1

Introductory

1.1 Effects of Magnetic Field

Electrical systems—power or control, use two basic elements, *viz.* rotating machine and transformer. Whereas the former, generally termed as rotating electro-mechanical energy-converter, enables conversion of energy from electrical to mechanical form (motor mode), or from mechanical to electrical form (generator mode), the latter is a static electromagnetic device and is not an energy-converter. The most wide use of transformer is for changing an alternating voltage from one level to another (voltage transformer), some other uses being current transformation (current transformer), isolating one electrical circuit from another (de-coupling transformer), and matching impedance of a source with a load for maximum transfer of power (matching transformer), etc. In a rotating machine, the process of conversion of energy is reversible though during this process a portion of energy is lost into heat. Similarly, in a transformer, the process of transformation of an alternating system (voltage or current) is generally reversible and in this process, some amount of energy is lost in heat.

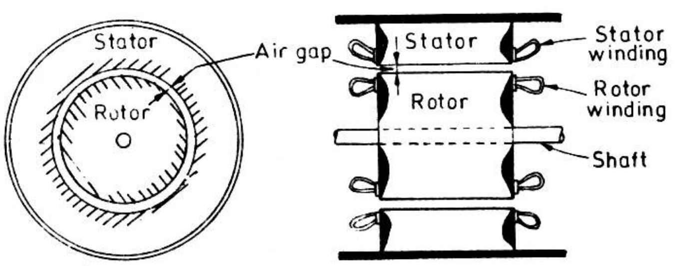


Fig. 1.1 Basic rotating electrical machine.

Basically, a rotating electromechanical energy converter operating on electromagnetic principle consists of two concentric cylinders of ferro-magnetic material separated by a gap (Fig. 1.1). One of the cylinders is stationary and is called the 'stator' ; and the other—a rotating member is called the 'rotor'. Magnetic material is used as a medium for the conversion of energy as a magnetic field has a much larger capacity to store energy in comparison to an electric field. The magnetic field can be developed either by an electrically excited winding or by permanent magnetism, housed on the stator or on the rotor.

[**Note :** The energy that can be stored with a magnetic field is about 25,000 times that with an electric field].

In a transformer, on the other hand, no gap is needed as there is no movement of one member with respect to another. As in rotating machines, ferro-magnetic material is also used as the core material in a transformer.

The basic principles involved in electromagnetic rotating machines and transformers are :

(*a*) **Magnetic induction.** When the flux linking a coil of N-turns increases by an amount $d\Phi$ in time dt, a voltage

$$e = -N\frac{d\Phi}{dt} \qquad ...(1.1)$$

is induced in the coil. The minus sign indicates that the direction of the induced voltage is such that the current due to it opposes the increase of flux.

Again, if from an external source, an instantaneous voltage e_t is impressed across a coil, a current i will flow in the coil so that,

$$e_t = e_o + i.r \qquad ...(1.2)$$

where, $e_o = -e$, is the balancing counter voltage ;

$$= N\frac{d\Phi}{dt},$$

$$= \frac{d\Psi}{dt}$$

Ψ wb-turns being the flux-linkage and r is the resistance of the coil.

Defining inductance as the flux-linkage per unit current,

$$\Psi = L.i \qquad ...(1.3)$$

we have, $$e_o = \frac{d}{dt}(L.i)$$

That is, $$e_t = \frac{d\Psi}{dt} + i.r \qquad ...(1.4a)$$

$$= \frac{d}{dt}(L.i) + i.r \qquad ...(1.4b)$$

[**Note.** Change of flux-linkage in the coil may be due to;

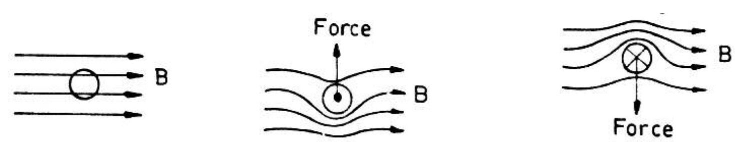
(*i*) Relative motion of the coil with respect to constant flux;

(*ii*) Variation in the magnitude of the flux, the coil being stationary with respect to the flux;

(*iii*) Relative motion of the coil with respect to flux which is time varying.

The change of flux-linkage under (*i*) and (*iii*) is associated with conversion of energy from electrical to mechanical form or *vice-versa*, as in rotating electrical machines ; case (*ii*) relates the phenomenon in a transformer].

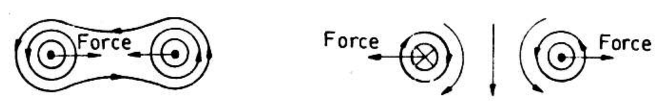
(*b*) **Interactive force.** A current-carrying conductor placed *suitably* in a magnetic field experiences a mechanical force. Fig. 1.2 (*a*) shows the direction of such a force as experienced by a conductor carrying current *at right angles* to the direction of the magnetic field.



(*i*)Conductor not carrying current (*ii*) Conductor current towards observer (*iii*) Conductor current into the paper.

(*a*) Force due to current-carrying conductor

Similarly, a mechanical force is exerted between two current-carrying conductors due to the interaction of their magnetic fields [Fig. 1.2 (*b*)].

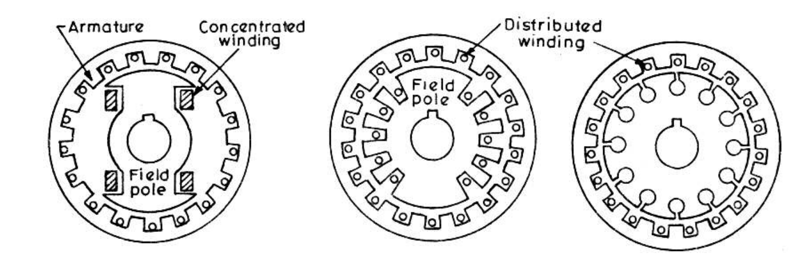


(*i*) Conductors carrying currents in the same directons. (*ii*) Conductors carrying currents in oppsoite directons.

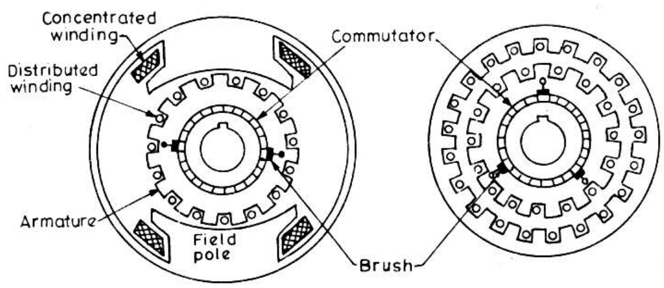
(*b*) Force between two current carrying conductors

Fig. 1.2 Production of electro-magnetic force.

The above processes are reversible because voltage is induced in a conductor or circuit when it undergoes motion in a magnetic field.



(*a*) Synchronous machine (salient pole) (*b*) Synchronous machine (cylindrical rotor) (*c*) Induction machine



(*d*) Commutator machine (direct current) (*e*) Commutator machine (alternating current)

Fig. 1.3 Types of electrical machines.

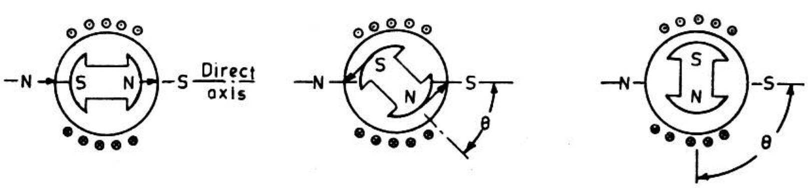
Whereas as mentioned above, the magnetic field is produced either by an excited winding or by permanent magnetism, say, on the stator, conductors are placed on the rotor so that the above mentioned effect can be utilised. The stator and rotor windings may be 'concentrated' or 'distributed' (Fig. 1.3). The distributed winding is housed in slots distributed around the gap-surface and the concentrated winding is on the 'salient' or projected structure.

In a transformer, interactive force of type as in Fig. 1.2 (*a*) is absent as there is no motion involved ; however, force is exerted between the primary and secondary windings as in Fig. 1.2 (*b*)-(*ii*) since the primary and secondary currents flow in opposite directions.

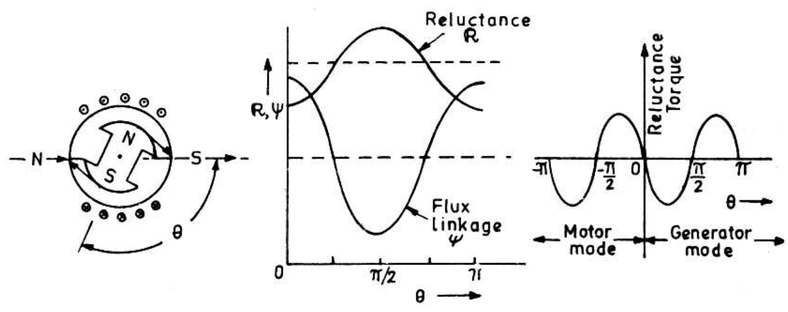
(*c*) **Reluctance effect.** A rotor of ferro-magnetic material placed in a magnetic field of non-uniform reluctance experiences a mechanical force tending to align itself to give minimum reluctance for magnetic flux.

The torque thus developed is known as the 'reluctance torque'. In a salient-pole machine in addition to the force due to current-carrying conductors placed in a magnetic field, there *may be* an additional force due to varying reluctance along the armature periphery.

A qualitative picture of the production of force due to reluctance effect can be obtained from the following analysis.



(*a*) $\theta = 0$; $T_e = 0$ (*b*) $0 < \theta < \frac{\pi}{2}$; $T_e = ve$ (*c*) $\theta = \frac{\pi}{2}$; $T_e = 0$



(d) $\frac{\pi}{2} < \theta < \pi$ (e) Spatial variation of (f) Reluctance torque

$T_e + ve$ reluctance & linkage for constant MMF

Fig. 1.4 Production of reluctance torque.

Consider a two-pole machine (Fig. 1.4) with the stator winding excited from a d.c. source and an unwound rotor with a salient structure. The axis of the stationary stator field is horizontal (known as the 'direct axis') and is chosen as the reference axis. As the rotor is rotated, the magnetic reluctance along the field axis varies with the instantaneous space-angle θ between the direct axis and the axis of the salient rotor structure. A typical variation of reluctance R (assumed sinusoidal) is shown in Fig. 1.4 (e). Again, assuming the stator ampere-turns constant, the flux-linkage Ψ with the stator winding will vary as shown in the figure, since, $\Psi \propto \frac{1}{R}$.

Let the rotor be rotated in the clockwise direction.

Case I. θ = zero.

Opposite poles are induced in the rotor, the axis of the induced rotor field coinciding with the direct axis. Forces of attraction (shown by arrow from the rotor to the stator) under the stator poles are radial, balancing each other. The developed torque is thus zero.

[**Note.** At this instant, the reluctance is minimum]

Case II. $0 < \theta < \frac{\pi}{2}$.

Forces of attraction between the stator and induced rotor poles could be resolved into radial and circumferential components [Fig. 1.4 (b)]. Whereas the radial components balance each other, the circumferential components develop torque in a direction opposite to that of rotation. The developed torque is thus negative.

[**Note.** The developed torque tends to align the rotor-axis with the direct-axis]

Case III. $\theta = \frac{\pi}{2}$.

Stator North and South poles have little effect on the rotor so that there is no induced rotor poles [Fig. 1.4 (*c*)], and the developed torque is zero.

[**Note.** The reluctance is maximum in this position]

Case IV. $\frac{\pi}{2} < \theta < \pi$.

Forces of attraction when resolved [Fig. 1.4 (*d*)] show radial components balance each other, and the circumferential components develop torque in the same direction as the rotor rotation. Reluctance torque is thus positive, trying to align the rotor-axis with the direct-axis.

Case V. $\theta = \pi$

The case is the same as the case I, and the reluctance torque is zero.

Table 1.1 Production of Reluctance Torque

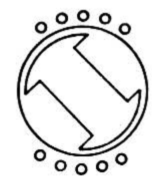
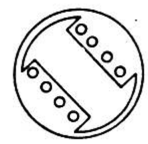
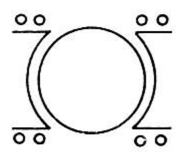
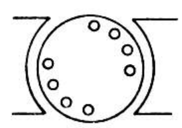
The exciting winding is on cylindrical stator structure ; saliency on the rotor. Magnetic reluctance offered to the developed magnetic field varies with rotor position. Reluctance torque is produced.	
The exciting winding is on the salient rotor structure, stator being cylindrical. The reluctance offered to the developed magnetic field is invariant with rotor position. No reluctance torque is produced.	
The exciting winding is on the salient stator structure, rotor being cylindrical. The reluctance offered to the developed magnetic field is invariant of rotor position. No reluctance torque is produced.	
The exciting winding is on the cylindrical rotor, saliency being with the stator. The reluctance offered to the developed magnetic field varies with the rotor position. Reluctance torque will be produced.	

Figure 1.4 (*f*) shows a typical variation of reluctance torque (assumed sinusoidal) with the space angle θ.

The process of production of the reluctance torque is also reversible. As the rotor rotates in a magnetic field of varying reluctance, the flux will change. A coil placed suitably to link this flux will have a voltage induced in it.

