circ}$ *ohm in series with the load. The load voltage and current are* V_L *and* I_L *respectively. Obtain expressions for* V_s *and* I_s *in the form* $V_s = A'V_L + B'I_L$ *and* $I_s = C'V_L + D'I_L$ *and evaluate these complex constants.*

Solution. An equivalent circuit is drawn in Fig. 1.36 showing the transmission line circuit and the transformer equivalent parameters.

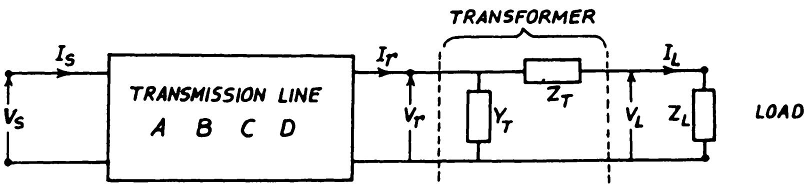


Fig. 1.36. Equivalent single-phase-circuit for Example 1.17.

For transmission line,

$$V_s = AV_r + BI_r$$

$$I_s = CV_r + DI_r$$

Now we are called upon to replace the $V_r - I_r$ terms by $V_L - I_L$ terms, and find the new constants, *viz.* A' B' C' D'.

Inspection of the equivalent circuit in Fig. 1.36 shows that

$$\left.\begin{aligned} V_r &= V_L + I_L Z_T \\ I_r &= I_L + V_r Y_T \end{aligned}\right\} \qquad \text{...(1.116)}$$

whence

$$\begin{aligned} I_r &= I_L + Y_T(V_L + I_L Z_T) \\ &= (Y_T)V_L + (1 + Z_T Y_T)I_L \qquad \text{...(1.117)} \end{aligned}$$

$$V_r = V_L + I_L Z_T$$

$$\begin{aligned} V_s &= AV_r + BI_r \\ &= A(V_L + I_L Z_T) + B[Y_T V_L + (1 + Z_T Y_T) I_L] \end{aligned}$$

Simplifying

$$\begin{aligned} V_s &= (A + BY_T)V_L + [AZ_T + B(1 + Z_L Y_T)]I_L \qquad \text{...(1.118)} \\ &= A' V_L + B' I_L \end{aligned}$$

whence

$$\left.\begin{aligned} A' &= A + BY_T \\ B' &= AZ_T + B(1 + Z_T Y_T) \end{aligned}\right\} \qquad \text{...(1.119)}$$

$$\begin{aligned} I_s &= CV_r + DI_r \\ &= C(V_L + I_L Z_T) + D[V_L Y_T + I_L(1 + Z_T Y_T)] \end{aligned}$$

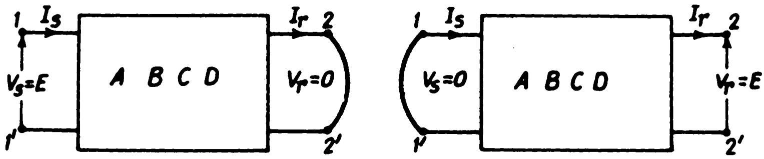
As $$I_r = -\frac{A}{B}E, \text{ we get}$$

$$-\frac{E}{B} = CE + D\,[-(A/B)E]$$

$$-\frac{1}{B} = C - \frac{AD}{B}$$

$$\left(\frac{AD}{B}\right) - C = \frac{1}{B},$$

whence $$AD - BC = 1.$$



(a) the receiving end (b) the sending end

Fig. 1.28. Two-terminal pair network with a short-circuit.

1.5.2. Transmission line with transformers at both ends

Fig. 1.29 shows an equivalent single phase network corresponding to a transmission system comprising two transformers, one at each end of a transmission line. The transformer T_1 at the sending end has an equivalent series impedance Z_{T_1} and negligible shunt admittance. The transmission line has the general constants A, B, C, D. The transformer T_2 at the receiving end has an equivalent series impedance Z_{T_2} and negligible shunt admittance.

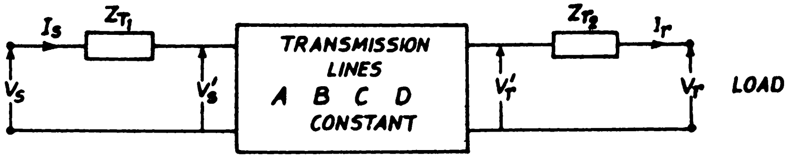


Fig. 1.29. Transmission system network for transmission line with terminal transformers.

The problem is to evaluate the constants A_0, B_0, C_0, D_0 for the overall transmission system. As the transformers are portrayed by merely the series impedances Z_{T_1} and Z_{T_2}, the supply current I_s is the same as the input current to the transmission line, and similarly, the output current of the line, I_r is the same as the load current. However, the voltage V_s (input to the sending end transformer) is different from the input voltage V_s' to the transmission line. Similarly the output voltage of the line V_r' is different from the voltage V_r

1.7. Ferranti Effect

Let us suppose that the sending end voltage V_s of a transmission line is maintained constant as desired under the prescribed load conditions and also when the load is thrown off. For convenience in analysis, if we assume that the resistance and leakance of the line are ignored, we have only the distributed series inductance and shunt capacitance in the line representation. When the load is thrown off, it does not result in cessation of current in the line. as the charging current will flow due to the presence of line capacitance. This current will flow through the series inductance and result in pressure rise at the receiving end of the line. Such a phenomenon is referred to as Ferranti effect. named after Ferranti who was the first to observe the phenomenon on Deptford Mains.

Fig. 1.37 depicts a long transmission line with length l km. consisting of distributed series inductances and shunt capacitances.

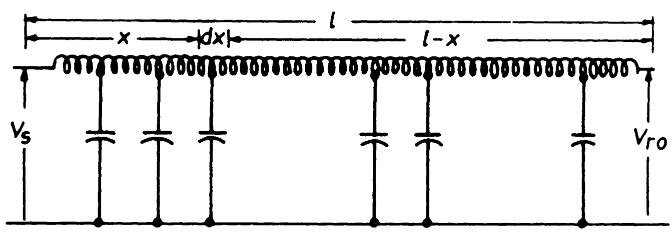


Fig. 1.37. Long transmission line represented by the distributed series inductance and shunt capacitances.

Let us consider a small length 'dx' of the line at a distance x from the sending end and $(l-x)$ from the receiving end. Let L_0 = inductance in Henry per km length and

C_0 = capacitance in Farad per km length of the line.

The elemental length 'dx' has an inductance of $L_0\, dx$ and carries a current equal to the sum of the charging currents over a length $(l-x)$. Let it be I_x.

Taking the average value of transmission voltage as V_s (ignoring the change in voltage from point to point, for the purpose of evaluating the current and the eventual pressure change at the receiving end) we have

$$I_x = j\, V_s\, \omega\, C_0\, (l-x)$$

Voltage drop in the elemental length 'dx'

$$= I_x\, (j\, \omega\, L_0\, dx)$$
$$= j\, V_s\, \omega\, C_0\, (l-x)\, j\, \omega\, L_0\, dx$$
$$= -\, V_s\, \omega^2\, L_0\, C_0\, (l-x)\, dx$$

The negative sign signifies that the drop in voltage is negative, meaning a rise in potential as we proceed from the sending to the receiving end. Total drop from the sending end to the receiving end is obtained by integrating the above from $x = 0$ to $x = l$.

Total drop $\quad v' = -V_s \omega^2 L_0 C_0 \int (l - x)\, dx$

$$v' = -\frac{1}{2} V_s \omega^2 L_0 C_0 l^2. \qquad ...(1.122)$$

The above negative drop indicates that there will be a pressure rise equal in magnitude to $\frac{1}{2} V_s \omega^2 L_0 C_0 l^2$

where V_s is the sending end voltage per phase. If V_s is in kV, the pressure rise is also in kV. If V_s is the line kV, the pressure rise corresponds to increase in line pressure. (line-to-line voltage rise).

Thus the pressure rise is given by

$$v = -v' = \frac{1}{2} V_s \omega^2 L_0 C_0 l^2 \qquad ...(1.123)$$

Total inductance per phase, $L = L_0 l$

and total capacitance to neutral, $C = C_0 l$

Hence the equation (1.123) can be rewritten as

$$v = \frac{1}{2} V_s \omega^2 LC \qquad ...(1.124)$$

Let us now consider a moderately long line represented by a nominal-π circuit, as shown in Fig. 1.38.

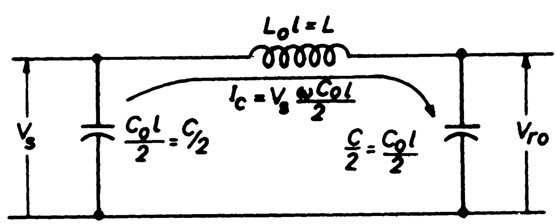


Fig. 1.38. Nominal-π circuit : Ferranti effect.

With no load at the receiving end and the voltage at the sending end, V_s held constant,

charging current flowing through the line is

$$I_c = V_s \frac{1}{j\left\{\omega L_0 l - \dfrac{2}{\omega C_0 l}\right\}}$$

As the reactance due to capacitance is considerably larger than the series inductive reactance of the line, the effect of inductance on the value of current may be justifiably ignored. Accordingly, we may write

$$I_c = V_s \left(j \frac{\omega C_0 l}{2} \right)$$

$$= \frac{1}{2} V_s j \omega C_0 l = j \frac{1}{2} V_s \omega C_0 l$$

This leading current flow through the inductive reactance $\omega L_0 l$ and causes a voltage drop given by

$$v' = I_C (j \omega L_0 l)$$

$$= (j \tfrac{1}{2} V_s j C_0 1) (j \omega L_0 l)$$

$$= -\frac{1}{2} V_s \omega^2 L_0 C_0 l^2 \qquad \text{...(1.125)}$$

As in the general case, voltage drop being negative, there is a pressure rise given by

$$v = -v' = \frac{1}{2} V_s^2 \omega L_0 C_0 l^2$$

where is the same as equation (1.123)

Consequently, the receiving end pressure on open circuit will be given by

$$V_{ro} = V_s + v = V_s + \frac{1}{2} V_s \omega^2 L_0 C_0 l^2$$

$$V_{ro} = V_s (1 + \tfrac{1}{2} V_s \omega^2 L_0 C_0 l^2) \qquad \text{...(1.126)}$$

Now let us consider a long line approximated by a nominal-T circuit as shown in Fig. 1.39.

