1

Force System

1.1 INTRODUCTION

Engineering mechanics is a basic subject which describes and predicts the effect of forces on rigid bodies. "Mechanics is the branch of science which deals with the physical state of rest or motion of bodies under the action of forces".

1.2 CLASSIFICATIONS

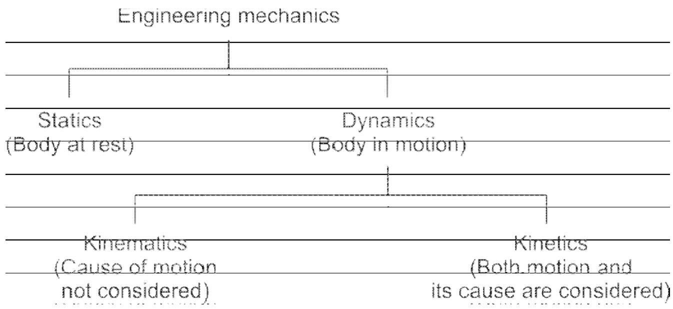
Mechanics is broadly classified into three categories:

(*a*) Mechanics of rigid body. (*b*) Mechanics of deformable body.

(*c*) Mechanics of fluids.

A rigid body is a substance which does not deformed when it is acted upon by a force system, while in case of deformable body it gets deformed before fracture when load is applied. Fluid is a substance which can not resist shear force.

Mechanics of rigid body is further classified into statics and dynamics. In statics we study the effect of forces on bodies at rest while dynamics deals with the effect of forces on bodies in motion.



Dynamics is further divided into kinematics and kinetics. Kinematics is concerned with the description of motion of objects without considering the cause of motion. In kinematics study is made of motion interrelationship among position, velocity, acceleration and time without taking into account of the forces causing motion. In kinetics both the motion and its cause are considered.

1.3 PHYSICAL QUANTITIES

A physical quantity is a physical property of a phenomenon body, or substance that can be quantified by measurement. A physical quantity can be expressed as the combination of a number, usually a real number and a unit or combination of units. Some basic physical quantities related with engineering mechanics are listed below:

1. **Time:** Time is a measure of succession of events. The unit of time is seconds.
2. **Mass:** It is a measure of inertia of body. Unit of mass if kg (kilogram).
3. **Length:** The linear extent or measurement of something from end to end. Its unit is meter.
4. **Force:** It is a physical quantity that changes or tries to change the state of rest or uniform motion of a body. Unit of force is newton N (*i.e.*, kg m/s^2).

1.4 FORCE AND SYSTEM OF FORCES

As discussed is above article force can also be defined as an external agency which produces or tends to produce, destroy or tends to destroy the motion. Force is a vector quantity defined completely by its:

- Magnitude
- Direction
- Point of application
- Line of action.

A force or system of forces when acting on a body may:

- Change its state of rest or motion
- Accelerate or retard its motion
- Change its shape or size
- Turn or rotate it, and
- Keep it in equilibrium.

When a body is acted upon by two or more number of forces simultaneously, it will form a system of forces. Considering the plane in which forces are applied and depending upon the position of line of action, system of forces may be categorized as:

1. Coplanar forces
2. Non-coplanar forces
3. Collinear forces
4. Non-collinear forces
5. Concurrent forces
6. Non-concurrent forces
7. Parallel forces.

1.4.1 Coplanar Forces

When all the forces acting on a body lie in a same plane, it is termed as coplanar force system. It is a two dimensional force system.

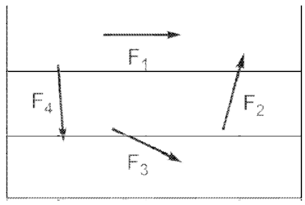


Fig. 1.1. Coplanar forces

1.4.2 Non-coplanar Forces

When all the forces acting on a particular body can not pass through a single plane; it is called non-coplanar forces. It is a three dimensional force system.

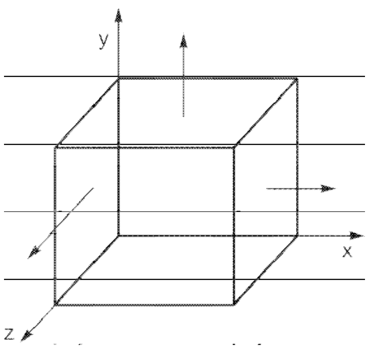


Fig. 1.2. Non-coplanar forces.

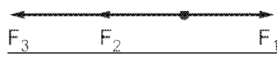


Fig. 1.3. Collinear forces.

1.4.3 Collinear Forces

In this type of force system the line of action of all forces lie along the same straight line.

1.4.4 Non-collinear Forces

If line of action of all the forces are not passing through a single straight line, it is called non-collinear force system.

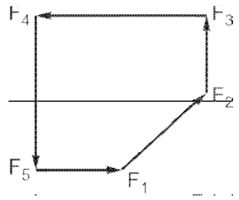


Fig. 1.4. Non-collinear forces.

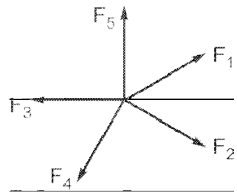


Fig. 1.5. Concurrent forces.

1.4.5 Concurrent Forces

If line of action of all the forces are passing through a single common point, it is called as concurrent force system. Point of intersection of all the forces is termed as point of concurrency.

1.4.6 Non-concurrent Forces

Forces are not passing through a common point in non-concurrent force system. Fig 1.5 shows a non-concurrent force system.

1.4.7 Parallel Force System

If line of action of all the forces are parallel to each other, it is called parallel force system. Depending on the direction of forces it may be

(*i*) **Like parallel forces:** Forces are parallel and in same direction.

(*ii*) **Unlike parallel forces:** Forces are parallel but in opposite direction.

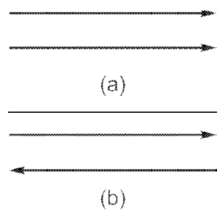


Fig. 1.6. Parallel forces.

1.5 BASIC PRINCIPLES

Some fundamental principle commonly used in mechanics are described in this article.

1.5.1 Newton's First Law

It states that every body tries to be in its state of rest or of uniform motion along a straight line unless it is acted upon by an external force. This law is also known as law of inertia.

1.5.2 Newton's Second Law

It states that the rate of change of momentum of a body is directly proportional to the force acting on it and takes place in the direction of the force.

1.5.3 Newton's Third Law

It states that every reaction has equal and opposite reaction.

1.5.4 Newton's Law of Gravitation

According to this every body in the universe attracts other body with a force whose magnitude is directly proportional to the product of the two masses of bodies and inversely proportional to square of the distance between them.

1.5.5 Principle of Transmissibility

According to this principle when the point of application of force acting on a body is shifted to any other point on the line of action of the force without changing its direction, there will be no change in the equilibrium state of the body. This implies that a force acting at any point on a body may also be considered to act at any other point along its line of action without changing its effect on the body.

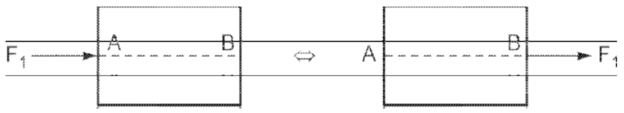


Fig. 1.7

1.5.6 Parallelogram Law of Forces

This law is used to find out the resultant of two forces acting at a point of a rigid body. It states that "If two forces acting simultaneously on a rigid body, can be represented in magnitude and direction by two adjacent sides of a parallelogram then their resultant may be represented in magnitude and direction by the diagonal of the parallelogram, passing through their point of intersection".

Let P and R are the two force acting at a point A. Parallelogram $ABCD$, as shown in Fig. 1.8, is completed using dotted lines. Drop a perpendicular from C to meet the extended line AB at point E.

Considering ΔACE, we can write

$$AC^2 = CE^2 + AE^2$$
$$AC^2 = CE^2 + (AB + BE)^2$$
$$R^2 = Q^2 \sin^2\theta + (P + Q \cos\theta)^2$$
$$= Q^2 \sin^2\theta + P^2 + Q^2 \cos^2\theta + 2PQ \cos\theta$$
$$= P^2 + Q^2 (\sin^2\theta + \cos^2\theta) + 2PQ \cos\theta$$
$$= P^2 + Q^2 + 2PQ \cos\theta.$$

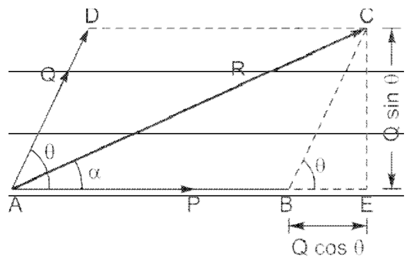


Fig. 1.8

or $$R = \sqrt{P^2 + Q^2 + 2PQ \cos\theta}$$

Angle of resultant R from force P is

$$\alpha = \tan^{-1} \frac{Q \sin\theta}{P + Q \cos\theta}$$

1.5.7 Triangular Law of Forces

It states that "If two forces acting simultaneously on a body are represented by the sides of a triangle taken in order, then their resultant is represented by the closing side of the triangle taken in opposite order."