[**Note.** Mere provision of varying reluctance with θ does not produce reluctance torque. For example (Table 1.1), if the exciting winding in Fig. 1.4 is placed on the rotor instead of the stator, the reluctance offered to the magnetic field developed by the winding is constant and no reluctance torque is produced.

Similarly, an exciting winding on the salient stator with a cylindrical rotor structure does not produce any reluctance torque.

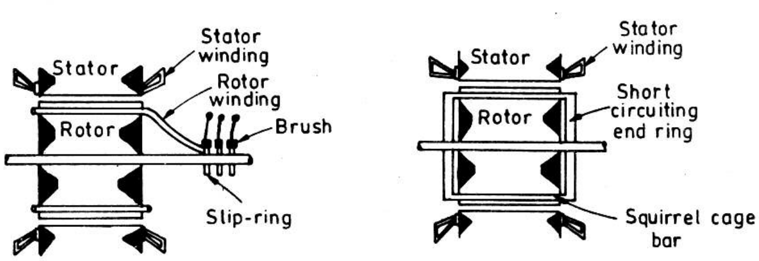
However, with the exciting winding on cylindrical rotor and saliency on the stator structure there will be reluctance torque developed.]

1.2 Types of Rotating Electrical Machines

Following are some important types of rotating electrical machines :

(*a*) **Synchronous Machine.** The field structure may either be salient-pole type or cylindrical-rotor type. In the former case, the field winding is concentrated and wound on poles either on the stator or on the rotor [Fig. 1.3 (*b*)]. The armature winding may be either polyphase or single phase, distributed uniformly in slots around the armature periphery, and carries alternating current. When run as a motor, on a constant frequency supply, the rotor rotates at constant speed called the 'synchronous speed' determined by the number of poles p and the frequency of the supply, f Hz as,

$$\text{Synchronous speed } n_s = 2\,\frac{f}{p} \text{ rev. per sec.} \qquad \text{...(1.5)}$$



(*a*) Wound rotor (*b*) Squirrel cage

Fig. 1.5 Induction machines.

As a generator in order to generate at a constant frequency, the rotor should be driven at a constant speed also determined by the equation (1.5).

(*b*) **Induction Machine.** The field system consists of a distributed winding on the stator, either single or polyphase, energised from an a.c. supply. The winding is housed in slots distributed throughout the stator periphery [Fig. 1.3 (*c*)]. Normally the armature or rotor consists of either a distributed winding of the same number of poles as the stator, wound in slots and terminating in slip-rings [called 'wound-rotor' type—Fig. 1.5 (*b*)] or a distributed winding consisting of bars placed in slots and short-circuited at each end by endrings [called 'squirrel-cage' type—Fig. 1.5 (*b*)]. The rotors of the later type are normally die-casted for machines of smaller ratings.

Generator mode of operation of induction machines is much less popular than its motor mode. The motor runs at a speed less than the synchronous speed.

(*c*) **Commutator Machines.** Commutator machines can be generally classified into three categories :

(*i*) *Direct current machines*—the field structure normally consists of salient poles housed in the stator; the exciting winding is concentrated, the exciting current being d.c. For machines of fractional h.p. size, sometimes the field structure is non-salient pole type, the exciting winding being distributed in slots around the stator periphery. The rotor or armature carries a uniformly distributed winding closed in itself, the armature current being conducted to the external circuit through commutator and brushes [Fig. 1.3 (*d*)].

(*ii*) *Alternating current machines*—the flied structure for thesemachines is generally non-salient pole type consisting of completely laminated cores and the exciting winding, single or three phase, is distributed in slots around the stator periphery. The rotor has winding similar to that of direct current machines, the current being conducted through commutator and brushes [Fig. 1.3 (*e*)].

(*iii*) *Universal motor*—this motor can be operated both on direct and alternating current supplies, and has, like alternating current commutator motor, laminated field and armature cores. Both the field and armature windings are distributed type, the later being a closed winding and connected to the external circuit through commutator and brushes. Normally these motors are of single-phase (on a.c. supply) and of a few watt-rating.

[**Note.** 1. An important feature in an electromagnetic machine is the losses that occur in the magnetic material. Whereas the stator cores of all a.c. machines are laminated to keep the core-losses to a minimum, the yoke and the pole-shoes of a normal d.c. machine are unlaminated as the flux in these is unidirectional. The rotor core of a turbo-alternator consists of cylindrical pieces of forged steel assembled together.]

Under the headings of each of the above three classifications, there are quite a large variety of motors and generators each having its own speciality and range of operation ; but as has been pointed out earlier, the unifying features of all these machines enable them to be broadly grouped under two heads :

(*a*) Singly-excited system ;
(*b*) Multiply-excited system.

Whereas most of the normal rotating electrical machines fall under the second category, an important example of singly-excited rotating electrical machine is the reluctance machine.

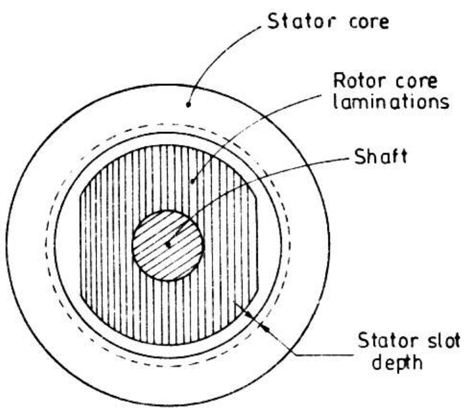


Fig. 1.6 (*a*) Two-pole Reluctance Motor.

Fig. 1.6 (*a*) shows one form of a two-pole reluctance motor [Art. 1.1 (*c*)]. The core is laminated with uniformly distributed slots carrying normally a three-phase distributed winding. The rotor construction shows gap length along the pole axis much smaller than that along the interpolar axis. The rotor core is laminated. To have more reluctivity along the interpolar axis the inter-lamination insulation can be increased.

2. Commutator machines, in general, possess desirable control characteristics. For example, under generator mode, direct current machines have volt-ampere characteristics through wide ranges attainable by variety of field winding connections with respect to armature, and smooth voltage control achievable by field excitation control. Again, both direct - and alternating-current motors enable smooth control of speed under stringent control conditions.

But such qualities are often subjected to important limitations, such as :

(*a*) the problem of commutation and sparking in general, and in particular for large load applications :

(*b*) relatively low maximum rotor speed limited by mechanical stress on the commutator ;

(*c*) need for regular maintenance of commutator and brushes;

(*d*) rotor relatively heavy and in consequence, have high inertia ;

(*e*) difficulty in producing a machine with 'totally-enclosed'—enclosure as needed for certain application under hazardous condition ;

(*f*) comparatively higher cost.

With major breakthroughs in the area of Power Electronics, direct current generators are being replaced in many applications by solid-state rectifiers on a firm a.c. supply and both direct- and alternating current commutator motors by induction and synchronous motors with various solid-state controllers.

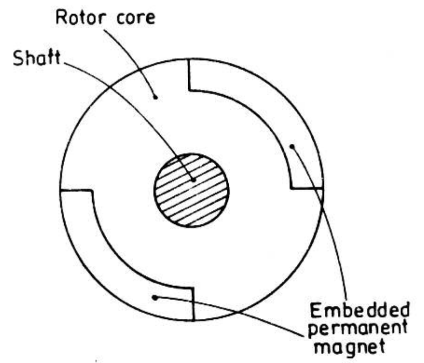
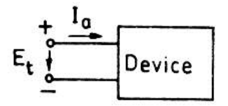


Fig. 1.6 (*b*) One type of rotor of Permanent Magnet motor.

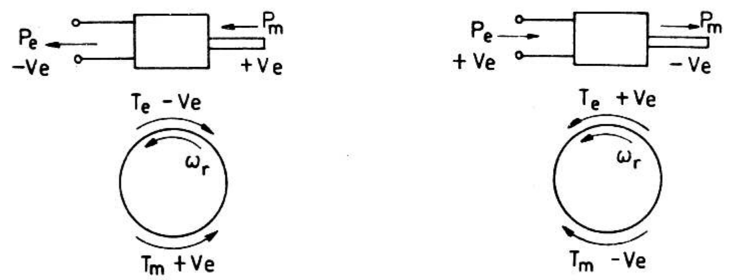
Indeed, this area has at present attracted researchers' attention in view of the intense global competitiveness demanding sophistication of process automation employing electric motors. The alternating current commutator motors have already become obsolete, and looking at the trend of success in power electronics research, one may conclude that direct current machines would follow the same path.

(*d*) Currently **Permanent Magnet Motors** (Fig. 1.6 *b*), **and reluctance motors** with permanent magnet excitation system have become the sources of attraction of the researchers in view of the development of newer permanent magnet material* and breakthrough in power electronic control of these motors. The development matches with the current concern with respect to energy saving. Though the basic cost of these motors is higher than induction motor of corresponding ratings, energy saving taking into account the life-cycle ultimately results in cost saving.

1.3 Signs and Conventions

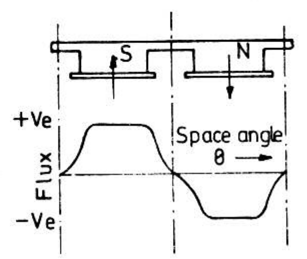


(*a*) Positive direction of current and positive polarity of voltage



(*b*) Motor Operation
Electrical Power Input P_e : + *ve*
Mechanical Power Output P_m : − *ve*
Electromagnetic Torque T_e : + *ve*
Mechanical Torque Output T_m : − *ve*

(*c*) Generator Operation
Mechanical Power Input P_m : + *ve*
Electrical Power Output P_e : − *ve*
Electromagnetic Torque T_e : − *ve*
Mechanical Torque Input T_m : + *ve*



1. Typical Field-form In A Salient-pole Machine
Fig. 1.7 Signs and conventions.

*Sagawa, M., Fugimura, S., Togawa, N., Yamamoto, H., and Maysuura, Y. : "New material for Permanent Magnets on a base of Nd and Fe." *Journal of Applied Physics, vol. 55, No. 6, pp. 2083-87, 1984.*

In developing equations, the following signs and conventions would be used :

(*a*) **Current, voltage and power.** The assumed direction of current is positive when the current flows into the device (machine or circuit). The choice implies that the armature current is positive for motor mode and negative for generator mode.

The polarity of voltage is such that positive voltage at the terminals will cause positive current through the device [Fig. 1.7 (*a*)].

Thus, power is positive as it flows into the device, that is, the electrical power P_e to a motor is positive and that from a generator is negative. Again, the mechanical power P_m to a generator is positive, and that from a motor shaft is negative [Fig. 1.7 (*b*) and (*c*)].

(*b*) **Flux.** Flux is assumed positive when it goes 'away' from the rotor. That is, a rotor 'North' pole or a stator 'South' pole develops positive flux. Typical flux distribution in space for an electrical machine is shown in [Fig. 1.7 (*d*)].

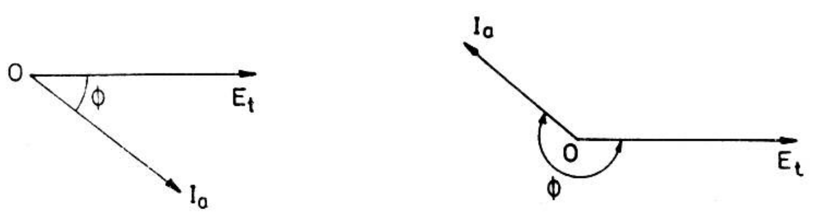
(*c*) **Mechanical rotation and torque.** Positive direction of rotations is counter-clockwise. Torque is positive when it acts to assist motion.

Thus, if ω'_r mechanical radians per second be the angular speed of the rotor, mechanical power

$$P_m = \omega r' \,.\, T_m \qquad ...(1.2a)$$

where, T_m is the mechanical torque.

[**Note.** Thus, for an alternating current motor, positive input power under the steady-state sinusoidal condition and at a lagging power-factor at the terminals leads to a phasor diagram of Fig. 1.8 (*a*) ; whereas, for an a.c. generator, output power (negative as per adopted convention) under the same conditions as above leads to the phasor diagram of Fig. 1.8 (*b*)].



(*a*) (*b*)

Fig. 1.8 Convention-Phasor diagram.

1.4 Basis of Analysis

The following discussions form the fundamental basis of analysis of all electromagnetic machines :

(*a*) **Reluctance.** In a magnetic circuit, the magnetomotive force M ampere-turns establishes a flux Φ Webers so that,

$$\text{flux } \Phi = \frac{\text{mmf. } M}{\text{reluctance, } R} \quad ...(1.6)$$

the reluctance R is determined by the dimensions of the magnetic circuit and the magnetic properties (specially the permeability) of the core material.

For a magnetic circuit consisting of several parts *a, b,* and *c in series,* the effective reluctance,

$$R = R_a + R_b + R_c \quad ...(1.7a)$$

and when the parts are *in parallel,*

$$\frac{1}{R} = \frac{1}{R_a} + \frac{1}{R_b} + \frac{1}{R_c} \quad ...(1.7b)$$

(*b*) **MMF distribution in space.** A full pitch coil (*i.e.* a coil of spread one pole pitch = π electrical radians) of N-turns carrying a current i placed in slots as shown in Fig. 1.9, develops an mmf of $\frac{1}{2}Ni$ ampere-turns per pole, the mmf distribution in space being rectangular.