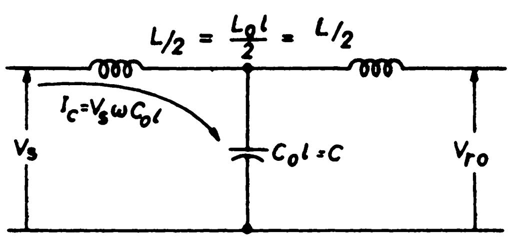


Fig. 1.39. Nominal T circuit : Ferranti effect.

$$L_2 = \frac{L_0 l}{2} = L_2$$

In this case a charging current I_c flow through an inductance $L_0\, l/2$.

$$I_c = j\, V_s\, \omega\, C_0\, l.$$

The charging current flows through $L/2$ and C whose equivalent series impedance is given by $j \left(\omega L_0\, l/2 - \dfrac{1}{\omega C_0 l} \right)$.

It should be noted that the total series impedance that controls the charging currant is merely the capacitive reactance as the inductive reactance becomes insignificant, whereas the line drop is essentially due to the inductive reactance, as the resistance is negligible in effect.

The voltage drop taking place in the line due to the charging current, as in the previous case is, therefore,

$$v' = (j\, V_s\, \omega\, C_0\, l)\,(j\, \omega\, L_0\, l/2) = -\tfrac{1}{2}\, V_s\, \omega^2\, L_0\, C_0\, l^2$$

From the negative pressure drop above, it is seen that the pressure rise at the receiving end will be given by

$$v = -v' = \tfrac{1}{2}\, V\, \omega\, L_0\, C_0\, l^2$$

which is the same as for nominal-π circuit.

Again, the pressure at the receiving end on open circuit is

$$V_{ro} = V_s + \tfrac{1}{2}\, V_s\, \omega^2\, L_0\, C_0\, l^2$$

$$= V_s\,(1 + \tfrac{1}{2}\, V_s\, \omega^2 L_0\, C_0\, l^2)$$

which is the same as equation (1.126).

It should be remembered that the above results are only approximate and would serve the purpose of estimating the pressure rise at the receiving end of a transmission line on open circuit. Of course in case of short lines, wherein the capacitance is insignificant, the receiving end voltage is equal to the sending end voltage itself.

In the general equation for long lines, length l, we have

$$V_s = A\, V_r + B\, I_r$$

where

$$A = \cosh \sqrt{ZY}$$

$$\dot{Z} = j\, \omega\, L_0\, l \quad \text{and} \quad \dot{Y} = j\, \omega\, C_0\, l$$

(again neglecting the effects of resistance and leakance)

Substituting for $\dot{Z}$ and $\dot{Y}$ and expanding,

$$A = \cosh \sqrt{\dot{Z}\dot{Y}} = 1 + \tfrac{1}{2}\, \dot{Z}\, \dot{Y} \text{ (approximately)}$$

$$= 1 + \tfrac{1}{2}\,(j\, \omega\, L_0\, l)(j\, \omega\, C_0\, l)$$

$$= 1 - \tfrac{1}{2}\, \omega^2\, L_0\, C_0 l^2 \qquad \text{...(1.127)}$$

At no load, $I_r = 0$

and $V_r = V_{ro}$

$$\dot{V}_s = \dot{A}\dot{V}_{ro}$$

Hence, receiving end voltage at no load,

$$V_{ro} = \frac{V_s}{A} = \frac{V_s}{1 - \frac{1}{2}\, \omega^2 L_0 C_0\, l^2} = V_s\,(1 + \tfrac{1}{2}\, \omega^2\, L_0\, C_0\, l^2)$$

This expression is exactly the same as equation (1.126). Of course, this is only approximate and is quite adequate for estimating approximately the pressure rise due to the effect of no load charging current flowing through the line inductance.

It is seen from the expression in eqn. (1.123) that the rise in pressure at the receiving end on no load is proportional to square of the distance from the sending end. It may be noted that the product L_0C_0 is a constant as shown below.

$$L_0 = \text{inductance in Henry per km length}$$
$$= \left(0.4606 \log_{10}\frac{D}{r}\right)10^{-3}$$
$$C_0 = \left(\frac{0.0241}{\log_{10}\frac{D}{r}}\right)10^{-6} = \text{capacitance in Farad per km}$$
$$L_0C_0 = 0.4606 \times 0.0241 \times 10^{-9}$$
$$= 1.11 \times 10^{-11}.$$

Hence, rise in pressure at the receiving end per phase is given by

$$v = \tfrac{1}{2}\, V_s\, \omega^2\, l^2 \times 1.11 \times 10^{-11}$$
$$= 0.555 \times 10^{-11}\, V_s\, \omega^2\, l^2 \text{ kV/phase}$$

if V_s is in kV per phase

If V_s is the line to line pressure in kV, the expression gives the increase in voltage at the receiving end between lines. It should be noted that the expression gives only approximately the excess in potential that appears at the receiving end of transmission line on no load, on the tacit assumption that the sending end voltage is maintained constant at the stipulated value.

As an example, consider a transmission line whose sending end voltage is constant at 220 kV, length of the line = 250 km and f = 50 Hz, ω = 314 rad/sec.

Line voltage rise at the receiving end due to Ferranti effect

$$= 0.555 \times 10^{-11} \times 220 \times 314^2 \times 250^2 = 7.53 \text{ kV}.$$

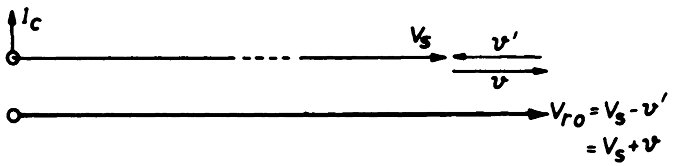
Hence $\qquad V_{ro} = 227.53$ kV

If the receiving end voltage was 200 kV on load, there would be a pressure rise of 227.52 – 200 kV = 27.52 kV between lines, if the load were thrown off. Thus the % rise in pressure at the receiving end

$$= \frac{27.52}{200} \times 100 = 13.76. \quad \text{(from the given load to no load).}$$

In fact, a small load may eliminate the Ferranti effect, and when there are transformers in the circuit at the receiving end, the magnetizing current will act in such a manner as to restrain the above effect.

Fig. 1.40 shows a phasor diagram to demonstrate how the pressure at the receiving end of a transmission line may rise on no load to a level beyond that at the sending end. The resistance and leakance are neglected for convenience.



$$V_{ro} = V_s - v' = V_s + v$$

$$v = \frac{1}{2} V_s \,\omega^2 LC$$

Fig. 1.40. Phasor diagram to demonstrate Ferranti effect : Resistance and leakance ignored.

1.8. Losses in Transmission Line on Open Circuit

As seen above, a long line draws a charging current even when open circuited at the receiving end, on account of the pronounced effect of capacitance. The magnitude of the charging current depends on the length and capacitance of the line. As a result of the current flowing through the line resistance possessed by the line, there will be inevitable transmission losses taking place.

Let I_c be the total charging current over the entire length l.

Approximately, $\quad |\dot{I}_c| = V_s \,\omega\, C_0\, l$ ...(1.128)

Charging current per unit length of the line

$$= i = V_s \,\omega\, C_0 \qquad ...(1.129)$$

Fig. 1.41 shows one phase of a three phase transmission circuit with an impressed voltage V_s (at the sending end). At a distance x from the sending end, the line charging current is

$$I_x = i(l - x)$$

$$= V_s \,\omega\, C_0 \,(l - x)$$

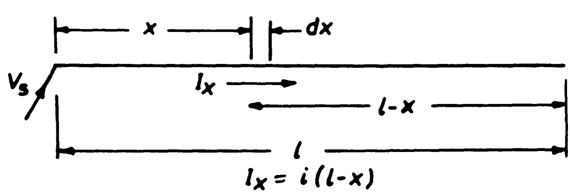


Fig. 1.41. Transmission circuit : Losses on open circuit.

Loss in the elemental length 'dx' is $I_x^2\,(r\,dx)$
where r is the resistance per unit length.

Total resistance $= R = rl$

Total power loss in transmission per phase is then

$$P_L = \int_{x=0}^{1} I_x^2\, r\, dx$$

$$= \int_0^1 [V_s\, \omega\, C_0\, (l - x)]^2\, r\, dx$$

$$= V_s^2\, \omega^2\, C_0^2\, r \int_0^l (l - x)^2\, dx$$

$$= \frac{V_x^2\, \omega^2\, C_0^2\, rl^3}{3}$$

$$= \frac{1}{3} V_s^2\, \omega^2\, C_0^2\, 1^2\, (rl)$$

$$P_L = \frac{1}{3} I_c^2 R \qquad \text{...(1.130)}$$

1.9. Tuned Power Transmission Lines

We have seen in the earlier articles that the voltage regulation of a transmission line depends on the line parameters which again are based upon the length and design of the line consistent with the operating voltage chosen. Voltage regulation and stability of the power system can be significantly improved by reducing the effect of line inductance and charging currents by taking steps for adequate compensation for the inductive voltage drops and capacitive currents. A line is said to be "TUNED" if the line design takes care to see that the receiving end voltage and current are equal in magnitude to the sending end values.

For a long line, the $ABCD$ parameters are given by the equations (1.34) reproduced below :

$$\left.\begin{aligned} A &= D = \cosh \sqrt{ZY} \\ B &= \sqrt{Z/Y} \sinh \sqrt{ZY} \\ C &= \sqrt{Y/Z} \sinh \sqrt{ZY} \end{aligned}\right\} \qquad \text{...(1.34)}$$

The relations for the sending end voltage and current, V_s and I_s, each in terms of the receiving end voltage and current V_r and I_r are given in equations (1.32) and (1.33) which describe the behaviour of a transmission line. They are reproduced below :

$$\left.\begin{aligned} V_B &= A\, V_r + B\, I_r \\ &= V_r \cosh \sqrt{ZY} + I_r \sqrt{Z/Y} \sinh \sqrt{ZY} \end{aligned}\right\} \qquad \text{...(1.132)}$$

$$\left.\begin{aligned} I_s &= C\,V_r + D\,I_r \\ &= V_r\sqrt{Y/Z}\,\sinh\sqrt{Z/Y} + I_r\cosh\sqrt{Z/Y} \end{aligned}\right\} \quad ...(1.33)$$

If the effects of resistance and leakance are ignored, we obtain

$$Y = j\,\omega\,C_0 l \quad \text{and} \quad Z = j\,\omega\,L_0\,l$$

where L_0 and C_0 are per unit length and l, the length of the transmission line. Hence the characteristic impedance is given by

$$Z_c = \sqrt{Z/Y} = \sqrt{L/C}$$

and

$$\sqrt{ZY} = j\,\omega\,l\sqrt{L_oC_o}$$

$$\cosh\sqrt{ZY} = \cosh\left(j\,\omega\,l\sqrt{L_oC_o}\right) = \cos\omega\,l\sqrt{L_oC_o}$$

$$\sinh\sqrt{ZY} = \sinh\left(j\,\omega\,l\sqrt{L_oC_o}\right)$$

$$= j\sin\left(\omega\,l\sqrt{L_oC_o}\right)$$

Substituting the above in equations (1.32) and (1.33), we get

$$V_s = V_r\cos\left(\omega\,l\sqrt{L_oC_o}\right. + j\,Z_c\,I_r\sin\left(\omega\,l\sqrt{L_oC_o}\right) \quad ...(1.131)$$

$$I_s = j\,(V_r/Z_c)\sin\left(\omega\,l\sqrt{L_oC_o}\right) + I_r\cos\left(\omega\,l\sqrt{L_oC_o}\right) \quad ...(1.132)$$

At no load, $I_r = 0$ and then the charging current is given by I_s as follows :

$$I_s = j(V_r/Z_c)\sin\left(\omega\,l\sqrt{L_oC_o}\right) \quad ...(1.133)$$

The sending end voltage when $I_r = 0$ becomes

$$V_s = V_r\cos\left(\omega\,l\sqrt{L_oC_o}\right) \quad ...(1.134)$$

It is clearly seen from equations (1.133) and (1.134) that

V_s and I_s are in quadrature.