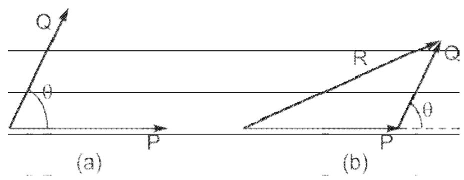


Fig. 1.9

1.6 EQUILIBRIUM, RESULTANT AND EQUILIBRANT

When two or more than two forces acting on a body in such a way that body remains in a state of rest or of uniform motion (no acceleration or retardation), then the system of forces is said to be in equilibrium. Conditions of equilibrium for different system of forces are listed below:

(*a*) Coplanar forces system:

— Concurrent force: $\Sigma F_x = 0$, $\Sigma F_y = 0$ — Non concurrent forces: $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$

(*b*) Non coplanar force system:

— Concurrent forces: $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$ — Non concurrent forces: $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$, $\Sigma M = 0$

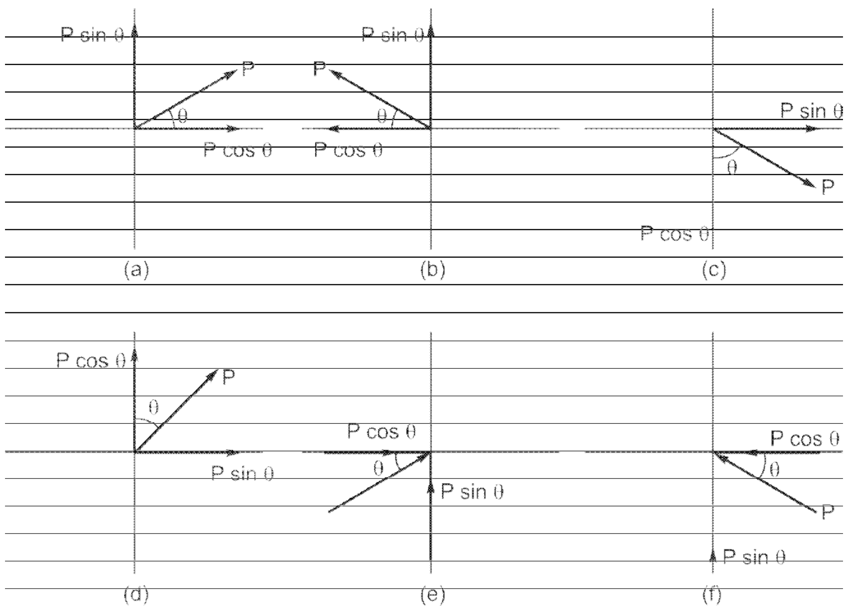
When a body is acted upon by a system of forces, then vectorial sum of all the forces is known as resultant. Hence resultant refers to a single force which produces the same effect as is done by the combined effect of several forces.

A number of forces may act on a body in such a manner, that the body is not in equilibrium. A single force which brings the body in equilibrium is known as equilibrant. Equilibrant is equal and opposite to the resultant of several forces acting on the body.

1.7 COMPOSITION AND RESOLUTION OF FORCES

Composition or compounding is the procedure to find out single resultant force of a force system, while resolution is the procedure of splitting up a single force into number of components without changing the effect of the same.

In orthogonal resolution a single force is splitted into two mutually perpendicular direction. Some examples are given below:



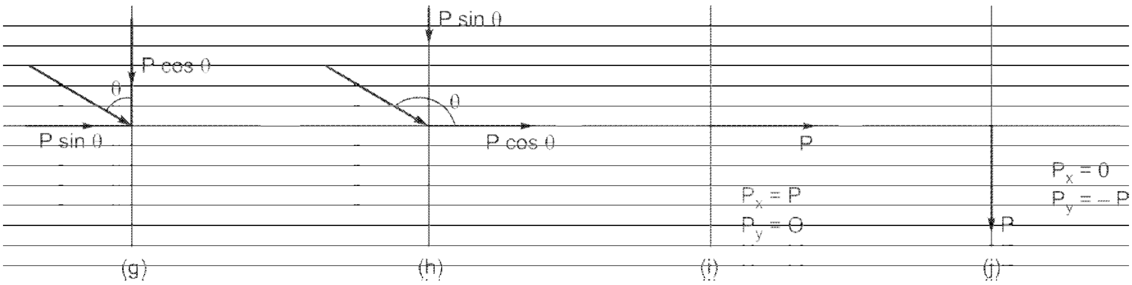


Fig. 1.10

1.8 METHODS OF COMPOSITION

For finding the resultant of forces either analytical or graphical methods can be used.

1.8.1 Analytical Method

Following steps are involved for finding the resultant for coplanar concurrent force system:

- Resolve all the forces in two mutually perpendicular directions (*i.e.* along horizontal and vertical direction).
- Take upward and rightward forces as positive and downward and leftward forces as negative.
- Find out alzebric sum of all the vertical [ΣF_y] and horizontal [ΣF_x] components.
- Calculate the magnitude of the resultant force as, $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$
- Its direction can be calculated by

$$\alpha = \tan^{-1}\left[\frac{\Sigma F_y}{\Sigma F_x}\right]$$

α is the angle of resultant from x-axis.

In case of non-concurrent forces magnitude and direction of force is not sufficient to define resultant. In this case it is required to find out the point of application of resultant force. Point of application is calculated using Verignon's theorem which will be discussed later.

1.8.2 Graphical Method

Graphical method for finding the resultant of coplanar concurrent forces is called *Law of Polygon*.

Law of polygon states that "If a number of concurrent forces acting simultaneously on a body are represented in magnitudes and direction by the sides of a polygon, taken in order, then the resultant is represented in magnitude and direction by the closing side of the polygon, taken in opposite order."

Consider forces P_1, P_2, P_3 and P_4 acting on a body at point O as shown in Fig. 1.11 (*a*). Draw a line *oa* to represent P_1, line *ab* to represent P_2, line *bc* to represent P_3 and line *cd* to represent P_4. The polygon is computed by drawing the closing line *od*. This closing line *od* represents the resultant of the given system in magnitude, line of action and direction.

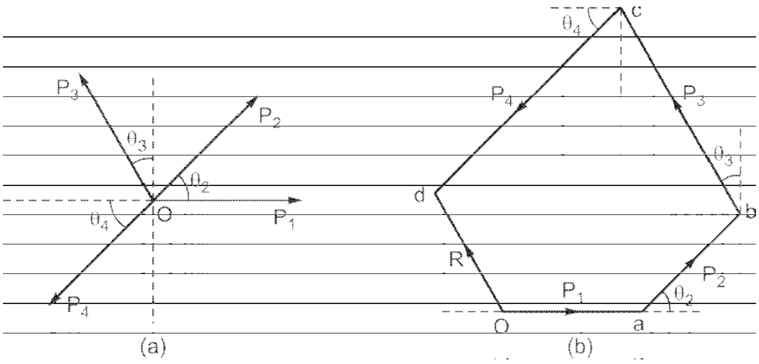


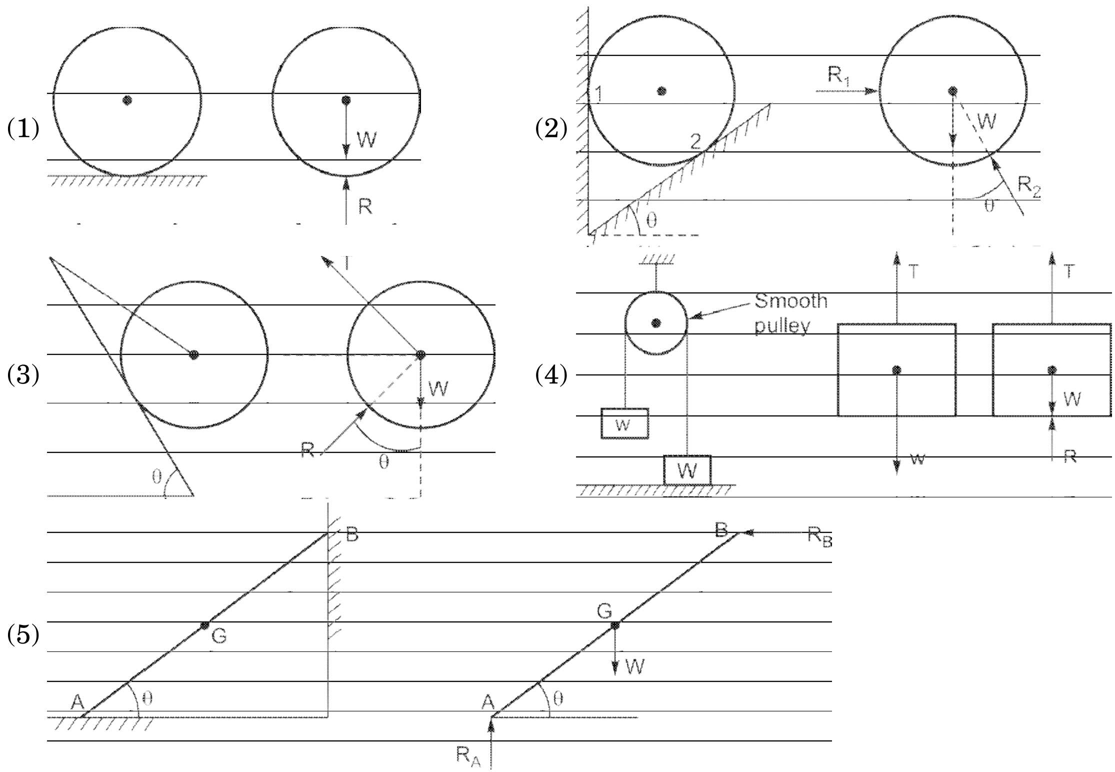
Fig. 1.11

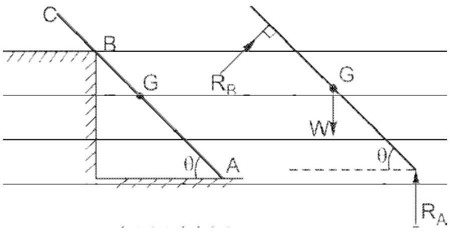
1.9 FREE BODY DIAGRAM

An isolated body separated from all other connected bodies or surface is called free body. For analysing the forces acting on a particular part of a system we need to draw *FBD* (free body diagram). For making FBD, detach the body from surrounding and draw it separately without changing its orientation. On this body following forces need to be draw according to situation:

1. Self weight, acting vertically downwards from the centre of gravity of the body.
2. Support or contact reactions, acts at the point of contact from any other surface or bodies.
3. External forces.