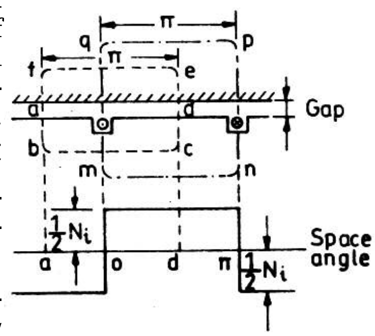


Fig. 1.9 M.m.f. Waveform—one full-pitch coil.

This may be determined by considering two paths *abcdef* and *mnpq* and assuming iron having infinite permeability. Since the mmf acting on any path is equal to the total current enclosed by the path :

(*i*) mmf for path $a - b - c - d - e - f = V_i$.

giving the magnetic potential difference at angles a and at $d = \frac{1}{2}Ni$, since there are two gaps in the path.

(*ii*) mmf for path $m - n - p - q$ = zero; the magnetic potential difference at angles zero and π = zero.

Choosing any other path, it can be shown that the magnetic potential difference at all space angles other than zero and $\pi = \frac{1}{2}Ni$.

[**Note.** The mmf waveform in space is rectangular if the slots and conductors are assumed to have very small width].

Example 1.1. *Determine the mmf waveform for the configurations shown in Fig. 1.10 (a) and (f). The coils AB, CD, EF, etc., each having N turns are connected in series and carry a current of i ampere.*

Solution. For each of the coils *AB, CD, EF,* the rectangular mmf wave-form is shown in Figs. 1.10 (*b*), (*c*) and (*d*) respectively. The resultant mmf wave is stepped as shown in Fig. 1.10 (*e*), the peak value of the mmf being $\frac{1}{2}Ni$.

[**Note.** If the slots are infinitely close to each other, the mmf wave is trapezoidal].

For the problem in Fig. 1.10 (*f*), a similar procedure as above will give a resultant mmf waveform shown in Fig. 1.10 (*g*), the peak mmf being $\frac{5}{2}Ni$.

With slots infinitely close to each other, the mmf waveform is triangular.

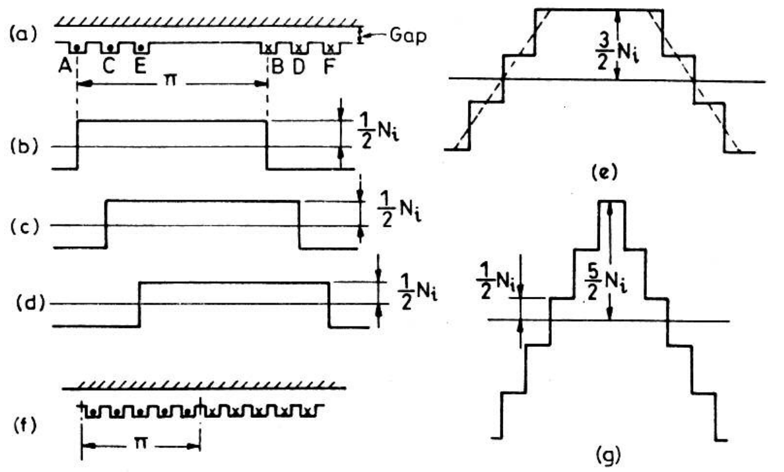


Fig. 1.10 Pertaining Example 1.1.

(*c*) **Losses.** In the process of transformation of an alternating system in a transformer or in the process of electromechanical energy conversion in a rotating machine, some amount of power is lost. The important losses are :

(*i*) **Electrical.** I^2r—losses in the windings due to the flow of electric current in them.

(*ii*) **Magnetic.** Core loss's in the portions of the magnetic circuit which are subjected to varying or alternating magnetic fields. The magnetic losses consist of 'hysteresis' and 'eddy-current' losses.

Apart from above, there are minor losses such as,

(*i*) stray loss due to e.m.fs. induced by stray fields in adjacent conductors ;

(*ii*) dielectric losses in the insulation ; and

(*iii*) in rotating machines, the mechanical losses due to bearing friction, brush friction and windage.

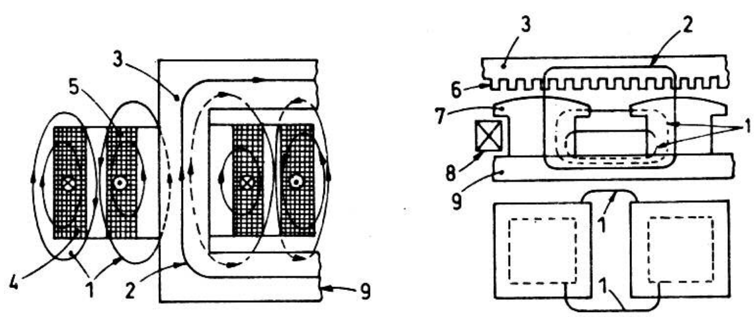
(*d*) **Leakage Flux and Saturation.** As mentioned earlier, the study of electromagnetic machines involves quantitative evaluation of magnetic fields under various operating conditions. Fluxes are produced by currents flowing in various coils and not all the fluxes contribute to the intended action of the machine. For example, in a transformer, only the flux which links both primary and secondary windings contribute to the process of transformation. The total flux developed due to the excitations of primary and secondary coils can thus be splitted up into three components (Fig. 1.11*a*) :

(*i*) Mutual or useful flux Φ_M due to the currents i_a and i_b in primary coil *A* and secondary coil *B* respectively ;

(*ii*) Primary leakage flux Φ_{ea} due to i_a in primary linking coil *A* and not coil *B* ;

(*iii*) Secondary leakage flux Φ_{lb} due to current i_b in secondary linking coil *B* but not coil *A*.

In a rotating machine, only that flux which crosses the gap so as to link both stator and rotor coils (in multiple-excited machine) contributes to the conversion of energy. Fig. 1.11 (*b*) shows some of the component leakage fluxes in a salient-pole machine.



(*a*)Transformer (*b*) Salient Pole Machine
1. Leakage Flux 2. Mutual Flux 5. L.V. Winding 6. Teeth 7. Pole
3. Core 4. H.V. Winding 8. Field Winding 9. Yoke

Fig. 1.11 Leakage flux in Machines.

An electromagnetic machine uses ferro-magnetic core as the medium for transformation or energy-conversion, and as such its performance is influenced by the presence of magnetic saturation in the core. The phenomenon of saturation is a function of core material and the working flux-density and affects the flux distribution in space in a rotating machine, and in time in a transformer. The overall effect of saturation in machine is generally taken using the open-circuit characteristic.

For simplified analysis of a rotating machine, the effect of saturation is generally neglected by assuming that the iron parts of the magnetic circuit has infinite permeability and the gap absorbs the whole mmf.

(*e*) **Electro-mechanical Energy conversion.** The conversion of energy from one form to another is based on the principle of conservation of energy which states that energy is neither created nor destroyed ; it can only be converted from one form into another.

The energy balance equation can thus be written as,

Total energy input to the machine

= Total energy stored in the machine + Total energy dissipated as heat.

(*i*) Since the inputs to an electromagnetic machine are of electrical or mechanical form,

Total energy input

= Electrical energy input W_{ei} + Mechanical energy input W_{mi} ;

(*ii*) The total stored energy

= Energy stored in the magnetic field W_f + the stored mechanical energy W_j ;

(*iii*) The total energy dissipated

= Energy dissipated in electrical losses, W_{le} + energy dissipated in mechanical losses W_{lm}.

Thus, $W_{ei} + W_{mi} = (W_f + W_j) + (W_{le} + W_{lm})$ ...(1.8)

For motor mode : electrical energy input W_{ei} is +ve ; mechanical energy input W_{mi} is – ve.

For generator mode : electrical energy input W_{ei} is – ve ; mechanical energy input W_{mi} is +ve.

Again, defining power as the rate of change of energy, equation (1.8) gives,

$$P_{ei} + P_{mi} = \left(\frac{d}{dt} W_f + \frac{d}{dt} W_j\right) + \left(\frac{d}{dt} W_{le} + \frac{d}{dt} W_{lm}\right) \qquad ...(1.9)$$

where, P_{ei} and P_{mi} are respectively the instantaneous electrical power input and the instantaneous mechanical power input.

Denoting, $\frac{d}{dt} W_{le} = P_{le}$, the instantaneous electrical power losses,

$\frac{d}{dt} W_{lm} = P_{lm}$, the instantaneous mechanical power losses ; and defining,

$P_e = (P_{ei} - P_{le})$ as the instantaneous electromagnetic power,

$P_m = (P_{mi} - P_{lm})$ as the instantaneous internal mechanical power,

we have, $$P_e + P_m = \frac{d}{dt} W_f + \frac{d}{dt} W_j \qquad ...(1.10)$$

1.4.1. Stored Magnetic Field Energy

In an electrical machine, magnetic field is established whenever a winding is excited. For example with reference to Fig. 1.6 (*d*), when the field windings of the salient poles are excited a flux-density wave (as shown) is established in space. As the exciting current takes certain time to build up from its initial zero to a certain steady value, the flux-density wave also takes time to establish itself and in that process magnetic field energy is stored in the system. As the current reaches its steady value, stored magnetic energy also reaches a steady value, and when the winding is disconnected from the supply, the field energy decreases to zero.

In common electrical machines, under steady-state condition, the magnetic field remains somewhat unaltered whether it is stationary in space or it rotates round the gap. The stored magnetic field energy thus remains substantially constant and $\left(\frac{d}{dt} W_f\right)$ is zero.

However, between one steady-state condition and another, the magnitude and spatial distribution changes and the stored field energy alters from one steady value to another.

1.4.2. Stored Mechanical Energy

The stored mechanical energy is associated with the moment of inertia of the rotor and its connected mass. Whenever there is a change in speed, the mechanical energy stored in the rotating mass alters, its magnitude depending on the rate of change of speed. For example, during the process of acceleration of the rotor, storage of mechanical energy takes place till the rotor reaches a steady speed. So long the rotor runs at a steady speed, the stored mechanical energy-remains constant at a value reached during the acceleration of the rotor. Again, as the rotor decelerates, mechanical energy is dissipated and at standstill, the stored mechanical energy is zero.

A mass of moment of inertia J_i rotating about its axis at an angular speed $\omega' r$ mechanical radian per second has a kinetic energy,

$$W_j = \frac{1}{2} J_i \, . \, \omega'^2_r \text{ joule}$$

stored in it, which it has acquired during its acceleration from rest to the above speed. If the inertia is constant,

$$\text{the energy-rate } P_j = \frac{d}{dt} W_j$$

$$= J_i . \omega'_r \, . \, \frac{d\omega'_r}{dt} \qquad \text{...(1.11)}$$

and hence, the inertial torque,

$$T_j = \frac{P_j}{\omega'_r}$$

$$= J_i \frac{d\omega'_r}{dt} \text{ N-m.} \qquad \text{...(1.12)}$$

Example 1.2. *Consider a simple magnetic circuit of a toroid [Fig. 1.12 (a)] which is a singly-excited system. Let the exciting current be increased from its initial zero value to I ampere in time t second. The flux-linkage thereby increases from initial zero value (assuming zero residual magnetism) to Ψ W b-turns. Deduce expressions for the stored magnetic field energy in terms of*

(*a*) *the effective reluctance of the magnetic circuit and Ψ ;*

(*b*) *I and ψ;*

and (*c*) *inductance L,*

Solution

Let i amp. be the instantaneous current in the coil.

The instantaneous electrical power input P_{ei}

$$= e_t \, . \, i \text{ watt}$$

$$= r.i^2 + i \frac{d\Psi}{dt} \text{ watt}$$

where, r is the resistance of the coil and Ψ, the instantaneous flux-linkage.

Since there is no mechanical power input, the total energy input to the toroid in time dt second

= electrical energy input in time dt

$$= r\, i^2 \, dt + i \, d\Psi \text{ joule} \qquad \text{...(1.13)}$$

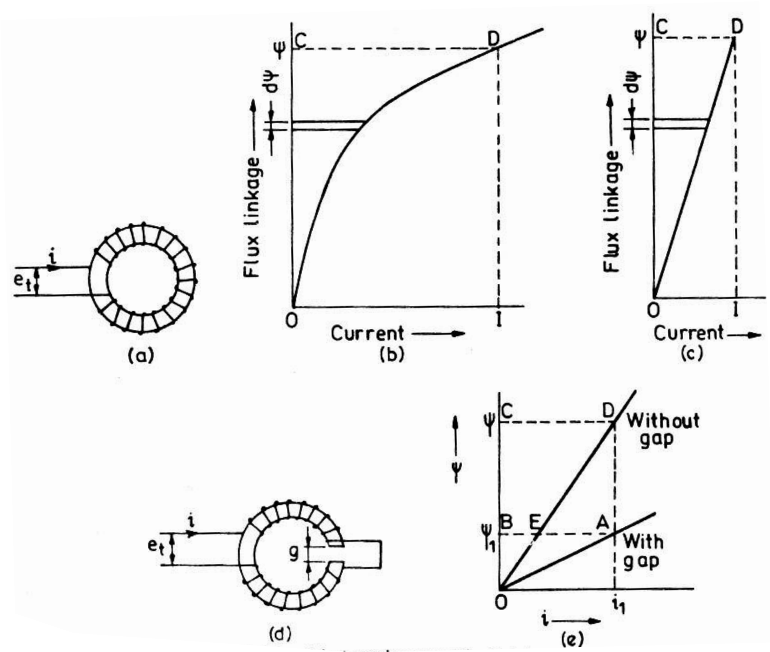


Fig. 1.12. Toroid stored magnetic-field energy.