If $\quad \omega\,l\sqrt{L_oC_o} = \pi$ or $2\,\pi$, $\quad$ ...(1.135)

$I_s = 0$, meaning that there will be no charging current on no load.

If the expression is equal to π,

$$\cos\omega\,1\sqrt{L_oC_o} = -1$$

and then, for load conditions, equations (1.131) and (1.132) become

$$V_s = -V_r \text{ and } I_s = -I_r.$$

If the expression becomes 2π, then we have

$$V_s = V_r \text{ and } I_s = I_r.$$

In both cases, the voltage and current at the receiving end are equal in magnitude to the voltage and current at the sending end respectively. Therefore, there is no loss of voltage or current on load. Such a line is said to be a Tuned Power Line, as indicated earlier.

We have already seen that the velocity of propagation of an electromagnetic wave in space along a lossless line is

$$v = \frac{1}{\sqrt{L_o C_o}} = \text{velocity of light}$$
$$= 3 \times 10^5 \text{ km ps.}$$

If the power frequency is 50 Hz, it is necessary that for tuning, the length of the line be equal to

$$l = \frac{\pi \text{ or } 2\pi}{\omega \sqrt{L_o C_o}}$$
$$= \frac{\pi \text{ or } 2\pi}{2\pi \times 50} (3 \times 10^5) \text{ km}$$
$$= (\tfrac{1}{2} \text{ or } 1)\, 6000 \text{ km}$$
$$= 3000 \text{ or } 6000 \text{ km}$$

However, if the length of the transmission line is fixed, tuning may be accomplished by appropriate choice of frequency. For example, if $l = 800$ km,

$$\omega = \frac{\pi \text{ or } 2\pi}{l \sqrt{L_o C_o}} = \frac{\pi \text{ or } 2\pi}{800/(3 \times 10^5)} \text{ rad/sec}$$
$$f = (\tfrac{1}{2} \text{ or } 1) \frac{3 \times 10^5}{800} \text{ Hz} = 187.5 \text{ or } 373 \text{ Hz}$$

However, it is not a good proposition to fix the frequency to suit the length of the line, as the length may vary.

If the line length and frequency are fixed, the line may be tuned by providing capacitance at regular intervals between the line and neutral as shown in Fig. 1.42.

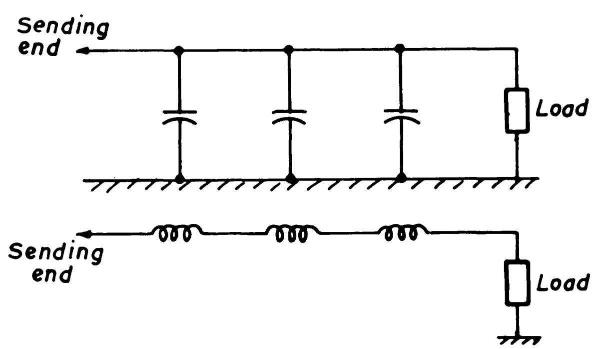


Fig. 1.42. Tuning by shunt capacitors and series inductors.

Provision of capacitors between line and neutral has the beneficial effect of increasing capacitance of the system to the desired value. As an alternative, inductive coils may be introduced in the series circuit of the line for the purposes of increasing the inductance in the transmission circuit to the required value, as shown in Fig. 1.42. This particular method is adopted in telephony, but not favoured in practice in power transmission network.

Another approach for line-tuning is depicted in Fig. 1.43 and is prevalent in some power systems. C' are capacitors in series with the line for

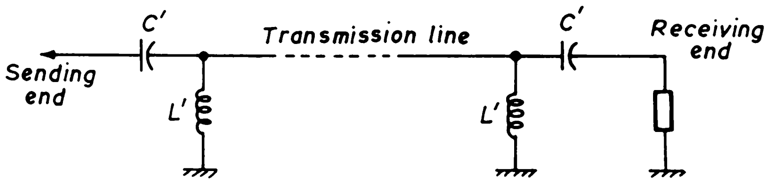


Fig. 1.43. Compensating sections used for tuning power transmission lines.

neutralizing the voltage drop due to line inductance and L' the inductors across the line for neutralizing the charging current due to the line capacitance. This mode of tuning is referred to as the "Method of tuning by application of compensating sections". Thus the total series connected capacitance is in effect equal to $\frac{1}{2}C'$ used for neutralizing the equivalent impedance Z'' of the equivalent π-circuit, which is given by the following expression. (Vide equation (1.47) See Fig. 1.26.

$$Z'' = \sqrt{Z/Y} \sinh \sqrt{ZY}$$

Neglecting the resistance and leakance of the line for simplicity and convenience, we have

$$Z = j\,\omega\,L \text{ and } Y = j\,\omega\,C$$

where L and C are the total inductance and capacitance per phase of the entire line. Accordingly, the impedance may be expressed as follows :

$$Z'' = j\,\sqrt{L/C}\,(\sin \omega\,\sqrt{LC}) \qquad ...(1.136)$$

Hence by full compensation, the total series impedance becomes zero.

$$Z'' + \frac{2}{j\omega C'} = 0$$

$$j\,\sqrt{L/C}\,\sin \omega\,\sqrt{LC} - j\,\frac{2}{\omega C'} = 0$$

whence

$$C' = \frac{2}{\omega\,\sqrt{L/C}\,\sin(\omega\,\sqrt{LC})} \qquad ...(1.137)$$

gives the capacitance at each end for compensation.

In a similar way, shunt inductance L' at each end of the transmission line must neutralize the shunt admittance $Y''/2$ of the equivalent-π circuit of Fig. 1.26. From equation (1.49),

$$\frac{Y''}{2} = \sqrt{Y/Z} \tanh\left(\tfrac{1}{2}\sqrt{ZY}\right)$$

$$= \sqrt{C/L}\; j \tan\left(\tfrac{1}{2}\,\omega\sqrt{LC}\right)$$

Again, we have, for compensation

$$-\frac{j}{\omega L'} + j\sqrt{C/L}\tan\left(\tfrac{1}{2}\,\omega\sqrt{LC}\right) = 0$$

from which, we obtain

$$L' = \frac{1}{\omega\sqrt{C/L}\tan\left(\tfrac{1}{2}\,\omega\sqrt{LC}\right)} \qquad \text{...(1.138)}$$

For compensating sections for long lines, the above expressions may be used, by making use of the equivalent circuit.

For moderately long lines, nominal circuit will suffice, in which case, the equivalent series capacitance C' of Fig. 1.43 is used for compensation of the total inductance L. As the total series capacitance is $\tfrac{1}{2}C'$, we have

$$-j\frac{2}{\omega C'} + j\,\omega L = 0$$

$$\omega^2 L\left(\tfrac{1}{2}C'\right) = 1$$

whence the compensating capacitance is given by

$$C' = \frac{2}{\omega^2 L} \qquad \text{...(1.140)}$$

Similarly the compensating inductance L' for the localized capacitance $C/2$ is related by the equation

$$-\frac{j}{\omega L'} + j\,\omega\, C/2 = 0$$

from which we get

$$L' = \frac{2}{\omega^2 C} \qquad \text{...(1.141)}$$

It will be seen in Chapter on stability (under concluding remarks) that the lines should have appropriately low value of reactance so as to raise the maximum transmissible power. It has been suggested that the power lines may be tuned with compensating sections *viz.*, series capacitors and shunt reactors at strategic points in power systems.

EXERCISES

1.18. (*a*) Show, with the aid of a Phasor diagram, how the voltage at the receiving end of a transmission line can be maintained constant by using a synchronous phase modifier.

(*b*) A three-phase overhead transmission line supplies a load of 50 MW at 0.707 lagging power factor at 66 kV. If the receiving end pressure is held constant and a synchronous motor is installed to improve the overall receiving end power factor to 0.866 lagging, while maintaining the line current constant in magnitude, determine the KVA rating and power factor of the synchronous motor.

1.19. A single phase 50 Hz generating station supplies a load of 8.0 MW at a lagging power factor 0.707 by means of a transmission line 20 km long. The resistance per km of each conductor is 0.015 ohm and the loop inductance is 0.75 mH/km. The receiving end pressure is held constant at 11 kV, and a capacitor is connected across the load in order to raise the overall receiver power factor to 0.9 lagging.

Calculate : (*a*) the capacitance and KVA rating of the capacitor. (*b*) the generating station voltage (*i*) when the capacitor is switched in (*ii*) when the capacitor is switched out. (*c*) the approximate pressure rise at the receiving end when the load is thrown off, if the station voltage is held at a value as in part (*b*) (*i*), and with the capacitor in the circuit.

1.20. (*a*) Explain the object of duplicating three-phase transmission lines and of erecting two such lines over different routes. Establish pertinent general expressions for determining the loads (both KVA and p.f.) shared by the parallel lines. Draw relevant phasor diagrams to portray the manner in which a given total load is divided between the two lines (in respect of KW and KVAR loadings).

(*b*) A total load of 12.5 MW at 33 kV and p.f. 0.9 lagging is delivered to a sub-station by two three-phase lines connected in parallel. One of the lines is an underground cable with a resistance of 1.5 ohm for each conductor and a reactance to neutral of 1.8 ohm, and delivers 6 MW at 0.8 p.f. lagging. What should be the resistance and reactance of the second line ?

1.21. Develop an expression for evaluating the voltage regulation of a short transmission line (*a*) by exact method ; (*b*) by approximate method, and deduce therefrom the receiver p.f. (approximately) at which the regulation is an extremum, for a given load KVA with fixed p.d.