Some examples of *FBD* are given below:



(6) 

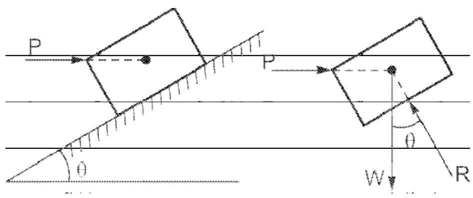
(7) 

Fig. 1.12

1.10 LAMI'S THEOREM

It states that if a body is in equilibrium under the action of three forces, which are concurrent, then each force is proportional to the 'sine' of the angle between other two forces.

If P, Q and R are the three forces acting on a body at point O. Angles between the forces are shown in Fig. 1.13. Then according to Lami's theorem,

$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma}$$

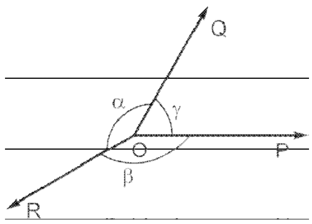


Fig. 1.13

Note: Before applying Lami's theorem, draw all the forces outward from the point of concurrency.

1.11 MOMENT OF A FORCE

It is a turning effect produced by a force on the body on which it acts. Moment of a force is equal to the product of the force and the perpendicular distance of the point, from the line of action of force, about which the moment is required.

Moments are considered as positive or negative depending upon its tendency to rotate the body in the clockwise (assuming (+) ve) or anticlockwise (assuming (–) ve) direction.

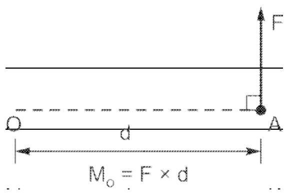


Fig. 1.14

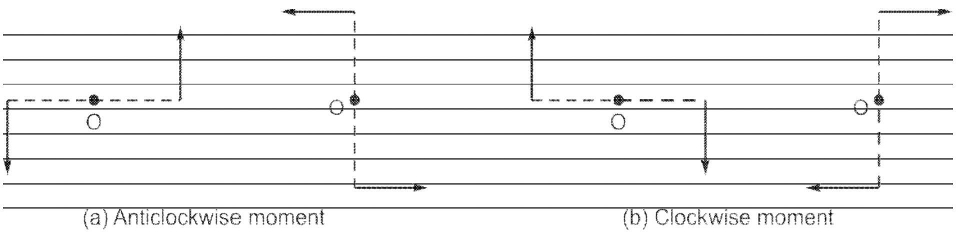


Fig. 1.15

Point about which moment is calculated is called moment centre and the perpendicular distance of the force from the point about which moment is calculated is termed as moment arm.

1.12 VERIGNON'S THEOREM

It states that the algebraic sum of moments due to all forces acting on an object about any point is equal to the moment because of their resultant about the same point.

Considering two force F_1 and F_2 whose resultant is R. Angle from y-axis and perpendicular distances from point A, about which moment have to be calculated, are shown in Fig. 1.16.

Moment due to resultant R about point A is

$$M_A = R \,.\, d_1 = R \,.\, (OA \cos\theta)$$
$$= OA\ (R \cos\theta) \qquad ...(1)$$

Moment due to force F_1 about point A is

$$M_{1A} = F_1 \,.\, d,$$
$$= F_1 \,.\, (OA \cos\theta_1)$$
$$= OA \,.\, (F_1 \cos\theta_1) \qquad ...(2)$$

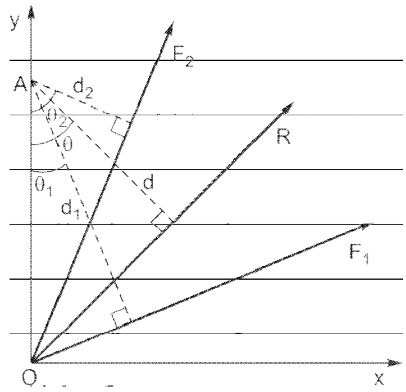


Fig. 1.16

Moment due to force F_2 about point A is

$$M_{2A} = F_2 \,.\, d_2 = F_2\ (OA \cos\theta_2)$$
$$= OA\ (F_2 \cos\theta_2) \qquad ...(3)$$

Adding equation (2) and (3)

$$M_{1A} + M_{2A} = OA\ [F_1 \cos\theta_1 + F_2 \cos\theta_2]$$

or $\qquad M_{1A} + M_{2A} = M_A \qquad$ [$\because$ $F_1 \cos\theta_1$ and $F_2 \cos\theta_2$] are the x-components of force F_1 and F_2 which must be equal to the x-component of resultant R *i.e.,* $R \cos\theta$.

1.13 COUPLE

Two unlike parallel, non-collinear forces having same magnitude form a couple.

Moment of a couple = Force × Distance between the forces, $M = P \,.\, d$.

Moment of a couple is independent of the moment centre. It can be understand by following description.

Moment about point O;

$$M_O = P \times d_1 + P \times d_2$$
$$= P\ (d_1 + d_2) = P \,.\, d$$

Moment about point A;

$$M_A = P \times d_4 - P \times d_3$$
$$= P\ (d_4 - d_3) = P \,.\, d$$

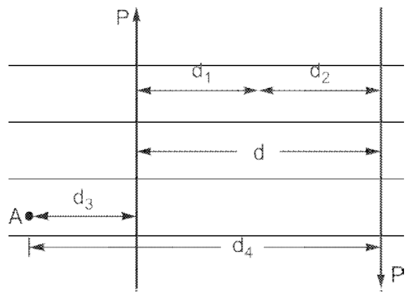


Fig. 1.17

1.13.1 Properties of a Couple

A couple has following properties:

- Two unlike parallel, non-collinear and same magnitude of forces form a couple.
- Resultant force of a couple is zero.
- A couple can not be balanced by a single force.
- The moment of couple is independent of moment centre.
- The translating effect of a couple in a body is zero.
- The effect of couple on a body remains unchanged if the couple is
  - (*i*) rotated through an angle
  - (*ii*) shifted to any other position
  - (*iii*) replaced by another pair of forces whose rotational effect is same.

1.14 EQUIVALENT FORCE COUPLE SYSTEM

A force F is applied to a rigid body at any point M can be replaced by an equal force applied at another point N together with a couple without changing the effect.

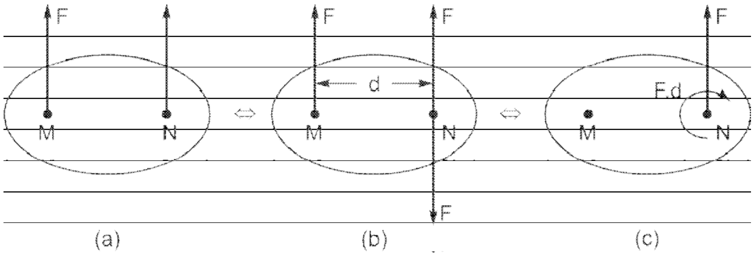


Fig. 1.18

Consider a force F acting at point M on a rigid body shown in Fig. 1.18 (a). Now apply two equal and opposite forces of magnitude same as F, at point N, Fig. 1.18 (b). Effect of forces remains same in case (a) and (b). Now two equal and opposite forces, one is at M and other is at N, will form a couple of moment $F.d.$ Therefore a force F and a couple having moment $F.d.$ is acted at point N as shown in Fig. 1.18 (c), having the same effect as it was in the previous two cases.