Comparing equations (1.8) and (1.13) and noting that there can neither be any storage of mechanical energy nor any dissipation of mechanical energy in the absence of any moving member in the system, we have,

$r\, i^2\, dt =$ the energy dissipated in electrical losses in time dt ;

and $id\Psi$ = the energy stored in the magnetic field in time dt.

That is, the stored magnetic field energy W_f in time t sec. when the current changes from zero to I ampere and the flux-linkage from zero to Ψ Wb-turn,

$$= \int_0^{\Psi} i.\, d\Psi \text{ joule} \qquad \text{...(1.14}a)$$

But, the instantaneous mmf $m = N.i$, N being the effective number of turns of the exciting coil; and the instantaneous flux $\phi = \frac{m}{R}$, R being the effective reluctance of the magnetic circuit.

That is, $\quad i = \frac{m}{N}$

$$= \frac{\phi}{\mathrm{N}} . R$$

$$= \frac{\Psi}{N^2} \cdot R$$

or $$W_f = \frac{R}{N^2} \int_0^{\Psi} \Psi d\Psi,$$

assuming constant permeability of the core material of the toroid ;

$$= \frac{1}{2} R \frac{\Psi^2}{N^2} \text{ joule}$$

$$= \frac{1}{2} I. \Psi \text{ joule} \qquad ...(1.14b)$$

since the exciting current $I = \frac{\Psi}{N^2} \cdot R$.

Defining inductance L henry as the flux-linkage per ampere $= \frac{\Psi}{I}$,

$$W_f = \frac{1}{2} L I^2 \text{ joule} \qquad ...(1.14c)$$

[**Note.** (*a*) Considering the core material of the toroid to be ferro-magnetic, the ψ *vs.i* characteristic is as shown in Fig. 1.12 (*b*). The stored magnetic field energy, from equation 1.14 (*a*) is equal to the area *ODCO*.

(*b*) If the permeability of the ferro-magnetic core be assumed constant, ψ *vs. i* characteristic is a straight line [Fig. 1.12 (*c*)], and stored magnetic field energy $W_f =$ area of triangle *ODC*

$$= \frac{1}{2} \Psi \cdot I \text{ joule}$$

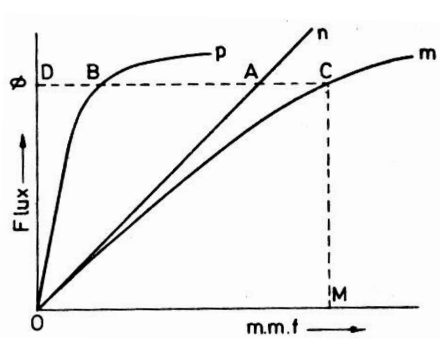


Fig. 1.13 Series Magnetic circuit.

(*c*) If there is a saw cut in the toroid [Fig. 1.12 (*d*)], the effective reluctance of the series magnetic circuit, $R(= R_{gap} + R_{iron})$ is increased. Hence, for any exciting current i_1, the flux-linkage Ψ with a saw cut in toroid is less than the flux-linkage Ψ without gap. The magnetising characteristic under linear condition is given by *OA* [Fig. 1.12 (*e*)] and the stored magnetic field energy due to an increase in current from zero to i_1 is,

$$W_f = \text{the area of the triangle } OAB$$

$$= \frac{1}{2} i_1 \,.\, \Psi_1 \text{ joule}$$

(*d*) In the series magnetic circuit of Fig. 1.12 (*d*), the resultant magnetising characteristic (*"om"* in Fig. 1.13) is due to two components: (*i*) due to gap, in which case flux-mmf relationship is a straight line *'on'*.

(*ii*) due to ferro-magnetic core in which case the flux-mmf relationship is given by the curve *'op'*.

The stored magnetic field energy required to establish a flux from zero value to Φ,

$$W_f = \text{the area } OCDO \text{ joule,}$$

has two components (assuming negligible leakage flux) :

(*i*) the area *OADO* due to the gap ;

and (*ii*) the area *OBDO* due to the ferro-magnetic core.

For fields in free space (or in air) the absolute permeability $\mu = \mu_o = 4\pi \times 10^{-7} = 0.125 \times 10^{-5}$, and for fields in ferro-magnetic materials, $\mu = \mu_o.\mu_r$ is much greater than μ_o since μ_r normally ranges between 10^3 and 10^5. That is, component (*ii*) is normally much smaller than the component (*i*) and it can be assumed for simplicity that all the field energy is stored in the airgap. This amounts to the assumption that the ferro-magnetic core has infinite permeability.

(*e*) The equations for the stored magnetic field energy can be summerised as below :

$$\begin{aligned} W_f &= \frac{1}{2}\,\Psi.I \text{ joule} \\ &= \frac{1}{2}\,M.\Phi \text{ joule} \\ &= \frac{1}{2}\,\Phi^2.R \text{ joule} \qquad \text{...(1.14)} \\ &= \frac{1}{2}\,M^2/R \text{ joule} \\ &= \frac{1}{2}\,L.I^2 \text{ joule]} \end{aligned}$$

(*f*) The concept of inductance introduced above enables treatment under the steady-state by phasor diagram and equivalent circuit approaches. However, such a concept will be meaningless unless the parameters could be computed from physical data and material characteristics (as shown in art 4.8) and could be measured*. However, there would always be certain divergences between computed and rest results in a practical machine, because of non-linearities due to magnetic saturation and hysteresis, and also leakage flux (which is quite small in large machines but high in small machines in comparison to the total flux produced and is difficult to measure accurately).

Example 1.3. *Fig. 1.14 shows the magnetic circuit of a doubly-excited machine. In the figure, subscripts s and r stand for stator and rotor respectively, l for leakage, and M for mutual. Thus,* Φ_{ls} *is the stator leakage flux, and* Φ_{rs} *is flux due to the stator current linking the rotor coil r. Similarly,* Φ_{lr} *is the rotor leakage flux, and* Φ_{sr} *is the flux due to the current in rotor coil r linking the stator coil s.*

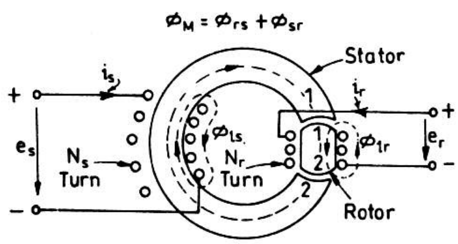


Fig. 1.14. (*a*) Doubly-excited machine.

With the rotor standstill in the position shown (rotor pole1 under stator pole 1) the stator and rotor exciting currents are simultaneously increased from zero value to i_s *and* i_r *respectively in a time t seconds. Assuming zero residual magnetism and negligible saturation and hysteresis, derive an experssion for the total stored magnetic field energy in terms of*

(*a*) *flux-linkages ;*

and (*b*) *inductance parameters.*

Consider the fluxes due to the two coils aiding each other so that the total mutual flux $\Phi_m = \Phi_{rs} + \Phi_{sr}$.

Solution.

The volt-ampere equations are :

$$e_s = i_s\, r_s + N_s \frac{d}{dt} (\Phi_{ls} + \Phi_M)$$

$$= i_s\, r_s + N_s \frac{d}{dt} (\Phi_{ls} + \Phi_{rs} + \Phi_{sr})$$

* Jones, C.V. : Unified Theory of Electrical Machines.

Defining self-flux of stator coil $\Phi_{ss} = \Phi_{ls} + \Phi_{rs}$,

$$= i_s\, r_s + N_s \frac{d}{dt}(\Phi_{ss} + \Phi_{sr})$$

and
$$e_r = i_r\, r_r + N_r \frac{d}{dt}(\Phi_{lr} + \Phi_m)$$

$$= i_r\, r_r + N_r \frac{d}{dt}(\Phi_{rr} + \Phi_{rs})$$

where, Φ_{rr} is the self-flux of rotor coil

$$= \Phi_{lr} + \Phi_{sr}$$

Defining Ψ_{ss} = self flux-linkage of stator coil

$$= N_s.\, \Phi_{ss};$$

and Ψ_{rr} = self flux-linkage of rotor coil

$$= N_r.\, \Phi_{rr};$$

and also
$$\Psi_{sr} = N_s.\, \Phi_{sr};$$

$$\Psi_{rs} = N_r.\, \Phi_{rs},$$

$$e_s = i_s.\, r_s + \frac{d}{dt}\Psi_{ss} + \frac{d}{dt}\Psi_{sr} \qquad \text{...(1.15)}$$

$$e_r = i_r\, .\, r_r + \frac{d}{dt}\Psi_{rr} + \frac{d}{dt}\Psi_{rs}$$

Total energy input in the two coils in time dt second,

$$= (i_s^2\, .\, r_s + i_r^2\, .\, r_r)\, dt + i_s\, .\, d\Psi_{ss} + i_r\, .\, d\Psi_{rr} + i_s\, .\, d\Psi_{sr} + i_r\, .\, d\Psi_{rs}\, \ldots. \qquad \text{...(1.16)}$$

In the above equation, the 1st term is the energy lost in heat. With the rotor at standstill, mechanical energy input, stored mechanical energy and the dissipation of mechanical energy are each zero. That is, the total stored magnetic field energy in dt second,

$$dW_f = i_s\, .\, d\Psi_{ss} + i_r\, .\, d\Psi_{rr} + i_s\, .\, d\Psi_{sr} + i_r\, .\, d\Psi_{rs}$$

$$= i_s\, .\, d\Psi_s + i_r\, .\, d\Psi_r$$

where, $\Psi_s = \Psi_{ss} + \Psi_{sr}$ is the total flux-linkage of coil s, in Wb-turn ; and $\Psi_r = \Psi_{rr} + \Psi_{rs}$ is the total flux-linkage of coil r, in Wb-turn. That is, the total stored magnetic field energy in t second,

$$W_f = \int_0^{\Psi_s} i_s.\, d\Psi_s + \int_0^{\Psi_r} i_r.\, d\Psi_r$$

But mmf of coil s due to a current i_s

$$= M_s = N_s.\, i_s; \text{ and}$$

that of coil r due to a current i_r

$$= M_r = N_r.\, i_r.$$

Also, $\Psi_s = \Phi_s.\, N_s$

and $\Psi_r = \Phi_r.\, N_r$

where Φ_s and Φ_r are the flux linked by coil s and coil r respectively.

Defining R_1 as the net reluctance of the flux-path with rotor in position as shown, (**Note** : reluctance is minimum in this position),

$$W_f = \frac{R_1}{N_s^2}\int_0^{\Psi_s} \Psi_s \,.\, d\Psi_s + \frac{R_1}{N_r^2}\int_0^{\Psi_r} \Psi_r.\, d\Psi_r$$

$$= \frac{1}{2}\, i_s.\, \Psi_s + \frac{1}{2}\, i_r.\, \Psi_r \text{ joule} \qquad \text{...(1.17}a\text{)}$$

$$= \frac{1}{2}\, M_s.\, \Phi_s + \frac{1}{2}\, M_r.\, \Phi_r \text{ joule} \qquad \text{...(1.17}b\text{)}$$

In terms of inductance parameters, we define,

$$\Psi_{ss} = L_s\, i_s$$

$$\Psi_{sr} = L_{sr}\, i_r$$

$$\Psi_{rr} = L_r\, i_r$$

$$\Psi_{rs} = L_{rs}\, i_s$$

so that,

$$\Psi_s = L_s \,.\, i_s + L_{sr} \,.\, i_r$$

$$\Psi_r = L_{rs} \,.\, i_s + L_r \,.\, i_r \qquad \text{...(1.18}a\text{)}$$

Equation (1.18a) can be represented in the following form :

$$\begin{bmatrix} \psi_s \\ \psi_r \end{bmatrix} = \begin{bmatrix} L_s & L_{sr} \\ L_{rs} & L_r \end{bmatrix} \cdot \begin{bmatrix} i_s \\ i_r \end{bmatrix} \qquad \text{...(1.18}b\text{)}$$

When saturation is negligibly small or under the same conditions of saturation, flux linking the stator coil per rotor ampere is equal to the flux linking the rotor coil per stator ampere. That is,

$$L_{sr} = L_{rs} = L_M \text{ (say)}.$$

or,

$$\begin{bmatrix} \psi_s \\ \psi_r \end{bmatrix} = \begin{bmatrix} L_s & L_M \\ L_M & L_r \end{bmatrix} \cdot \begin{bmatrix} i_s \\ i_r \end{bmatrix} \qquad \text{...(1.18}c\text{)}$$

Thus,

$$W_f = \frac{1}{2}\, L_s.i_s^2 + \frac{1}{2}\, L_r.i_r^2 + L_M \,.\, i_s \,.\, i_r \text{ joule} \qquad \text{...(1.19}a\text{)}$$

It is of interest of note that equation (1.19a) can be represented as,

$$[W_f] = 1/2 \cdot \begin{bmatrix} i_s & i_r \end{bmatrix} \cdot \begin{bmatrix} L_s & L_M \\ L_M & L_r \end{bmatrix} \begin{bmatrix} i_s \\ i_r \end{bmatrix}$$

$$= 1/2 \, [i_t] \cdot [L] \cdot [i] \qquad \text{...(1.19}b\text{)}$$

where, $[i]$ is the current matrix,

$[i_t]$, transpose of $[i]$,

and $[L]$, is the inductance matrix $= \begin{bmatrix} L_s & L_M \\ L_M & L_r \end{bmatrix}$

Matrix representation of equations leads to two-fold advantage:

(*a*) the inductance matrix can be written down straightway through observation of configuration of core and windings, thus,

[inductance] = matrix

stator complete inductance	stator-rotor mutual inductance
stator-rotor mutual inductance	rotor complete inductance

...(1.19*c*)

[current matrix] = $\begin{bmatrix} \text{stator current} \\ \text{rotor current} \end{bmatrix}$

(*b*) the above equations are easily amenable for use in a computer.