Explain how full-load regulation diagrams can be drawn for a short transmission line for (*i*) V_s fixed, V_r varied ; and (*ii*) V_r fixed, V_s varied.

1.22. The constants per km of a 250 km long three-phase line are given as follows : resistance = 0.15 ohm, inductance 1.2 mH, capacitance to neutral 0.009 μF. A balanced three-phase load of 40 MVA at 0.8 p.f. lagging is connected to the receiving end and a synchronous capacitor operating at zero p.f. is connected to the mid-point of the line. The frequency is 50 Hz. If the voltage across the load is 120 kV (line to line), determine the KVA rating of the synchronous capacitor in order that the voltage at the sending end may be equal in magnitude to that at the middle of the line. Use nominal *T*-circuit for the calculations.

Hint. Taking V_r or I_r as reference, find the voltage at the middle of the line, say V_m, which is also the voltage across the synchronous capacitor connected at the middle of the line. [As the sending end voltage is to be numerically equal to V_m, obtain a relation for sending end voltage in terms of the unknown value of the modifier (synchronous capacitor) current I_m, and solve the quadratic equation for I_m].

1.23. A three phase 50 Hz long transmission line has a total series impedance of $(25.3 + j66.5)$ ohm per phase and a shunt admittance of 4.42×10^{-4} mho/phase. It delivers a load of 50 MW at 220 kV at 0.8 p.f. lagging. Determine the sending end voltage

(*a*) by short line approximation.

(*b*) by nominal π approach.

(*c*) by rigorous solution for long lines.

(*d*) Calculate the displacement angle, *viz.* angle between V_s and V_r, in each case.

1.24. Explain why the voltage at the receiving end of a long overhead unloaded transmission line may exceed that at the sending end. Draw a phasor diagram illustrating this condition. Stating the assumptions made, derive a formula from which the pressure rise can be computed assuming one-half of the line capacitance to be concentrated at the receiving end.

The potential difference at the sending end of a three-phase 50 Hz line 250 km long is 110 kV. Calculate the voltage at the receiving end on open circuit.

[**Note.** The product of C in farad per km and inductance in Henry per km may be taken as 1.15×10^{-11}].

1.25. (*a*) Explain the physical significance of *ABCD* constants of two-pair-terminal networks and state how these constants may be determined experimentally on a medium length transmission line. (*Madurai University, Nov., 1970*)

(*b*) The *ABCD* constants of a symmetrical π network are

$$A = 0.95 \underline{/0^\circ}\ ; B = 50 \underline{/90^\circ} \text{ ohm};$$

$$C = 0.0025 \underline{/90^\circ} \text{ mho}$$

Determine the sending end voltage and the torque angle (displacement angle) for a receiving end power of 100 MW at 200 kV and 0.8 p.f. lagging.

1.26. (*a*) Draw and explain the phasor diagram for a transmission line assuming that half the line capacitance is localized at either end of the line.

(*b*) The constants of a three phase 100 km long transmission line are resistance, 0.1 ohm per km per conductor ; inductive reactance, 0.3 ohm per km, line to neutral ; capacitive susceptance, 0.4×10^{-5} mho per km, line to neutral ; leakance negligible . (*i*) Find the sending end voltage, current and power factor and also the efficiency of transmission when the line delivers 24 MW at 0.8 p.f. lagging at 66 kV. (*ii*) Estimate for this load the approximate rating of the Phase Modifier apparatus installed at the receiving end to enable the sending end voltage to be maintained at 70 kV.

Use split-capacitor (Nominal π)—circuit for calculations.

1.27. A three-phase transmission line has an impedance per phase of $(1.5 + j\,3.5)$ ohm. It supplies a load at a constant pressure of 11.5 kV. A synchronous phase modifier is connected in parallel with the load and its excitation so adjusted that with a load of 2500 KW at 0.8 p.f. lagging, the voltage at the sending end is also 11.5 kV. Determine the KVA rating of the modifier if its losses are 130 kW.

1.28. A three-phase transmission line, 600 km long, is to be operated on a constant voltage system, both sending and receiving end pressures being maintained constant at 150 kV. Find the rated capacity of a synchronous phase modifier when a load of 60 MVA is to be delivered at a p.f. of 0.9 lagging. The modifier is also utilized to supply an additional load of 10 MW.

Data. $\dot{V}_s = (0.87 + j\,0.035)\,\dot{V}_r + (47.9 + j\,180.8)\,\dot{I}_r$

where V_s, V_r and I_r are the sending end voltage, receiving end voltage and receiving end current respectively.

1.29. Two four-terminal networks *a* and *b* with general circuit constants A_a, B_a, C_a, D_a and A_b, B_b, C_b, D_b respectively are, connected in parallel. Determine the equivalent π-circuit parameters Y_s, Z and Y_r for the above parallel combination in terms of the general circuit constants.

1.30. (*a*) Show that in a linear bilateral two terminal pair network, the generalized circuit constants A, B, C, D satisfy the relationship $AD—BC = 1$.

(*b*) Two feeders having generalized constants A_1, B_1, C_1, D_1 and A_2, B_2, C_2, D_2 are paralleled at both ends. Obtain the sending end impedance of the resulting network, with the receiving end open.

(Madurai University, Nov. 1980)

1.31. A single-phase line has a total resistance of 2 ohm and reactance of 3 ohm. Voltage at the sending end is held at 2.2 kV. (*a*) Estimate the maximum theoretical value of power that can be delivered to a receiver circuit of power factor 80% lagging. (*b*) Determine also the equivalent load constants (resistance and reactance), load current and receiver voltage. (*c*) What is the efficiency of transmission ? (*d*) If the receiver power factor be unrestricted, what would be the maximum power delivered ? What would then be the revised. Values for (*b*) and (*c*) ?

1.32. A three phase transmission line with resistance and reactance per phase of 5 and 12 ohm respectively and with negligible shunt admittance delivers a load of 1600 KW at 0.8 lagging power factor. Voltage at the sending end is 13.2 kV, at 50 Hz. Estimate the power factor at the sending end and voltage at the receiving end.

Example 1.33. *A three-phase load of 1 MW at power factor 0.8 lagging is supplied over a line of impedance 25 + j 12 ohm per phase. Calculate the supply voltage when the load voltage is (a) 30 kV ; (b) 10 kV obtained by using a 30/10 kV (star-star connected) transformer. The equivalent resistance and reactance of the transformer referred to 10 kV side are 0.8 and 2.5 ohm per phase respectively.*

Solution. (*a*) Equivalent single phase circuit for the three phase system is shown in Fig. 1.44 (*a*).

Taking the receiving end voltage as reference,

$$\dot{V}_r = \frac{30}{\sqrt{3}} + j\,0 \text{ kV to neutral}$$

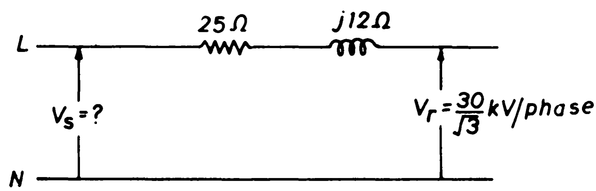


Fig. 1.44. (*a*) For Example 1.33 (*a*).

Line (phase) current $= \dfrac{1000}{\sqrt{3} \times 30 \times 0.8} = 24.06$ A

Voltage drop in the transmission line

$$= \dot{I}\,\dot{Z}_L = 24.06\,(0.8 - j\,0.6)(25 + j\,12)$$

$$= 654.53 - j\,130 \text{ V}$$

Voltage at the sending end of the transmission line,

$$\dot{V}_s = \dot{V}_r + \dot{I}\,\dot{Z}_L = 17321 + 654.53 - j\,130 \text{ V}$$

$$|\,\dot{V}_s\,| = \sqrt{17975.53^2 + 130^2} \text{ V} = 17.976 \text{ kV}$$

Line voltage at the sending end = $17.976\sqrt{3} = 31.13$ kV.

(*b*) Ratio of transformation of the transformer, $k = 30/10 = 3$

Transformer equivalent impedance referred to 30 kV side

$$\dot{Z}_t' = k^2 Z_{eq} \text{ referred to Lower voltage side (10 kV)}$$

$$= 3^2\,(0.8 + j\,2.5)$$

$$= 7.2 + j\,22.5 \text{ ohm per phase}$$

The equivalent circuit of the transmission system referred to 30 kV side is depicted in Fig. 1.44 (*b*).

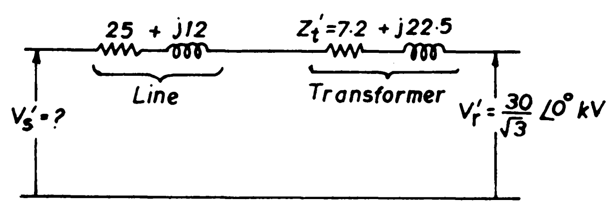


Fig. 1.44. (*b*) For Example 1.33 (*b*).

Total series impedance per phase, referred to H.V. side

$$\dot{Z} = \dot{Z}_L + \dot{Z}_t'$$

$$= 32.2 + j\,34.5 \text{ ohm}$$

Sending end voltage,

$$\dot{V}_s' = 17321 + (19.25 - 14.44)(32.2 + j\,34.5)$$
$$= 18439.03 + j\,199.16 \text{ V}$$
$$|\dot{V}_s'| = 18440 \text{ V}$$

Line voltage at the sending end

$$= 18.44\sqrt{3} = 31.9 \text{ kV}$$

It is seen from the results of (*a*) and (*b*) that a higher voltage is required to be maintained at the sending end in case (*b*) than in case (*a*) on account of the additional series impedance due to the impedance of the transformer, if the load voltage is to be the same at the receiving end : that is, 30 kV as referred to the H.V. side.

Example 1.34. *A three phase transmission line of impedance (16 + j 24) ohm per phase is fed through a 1 : 3 transformer whose equivalent impedance referred to secondary side is 2 + j 8 ohm. The load current is 100 A at power factor 0.8 lagging ; while the line voltage at the mid-point of the line is 33 kV. Find (a) the supply voltage on the low voltage side ; (b) the equivalent resistance and reactance of each phase of the load.*

Solution. The transmission system is shown by a single line diagram in Fig. 1.45.

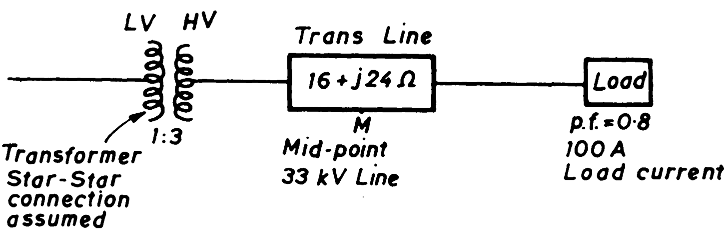


Fig. 1.45. Single line diagram for the transmission system : Example 1.34.