Example 1.1. *Two forces of equal magnitude P, act at an angle* θ *to each other. What will be their resultant?*

Solution. Using parallelogram law of forces

$$R^2 = P^2 + P^2 + 2P.\ P.\ \cos\theta. = 2P^2 + 2P^2 \cos\theta$$

$$= 4P^2 \left(\frac{1+\cos\theta}{2}\right) = 4P^2 \cos^2\frac{\theta}{2} \quad \text{or} \quad R = 2P\cos\frac{\theta}{2}$$

Example 1.2. *Resultant of two equal forces is equal to either of them. Determine the angle between the forces.*

Solution. From the solution of example 1.1

$$P = 2P\cos\frac{\theta}{2} \quad \text{or} \quad \cos\frac{\theta}{2} = \frac{1}{2},\ \frac{\theta}{2} = 60^\circ \qquad [\because\ P = Q = R]$$

$$\theta = 120^\circ$$

Example 1.3. *Two locomotives on opposite bank of a canal pull a vessel moving parallel to the banks by means of two horizontal ropes. The tensions in these ropes have been measured to be 20 kN and 24 kN while the angle between them is 60°. Find the resultant pull on the vessel and the angle between each of the ropes and the sides of the canal.*

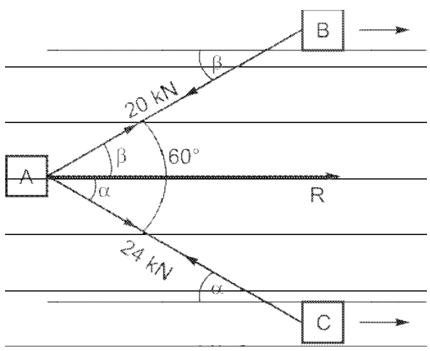


Fig. 1.19

Solution. Vessel A is attached with the locomotive B and C as shown in Fig. 1.19.

Let $P = 24$ kN

$Q = 20$ kN

$\theta = 60°$

$$R = \sqrt{24^2 + 20^2 + 2 \times 24 \times 20 \times \cos 60^\circ} = 38.16 \text{ N.}$$

Inclination of resultant R with force P = 24 kN is

$$\alpha = \tan^{-1} \frac{20 \sin 60}{24 + 20 \cos 60}$$

$$= \tan^{-1} 0.5094 = 27°$$

$$\therefore \quad \beta = 60 - 27 = 33°.$$

Example 1.4. *Determine the horizontal force and a force inclined at an angle of 60° with the vertical whose resultant equals a vertical force of 60 kN.* (See in Fig. 1.20)

Solution. Using law of parallelogram

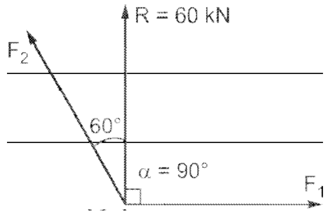


Fig. 1.20

$$P = F_1$$
$$Q = F_2$$
$$\alpha = 90°$$
$$\theta = 90 + 60 = 150°$$
$$R = 60 \text{ kN}$$

$$\therefore \quad \alpha = \tan^{-1} \frac{F_2 \sin 150}{F_1 + F_2 \cos 150}$$

or $\quad F_1 + F_2 \cos 150 = 0 \qquad [\because \quad \tan \alpha = \tan 90 = \infty]$

Upon squaring the above equation

$$F_1^2 + F_2^2 \cos^2 150 + 2F_1 F_2 \cos 150 = 0$$

$$F_1^2 + 2F_1F_2 \cos 150 = -F_2^2 \cos^2 150° \qquad ....(i)$$

But Resultant $\quad R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$, gives

$$R^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos 150$$

or $$R^2 - F_2^2 = F_1^2 + 2F_1F_2 \cos 150 \qquad ..(ii)$$

From equation (*i*) and (*ii*)

$$R^2 - F_2^2 = -F_2^2 \cos^2 150$$

$$R^2 = F_2^2 (1 - \cos^2 150) = F_2^2 \sin^2 150 = 0.25 F_2^2$$

or $$F_2 = \sqrt{\frac{R^2}{0.25}} = 2R = 2 \times 60 = 120 \text{ kN}$$

and $$F_1 = -F_2 \cos 150 = -120 \left(-\frac{\sqrt{3}}{2}\right) = 60\sqrt{3}$$

Example 1.5. *Two cables which have known tensions of 40 N and 60 N are attached to the top of a tower PQ. What tensions will be induced in the wire PR if the resultant of the forces exerted at the top P, by the cables acts vertically downwards?*

Solution. Given that the resultant of the forces at the top P, acts vertically downwards, means $\Sigma F_x = 0$, it gives

$\Rightarrow \quad 40 \cos 15 - 60 \cos 30 + T \cos \theta = 0$

$\Rightarrow \quad 40 \cos 15 - 60 \cos 30 + T \cos 56.31 = 0$

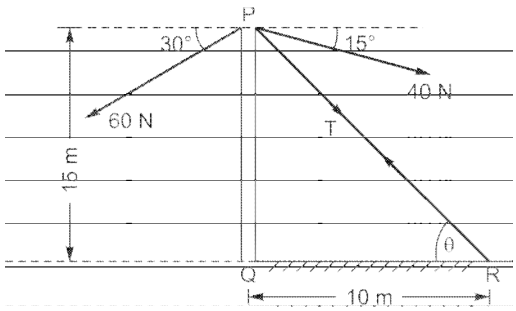


Fig. 1.21

$$T = 24 \text{ N} \qquad \theta = \tan^{-1} \frac{15}{10} = 56.31°$$

Example 1.6. *Determine the resultant of the force system shown in Fig. 1.22.*

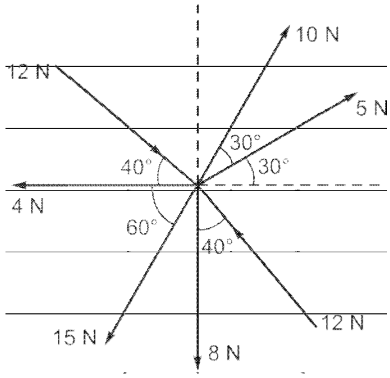
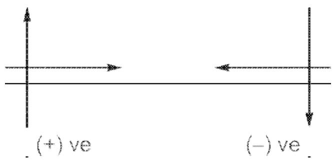


Fig. 1.22

Solution: Resolving all the forces along x and y direction and finding the algebric sum of forces along x and y direction assuming following sign convention;



$$\Sigma F_x = 5 \cos 30 + 10 \cos 60 + 12 \cos 40 - 4 - 15 \cos 60 - 12 \sin 40$$
$$= -0.69 \text{ N (leftward)}$$
$$\Sigma F_y = 5 \sin 30 + 10 \sin 60 - 12 \sin 40 - 15 \sin 60 - 8 + 12 \cos 40$$
$$= -8.35 \text{ N (downward)}$$

Magnitude of resultant $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$$= \sqrt{(0.69)^2 + (8.35)^2}$$

Direction of resultant $\alpha = \tan^{-1} \dfrac{\Sigma F_y}{\Sigma F_x} = \tan^{-1} \dfrac{8.35}{0.69}$

$$= 85.28° \text{ (From } x\text{-axis in III}^{rd} \text{ quadrant)}$$

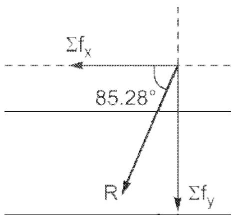


Fig. 1.23

Example 1.7. *Determine the resultant of the four forces acting on the body as shown in Fig. 1.24.*

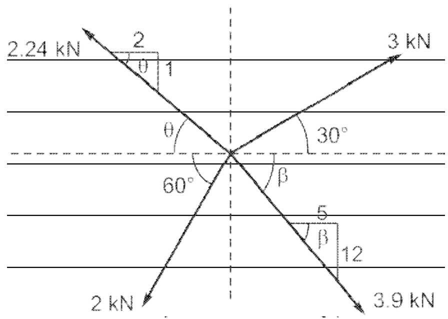


Fig. 1.24

Solution. $\tan \theta = \dfrac{1}{2}$

$$\theta = 26.56°$$

$$\tan \beta = \frac{12}{5}$$

$$\beta = 67.38°$$

Resolving all the forces along x and y directions

$$\Sigma F_x = 3 \cos 30° - 2.24 \cos 26.56° - 2 \cos 60° + 3.9 \cos 67.38° = 1.094 \text{ kN}$$

$$\Sigma F_y = 3 \sin 30° + 2.24 \sin 26.56° - 2 \sin 60° - 3.9 \sin 67.38° = -2.83 \text{ kN.}$$

$\therefore$ $R = \sqrt{(1.094)^2 + (2.83)^2} = 3.034 \text{ kN}$

$$\alpha = \tan^{-1} \frac{2.83}{1.094}$$

$$= 68.86° \text{ [From horizontal in IV}^{th} \text{ quadrant]}$$

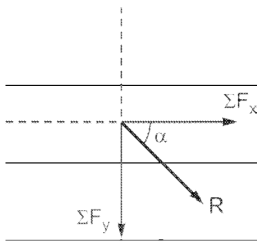


Fig. 1.25

Example 1.8. *Find the component of 1000 N force along the axis shown in Fig. 1.26.*

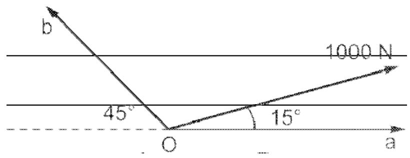


Fig. 1.26

Solution. Since force along *oa* and *ob* will be the component of 1000 N force, then these forces must be in equilibrium with equilibrant of 1000 N force.