[**Note.** (*a*) If the rotor is stationary after it has turned through $\frac{\pi}{2}$ radians from its position in Fig. 1.14 (*a*), the effective reluctance of the flux-path increases to its maximum value R'. Inductance L_s and L_r have their minimum values in this position. Further, there is no mutual linkage between the stator and rotor coils since their axes are at perpendicular to each other ; *i.e.*

$$L_{sr} = L_{rs} = \text{zero.}$$

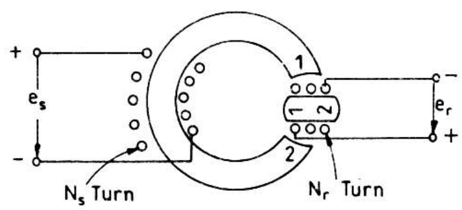
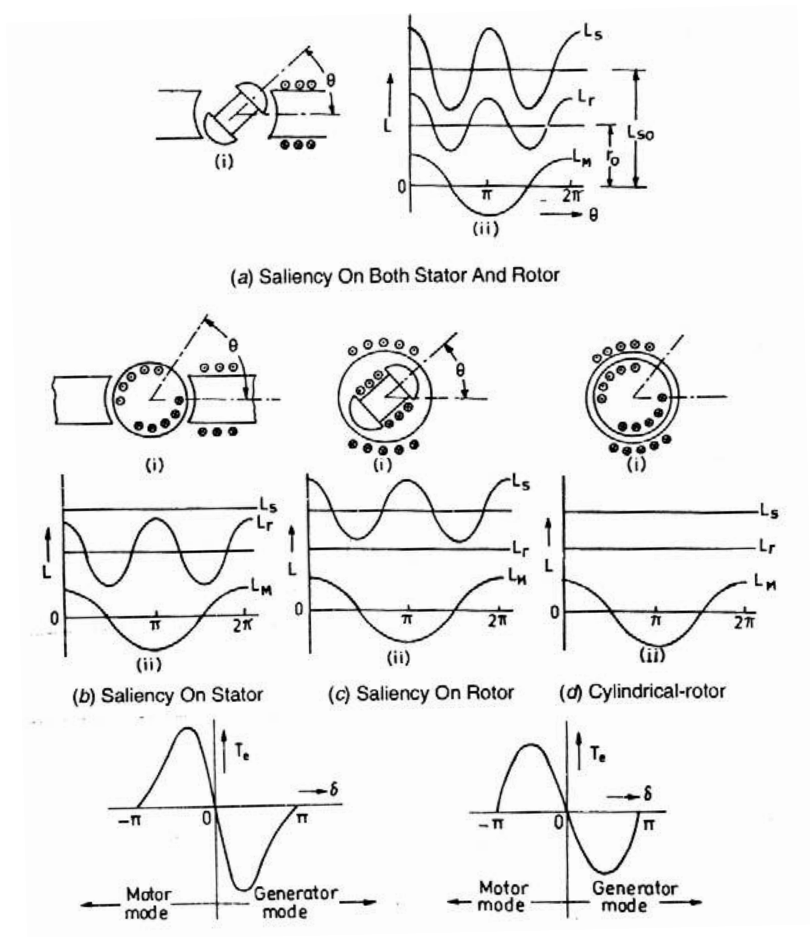


Fig. 1.14 (*b*) Doubly-excited machine.

The stored magnetic energy in this position is,

$$W_f = \frac{1}{2} L_s \cdot i_s^2 + \frac{1}{2} L_r \cdot i_r^2 \text{ joule} \qquad \text{...(1.19}d\text{)}$$

(*b*) If the rotor is turned through another $\frac{\pi}{2}$ radians so that rotor pole 1 is under the stator pole 2 and is left stationary in that position, the effective reluctance of the flux-path is *R,* its minimum value, and L_s and L_r have their maximum values. But, since the mutual fluxes of the two coils oppose each other, L_M is negative at the same value of Fig. 1.14 (*a*).



(*i*) SALIENT-POLE (*ii*) CYLINDRICAL-ROTOR

(*e*) Torque-angle Characteristics

Fig. 1.15 Doubly-excited machine.

(*c*) L_s, L_r and L_M can be measured experimentally* and the following equations can be written :

$$L_s = L_{so} + L_{s1} \cos 2\theta$$

$$L_r = L_{ro} + L_{r1} \cos 2\theta$$

$$L_M = L_m \cos \theta \qquad ...(1.20a)$$

where, θ is the space-angle in radians between the axes of stator and rotor [Fig. 1.15 (*a-i*)]. Typical variations of the inductance parameters L_s, L_r and L_M are illustrated in Fig. 1.15 (*a-ii*).

(*d*) A normal salient-pole machine has saliency either on the stator or on the rotor.]

Case I. When saliency is only on the stator [Fig. 1.15 (*b*)], L_s is unaffected by change in rotor position ; L_r can be expressed in terms of the maximum inductance L_d along the pole-axis or the direct-axis at θ = zero, and the minimum inductance L_q along the interpolar or the quadrature axis at $\theta = 90^\circ$, as,

$$L_r = \frac{1}{2}(L_d + L_q) + \frac{1}{2}(L_d - L_q) \cos 2\theta \qquad ...(1.20b)$$

$$L_r = L_s + L_D \cos 2\theta$$

where, $L_s = 1/2\ (L_d + L_q)$; $L_D = 1/2\ (L_d - L_q)$

Case II. If saliency is only on the rotor (Fig. 1.15 *c*),

$$L_s = \frac{1}{2}(L_d + L_q) + \frac{1}{2}(L_d - L_q) \cos 2\theta \qquad ...(1.20c)$$

and L_r is appreciably constant.

(*e*) In a non-salient pole machine [Fig. 1.15 (*d*)], L_s and L_r are both unaffected by rotor position.]

Example 1.4. *With reference to Fig. 1.15 (a), the following data are given :*

$$i_s = 1 \text{ amp}; \quad i_r = 0.5 \text{ amp};$$

$$N_s = 100 \text{ turns}; \quad N_r = 150 \text{ turns}.$$

The effective reluctance of the flux-path is such that an mmf of 100 ampere-turns produces a self-flux of 0.5 mWb.

Assuming a coupling coefficient (defined as the ratio of the stator flux linking rotor coil to the stator self-flux, and similarly as the ratio of rotor quantities), of 0.8 between the stator and rotor coils, and neglecting saturation and other non-linearities, compute,

(*i*) L_s, L_r and L_M;

(*ii*) *the total stored magnetic field energy.*

*Jones, C.V. : Unified Theory of Electrical Machines (Butterworth), 1967, pp. 15.

Solution.

From the given data, stator self-flux linkage,

$$\Psi_{ss} = 100 \times 0.5 = 50 \text{ mWb–turns.}$$

Again, Rotor coil ampere-turns

$$= 0.5 \times 150 = 75,$$

producing a rotor self-flux

$$(\Phi_{ir} + \Phi_{sr}) = \frac{75}{100} \times 0.5 = 0.375 \text{ mWb.}$$

or rotor self-flux linkage

$$\Psi_{rr} = 0.375 \times 150 = 56.25 \text{ mWb–turns.}$$

Since, the coupling coefficient

$$= 0.8$$

$$\frac{\Phi_{rs}}{\Phi_{ls} + \Phi_{rs}} = 0.8 \text{ and also } \frac{\Phi_{sr}}{\Phi_{lr} + \Phi_{sr}} = 0.8.$$

That is, $\Phi_{rs} = 0.8 \times 0.5 = 0.4$ mWb

and $\Phi_{sr} = 0.8 \times 0.375 = 0.3$ mWb

or $\Psi_{rs} = N_r \,.\, \Phi_{rs} = 150 \times 0.4 = 60$ mWb–turns;

and $\Psi_{sr} = N_s \,.\, \Phi_{sr} = 100 \times 0.3 = 30$ mWb–turns.

(*i*) $$L_{sr} = \frac{\Psi_{sr}}{i_r} = \frac{30}{0.5} = \mathbf{60 \text{ mH}}$$

$$L_{rs} = \frac{\Psi_{rs}}{i_s} = \frac{60}{1} = \mathbf{60 \text{ mH.}}$$

[**Note.** $L_{sr} = L_{rs} = L_M$ as expected since non-linearities are neglected.]

$$L_s = \frac{\Psi_{ss}}{i_s} = \frac{50}{1} = \mathbf{50 \text{ mH}}$$

$$L_r = \frac{\Psi_{rr}}{i_r} = \frac{56.25}{0.5} = \mathbf{112.5 \text{ mH.}}$$

(*ii*) Total stored magnetic field energy,

$$W_f = \frac{1}{2}\Psi_s.i_s + \frac{1}{2}\Psi_r.i_r$$

$$= \frac{1}{2}(\Psi_{ss} + \Psi_{sr})\, i_s + \frac{1}{2}(\Psi_{rr} + W_{rs}) \,.\, i_r$$

$$= \frac{1}{2}(50 + 30) \times 1 + \frac{1}{2}(56.25 + 60) \times 0.5$$

$= 69.06$ mJ

Check.

$$W_f = 1/2 \begin{bmatrix} 1 & 0.5 \end{bmatrix} \cdot \begin{bmatrix} 50 & 60 \\ 60 & 112.5 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

$= 69.06$ mJ.

Example 1.5. *Consider the doubly-excited machine in Fig. 1.14 (b) having two rotor windings 'a' and 'b' in quadrature as indicated in Fig. 1.16. Obtain expressions for complete self inductances L_{aa}, and L_{bb}, mutual inductances L_{af}, L_{bf}, and L_{ab}.*

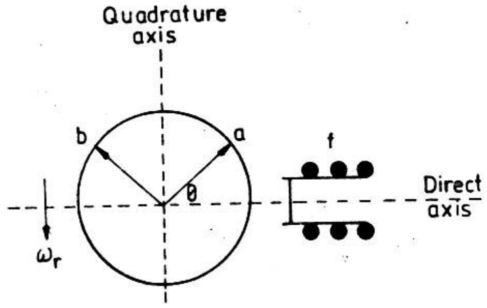


Fig. 1.16 Pertaining Example 1.5.

Solution. Reference equation (1.20 *b*), $L_{aa} = L_S + L_D \cos 2\theta$; and putting $\theta = -(90^\circ - \theta)$ in the same equation, $L_{bb} = L_S - L_D \cos 2\theta$

Again, from equation (1.20*a*), $L_{af} = L_m \cos \theta$; and putting $\theta = (90^o + \theta)$ in the same equation, $L_{bf} = -L_m \sin \theta$

Mutual inductance between '*a*' and '*b*' windings on the rotor : Occurs due the saliency on the stator.

When any of the two windings is along or opposite to the direct axis, *i.e.,* at $\theta = 0^\circ, 90^\circ, 180^\circ$, ..., L_{ab} = zero.

Hence, L_{ab} should be represented by a *sine series.* That is,

$$L_{ab} = B \sin \theta + C \sin 2\theta + ...$$

But, since, L_{ab} = zero, when θ = zero, $L_{ab} = C \sin 2\,\theta$.

Again, when $0 < \theta < 90^\circ$, components of i_a and i_b along the direct axis are in the opposite directions. Hence, *C must be negative*.

That is, $L_{ab} = -C \sin 2\theta$.

For a cylindrical rotor machine, L_{ab} *is always zero leading to* $L_d = L_q$. Thus C must be numerically equal to L_D. Or,

$$L_{ab} = -L_D \sin 2\theta.$$

1.5 Mechanism of Electro-Mechanical Energy Conversion

Based on the discussion in the later part of the article 1.4, the following picture (Fig. 1.17) with respect to the mechanism of electro-mechanical energy conversion may be developed.

A singly excited machine is considered which is shown as a link between the electrical and mechanical systems.

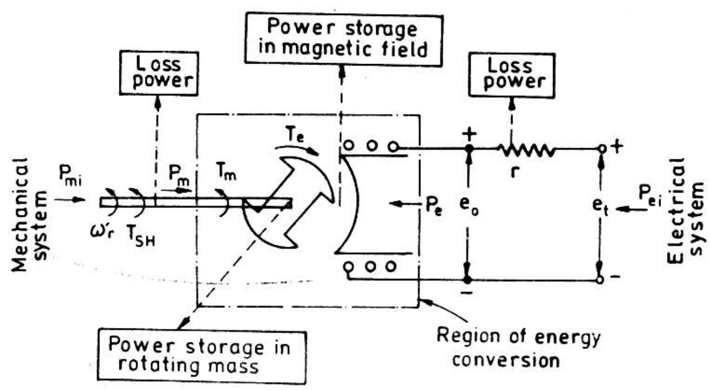


Fig. 1.17 Electromechanical Energy Conversion.

The instantaneous electrical power input, $P_{ei} = e_{t.i}$; and the instantaneous mechanical power input, $P_{mi} = T_{sh} \cdot \omega'_r$ at the machine shaft.

The developed electromagnetic torque T_e opposes the mechanical torque T_m, whereas the induced voltage in the winding e_0 opposes the input voltage e_t.