The equivalent single phase diagram of the three phase system (referred to H.V. side) is depicted in Fig. 1.46.

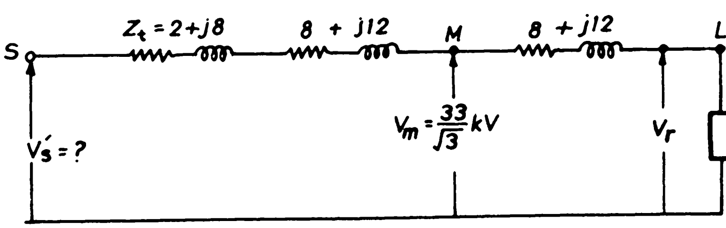


Fig. 1.46. Equivalent single phase circuit for Example 1.34.

As a 1 : 3 ratio step up transformer is used at the sending end of the transmission line, we have

$$V_s' = 3\ V_s,$$

where V_s is the sending end voltage on the LV side

and V_s' is the sending end voltage referred to HV side.

$$V_s' = V_r + \dot{I}\,(2 + j\,8 + 16 + j\,24)$$

Taking V_r as reference, let $\dot{V}_r = V_r \angle 0°$

$$\dot{I} = 100\,(0.8 - j\,0.6) = 80 - j\,60 \text{ A}$$

Impedance drop from the middle of the line to the load (from M to L in Fig. 1.46)

$$= (80 - j\,60)\,(8 + j\,12)$$
$$= 1360 + j\,480 \text{ V}$$

Hence the voltage at M is $V_M = V_r + 1360 + j\,480$

The magnitude of V_M is given as 33 kV (line) ; hence it is $33/\sqrt{3}$ kv/phase *i.e.*, 19053 V/phase

Thus $(V_r + 1360)^2 + 480^2 = V_M^2 = 19053^2$

Solving, we get $V_r = 17.693$ kV/phase

The equivalent load impedance is given by the ratio of the terminal voltage to the load current. Let Z_{eq} be the load equivalent impedance.

$$\dot{Z}_{eq} = \frac{17693}{100 \angle -36.9°}$$

[as the load p.f. angle is $\cos^{-1}(0.8) = 36.9°$]

$$= 176.93\,(0.8 + j\,0.6)$$
$$= 141.5 + j\,106.2 \text{ ohm per phase}$$

To find the supply voltage :

$$V_s' = V_r + I(2 + j\,8 + 16 + j\,24)$$
$$= 17693 + (80 - j\,60)\,(18 + j\,32)$$
$$= 21053 + j\,1480 \text{ V/phase}$$
$$|\,V_s'\,| = 21.10 \text{ kV/phase}$$

Supply voltage (line) on the LV side of the transformer

$$V_s = \frac{V_s'}{3}\sqrt{3}$$
$$= \frac{21.10}{\sqrt{3}} = 12.18 \text{ kV.}$$

Example 1.35. *The voltages of a switching station, transmission line and sub-station have nominal ratios corresponding to line values of 66, 132 and 22 kV. The sub-station load is 20 MVA at 20.5 kV (line) and power factor 0.85 lagging. The transformer at each end of the transmission line have equivalent impedance of 9 + j 36 ohm per phase, referred to the high voltage* *side. The line to neutral impedance of the line is 21 + j 60 ohm. Find the voltage at the switching station and the overall efficiency of transmission.*

Solution. Fig. 1.47 depicts the single line diagram of the power system.

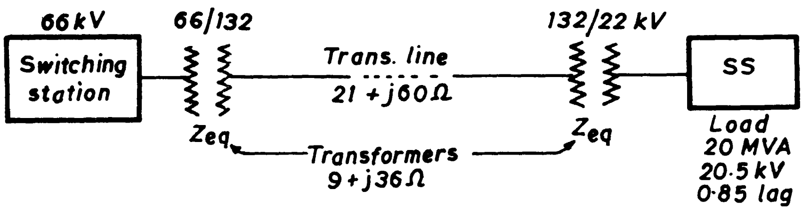


Fig. 1.47. Single line diagram for Example 1.35.

The corresponding single phase equivalent circuit is shown in Fig. 1.48.

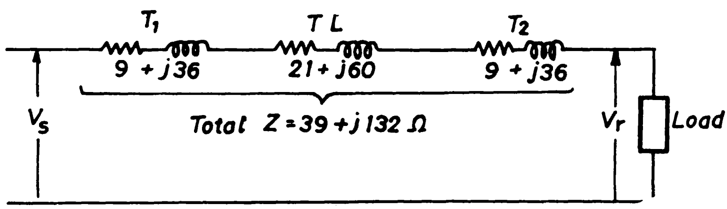


Fig. 1.48. Single phase equivalent circuit for the power system of Example 1.35.

Referred to 132 kV side, voltage at the sub-station ($L-L$) is

$$V_r = \frac{132}{22} - \times 20.5 = 123 \text{ kV}$$

Voltage per phase, $V_r = 123/\sqrt{3} = 71.02\underline{/0°}$ kV

(taken as reference phasor)

Equivalent load current referred to HV side)

$$I = \frac{20000}{\sqrt{3} \times 123} = 93.88 \text{ Amp}$$

$$\dot{I} = 93.88\,(0.85 - j\,0.52)$$

$$= 79.80 - 49.47 \text{ A}$$

Total impedance drop in the transmission line and transformers

$$= \dot{I}\dot{Z} = (79.80 - j\,49.47)\,(39 + j\,132)$$

$$= 9642.24 + j\,8604.27 \text{ V}$$

$$\dot{V}_s = \dot{V}_r + \dot{I}\dot{Z}$$

$$= 71020 + 9642.24 + j\,8604.27 \text{ V}$$

$$= 80.662 + j\,8.604 \text{ kV}$$

V_s = 81.12 kV per phase, referred to 132 kV side (transformer)

Hence, the switching station voltage per phase

$$= 81.12 \times \frac{66}{132} = 40.56 \text{ kV}$$

Line voltage at the switching station

$$= \sqrt{3} \times 40.56 = 70.25 \text{ kW}$$

Efficiency of transmission

Total loss in power transfer (including line and transformers)

$$= 3\, I^2 R_{eq} \text{ (total)}$$
$$= 3 \times 93.88^2 \times 39 \text{ W}$$
$$= 1031.17 \text{ KW}$$

Power supplied to load

$$= 20 \times 0.85 \text{ MW} = 17000 \text{ KW}$$

Hence efficiency of transmission

$$= \frac{17000}{17000 + 1031.17} = 0.943 = 94.30\%.$$

Example 1.36. *A 10 MW load, power factor 0.8 lagging, is received in a sub-station at 30 kV through two three-phase overhead transmission lines operating in parallel. Current supplied by line A is 100 A and the power delivered by line B is 5.5 MW. If each of the B-lines has resistance and reactance of 8.0 and 12.0 Ohm respectively, what are the corresponding parameters (R and X) of line A ?*

Solution. Fig. 1.49 shows a single line diagram of the parallel operating lines A and B feeding a sub-station.

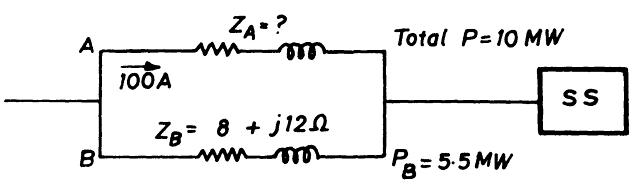


Fig. 1.49. Single line diagram for Example 1.36.

Power delivered by line A = $10 - P_B$ (delivered by line B)

$$= 10 - 5.5 = 4.5 \text{ MW}$$
$$= 4500 \text{ KW}.$$

$$I_A = 100 \text{ A}.$$

Power factor of the load delivered by line A

$$= \frac{4500}{\sqrt{3} \times 30 \times 100} = 0.866 \text{ lagging}.$$

Taking $\dot{V}_r$ as reference phasor, let $\dot{V}_r = V_r \angle 0°$

$$V_r = 30/\sqrt{3} \text{ kV}$$

$$\dot{I}_A = 100\,(0.866 - j\,0.5)$$

$$= 86.6 - j\,50 \text{ A}$$

Total load current $$I_r = \frac{10000}{\sqrt{3} \times 30 \times 0.8} = 240.57 \text{ A}$$

$$\dot{I}_r = 240.57\,(0.8 - j\,0.6)$$

$$= 192.46 - j\,144.34 \text{ A}$$

The current in line B is

$$\dot{I}_B = \dot{I}_r - \dot{I}_A = 105.86 - j\,94.34 \text{ A}$$

As the two lines are in parallel, their impedance drops are equal.

$$\dot{I}_A \dot{Z}_A = \dot{I}_B \dot{Z}_B$$

$$\dot{Z}_A = \frac{\dot{I}_B \dot{Z}_B}{\dot{I}_A} = \frac{(105.86 - j94.34)(8 + j\,12)}{86.6 - j\,50}$$

Simplifying, $$\dot{Z}_A = 14.56 + j\,14.36 \text{ ohm}$$

$$= R_A + j\,X_A$$

Hence the transmission line A has resistance of R_A = 14.56 ohm and reactance of X_A= 14.36 ohm.

Example 1.37. *The total power supply delivered to a three phase overhead line in parallel with a three phase underground cable is 250 A at 3.3 kV, power factor 0.8 lagging. Calculate the current distribution between the line and the cable and the overall power factor of the total load supplied by the combination. The impedances of the line and the cable per phase are 4 + j 6 and 3 + j 2 ohm respectively.*

Solution. A single, line diagram depicting the overhead line and underground cable sharing the transmission system and a common load is indicated in Fig. 1.50.

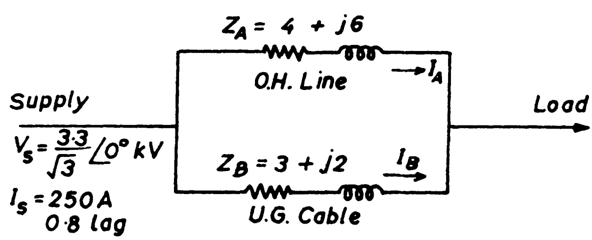


Fig. 1.50. For Example 1.37.

It should be carefully noted that the power factor of 0.8 lagging refers to the power input to the transmission system.