Applying Lami's theorem

$$\frac{1000}{\sin(180-45)} = \frac{F_{oa}}{\sin(45+15)} = \frac{F_{ob}}{\sin(180-15)}$$

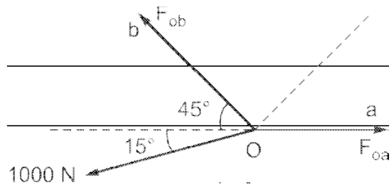


Fig. 1.27

$$F_{oa} = \frac{1000}{\sin(135)} \times \sin 60$$

$$= 1224.75 \text{ N}$$

$$F_{ob} = \frac{1000}{\sin(135)} \times \sin 165$$

$$= 366.03 \text{ N.}$$

Example 1.9. *The force F acting on the frame has a magnitude of 500 N and it is to be resolved into two components acting along strut. AB and AC. Determine the angle θ so that component F_{AC} is directed from A towards C and has a magnitude of 400 N, see Fig. 1.28.*

Solution. Considering joint *A*, applying Lami's theorem

$$\frac{500}{\sin(60)} = \frac{F_{AB}}{\sin(\theta)} = \frac{F_{AC}}{\sin(360-60-\theta)}$$

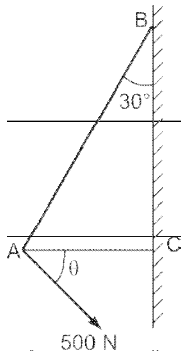


Fig. 1.28

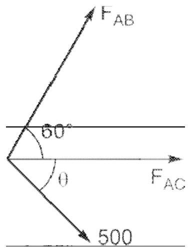


Fig. 1.29

$\because$ $F_{AC} = 400$

$\therefore$ $$\frac{500}{\sin 60} = \frac{400}{\sin(300-\theta)}$$

$\Rightarrow$ $$\sin(300-\theta) = \frac{4}{5}\sin 60$$

$\Rightarrow$ $$\sin(300-\theta) = 0.6928$$

or $$300-\theta = 43.87 \quad \text{or} \quad (180+43.87)$$

$$\theta = 256.13° \text{ or } 76.13°$$

Example 1.10: *An electric light fixture weighing 50 N hangs from point C by two strings AC and BC as shown in Fig. 1.30 (a). Determine forces in string AC and BC.*

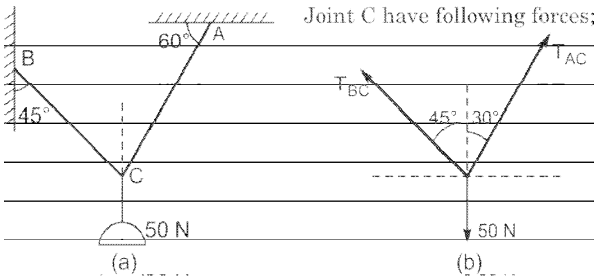


Fig. 1.30

Solution. Applying Lami's theorem by considering forces on joint C as shown in Fig. 1.30 (*b*).

$$\frac{T_{AC}}{\sin(180-45)} = \frac{T_{BC}}{\sin(180-30)} = \frac{50}{\sin(45+30)}$$

$$\Rightarrow \qquad T_{AC} = \frac{50}{\sin 75} \times \sin 135 = 36.594 \text{ N}$$

$$\Rightarrow \qquad T_{BC} = \frac{50}{\sin 75} \times \sin 150 = 25.88 \text{ N.}$$

Example 1.11. *A string ABCDE whose extremity A is fixed has weights W_1 and W_2 attached to it at B and C, and passes round a smooth peg at D carrying a weight of* 800 *N at the free end E. If in a state of equilibrium, BC is horizontal and AB and CD make angles of* 150° *and* 120° *respectively with BC, make calculations for*

(*a*) *tensions in the portions AB, BC, CD and DE.*

(*b*) *value of weight W_1 and W_2*

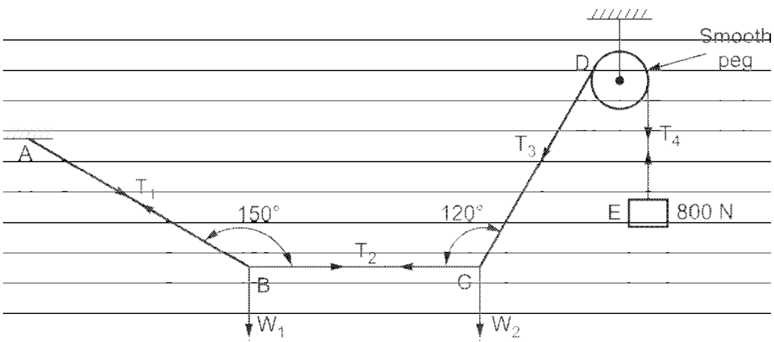


Fig. 1.31

Solution. Let T_1, T_2, T_3 and T_4 are the tensions in portion AB, BC, CD and DE respectively. At D there is a smooth peg, therefore

$$T_3 = T_4 = 800 \text{ N}$$

Applying Lami's theorem at joint C

$$\frac{T_3}{\sin(90)} = \frac{T_2}{\sin(270-120)} = \frac{W_2}{\sin(120)}$$

$$\therefore \quad T_2 = \frac{T_3}{\sin 90} \times \sin 150 = \frac{800}{1} \times \sin 150 = 400 \text{ N.}$$

$$W_2 = \frac{T_3}{\sin 90} \times \sin 120 = 800 \times \sin 120 = 692.8 \text{ N.}$$

Now applying Lami's theorem at point B

$$\frac{T_2}{\sin (270 - 150)} = \frac{T_2}{\sin (90)} = \frac{W_1}{\sin (150)}$$

$$\therefore \quad T_1 = \frac{T_2}{\sin 120} \times \sin 90 = 461.89 \text{ N}$$

$$W_1 = \frac{T_2}{\sin 120} \times \sin 150 = 230.95 \text{ N.}$$

Example 1.12. *A roller of weight 500 N rests on a smooth inclined plane and is kept free from rolling down by a string as shown in Fig. 1.32. Calculate tension in the string and reaction at the point of contact.*

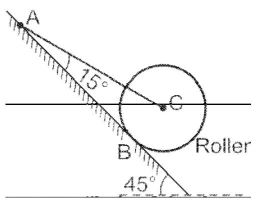


Fig. 1.32

Solution. Considering *FBD* of roller.

Applying Lami's theorem,

$$\frac{T}{\sin (180 - 45)} = \frac{R_B}{\sin (90 + 30)} = \frac{500}{\sin (90 + 45 - 30)}$$

$$\therefore \quad T = \frac{500}{\sin 105} \times \sin 135 = 365.94 \text{ N}$$

$$R_B = \frac{500}{\sin 105} \times \sin 120 = 448.24 \text{ N.}$$

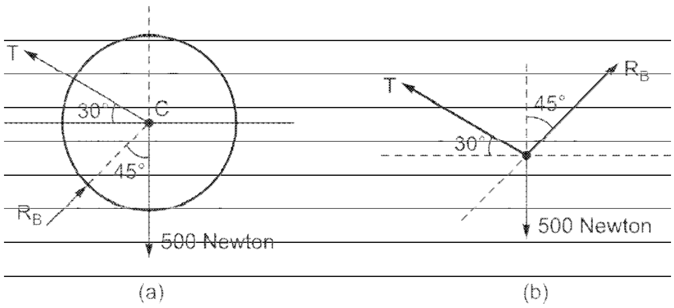


Fig. 1.33

Example 1.13. *A uniform bar of mass m = 2 kg is resting as shown in Fig. 1.34. Find angle θ for equililbrium of bar.*

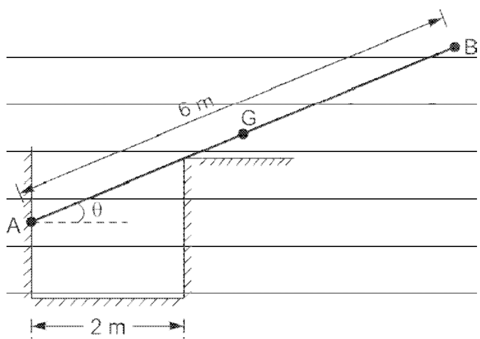


Fig. 1.34

Solution. Considering *FBD* of bar:

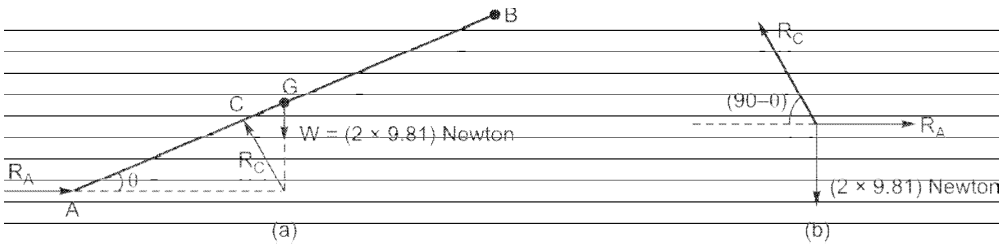


Fig. 1.35

Since bar is kept in equilibrium under the action of three non-parallel forces, they must be concurrent.