As a generator, a part of the input mechanical power is expended as mechanical losses and the remaining power P_m is available for conversion to an equivalent electrical power P_e. The induced voltage e_0 in the winding exceeds the terminal voltage e_t and the current reverses so as to provide *positive electrical power output.*

In the motor mode of operation, a part of the input electrical power P_{ei} is dissipated as heat and the remaining P_e is converted into mechanical power. The mechanical power input P_m is negative (thus providing positive output power). The power at the shaft is obtained from the above output (normally termed as the *internal mechanical power*) after subtracting the mechanical loss power.

In the region of energy conversion, there are storages of power —in the magnetic field, and in the rotating mass. Under steady operating condition, the time rate of change of stored magnetic field energy as well as that of the stored mechanical energy are zero, and equation (1.10) yields,

$$P_e + P_m = 0.$$

But, $$P_e = P_{ei} - i^2 . r = e_{t.} \, i - i^2 . r = e_0 . i$$

$$= i . \frac{d\Psi}{dt}.$$

Hence, $$P_m = -P_e = -i . \frac{d\Psi}{dt} \qquad ...(1.21)$$

Considering an elementary coil moving through $d\theta'$ mechanical radians in time dt second, the angular speed of rotation of the elementary coil,

$$\omega'_r = \frac{d\theta'}{dt} \text{ mechanical radian per second} \qquad ...(1.22)$$

That is, the incremental mechanical torque associated with the elementary coil,

$$dT_m = \frac{P_m}{\omega'_r} = \frac{1}{\omega'_r}\left(-i . \frac{d\Psi}{d\theta'}\right)\frac{d\theta'}{dt} = -i . \frac{d\Psi}{d\theta'}$$

$$= -\frac{p}{2} . i . \frac{d\Psi}{d\theta} \qquad ...(1.23)$$

where, θ is in electrical radians. The factor $(p/2)$ converts mechanical radians into its equivalent electrical radians, so that $\theta = (p/2).\theta'$. To obtain the total internal mechanical torque, the summation of all increments, as given by equation (1.23), over a pole-pitch π electrical radians for all the p-poles is to be made. That is,

$$T_m = \frac{p}{2}\int_0^{\pi} dT_m \qquad ...(1.24)$$

[**Note.** (*a*) A feature of considerable interest in the analysis shown above is that the instantaneous torque is dependent on the variation of flux-linkage with the instantaneous rotor position, whereas the instantaneous voltage induced in the rotor coil is given by the time-rate of change of flux-linkage.

(*b*) In an electro-mechanical energy converter, the generator and motor modes of operation occur side by side. In the generator mode, the rotor conductors rotate in a magnetic field developed by the stator excitation and a voltage is induced in the rotor winding. If the rotor winding is closed, a current flows in it and an electromagnetic torque is developed which opposes the mechanical torque at the generator shaft.

Similarly, in the motor mode, the current carrying rotor conductors when placed in a magnetic field develops an electromagnetic torque. When the developed torque is larger than the opposing torques such as static friction, the rotor rotates. A 'counter voltage' is induced in the rotor winding as it rotates in the magnetic field, the voltage being in the opposite sense to the applied voltage.

Defining the incremental electro-magnetic torque as,

$$dT_e = \frac{P_e}{\omega'_r} \qquad ...(1.25)$$

we have $$dT_e = \frac{p}{2} \cdot i \cdot \frac{d\Psi}{d\theta} = -\, dT_m \qquad ...(1.26)$$

(*c*) It is advantageous to represent the average torque, under linear condition, in terms of inductance parameters. Such representation enables magnetic effects to be stated in terms of voltage and current in the associated electric circuit. Inclusion of equivalent resistance parameter r so as to represent the loss power i^2r leads to the electric circuit approach to the analysis of electro-magnetic machines. Further, inductance parameters are generally directly measurable or can be calculated from the flux-plots with sufficient accuracy.

Considering that the magnetic field is due to an exciting current i_s in the stator winding, we have,

$$\Psi = L_M . i_s \qquad ...(1.27)$$

Noting that under linear condition, L_M is independent of current i_s and is a function of the space-angle θ,

$$dT_e = \frac{p}{2} . i_s . \, i_r \, . \frac{dL_M}{d\theta} \qquad ...(1.28)$$

where, i_r is the current in the elementary coil.

(*d*) Equations (1.23) and (1.28) give the incremental torque for a non-salient pole machine and does not include the component due to the reluctance effect as present in salient-pole machines.

(*e*) The energy approach as discussed leads to **Unified Theory** of electrical machines.

Quite often, a **force - on - current formula,**

$$f = B.i.L$$

is used for the determination of torque in an electrical machine. It must be appreciated that this formula is not a generalised one and is not applicable to even simple electromagnetic devices such as, the coil and plunger device. Energy principle on the other hand, is universal and valid for all electro-mechanical devices. However, if we express force as the space derivative of stored magnetic field energy, **the force-on-current formula can be shown to be a special case of the unified approach.**

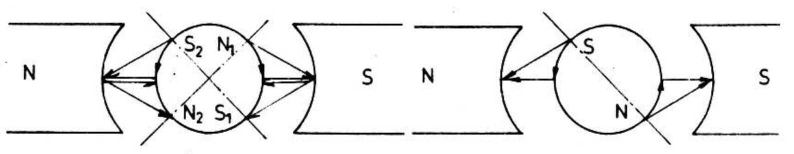
Example 1.6. *Show that for the production of uni-directional electromagnetic torque, the number of stator poles must be equal to the number of rotor poles.*

Solution.

Fig. 1.18 (*a*) illustrates a two-pole stator with a four-pole rotor, the axes of rotor field being at an angle with the axis of the stator field.

(*a*) The force of attraction between the rotor north-pole N_1 and the stator south-pole *S,* shown by arrow from the rotor to the stator, can be resolved into radial and circumferential components as shown in the figure.

(*b*) The force of repulsion between the stator south-pole S and the rotor south-pole S_1, shown by arrow from the stator to the rotor, gives a radial component which cancels the radial component of (*a*), but the circumferential component is in the same direction as in (*a*), *i.e.* clockwise.



(*a*) No. of rotor poles unequal to the no. of stator poles : torque zero

(*b*) No. of rotor poles equation to that of stator poles : unidirectional torque results

Fig. 1.18 Production of torque.

(*c*) Similarly, resolving the forces between the stator north-pole N and the rotor north-pole N_2, and also the rotor south pole S_2, the resultant is circumferential, but in the counter clockwise direction.

Hence, no average torque results.

Following the same procedure as above with respect to Fig. 1.18 (*b*), where the number of stator and rotor poles are equal, resolution of forces of repulsion gives radial components which balance each other, but the circumferential components are in the same direction resulting in uni-directional torque.

1.6. Torque Equation—Field-energy Approach

An alternative method of evaluation of torque can be based on field-energy considerations. It has already been discussed that in the process of electro-mechanical energy conversion, the magnetic field stores energy, and the field energy can be established by one or more exciting coils and sources; and may be expressed as, Single excitation : $W_f = 1/2\, i\,.\, \psi = 1/2\, L\,.\, i^2$

In matrix form,

$$[W_f] = 1/2\, [i]\,.\,[L]\,.\,[i] \qquad ...(1.29a)$$

double excitation : $W_f = 1/2\, i_s\,.\,\psi_s + 1/2\, i_r\,.\,\psi_r$

$$= 1/2\, L_s\, i_s + 1/2\, L_r\, i_r^2 + L_M\, i_s\, i_r$$

In matrix form,

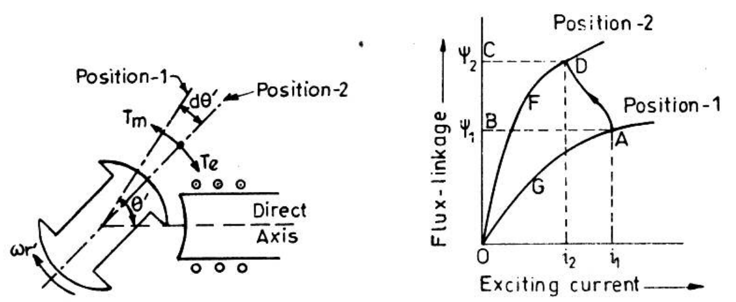
$$[W_f] = 1/2 \begin{bmatrix} i_s & i_r \end{bmatrix} . \begin{bmatrix} \psi_s \\ \psi_r \end{bmatrix}$$

$$= 1/2 \begin{bmatrix} I_S & I_r \end{bmatrix} . \begin{bmatrix} L_S & L_M \\ L_M & L_r \end{bmatrix} . \begin{bmatrix} I_S \\ I_r \end{bmatrix} \qquad ...(1.29b)$$

We consider the following two cases :

1.6.1. Singly-excited Machine

To investigate the torque production, let us consider two positions of the rotor (Fig. 1.19 a). In position 1, Ψ vs. i characteristic is given by curve *OGA,* and in position 2, the characteristic is given by the curve *OFD.* Let A be the initial point of operation. As the rotor moves through $d\theta'$ mechanical radians, the point of operation shifts from A to D (Fig. 1.19 (b)) taking an arbitrary path AD which depends on the characteristics of the magnetic circuit and the exciting coil.



(a) Singly excited machine. (b) Ψ vs. i-characteristic-general condition.

Fig. 1.19 Pertaining singly excited machine.

Following the discussion in example 1.2, the stored magnetic field energy in **position 1** = area *OABO*.

As the rotor moves to position 2, the reluctance of the flux-path decreases and the flux-linkage increases to Ψ_2. The induced voltage in the exciting coil increases and to balance it, an additional electrical energy input ΔW_e is necessary, given by,

$$\Delta W_e = \text{area } ABCD.$$

In position 2, the stored magnetic field energy = area *ODCO,* and thus, the change in the stored field energy,

$$\Delta W_f = \text{area } ODCO - \text{area } OABO.$$

But, from the energy-balance equation, assuming constant rotor speed, and negligible losses (*i.e.* W_j = zero, and $W_{le} = W_{lm}$ = zero) for simplicity,

$$\Delta W_e + \Delta W_m = \Delta W_f;$$

or, the change in the mechanical energy input,

$$\begin{aligned}\Delta W_m &= \Delta W_f - \Delta W_e \\ &= \text{area } ODCO - (\text{area } OABO + \text{area } ABCD) \\ &= -\text{ area } OGADFO.\end{aligned}$$

The electromagnetic torque during the movement of the rotor is defined as,

$$\begin{aligned}T_e &= \frac{\text{Energy to mechanical work}}{\text{Angular movement}} \\ &= \frac{-\text{ Change in mechanical energy input}}{-\,d\theta'} \\ &= \frac{-\text{ Area } OGADFO}{d\theta'} \text{ N–m.} \qquad \text{...(1.30a)}\end{aligned}$$

[**Note.** $d\theta'$ is negative since θ' decreases as $d\theta'$ increases]

Since the path AD is arbitrary, determination of area *OGADFO* cannot be made accurately. However, an estimation of the area can be made basing on the following two extreme cases. For simplicity, we assume that,

(*i*) the magnetic saturation i negligible, *i.e.* Ψ/i – characteristic is a straight line, and thus $\Psi = L.i$, where, L is a constant ;

(*ii*) all the field energy is stored in the gap, *i.e.* iron part of the magnetic circuit has infinite permeance.

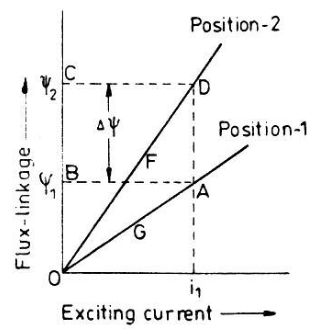


Fig. 1.19 (*c*) Ψ vs. *i*-characteristic—constant current condition.

Case I. The rotor movement is very slow so that the exciting current remains substantially constant at i_1 while the flux-linkage changes from Ψ_1 to Ψ_2. Or, in other words, the path AD is parallel to the ordinate (Fig. 1.19 (*c*)).

The additional electrical energy input,

ΔW_e= area of the rectangle *ABCD*

$= (\Psi_1 + \Delta\, \Psi - \Psi_1)\, .\, i_1$

$= i_1\, .\, \Delta\, \Psi$

Energy to mechanical work

= area *OGADFO*

= area of the triangle *OAD*

$= \frac{1}{2}$ area of the rectangle *ABCD*

$= \frac{1}{2}\, i_1\, .\, \Delta\, \Psi$

and $\Delta\, W_f$ = Area of the triangle *ODC* – Area of the triangle *OAB*

$= \frac{1}{2}\, i_1\, .\, \Delta\, \Psi$

[**Note.** The additional electrical energy input is equally divided into the stored magnetic field energy and the energy to mechanical work. This is known as "50-50 rule"].

Thus, $T_e = \frac{-\,\Delta\, W_f}{\Delta\, \theta'}$

$= -\frac{d\, W_f}{d\theta'}$

reducing $\Delta\theta'$ into the infinitesimal differential $d\theta'$. ...(1.30 (*b*))

$$= -\frac{1}{2}\, .\, i_1\, .\, \frac{d\Psi}{d\theta} \quad \text{...(1.30 (c))}$$

Case II. The rotor movement is instantaneous from position 1 to position 2 so that the flux-linkage remains constant*.

Since the effective reluctance in position 2 is less than that in position 1, the current changes from i_1 to $(i_1 + \Delta\, i)$; or, in other words the path. *AD* is parallel to the abscissa (Fig. 1.19 (*d*)).