Let The sending end voltage V_s be the reference phasor.

$$\dot{V}_s = \frac{3.3}{\sqrt{3}} \underline{/0^\circ} \text{ kV/phase}$$
$$= 1905.3 + j\,0 \text{ V/phase.}$$

Input current, $\dot{I}_s = 250\,(0.8 - j\,0.6) = 200 - j\,150$ A

The currents in lines A and B, *viz.*, I_A and I_B may be expressed in terms of the total current I_s as follows :

$$\dot{I}_A = \dot{I}_s \frac{Z_B}{Z_A + Z_B} \,;\, \dot{I}_B = \dot{I}_s \frac{Z_A}{Z_A + Z_B}$$

where $Z_A = 4 + j\,6$ ohm ; $Z_B = 3 + j\,2$ ohm

$$\dot{Z}_A + \dot{Z}_B = 7 + j\,8 = 10.63 \underline{/48.81^\circ} \text{ ohm}$$

$$\dot{Z}_A = 7.21 \underline{/56.31^\circ} \text{ ohm} \,;\, \dot{Z}_B = 3.61 \underline{/33.69^\circ} \text{ ohm}$$

$$\dot{I}_A = (250 \underline{/-36.9^\circ}) \frac{3.61 \underline{/33.69^\circ}}{10.63 \underline{/48.81^\circ}}$$
$$= 84.90 \underline{/-52.02^\circ} \text{ A}$$

Power factor at the input of line $A = \cos \phi_A = \cos 52.02^\circ =$ **0.615 lagging**

$$\dot{I}_B = (250 \underline{/-36.9^\circ}) \frac{7.21 \underline{/56.31^\circ}}{10.63 \underline{/48.81^\circ}}$$
$$= 169.57 \underline{/-29.4^\circ} \text{ A}$$

Power factor at the input of line $B = \cos \phi_B = \cos 29.4^\circ =$ **0.87 lagging.**

To find the overall power factor of the total power delivered by the overhead line and underground cable :

Receiving end voltage is calculable from the voltage at the sending end and voltage drop in one of the lines.

Accordingly, $\dot{V}_r = \dot{V}_s - \dot{I}_A \dot{Z}_A$

$$= 1905.3 + j\,0 - (84.90 \overline{/-52.02^\circ})(7.21 \underline{/56.31^\circ})$$
$$= 1294.9 - j\,45.80 = 1295 \underline{/-2.03^\circ}$$

This indicates that the receiving end voltage lags behind the sending end voltage by 2.03°. A phasor diagram shown in Fig. 1.51 depicts the phase relations of V_s, V_r and I_r.

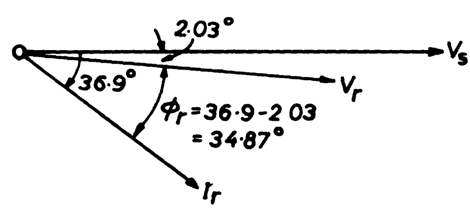


Fig. 1.51. For Example 1.37 (Phasor diagram).

It is seen that the load current lags on the voltage at the receiving end by 36.9 – 2.03 = 34.87° which is the overall power factor angle.

Overall power factor of load at the receiving end

$$= \cos 34.87°$$

$$= \mathbf{0.82 \text{ lagging.}}$$

Example 1.38. *Two 11 kV three phase sub-stations are connected by a feeder of impedance 0.2 + j 0.6 ohm per phase in parallel with a 33 kV feeder of impedance 1 + j 5 ohm per phase. At each end of the 33 kV feeder is a transformer rated 15 MVA, 33/11 kV with 10% reactance. If a load of 20 MW at 0.8 lagging power factor is supplied to one sub-station, what will be the output at the other sub-station and the current in each feeder ?*

Solution. Fig. 1.52 shows the sub-stations SS_1 and SS_2 linked by the two feeders in parallel.

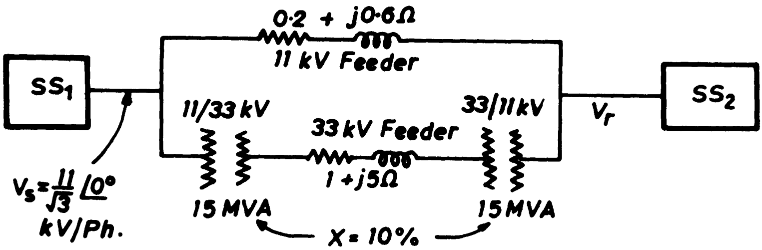


Fig. 1.52. Sub-stations linked by parallel feeder for Example 1.38.

Taking 15 MVA base, base impedance on 11 kV side becomes

$11^2/15 = 8.067$ ohm

Referred to the 11 kV side, the impedance of the 33 kV feeder

$$= \frac{1}{3^2}(1 + j\,5) = 0.111 + j\,0.556 \text{ ohm}$$

Each transformer has 10% reactance and the ohmic value as referred to 11 kV side is equal to 0.1 × 8.067 = 0.807 ohm.

Hence the total series impedance of the 33 kV feeder and transformers would be $j\,0.807 + (0.111 + j\,0.556) + j\,0.807 = \mathbf{0.111 + j\,2.17 \text{ ohm.}}$

The impedance values of both the feeder circuits are indicated in Fig. 1.53.

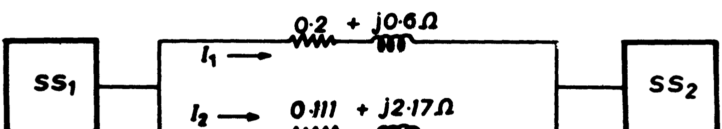


Fig. 1.53. For Example 1.38.

We shall assume that power is transmitted from the SS_1 *via.* the parallel feeder to the SS_2.

Taking the voltage at the sending end as reference phasor,

$$\dot{V}_s = \frac{11}{\sqrt{3}} + j\,0\text{ kV}$$

Total power input to the parallel feeder is $P_1 = 20$ MW. We have to find the power output at the other end, say P_2.

Total current $I = \dfrac{20000}{\sqrt{3} \times 11 \times 0.8} = 1312.20$ A

We are also called upon to find the current distribution in the two feeders. Let them be I_1 and I_2.

Let $\dot{Z}_1$ and $\dot{Z}_2$ be the impedances of the two feeders.

$$\dot{Z}_1 = 0.2 + j\,0.6 = 0.632\,\underline{/71.57^\circ}\text{ ohm}$$

$$\dot{Z}_2 = 0.111 + j\,2.17 = 2.173\,\underline{/87.07^\circ}\text{ ohm}$$

$$\dot{Z}_1 + \dot{Z}_2 = 0.311 + j\,2.77 = 2.787\,\underline{/83.59^\circ}\text{ ohm}$$

The equivalent impedance of the parallel combination of feeders

$$Z_{eq} = \frac{\dot{Z}_1\dot{Z}_2}{(\dot{Z}_1 + \dot{Z}_2)}$$

$$= \frac{0.632 \times 2.173}{2.787}\,\underline{/71.57^\circ + 87.07^\circ - 83.59^\circ}$$

$$= 0.493\,\underline{/75.05^\circ}\text{ ohm}$$

Equivalent resistance

$$R_{eq} = 0.493 \cos 75.05^\circ = 0.127\text{ ohm}$$

Total loss of power in transmission is given by

$$= \frac{3 \times I^2 R_{eq}}{1000}\text{ KW}$$

$$= \frac{3 \times 1312.2^2 \times 0.127}{1000}\text{ KW}$$

$$= 655.83\text{ KW}$$

Power at SS_2 $= 20000 - 655.83$ KW

$= 19344$ KW.

Thus P_2 = Output at SS_2 = 19.34 MW

To find the currents I_1 and I_2

$$\dot{I}_1 = \frac{\dot{Z}_2}{\dot{Z}_1 + \dot{Z}_2}\dot{I} = \frac{2.173\,\underline{/87.07^\circ}}{2.787\,\underline{/83.59^\circ}} \times 1312.20\,\underline{/-36.9^\circ}$$

$$= 1023\,\underline{/-33.42^\circ}\text{ Amp}$$

= current in the 11 kV feeder

Current in the second (33 kV) feeder, referred to 11 kV side,

$$\dot{I}_2 = \frac{\dot{Z}_1}{\dot{Z}_1 + \dot{Z}_2}\dot{I} = \frac{0.632\,\underline{/71.57^\circ}}{2.787\,\underline{/83.59^\circ}} \times 1312.20\,\underline{/-36.9^\circ}$$

Actual current in the 33 kV feeder would be $\frac{1}{3}$ of the above value.

Hence the current would be $\frac{1}{3}\frac{\dot{Z}_1}{\dot{Z}_1 + \dot{Z}_2}\dot{I}$ = **99.20 $\underline{/-48.92^\circ}$ Amp**

To find the power factor of the SS_2 load

$$\dot{V}_r = \dot{V}_s - \dot{I}_1\dot{Z}_1$$

$$= \frac{11000}{\sqrt{3}} + j\,0 - (1023\,\underline{/-33.42^\circ})(0.632\,\underline{/71.57^\circ})$$

$$= 5842.8 - j\,399.60$$

Phase angle of the voltage at sub-station 2 is

$$\tan^{-1}(-399.60/5842.8) = -3.912^\circ$$

V_r lags on V_s by an angle of 3.912°

The phase relations of V_s, V_r and I (total current) are shown in Fig. 1.54.

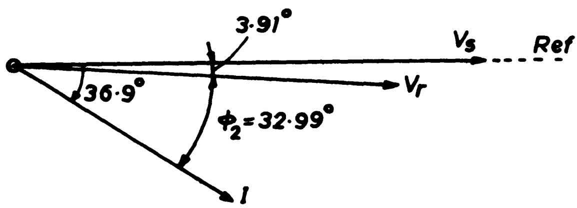


Fig. 1.54. Phasor diagram, for Example 1.38.

Power factor of the sub-station 2 = Cosine of the angle between V_r and I = cos (36.9° – 3.912°) = cos 32.99° = **0.84 lagging.**

Example 1.39. *(a) Draw and explain the vector diagram for a transmission line assuming that half the line capacitance is concentrated at each end of the line.*

(b) A 50 Hz three-phase transmission line delivers a load of 40 MVA at 110 kV and a lagging power factor of 0.7. The line constants (line to neutral) are $R = 11$ ohm ; $X = 38$ ohm ; $B = 3 \times 10^{-4}$ mho ; leakage negligible. Find the sending end voltage, current ; power factor and power input to the transmission line by nominal-π method.

Solution. (*a*) Refer to Article 1.3.2 for nominal—π method
and to Article 1.3.3 for the phasor diagram.

(*b*) Fig. 1.55 shows the nominal-π circuit on single phase basis for the three-phase transmission line.

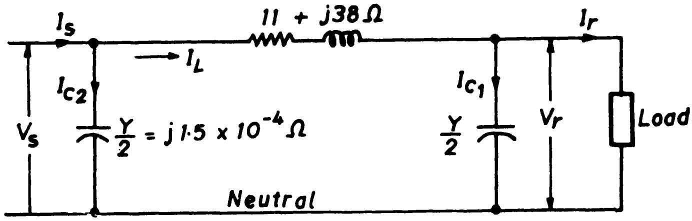


Fig. 1.55. Nominal-π circuit for Example 1.39.