Applying Lami's theorem

$$\frac{R_A}{\sin(180-\theta)} = \frac{(2\times 9.81)}{\sin(180-90+\theta)} = \frac{R_C}{\sin 90}$$

$$\Rightarrow \quad \frac{R_A}{\sin\theta} = \frac{(2\times 9.81)}{\cos\theta} = \frac{R_C}{1}$$

or $$R_A = 2\times 9.81 \cdot \frac{\sin\theta}{\cos\theta},\ R_C = \frac{2\times 9.81}{\cos\theta}$$

Taking moment about point A

$$\Sigma M_A = 0$$

$$\Rightarrow \quad R_C \times AC - (2\times 9.81)\times \frac{6}{2}\cos\theta = 0$$

$$\Rightarrow \quad \frac{(2\times 9.81)}{\cos\theta}\cdot\frac{2}{\cos\theta} = (2\times 9.81)\times\frac{6}{2}\cos\theta \qquad \left[\because AC = \frac{2}{\cos\theta}\right]$$

$$\Rightarrow \quad \cos^3\theta = \frac{2}{3}$$

$$\theta = 29.12°.$$

Example 1.14. *Two identical spheres of weight 50 N each and radius 0.3 m are placed between two vertical walls as shown in Fig. 1.36. Find reactions at point of contact.*

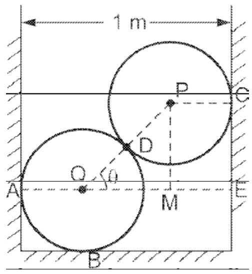


Fig. 1.36

Solution. From ΔPQM

$$\cos\theta = \frac{QM}{PQ} = \frac{1-0.3-0.3}{0.3+0.3} = \frac{0.4}{0.6} = \frac{2}{3}$$

$$\theta = 48.19°$$

Sphere P is in equilibrium under the action of three forces, so applying Lami's theorem

$$\frac{R_C}{\sin(90+48.19)} = \frac{R_D}{\sin(90)} = \frac{50}{\sin(180-48.19)}$$

$$\therefore \quad R_C = 44.72 \text{ N}$$

$$R_D = 67.08 \text{ N}.$$

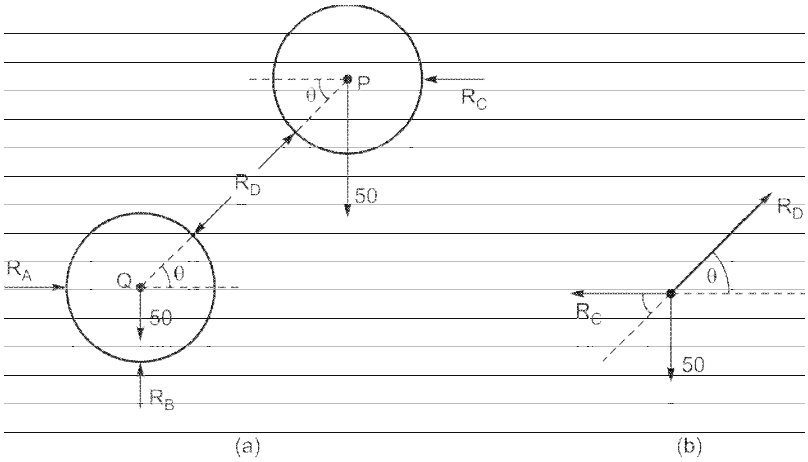


Fig. 1.37

Applying equilibrium equations for sphere Q

$\Sigma F_x = 0 \quad \Rightarrow \quad R_A - R_D \cos\theta = 0$

$\Rightarrow \quad R_A = 67.08 \cos 48.19$

$\Rightarrow \quad R_A = 44.72$ N

$\Sigma F_y = 0 \quad \Rightarrow \quad R_B - 50 - R_D \sin\theta = 0$

$\Rightarrow \quad R_B = 50 + 67.08 \sin 48.19$

$\Rightarrow \quad R_B = 100$ N.

Example 1.15. *Two spheres rests in a smooth trough as shown in Fig. 1.38. Find forces at all point of contact. Smaller sphere P has radius* 200 *mm and weight 200 N while sphere Q has radius 250 mm and weight 500 N.*

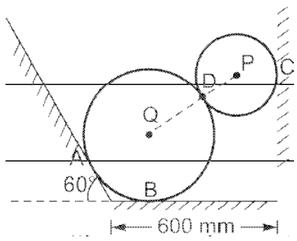


Fig. 1.38

Solution. From Fig. 1.39 (a),

$$PQ = 250 + 200 = 450 \text{ mm}$$

Let $\quad QM = x = BN$

$$EB = 600 - (200 + x) = 400 - x$$

Similar triangles, ΔQAE and ΔQBE gives

$$\angle QEA = \angle QEB = \theta = 60° \qquad [\because \quad 2\theta + 60 = 180°]$$

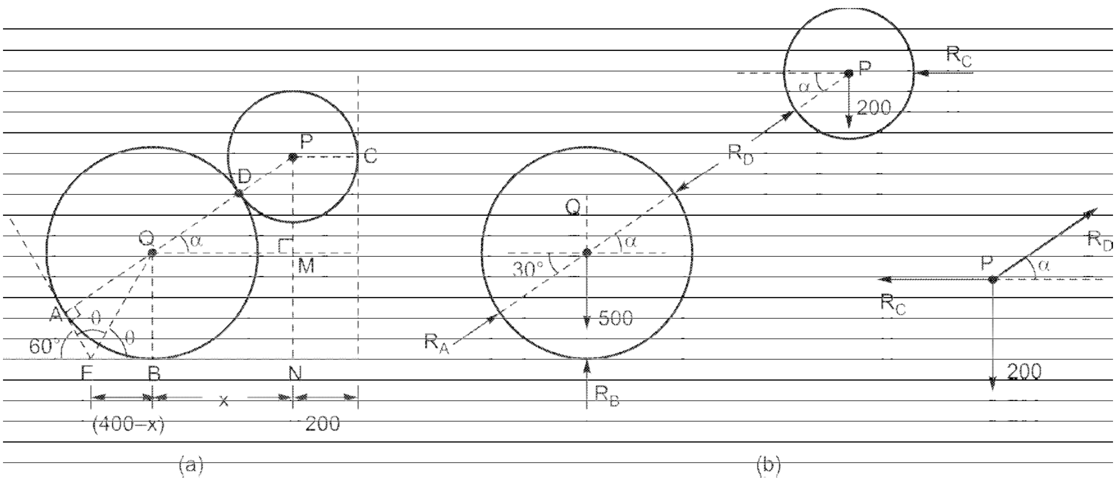


Fig. 1.39

$\therefore$ ΔQBE gives, $\tan 60 = \dfrac{250}{400 - x}$ or $x = 255.66$ mm

$\therefore$ ΔPMQ gives, $\cos \alpha = \dfrac{QM}{PQ}$ or $\cos \alpha = \dfrac{255.66}{450}$ or $\alpha = 55.38°$

Now applying Lami's theorem for sphere P [Fig. 1.39 (b)]

$$\frac{R_C}{\sin(90+\alpha)} = \frac{R_D}{\sin(90)} = \frac{200}{\sin(180-\alpha)}$$

$\Rightarrow$ $R_C = 138.07$ N

$R_D = 243.03$ N

Applying equilibrium equations for sphere Q

$\Sigma F_x = 0 \Rightarrow R_A \cos 30 - R_D \cos 55.38 = 0$

$\Rightarrow R_A = 159.43$ N

$\Sigma F_y = 0 \Rightarrow R_A \sin 30 + R_B - 500 - R_D \sin \alpha = 0$

$\Rightarrow R_B = 620.28$ N.

Example 1.16. *Two cylinders of masses 100 kg and 50 kg are connected by a rigid bar of negligible weight hinged at the centre of each cylinder. Determine magnitude of force P for equilibrium.*

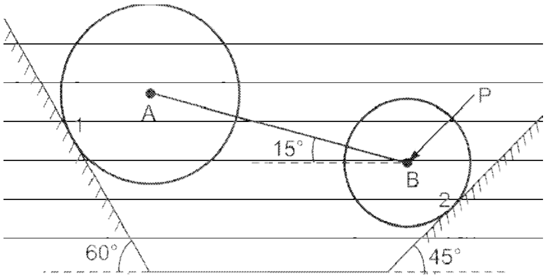


Fig. 1.40

Solution. Considering *FBD* of each cylinder

Applying Lami's theorem for cylinder A

$$\frac{R_1}{\sin(90+15)} = \frac{C}{\sin(90+13)} = \frac{981}{\sin(180-30-15)}$$

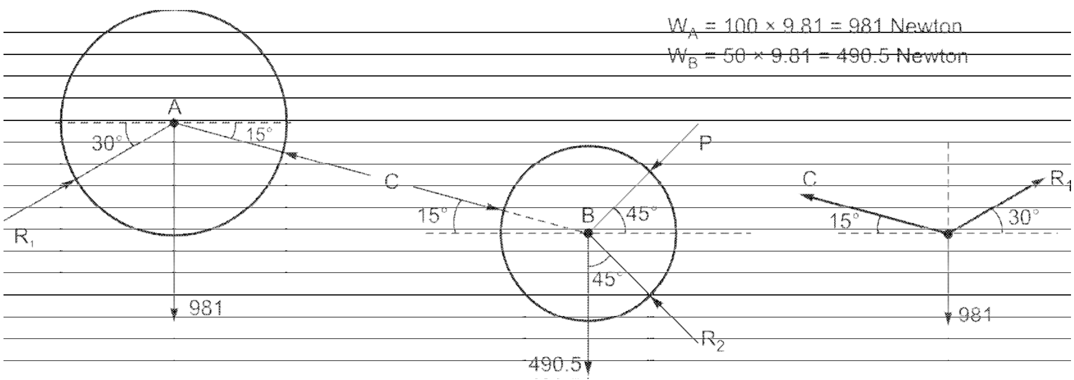


Fig. 1.41

$\therefore \qquad R_1 = 981 \times \dfrac{\sin 105}{\sin 135} = 1340.07$ N

and $\qquad C = 981 \times \dfrac{\sin 120}{\sin 135} = 1201.47$ N (compressive)