There is no additional electrical energy input during the rotor movement as in case I, since with constant flux-linkage the induced voltage is zero.

Thus, $\Delta\, W_e$ = zero ;

$\Delta\, W_f$ = Area of triangle *ODB* – Area of triangle *OAB*

*This is known as 'Constant Flux—linkage Theorem'.

$$= \frac{1}{2} \Psi_1 . \Delta i$$

[**Note.** Δi is negative, and hence, W_f decreases with increase in Δi]

That is, energy to mechanical work

$$= -\Delta W_m$$
$$= \Delta W_c - \Delta W_f$$
$$= -\Delta W_f$$
$$= -\frac{1}{2} \Psi_1 . \Delta i$$

[**Note.** The energy to mechanical work is obtained from an *equal* reduction in the stored magnetic field energy].

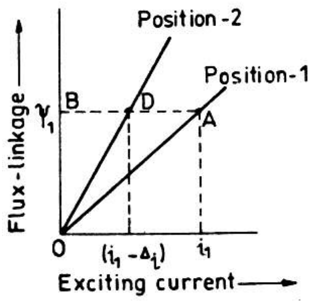


Fig. 19 (*d*) Ψ vs. *i*-characteristics—Constant flux condi-

The average electromagnetic torque

$$T_e = -\frac{\Delta W_f}{\Delta \theta'}$$
$$= \frac{1}{2} . \Psi_1 . \frac{di}{d\theta'} \quad \text{...(1.31)}$$

In the system of Fig. 1.19 (*a*) both Ψ and i vary with the movement of the rotor and neither of the above two cases apply consistently. Area *OGADFO* in equation (1.30 (*a*)) can be determined by graphical method after the path *AD* is found out from machine configuration and parameters. However, the energy to mechanical work will correspond to the magnitude of the stored magnetic field energy, and thus, the torque under the general condition can be written as,

$$T_e = \frac{dW_f}{d\theta'} \text{ N–m.} \quad \text{...(1.32}a\text{)}$$

From equation (1.14),

$$W_f = \frac{1}{2} i . \Psi$$
$$= \frac{1}{2} L . i^2,$$

L being independent of i under linear condition.

Hence,
$$T_e = \frac{1}{2} . i . \frac{d\Psi}{d\theta'} \quad \text{...(1.32}b\text{)}$$
$$= \frac{1}{2} . i^2 . \frac{dL}{d\theta'} \text{ N—m.} \quad \text{...(1.32}c\text{)}$$

For a p-pole machine, considering it as an assembly of successive 2-pole machines,

$$T_e = \frac{1}{2} \cdot \frac{p}{2} \cdot i^2 \frac{dL}{d\theta} \text{ N–m.} \qquad \text{...(1.32}d\text{)}$$

where, θ is in electrical radian, and

$$\theta = \frac{p}{2} \cdot \theta'$$

[**Note.** (*a*) It is of considerable interest that the average torque during the movement of the rotor in an electrical machine can be directly related to the stored magnetic field energy. This leads to an advantage that the average torque can be expressed in terms of the inductance parameters of the machine.

(*b*) Equations (1.32) indicate that the electromagnetic torque T_e acts in a direction from position 1 to position 2, that is, to reduce the space angle θ′. In other words, it will tend to align the rotor along the position of minimum reluctance.

(*c*) An expression for the inductance can be determined as below :

$$L = \frac{\Psi}{i} = N \cdot \frac{\Phi}{i} = \frac{N}{i} \cdot \frac{M}{R} = \frac{N}{i} \cdot \frac{N.i}{R}$$
$$= \frac{N^2}{R} \qquad \text{...(1.33)}$$

The effective reluctance R is a function of the space angle θ′. Assuming infinite permeance of the core material, the reluctance is due to the two gaps, and can be expressed as,

$$R = \frac{2g}{\mu_0 A} \qquad \text{...(1.34)}$$

where, g is the gap length ; $\mu_0 = 4\pi \cdot 10^{-7} = 0.125 \times 10^{-5}$; and A is the effective gap area through which the flux passes. Noting that, A is a function of the space angle θ being,

maximum, when $\theta = 0, \ \pi, \ 2\pi....,$

and minimum, when $\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} ...,$

we have,
$$L = \frac{N^2 \mu_0 \cdot A}{2g}$$
$$= 0.625 \frac{N^2 \cdot A}{g} 10^{-6} \qquad \text{...(1.35)}$$

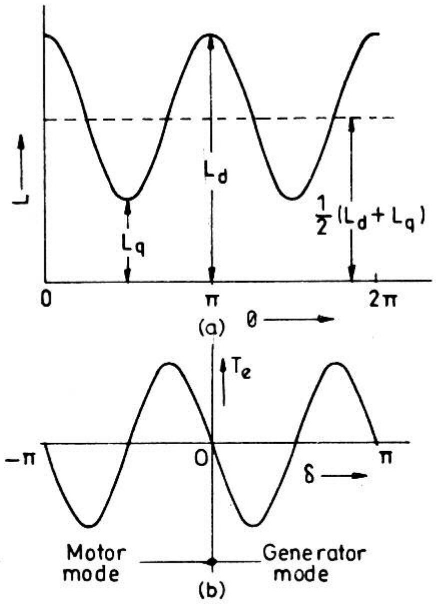
is maximum at $\theta = 0, \pi, 2\pi,....,$ and minimum at $\theta = \frac{\pi}{2}, \frac{3\pi}{2},....$

Since, L is always positive, it can be expressed as,

$$L = \frac{1}{2}(L_d + L_q) + \frac{1}{2}(L_d - L_q)\cos 2\theta \qquad ...(1.36a)$$

$$= L_S + L_D \cos 2\theta \qquad ...(1.36b)$$

where, $\quad L_S = 1/2\,(L_d + L_q)\,;\; L_D = 1/2\,(L_d - L_q)$



(*a*) Inductance characteristic
(*b*) Torque characteristic.
Fig. 1.20 Single Excited Machine.

assuming sinusoidal variation [Fig. 1.20 (*a*)].

The electromagnetic torque in a *p*-pole machine,

$$T_e = 1/2\,.\,\frac{p}{2}\,i^2\,\frac{d}{d\theta}\,[L_S + L_D \cos 2\theta]$$

$$= -\frac{p}{2}.\,i^2\,L_D \sin 2\theta \qquad ...(1.37)$$

(*a*) With steady exciting current *I*,

$$T_e = -\frac{p}{2}\,I^2\,.\,L_D \sin 2\theta \qquad ...(1.38)$$

indicating that the average torque over a complete revolution is zero.

(*b*) With the rotor running at an uniform angular speed ω_r electrical radians per second, so that, $\theta = \omega_r.\,t$, and with instantaneous exciting current

$$i = I_{\max} \cos(\omega_r t + \delta),$$

the instantaneous electromagnetic torque,

$$T_e = -\frac{p}{2} I_{\max}^2 L_D \cos^2(\theta + \delta).\ \sin 2\theta \qquad ...(1.39)$$

The average torque for a rotation through one pole-pitch,

$$T_e\,(av) = -\frac{1}{\pi} I_{\max}^2 L_D \int_0^{\pi} \cos^2(\theta + \delta).\ \sin 2\theta.\ d\theta$$

$$= \frac{1}{4} I_{\max}^2 L_D \sin 2\delta \text{ N.m.} \qquad ...(1.40)$$

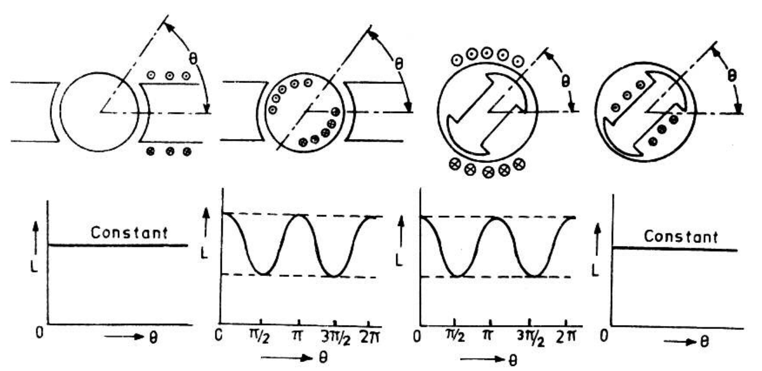
Typical variation of average electromagnetic torque with δ is shown in [Fig. 1.20 (*b*)]. Note that the torque is maximum at $\delta = \pi/4$.

(*c*) In (*b*) above, if the rotor is running at an uniform angular speed ω_r radians per second, with an instantaneous exciting current $i = I_{\max} \cos(\omega_s t + \delta)$, where, $\omega_s \neq \omega_r$, it can be shown following the procedure as above, that the average torque is zero.

Thus, for the production of reluctance torque,

(*i*) the rotor speed must be synchronous ;

(*ii*) there must be a space-rate of change of inductance, as shown below :



(*a*) winding on salient stator-pole ; rotor—cylindrical (*b*) winding on cylindrical saliency on stator (*c*) winding on cylindrical stator saliency on rotor (*d*) winding on salient rotor-pole ; stator-cylindrical

Fig. 1.21. Variation of L with θ in a Reluctance Machine.

(*a*) With the saliency only on the stator [Fig. 1.21 (*a*)], the inductance is constant with θ (neglecting the effect of slots) and no torque results.

However, in such a case, if the winding is on the rotor [Fig. 1.21 (*b*)], L varies with θ, and electromagnetic torque results.

(*b*) With saliency only on the rotor [Fig. 1.21 (*c*)], exciting winding on the stator will ensure a variation of L with θ. If the winding is on the rotor instead, L is independent of θ [Fig. 1.21 (*d*)], and the electromagnetic torque is zero.

(*c*) In a cylindrical-rotor machine, L is independent of θ, and there is no torque due to reluctance effect.]

Example 1.7. *Fig. 1.22 shows the magnetisation curves for positions 1 and 2 of the rotor of a transducer (Fig. 1.19 (a)). Assuming the exciting current constant at 3.0 amperes during the motion of the rotor, compute.*

(*i*) *increase in the stored magnetic field energy,*

(*ii*) *the energy expended to mechanical work, as the rotor moves from position 1 to position 2.*

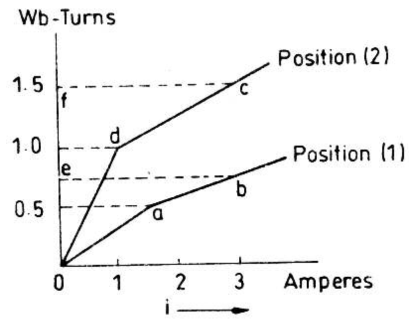


Fig. 1.22 Pertaining example 1.7.

Solution :

(*i*) The increase in the stored magnetic-field energy, ΔW_f

$= \text{area } odcfo - \text{area } oabeo$

$$= \left[\frac{1}{2}\, 1 \times 1 + \frac{1}{2}\, (1 + 3)\, 0.5\right] - \left[\frac{1}{2}\, 1.5 \times 0.5 + \frac{1}{2}\, (1.5 + 3)\, 0.25\right]$$

$= 1.50 - 0.938$

$= 0.562$ joule.

With saturation, $\Delta W_f = 0.675$ joule.

(*ii*) Energy expended to mechanical work

$= \text{area } oabcdo$

$= \text{area } oabeo + \text{area } bcfe - \text{area } odcfo$

$= 0.938 + 3 \times 0.75 - 1.50$

$= 1.688$ joules.

With saturation, energy = 1.727 joule

1.6.2. Doubly-excited Machine

Following the same procedure as in art. 1.6.1, it can be shown that equation (1.32 *a*) is equally applicable to a doubly-excited machine. Putting W_f from equation (1.19 *a*) into equation (1.32 *a*), and assuming linearity,

$$T_e = \frac{d}{d\theta'}\left[\frac{1}{2} L_s . i_s^2 + \frac{1}{2} L_r . i_r^2 + L_M . i_s . i_r\right] \text{N–m.}$$

$$= \frac{1}{2}, i_s^2 \frac{dL_s}{d\theta'} + \frac{1}{2} i_r^2 \frac{dL_r}{d\theta'} + i_s . i_r \frac{dL_M}{d\theta'} \text{ N–m} \quad ...(1.41a)$$

For a *p*-pole machine,

$$T_e = \frac{1}{2} . \frac{p}{2}\left[i_s^2 \frac{dL_s}{d\theta} + i_r^2 \frac{dL_r}{d\theta} + 2i_s . i_r \frac{dL_M}{d\theta}\right] \text{N–m.} \quad ...(1.41b)$$

As before, equation (1.41 *b*) can be represented as,

$$[T_e] = 1/2\ (p/2) \boxed{i_s \mid i_r} . \frac{d}{d\theta} \begin{array}{|c|c|} \hline L_s & L_M \\ \hline L_M & L_r \\ \hline \end{array} . \begin{array}{c} I_s \\ \hline I_r \end{array}$$

$$= 1/2\ (p/2)\ [i_t] . [G] . [i] \quad ...(1.41c)$$

where, $[G]$ is known as the **torque matrix**

$$= \frac{d}{d\theta} \begin{array}{|c|c|} \hline L_s & L_M \\ \hline L_M & L_r \\ \hline \end{array}$$

[**Note :** $(d/d\ \theta)$—operates not only on the inductance elements of *G*-matrix but also on the current-matrix. In practical electrical machines, the currents are independent of spatial position. However, if in any device, current varies with the space-angle θ, equation (1.41 *c*) would take that into account.]