Taking the receiving end voltage (per phase) as reference, we have

$$\dot{V}_r = \frac{110}{\sqrt{3}} + j\,0 \text{ kV.}$$

$$= 63510 + j\,0 \text{ V/phase}$$

Receiving end current (load current)

$$\dot{I}_r = \frac{40000}{\sqrt{3} \times 110} = 210 \text{ A}$$

$$\dot{I}_r = 210 \,\underline{/-45.57^\circ} \text{ A}\,; \quad (\cos\phi = 0.7, \sin\phi = 0.714)$$

$$\dot{I}_r = 210\,(0.7 - j\,0.714) = 147 - j\,149.94 \text{ A}$$

Charging current at the receiving end,

$$\dot{I}_{c1} = \dot{V}_r \dot{Y}/2 = 63510\,(j\,1.5 \times 10^{-4})$$

$$= j\,9.527 \text{ A}$$

Line current, $\dot{I}_L = \dot{I}_r + \dot{I}_{c1}$

$$= 147 - j\,140.41 \text{ A}$$

$$|\dot{I}_L| = 203.28 \text{ A}$$

Line impedance drop, $\dot{I}_L \dot{Z} = (147 - j\,140.41)(11 + j\,38)$

Simplifying, $\dot{I}_L \dot{Z} = 6952.58 + j\,4041.49$ V

Sending end voltage,

$$\dot{V}_s = \dot{V}_r + \dot{I}_L Z$$

$$= 63510 + 6952.58 + j\,4041.49 \text{ V}$$

$$= 70.463 + j\,4.041 \text{ kV}$$

$$= 70.58 \,\underline{/3.28^\circ} \text{ kV/phase.}$$

Line voltage at the sending end

$$= \sqrt{3} \times 70.58 \text{ kV} = \mathbf{122.24 \text{ kV}}$$

Charging current at the sending end,

$$\dot{I}_{c2} = \dot{V}_s \dot{Y}/2 = (70.58 \,\underline{/3.28^\circ})\,(1.5 \times 10^{-4} \,\underline{/90^\circ})\,10^3$$

$$= 10.59 \,\underline{/93.28^\circ} \text{ A}$$

$$= -0.635 + j\,10.57 \text{ A}$$

Current at the sending end of the transmission line,

$$\dot{I}_s = \dot{I}_L + \dot{I}_{c2}$$
$$= 147 - j\,140.41 - 0.635 + j\,10.57$$
$$= 146.37 - j\,129.84 \text{ A}$$
$$= 195.66 \underline{/-41.58^\circ} \text{ A}.$$

As the sending end voltage leads the reference by 3.28° and the current lags on the reference by 41.58°, the phase angle between $\dot{V}_s$ and $\dot{I}_s$ is equal to $\phi_s = 3.28 + 41.58 = 44.86°$ (current lagging).

Power factor at the sending end is $\cos\phi_s = \cos 44.86° =$ **0.709 lagging**

Power input to the transmission line is equal to the power delivered at the receiving end Plus the transmission losses.

Power input $= 40000 \times 0.7 + 3\, I_L^2\, R$

$= 28000 + 3 \times 203.28^2 \times 11/1000$ KW

= 29364 KW

Fig. 1.56 shows the phasor diagram for voltages and current.

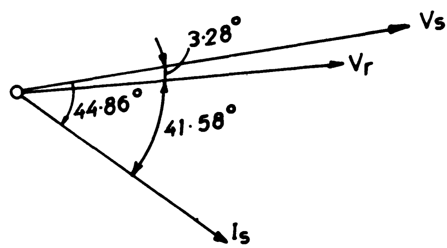


Fig. 1.56. For Example 1.39.

Example 1.40. *Find the regulation and efficiency of an 80 km three phase 50 Hz transmission line delivering 24 MVA at a power factor of 0.8 lagging and 66 kV to a balance load. The conductors are of copper, each having a resistance of 0.12 ohm per km, 1.5 cm outside diameter, spaced equilaterally 2.5 m between centres. Neglect leakance and use Nominal-π method.*

Solution. To find regulation and efficiency of the transmission line, first of all we shall evaluate the line constants, *i.e.*, inductance and capacitance, as the resistance is already given.

Resistance per phase, $R = 0.12 \times 80 = 9.6$ ohm.

$$\text{Spacing to radius ratio} = D/r = \frac{250}{0.75} = 333.33$$

$$\log_{10}\frac{D}{r} = 2.523$$

$$\text{Inductance per phase} = \left(0.05 + 0.4606 \log_{10} \frac{D}{r}\right) \text{mH per km}$$

Total inductance/phase,

$$L = (0.05 + 0.4606 \times 2.523) \times 80 \times 10^{-3} \text{ H}$$
$$= 96.97 \times 10^{-3} \text{ H}$$

Inductive reactance per phase,

$$X = 2\pi f L = 314 \times 96.97 \times 10^{-3}$$
$$= \mathbf{30.46 \ ohm}$$

Total capacitance per phase,

$$C = \frac{0.0241}{\log_{10} \dfrac{D}{r}} \times 80 \times 10^{-6} \text{ Farad}$$
$$= 0.764 \times 10^{-6} \text{ Farad}$$

Shunt admittance, $Y = j\,\omega\, C = j\,314.3 \times 0.764 \times 10^{-6}$

$$= \mathbf{j\ 2.40 \times 10^{-4}\ mho}$$

Series impedance, $Z = R + jX = 9.6 + j\,30.46$ ohm per phase.

The nominal-π circuit of the transmission line is shown in Fig. 1.57.

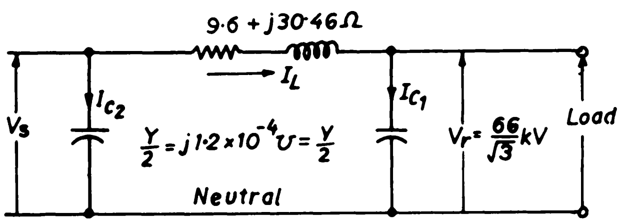


Fig. 1.57. Nominal-π circuit for Example 1.40.

Load current, $I_r = \dfrac{24000}{\sqrt{3} \times 66} = 210$ A

$$I_r = 210\,(0.8 - j\,0.6)$$
$$= 168 - j\,126 \text{ A}$$

Charging current at the receiving end,

$$I_{c1} = V_r Y/2 = 38100\,(j\,1.2 \times 10^{-4}) = j\,4.572 \text{ A}$$

Line current, $I_L = I_r + I_{c1} = 168 - j\,121.43$ A

$$|I_L| = 207.30 \text{ A}$$

Transmission line loss

$$= 3\,I_L^2 R = \frac{3 \times 207.3^2 \times 9.6}{1000} \text{ KW}$$

$$P_L = 1237.63 \text{ KW}$$

Efficiency of transmission,

$$\eta = \frac{P_r}{P_r + P_L}$$

$$= \frac{24000 \times 0.8}{24000 \times 0.8 + 1237.63} \times 100\%$$

$$= 93.9\%$$

To find regulation : The general relation is given by

$$\dot{V}_s = \dot{A}\ \dot{V}_r + \dot{B}\ \dot{I}_r$$

When the load is thrown off, $I_r = 0$. Let $V_r = V_{ro}$ (on no load)

$$\dot{V}_s = A\ V_{ro}$$

$$\dot{V}_{ro} = \dot{V}_s/A$$

Regulation $= (V_{ro} - V_r)/V_r$

For nominal-π circuit,

$$\dot{A} = 1 + \frac{1}{2}\dot{Z}\,\dot{Y} \qquad \text{[Vide equation (1.22)]}$$

In this example, $\dot{A} = 1 + \frac{1}{2}(9.6 + j\,30.46)\,j\,2.4 \times 10^{-6} \approx 0.9963 \underline{/0^\circ}$

The numerical value of $A = 0.9963$

We shall assume that the voltage at the sending end is maintained constant.

$$\dot{V}_s = A\ \dot{V}_r + \dot{B}\,I_r \quad \text{where } \dot{B} = \dot{Z} = 9.6 + j\,30.46$$

$$= 0.9963 \times 38100 + (9.6 + 30.46)\,(168 - j\,126)$$

Simplifying, the magnitude of $V_s = 43.59$ kV

$$V_{ro} = 43.59/0.9963 = 43.75 \text{ kV}$$

Voltage regulation $\varepsilon = (V_{ro} - V_r)/V_r = \dfrac{43.75 - 38.10}{38.10} = \mathbf{14.8\%}$.

Example 1.41. *A single phase transmission line delivers 1 MVA at power factor of 0.71 lagging, 22 kV, 50 Hz. The loop resistance is 15 ohm, the loop inductance 0.2 H and the capacitance, 0.5 micro-farad. Find (a) the voltage, (b) the current and (c) the power factor at the sending end. Use the nominal-π method. (d) If the sending end voltage be maintained constant, to what value would the receiving end voltage rise on no load* ?

Solution. Fig. 1.58 indicates the nominal-π circuit for the single phase transmission line.

Loop reactance $= X = 2\,\pi f L = 314 \times 0.2 = 62.8$ ohm

Shunt admittance $\dot{Y} = j\,\omega\,C = j\,314 \times 0.5 \times 10^{-6}$

$$= j\,157 \times 10^{-6} \text{ mho}$$

Localized admittance at each end

$$= \dot{Y}/2 = j\,7.85 \times 10^{-5} \text{ mho}$$

Series impedance $Z = 15 + j\,62.80$ ohm

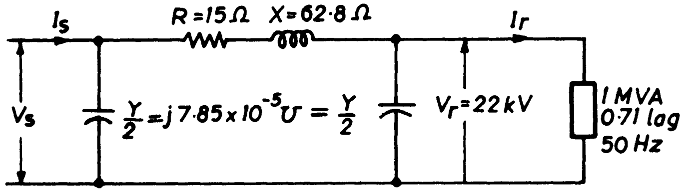


Fig. 1.58. Nominal-π circuit for the single-phase line : Example 1.41.