Applying equilibrium equations for cylinder B

$\Sigma F_x = 0 \Rightarrow \quad C \cos 15 - R_2 \sin 45 - P \cos 45 = 0$

$\Rightarrow \quad P + R_2 = 1641.24 \qquad$ ...(1)

$\Sigma F_y = 0 \Rightarrow \quad -490.5 + R_2 \cos 45 - P \sin 45 - C \sin 15 = 0$

$\Rightarrow \quad R_2 \cos 45 - P \sin 45 = 801.46$

$\Rightarrow \quad R_2 - P = 1133.435 \qquad$ ...(2)

Solving equation (1) and (2), we get

$$P = 253.9 \text{ N.}$$

Example 1.17. *A 600 N cylinder is supported by the frame BCD as shown in Fig. 1.42. Frame is hinged at D. Determine the reaction at A, B, C and D.*

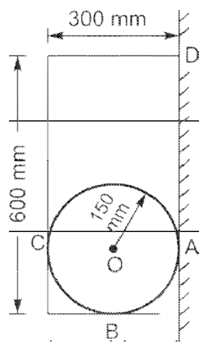


Fig. 1.42

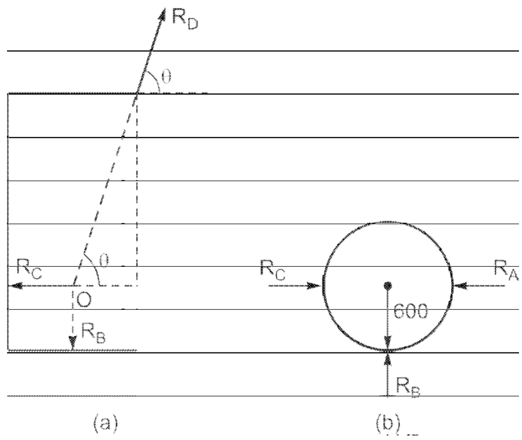


Fig. 1.43

Solution. Considering *FBD* of sphere [Fig. 1.42]

$\Sigma F_x = 0 \Rightarrow \quad R_A = R_C$

$\Sigma F_y = 0 \Rightarrow \quad R_B = 600$ N

As the frame is in equilibrium under the actions of three non-parallel forces only, they must be concurrent. Finding the direction of R_D as

$$\tan \theta = \frac{600 - 150}{150}, \ \theta = 71.565°$$

$\Sigma F_x = 0 \Rightarrow \quad R_C = R_D \cos \theta$

$\Sigma F_y = 0 \Rightarrow \quad R_D \sin \theta = R_B$

$\Rightarrow \quad R_D = \dfrac{600}{\sin 71.565} = 632.456$ N

$\therefore \quad R_C = 632.456 \times \cos 71.565 = 200 \text{ N} = R_A.$

Example 1.18. *A roller of radius r = 300 mm and weight 2000 N is to be pulled over an obstacle of height 150 mm, by a horizontal force P applied to the end of a string wound tightly* *around the circumference of the roller. Find the magnitude of P required to start the roller move over the obstacle. What is the least pull P through the centre of the wheel to just turn the roller over the obstacle?*

Solution. When roller is about to turn over the obstacle, the contact with the floor is lost and hence there is no reaction from the floor. Remaining three forces on the roller must be concurrent for maintaining equilibrium.

Case I. Force P applied horizontally from the top of the roller.

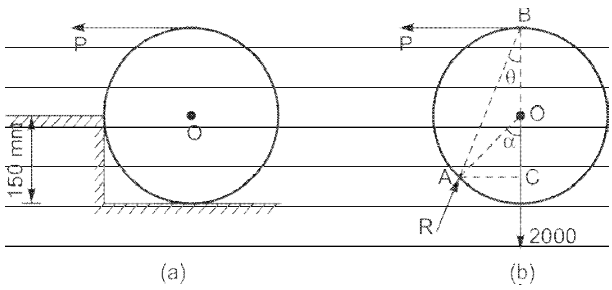


Fig. 1.44

$$\cos \alpha = \frac{OC}{OA} = \frac{300 - 150}{300}, \ \alpha = 60°$$

Now $AC = OA \sin 60 = 259.8$ mm

$$\therefore \qquad \tan \theta = \frac{AC}{BC} = \frac{259.8}{600 - 150} = 0.577$$

$$\theta = 30°$$

Now applying equilibrium equations

$$\Sigma F_x = 0 \quad \Rightarrow \quad R \sin 30 - P = 0 \qquad ...(1)$$

$$\Sigma F_y = 0 \quad \Rightarrow \quad R \cos 30 - 2000 = 0$$

$$\Rightarrow \qquad R = \frac{2000}{\cos 30} = 2309.40 \text{ N}$$

From equation (1), $\quad P = R \sin 30 = 2309.4 \sin 30 = 1154.70$ N.

Case II. Least force P applied through the centre of the roller.

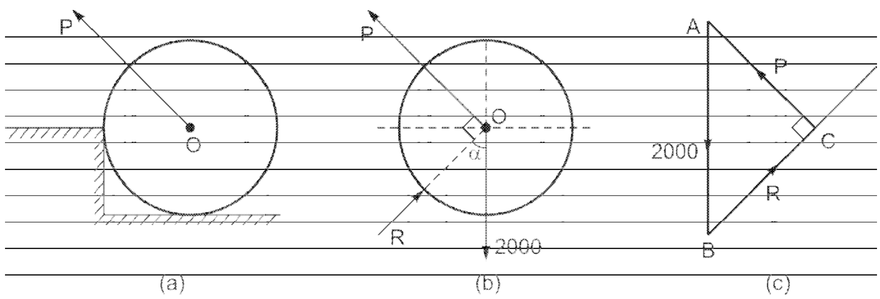


Fig. 1.45

If the force triangle ABC is constructed, Fig. 1.45 (*c*), representing self weight by AB, reaction R by BC and pull P by AC, it may be observed that for pull P (*i.e.* AC) to be least, it should be perpendicular to BC. In other words P makes an angle of 90° with R.

$\Sigma F_x = 0 \quad \Rightarrow \quad R \sin \alpha - P \cos \alpha = 0$ ...(1)

$\Sigma F_y = 0 \quad \Rightarrow \quad P \sin \alpha - 2000 + R \cos \alpha = 0$

$\Rightarrow \quad P \sin \alpha + R \cos \alpha = 2000$

$$\Rightarrow \quad P \tan \alpha + R = \frac{2000}{\cos \alpha}$$

$$\Rightarrow \quad R = \frac{2000}{\cos 60} - P.\tan 60 \qquad [\because \alpha = 60]$$

$\Rightarrow \quad R = 4000 - 1.732\, P$

Putting value of R in equation (1)

$\Rightarrow \quad (4000 - 1.732\, P) \times 0.866 - 0.5P = 0$

$$\Rightarrow \quad P = \frac{4000 \times 0.866}{1.999} = 1732.8 \text{ N.}$$

Example 1.19. *Determine the resultant of four forces tangent to the circle of radius 3 m shown in Fig. 1.46. What will be its location with respect to the centre of the circle ?*

Solution.

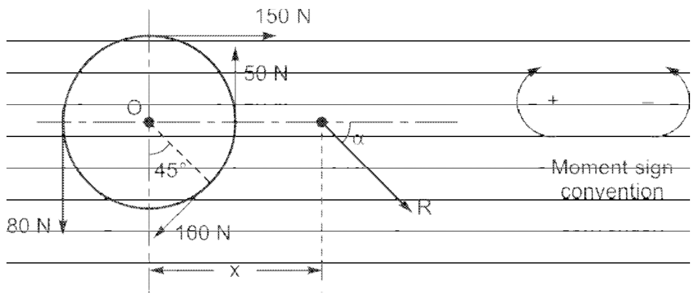


Fig. 1.46 (*a*)

Finding the algebraic sum of forces along horizontal and vertical directions.

$$\Sigma F_x = 150 - 100 \cos 45 = 79.29 \text{ N}$$

$$\Sigma F_y = 50 - 80 - 100 \sin 45 = -100.7 \text{ N}$$

$\therefore$ Magnitude of resultant

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$= \sqrt{(79.29)^2 + (100.7)^2} = 128.17 \text{ N}$$

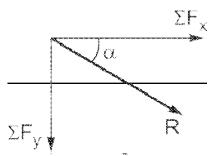


Fig. 1.46 (*b*)

Direction of resultant from x-axis

$$\alpha = \tan^{-1} \frac{100.7}{79.29} = 51.78° \text{ [IV}^{\text{th}} \text{ quadrant]}$$

Location of resultant can be find out using **Verignon's theorem**, considering moment about centre

$\Rightarrow \quad R \sin \alpha \times x = 150 \times 3 - 50 \times 3 + 100 \times 3 - 80 \times 3$