Referring to Fig. 1.15, consider the following cases :

Case I. When saliency is only on the stator.

L_s is independent of θ, and using equations (1.20 *a* and *b*),

$$T_e = -\frac{p}{2} [i_r^2 L_D \sin 2\theta + i_s . i_r . L_m \sin \theta] \text{ N–m.} \quad ...(1.41c)$$

[**Note.** A normal direct current machine has saliency only on the stator, the armature being cylindrical. As will be discussed later, the axis of the armature flux is fixed by the brush position and the armature flux (and mmf) is stationary in space. L_r is thus independent of rotor position θ_r, and there is no reluctance torque in the machine. The instantaneous torque is thus,

$$T_c = -\frac{p}{2} i_s . i_r . L_m \sin \theta \text{ N–m.} \quad ...(1.41d)$$

When the brushes are along the interpolar axis, $\theta = \frac{\pi}{2}$ electrical radians, and

$$T_c = -\frac{p}{2}, i_s . i_r . L_m \text{ N–m.} \qquad ...(1.41e)$$

Case II. When saliency is only on the rotor, L_r is independent of θ,

and $$T_e = -\frac{p}{2} \cdot \frac{1}{2} [i_s^2 (L_d - L_q) \sin 2\theta + 2\, i_s . i_r . L_m \sin \theta) \text{ N–m.} \qquad ...(1.41f)$$

Case III. In a non-salient pole machine, both L_s and L_r are independent of θ,

and $$T_e = -\frac{p}{2} . i_s . i_r . L_m \sin \theta \text{ N–m.} \qquad ...(1.41g)$$

Fig. 1.14 (*e*) shows typical torque-angle characteristics of a doubly-excited machine.

1.7. Per Unit System

It is often advantageous to express various quantities in ***per units*** rather than in actual units. Most important advantage is in design calculations, comparisons of transformers and machines of different ratings are easier.

1.7.1. Transformer

(*a*) Primary winding : 1.0 p.u. voltage may be chosen to be equal to the rated primary volts per phase (which is known as *base value of voltage*).

Similarly,

1.0 p.u. current = rated per phase primary amps (the *base value of current*).

Thus, the *base value of power* (which is automatically fixed once the base values of voltage and current are assigned) = rated primary volt per phase X rated primary amps. per phase.

1.0 p.u. resistance/inductance/impedance = rated volts per phase / rated amps per phase.

(*b*) Secondary winding : The base value of voltage is the rated *secondary* voltage per phase so that,

1.0 p.u. voltage = the rated secondary volts per phase.

Similarly, base value of secondary current = rated secondary amps. per phase.

These yield, 1.0 p.u. power = rated secondary volts per phase *X* rated secondary amps. and

1.0 p.u. resistance/inductance/impedance = rated volts per phase / rated amps per phase.

[**Note.** (*a*) If N_a and N_b are respectively the effective primary and secondary turns per phase,

1.0 p.u. secondary voltage = (N_b/N_a). 1.0 p.u. primary voltage ;

1.0 p.u. secondary current = (N_a/N_b). 1.0 p.u. primary current ;

Thus,

The base value of power on the primary side = the base value of power on the secondary side.

Also, 1.0 p.u. secondary resistance/inductance/impedance = $(N_b/N_a)^2$. 1.0 p.u. primary resistance/inductance/impedance.

(*b*) In terms of *rated power,*

The base value of power (in kW) = ***rated kVA*** *for a single phase machine, and =* ***1/3 rated kVA*** *for a three phase machine.*

This analysis holds good for a.c. rotating machines also. It is to be noted that the base value of power is not the actual kW-rating per phase of the machine. Thus, for an a.c. machine of rating 100 kVA, *single phase,* 0.8 p.f., the base value of power = 100 kW, but the rated power = 100 × 0.8 = 80 kW.

For a 100 kVA, *3-phase,* 0.8 p.f. machine,

the base value of power = 1/3 × 100 = 33.3 kW, but the rated power = 100 × 0.8 = 80 kW.

1.7.2. Rotating Machines

(*a*) Armature winding : 1.0 p.u. voltage may be chosen to be equal to the rated armature volts for d.c. machine, and rated armature volts per phase for a.c. machine. Similarly, for the current.

With the above choice of base values,

1.0 p.u. resistance = rated volts/rated amps for d.c. machine, and 1.0 p.u. resistance/inductance/impedance for a.c. machine = rated volts per phase / rated amps per phase.

(*b*) Field winding : 1.0 p.u. field current corresponds to the amount of field amperes which develops 1.0 per unit of armature voltage at rated speed on no-load, neglecting saturation.

The method for determining the per unit field current and voltage for a direct current machine is discussed in art. 15.3, and for alternating current machine, in art. 15.5.

(*c*) Speed : The base value of speed may be chosen *as the speed in electrical radians per second* (see appendix A.I). For a d.c. 2-pole

1500 r.p.m. machine, the base value $= (2/2) \times 2\pi\,(1500/60) = 314.16$ elec.rad.per sec ;

for a 4-pole, 1500 r.p.m. the base value $= (4/2) \times 2\pi(1500/60) = 628.32$ elec.rad.per sec.

For an a.c. machine, the base speed is the *synchronous speed of the equivalent two-pole machine.*

[Note : Once the base value of speed is chosen, *base values of time and angle* are automatically fixed. With the above definition of speed in electrical radians per second, the base value of time is one second, and the base value of angle is one electrical radian.]

(*c*) Torque : The base value of torque is the torque produced by base power at base speed. Thus, 1.0 p.u. torque = 1.0 p.u. power / 1.0 p.u. speed.

QUESTIONS

1.1. Define electro-mechanical energy conversion.

1.2. Why is magnetic material preferred as a medium for energy conversion ?

1.3. What are the three basic principles involved for electro-mechanical energy conversion ?

1.4. What is a reversible energy conversion process ?

1.5. Mention three ways by which the change of flux-linkage in a coil is possible.

1.6. What are the two ways of development of inter-active force ?

1.7. Write down the energy balance equation.

1.8. Define, stored magnetic field energy and stored mechanical energy.

1.9. What is reluctance torque ?

1.10. Why there is no reluctance torque developed in a direct current machine ?

1.11. Distinguish between (*i*) distributed and concentrated winding, (*ii*) singly-excited and doubly-excited machines, (*iii*) wound-rotor and squirrel-cage rotor.

1.12. Why in d.c. machine the yoke and pole-core are normally not laminated ?

1.13. Why are the stator and rotor cores of an induction motor normally laminated ?

1.14. Why is a synchronous machine (generator and motor) called 'Synchronous' ?

1.15. What is universal motor ?

1.16. When a unit is operating on motor mode, in which direction will the generator action take place ?

1.17. Enumerate the losses in electrical machines.

1.18. What is leakage flux ? Show the leakage flux-pattern in (*i*) a transformer, (*ii*) salient-pole machine. Define coupling co-efficient ?

1.19. What is magnetic saturation due to ?

1.20. Why in a non-salient-pole machine, torque is only due to mutual inductance between stator and rotor windings ?

1.21. With reference to the two-pole machine in Example 1.5, develop equations for complete self inductances L_{aa}, L_{bb}, L_{cc}, *and mutual inductances* L_{af}, L_{bf}, L_{cf}, L_{ab}, L_{bc}, L_{ca}, where there are three windings at $2\pi/3$ radians from each other on the rotor.

1.22. What is internal mechanical power in a rotating machine?

1.23. How the generator and motor action occur side by side ?

1.24. Why is it advantageous to represent torque in terms of inductance parameters ?

1.25. What is '50-50' rule ? Under what condition is this rule satisfied ?

1.26. State 'true' or 'false' :

(*a*) The rate of change in stored mechanical energy is zero when the machine operates at constant speed.

(*b*) At standstill, the stored mechanical energy in a machine is not zero.

(*c*) At any constant speed the mechanical energy stored in a rotating mass is proportional to the speed.

(*d*) The rate of change of stored magnetic field energy is zero when there is no change in magnitude and spatial distribution of flux in a machine.

(*e*) A machine with 4-pole stator and 2-pole rotor will not develop mean electromagnetic torque.

(*f*) A concentrated winding of N-turns carrying a current is on the stator or rotor of an electrical machine develops a m.m.f. of peak value equal to $N.i.$

1.27. What is the difference between gross developed torque and net output torque ?

1.28. What is the speed in **SI**-system of a 60–H_z, 12-pole synchronous motor ?

1.29. What is the advantage of using *per unit values* instead of *actual values* of parameters in the analysis of electrical machines ?

1.30. Show that for a transformer, the base value of power on the primary side equals that on the secondary side.

1.31. What is the per unit value of speed in terms of actual speed in revolutions per minute ?

1.32. State the difference between *electrical radians* and *mechanical radians* in relation to a (*a*) 2-pole machine, (b) 4-pole machine.

OBJECTIVE TYPE

1. The energy that can be stored with a magnetic field is,

(*a*) less than,
(*b*) equal to,
(*c*) larger than, that with an electrical field.

2. In an electromechanical energy converter, reluctance torque is produced when,

(*a*) the exciting winding is on the salient stator structure and the rotor is cylindrical ;
(*b*) the exciting winding is on cylindrical stator structure with saliency on the rotor,
(*c*) the exciting winding is on the salient rotor structure with stator cylindrical,

3. The most popular use of induction generator is for generation of electrical energy in a,

(*a*) steam power station, (*b*) wind power station,
(*c*) hydro power station, (*d*) nuclear power station,
(*e*) none of the above.

4. In Fig. Obj. 1.1, the Mutual and Leakage fluxes are identified as,

(*a*) 1 - armature leakage, 2 - mutual, 3 - field leakage ;
(*b*) 1 - mutual, 2 - armature leakage, 3 - field leakage ;
(*c*) 1 - field leakage, 2 - armature leakage, 3 - mutual ;

5. A 32-pole 50–*Hz* water-wheel generator has a synchronous speed.

(*a*) 750 r.p.m. (*b*) 375 r.p.m.
(*c*) 187.5 r.p.m. (*d*) 93.75 r.p.m.

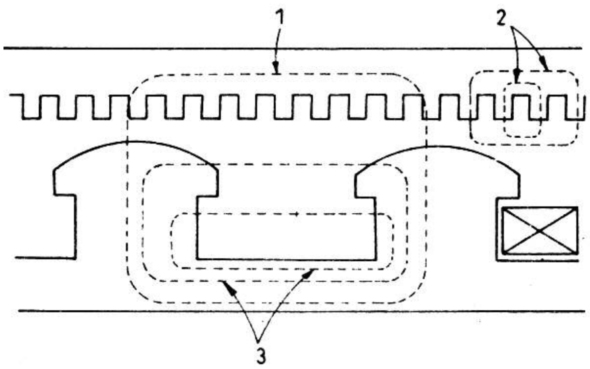


Fig. Obj. 1.1. Pertaining Question 1.1.

6. No reluctance torque is produced when,

(*a*) the excitation winding is on non-salient stator structure and rotor has saliency,

(*b*) excitation winding is on non-salient rotor with stator having saliency,

(*c*) none of the above.

7. A 6-pole synchronous generator driven at 250 r.p.m. will generate voltage at a frequency,

(*a*) 25 Hz. ; (*b*) 12 1/2 Hz. ;
(*c*) 10 Hz. ; (*d*) none of these.

8. The speed in mechanical radians per second of a 4-pole machine rotating at 1500 r.p.m. is,

(*a*) 100 π ; (*b*) 50 π ;
(*c*) 25 π ; (*d*) 12.5 π ;

9. The speed in electrical radians per second of a 4-pole machine rotating at 1500 r.p.m. is,

(*a*) 100 π ; (*b*) 50 π ;
(*c*) 25 π ; (*d*) 12.5 π.

10. The mechanical power developed by a 4-pole rotating electrical machine at 10 r.p.s. and 0.10 N.m. mechanical torque is,

(*a*) 1/2 π watt ; (*b*) π watt ;
(*c*) 2 π watt ; (*d*) none of these.

11. A 32-pole direct current motor with 100 watts mechanical power at a speed 25 r.p.s. develops an electromagnetic torque (in N.m.) of,

(*a*) $32 / \pi$; (*b*) $2 / \pi$;
(*c*) $\pi / 8$; (*d*) none of these.

12. Typical field form due to excitation of a salient pole field is,

(*a*) rectangular ; (*b*) sinusoidal ;
(*c*) trapezoidal ; (*d*) triangular.

13. A coil of N turns carrying a current i develops a rectangular mmf. of peak magnitude

(*a*) 1/2 n i ; (*b*) n. i ;
(*c*) none of above.

14. Stored magnetic field energy is associated with,

(*a*) conductance ; (*b*) capacitance ;
(*c*) inductance ; (*d*) none of above.

15. The mechanical energy stored in a mass of moment of inertia J, rotating at a constant speed of 50π mechanical radians per second is given by, in joules,

(*a*) 50π J ; (*b*) $25 \pi J$;
(*c*) J/25 π ; (*d*) J/50 π ;

Answers

1. (*c*) ;	2. (*b*) ;	3. (*b*) ;
4. (*b*) ;	5. (*c*) ;	6. (*c*) ;
7. (*b*) ;	8. (*b*) ;	9. (*a*) ;
10. (*b*) ;	11. (*a*) ;	12. (*c*) ;
13. (*a*) ;	14. (*c*) ;	15. (*b*).