Referring to equations (1.22), we have, for nominal- π circuit,

$$V_s = A V_r + B I_r, \text{ where } A = 1 + \tfrac{1}{2} Z Y \text{ and } B = Z$$

and
$$I_s = C V_r + D I_r, \text{ where } C = Y(1 + \tfrac{1}{4} Z Y) \text{ and}$$

$$D = A = 1 + \tfrac{1}{2} Z Y$$

For the given load, $V_r = 22000 \angle 0°\ V$ (chosen as reference phasor)

$$I_r = \frac{1000}{22} = 45.45 \text{ A}$$

$$I_r = 45.45 \angle -44.77° \text{ (as } \cos \phi_r = 0.71)$$

$$\phi_r = -44.77°$$

$$B = Z = 64.57 \angle 76.57°$$

$$A = 1 + \tfrac{1}{2} Z Y$$

$$= 1 + (64.57 \angle 76.57°)(7.85 \times 10^{-5} \angle 90°)$$

$$= 0.995 \angle 0° \text{ (approx.)}$$

(*a*) **Sending end voltage :**

$$V_s = A\ V_r + B\ I_r$$

$$= 0.995 \times 22000 + 64.57 \times 45.45 \angle 76.57 - 44.77°$$

$$= 24384.5 + j\,2092.44 \text{ V}$$

$$V_s = 24.47 \angle 4.9° \text{ kV.}$$

(*b*) **Sending end current :**

$$I_s = C\ V_r + D\ I_r$$

$$C = Y(1 + \tfrac{1}{4} Z Y)$$

$$= j\,157 \times 10^{-6}\,[1 + \tfrac{1}{4}(64.57) \times 157 \times 10^{-6} \angle 166.57°]$$

$$= j\,156.6 \times 10^{-6} \text{ (approx.)}$$

$$D = A = 0.995 \angle 0°$$

Substituting, $I_s = j\,156.6 \times 10^{-6} \times 22000$

$$+ 0.995 \times 45.45 \underline{/ -44.77^\circ}$$

$$= 32.10 - j\,28.40$$

$$= 42.86 \underline{/ -41.5^\circ} \text{ Amp}$$

(*c*) **Power factor at the sending end :**

Phase angle between V_s and $I_s = \phi_s = (4.90 + 41.50) = 46.40°$

Power factor $= \cos \phi_s = \cos 46.40° =$ **0.70 lagging**

(*d*) Voltage at the receiving end on no load,

$$V_{ro} = V_s/A = 24.47/0.995$$

$$= \mathbf{24.59\ kV.}$$

Example 1.42. (*a*) *Show that AD – BC = 1 for any transmission line, where A, B, C, D are generalized network constants.*

(*b*) *A 40 MVA generating station is connected to a three-phase line having series impedance Z = 250* $\underline{/60^\circ}$ *ohm and shunt admittance Y = 20 ×* 10^{-4} $\underline{/90^\circ}$ *mho. The power at the generating station is 40 MVA at unity power factor, at 120 kV. There is a load of 8 MW at unity power factor at the mid-point of the line. Calculate the voltage and load at the far end of the line. Use nominal-T model for the line.*

Solution. (*b*) Fig. 1.59 depicts the nominal-*T* circuit for the three phase line.

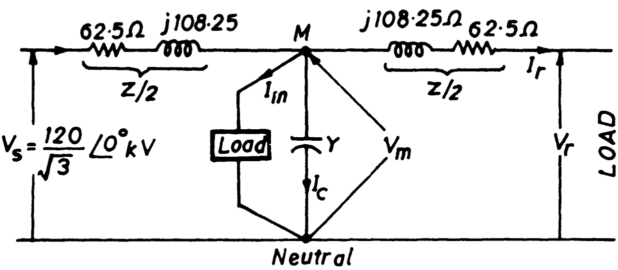


Fig. 1.59. Nominal-*T* circuit for Example 1.42.

As the sending end voltage is given and we have to find the voltage at the receiving end, we shall take V_s as the reference phasor.

$$V_s = \frac{120}{\sqrt{3}} \underline{/0^\circ} \text{ kV/phase}$$

$$= 69284 \underline{/0^\circ} \text{ V}$$

As the power at the sending end is 40 MVA at unity power factor, we have

$$I_s = \frac{40000}{\sqrt{3} \times 120} \underline{/0^\circ} \text{ A} = 192.46 \underline{/0^\circ} \text{ A}$$

Voltage drop at the sending end (half the line impedance)

$$= \dot{I}_s \dot{Z}/2$$
$$= 192.46\,(62.5 + j\,108.25)$$
$$= 12028.75 + j\,20833.80$$

Voltage at the middle (M) of the line

$$\dot{V}_m = \dot{V}_s - \dot{I}_s \dot{Z}/2$$
$$= 69284 - 12028.75 - j\,20833.80 \text{ V}$$
$$= 57.28 - j\,20.83 \text{ kV/phase}$$
$$= 60.93 \underline{/-20^\circ} \text{ kV/phase}$$

(Middle) Capacitor current,

$$\dot{I}_c = j\,20 \times 10^{-4}\,(57.28 - j\,20.83)\,10^3 \text{ A}$$
$$= 41.67 + j\,114.51 \text{ A}$$

Mid-point load current,

$$\dot{I}_m = \frac{8000}{3 \times 60.93} \underline{/-20^\circ} \text{ A}$$

(As the power factor of load is unity, $\dot{I}_m$ is in phase with $\dot{V}_m$)

$$\dot{I}_m = \mathbf{43.77} \underline{/\mathbf{-20^\circ}} \text{ A}$$

Receiving end current,

$$\dot{I}_r = \dot{I}_s - \dot{I}_m - \dot{I}_c$$
$$\dot{I}_s = 192.46 + j\,0 \text{ A}$$
$$\dot{I}_m = 41.13 - j\,14.97 \text{ A}$$

Substituting and simplifying,

$$\dot{I}_r = 109.66 - j\,99.54 \text{ A}$$
$$= 148.10 \underline{/-42.23^\circ} \text{ A} \qquad \ldots(i)$$

Voltage drop in the second half of the transmission line (receiving end side)

$$= \dot{I}_r \dot{Z}/2$$
$$= (109.66 - j\,99.54)\,(62.5 + j\,108.25)$$
$$= 17628.96 + j\,5649.45 \text{ V}$$

Receiving end voltage, $\dot{V}_r = \dot{V}_m - \dot{I}_r \dot{Z}/2$

Substituting and simplifying,

$$\dot{V}_r = 39.63 - j\,26.48 \text{ kV/phase}$$
$$= 47.66 \underline{/-33.75^\circ} \text{ kV} \qquad \ldots(ii)$$

Phasor diagram for V_s, V_r and I_r is shown in Fig. 1.60.

It is seen from (*i*) and (*ii*) and from the phasor diagram that the phase angle between the voltages and current, V_r and I_r is

$$\phi_r = 42.23 - 33.75 = 8.48^\circ \qquad \text{(Current lagging)}$$

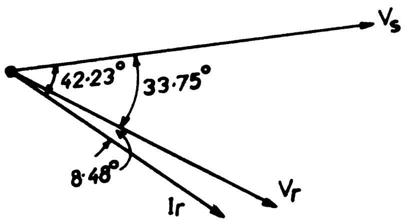


Fig. 1.60. Phasor diagram, Example 1.42.-

Power factor of the load is, therefore,

$$\cos \phi_r = \cos 8.48° = \mathbf{0.989\ lagging}$$

Voltage at the receiving end (Line)

$$= \sqrt{3} \times 47.66 = \mathbf{82.55\ kV}$$

Load delivered at the receiving end,

$$P_r \text{ (three phase)} = \frac{3 \times 47.66 \times 148.10 \times 0.989}{1000} \text{ MW}$$

$$= \mathbf{20.942\ MW}$$

That is, **21.175 MVA** at **0.989 lagging power factor.**

Example 1.43. *A long three-phase transmission line is supplied from a transformer at the sending end and a similar transformer is connected to the line at the receiving end. Each transformer has a reactance drop and resistance drop of 5% and 0.7% respectively of the normal voltage of 60000 V on full load current of 80 A. The load at the receiving and sending ends of the line is respectively 58000 $\underline{/26°}$ V, 80 $\underline{/0°}$ A and 62000 $\underline{/19°}$ V, 77 $\underline{/16°}$ A. Calculate the overall percentage voltage drop of the combined line and transformers as a % of the sending voltage. Neglect the magnetizing current and capacitance.*

Solution. Fig. 1.61 shows a single line diagram depicting the transmission line, at each end of which is connected a transformer.

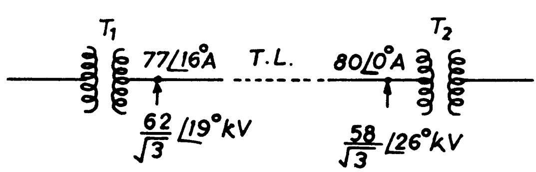


Fig. 1.61. Transmission line and transformers forming the power system for Example 1.43.

Given that the transformers have each an impedance of 0.7 + j 5.0% on the basis of normal voltage of 60 kV and current of 80 A, the ohmic impedance of each transformer can be evaluated as follows :

Base MVA $= \dfrac{\sqrt{3} \times 60 \times 80}{1000} = 8.314$

Base impedance $= \dfrac{60^2}{8.314} = 433$ ohm

The equivalent impedance of each transformer, referred to transmission line would, therefore, be equal to

$$Z_{eq} = 433 \times \frac{0.7 + j\,5.0}{100}$$
$$= 3.031 + j\,21.65 \text{ ohm}$$
$$= 21.861 \underline{/82.03^\circ}$$

It is required to find the overall voltage drop of the sending end transformer, line and the receiving end transformer.

Voltage drop in the transformer T_1 :

$$v_1 = (77.\,\underline{/16^\circ})\,(21.861\,\underline{/82.03^\circ})$$
$$= 1683.30\,\underline{/98.03^\circ} \text{ V}$$
$$= -235.16 + j\,1666.80 \text{ V} \qquad ...(1)$$

Voltage drop in transformer T_2

$$v_2 = (80\,\underline{/0^\circ})\,(21.861\,\underline{/82.03^\circ})$$
$$= 1748.88\,\underline{/82.03^\circ} \text{ V}$$
$$= 242.50 + j\,1731.99 \text{ V} \qquad ...(2)$$

Voltage drop in transmission line, TL :

v_3 = Difference between the voltages at the sending and receiving ends

$$= \frac{62000}{\sqrt{3}}\,\underline{/90^\circ} - \frac{58000}{\sqrt{3}}\,\underline{/26^\circ}$$
$$= 35800\,(0.946 + j\,0.326) - 33487\,(0.899 + j\,0.438)$$
$$= 3762 - j\,2996 \qquad ...(3)$$

Adding (1), (2) and (3), we obtain the overall voltage drop :

$$v_1 + v_2 + v_3 = 3769 - j\,403 \text{ V}$$

Magnitude of the overall voltage drop

$$= \sqrt{3769^2 + 403^2} \text{ V}$$
$$= 3.79 \text{ kV/phase.}$$

Sending end voltage/phase

$$= 62/\sqrt{3} = 35.8 \text{ kV.}$$

Hence the percentage voltage drop

$$= \frac{3.79}{35.80} \times 100 = \mathbf{10.60.}$$