$\Rightarrow \quad 128.17 \times \sin 51.78 \times x = 360$

$$\Rightarrow \quad x = \frac{360}{128.17 \times \sin 51.78} = 3.575 \text{ m}$$

Perpendicular distance (d) of the force from centre O is,

$$\frac{d}{x} = \sin \alpha, \; d = x \sin \alpha = 2.809 \text{ m.}$$

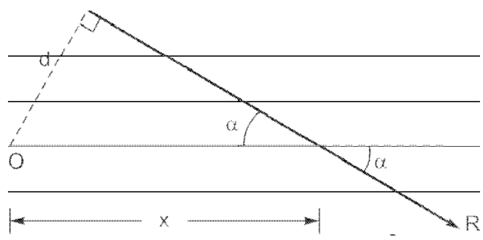


Fig. 1.46 (*c*)

Example 1.20. *Find the resultant of the force system shown in Fig. 1.47, acting on a Lamina of equilateral triangular shape.*

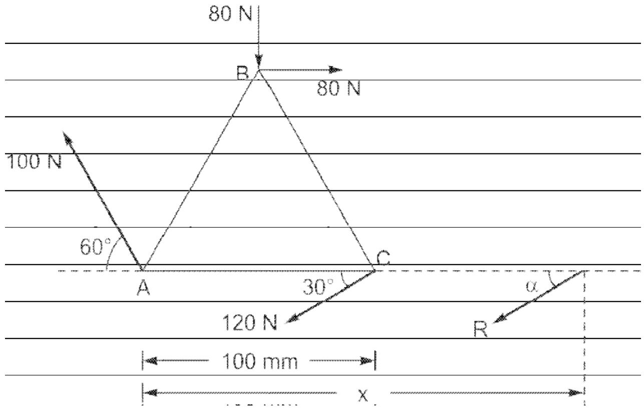


Fig. 1.47

Solution. Finding algebraic sum of forces along x and y directions.

$$\Sigma F_x = 80 - 120 \cos 30 - 100 \cos 60 = -73.92 \text{ N}$$

$$\Sigma F_y = -80 - 120 \sin 30 + 100 \sin 60 = -53.40 \text{ N}$$

Magnitude of resultant $R = \sqrt{(73.92)^2 + (53.40)^2} = 91.19$ N

Direction of resultant from x-axis

$$\alpha = \tan^{-1} \frac{53.40}{73.92} = 35.84° \qquad [\text{III}^{\text{rd}} \text{ quadrant}]$$

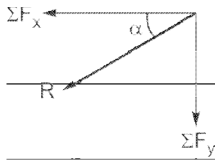


Fig. 1.47 (*a*)

Considering moment about point A, using Verignon's theorem

$$R \sin \alpha \,.\, x = 80 \times 50 + 80 \times 100 \sin 60 + 120 \sin 30 \times 100$$

$$x = 317.008 \text{ mm.}$$

Example 1.21. *Find the resultant of a set of coplanar forces acting on a Lamina as shown in Fig. 1.48. Each square has side of 10 mm.*

Solution.

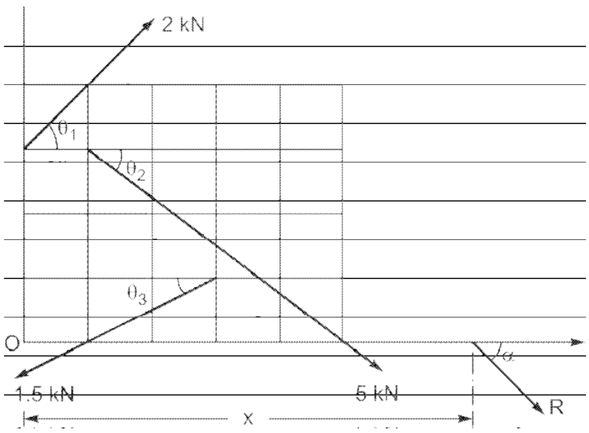


Fig. 1.48

Let θ_1, θ_2 and θ_3 are the slopes of forces as shown.

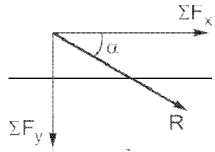


Fig. 1.48 (a)

$$\theta_1 = \tan^{-1} 1 = 45°$$

$$\theta_2 = \tan^{-1} \frac{30}{40} = 37°$$

$$\theta_3 = \tan^{-1} \frac{10}{20} = 30°$$

$$\Sigma F_x = 2 \cos \theta_1 + 5 \cos \theta_2 - 1.5 \cos \theta_3 = 4.0726 \text{ kN.}$$

$$\Sigma F_y = 2 \sin \theta_1 - 5 \sin \theta_2 - 1.5 \sin \theta_3 = -2.26 \text{ kN}$$

$$R = \sqrt{(4.0726)^2 + (2.26)^2} = 4.66 \text{ kN} \qquad (\text{IV}^{\text{th}} \text{ quadrant})$$

$$\alpha = \tan^{-1} \frac{2.26}{4.0726} = 28.99°$$

Taking moment about O, using Verignon's theorem

$$R \sin \alpha \,.\, x = 2 \cos \theta_1 \times 30 + 5 \cos \theta_2 \times 30 + 5 \sin \theta_2 \times 10$$
$$- 1.5 \cos \theta_3 \times 10 + 1.5 \cos \theta_3 \times 30$$

$$x = \frac{199.13}{4.66 \times \sin 28.99°}$$

$$= 88.169 \text{ mm.}$$

Perpendicular distance of resultant from O,

$$d = x \sin \alpha$$
$$= 88.169 \times \sin 28.99$$
$$= 42.73 \text{ mm.}$$

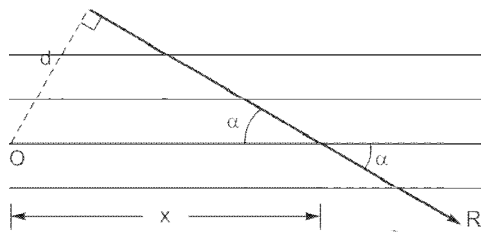


Fig. 1.48 (*b*)

Example 1.22: *A hollow right circular cylinder of radius 800 mm, is open at both ends and rests on a smooth horizontal plane as shown in Fig. 1.49. Inside the cylinder there are two spheres having weights 1 kN and 3 kN and radii 400 mm and 600 mm respectively. The lower sphere also rests on the horizontal plane. Neglecting friction find the minimum weight W of the cylinder for which it will not tip over.*

Solution: Considering $\Delta O_1, O_2 M$, Fig. 1.49 (*a*)

$$\cos \alpha = \frac{1600 - 400 - 600}{400 + 600} = 0.6, \ \alpha = 53.13°$$

$$\therefore \qquad \sin \alpha = 0.8$$

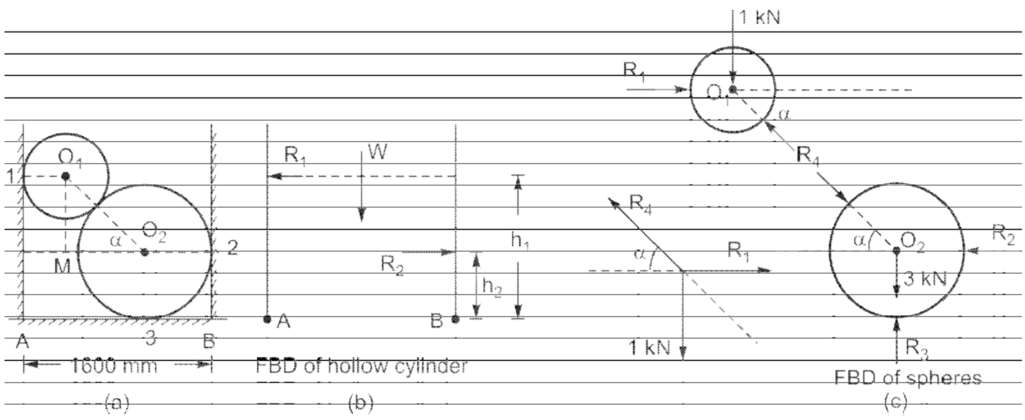


Fig. 1.49

Applying Lami's theorem for sphere O_1

$$\frac{R_1}{\sin(90+\alpha)} = \frac{R_4}{\sin(90)} = \frac{1}{\sin(180-\alpha)}$$

$$R_1 = 1 \times \frac{\sin(90+53.13)}{\sin(180-53.13)} = 0.75 \text{ kN}$$

$$R_4 = 1.25 \text{ kN}$$

Equilibrium equations for sphere O_2

$\Sigma F_x = 0 \Rightarrow \quad R_4 \cos\alpha - R_2 = 0$

$\Rightarrow \quad R_2 = 1.25 \cos 53.13$

$\Rightarrow \quad R_2 = 0.75$ kN

$\Sigma F_y = 0 \Rightarrow \quad R_3 - 3 - R_4 \sin\alpha = 0$

$\Rightarrow \quad R_3 = 3 + 1.25 \sin 53.13$

$\Rightarrow \quad R_3 = 4$ kN.

Now considering the equilibrium of cylinder, when it is about to tip over A, then there will be no reaction from ground at B.

$\Sigma M_A = 0 \Rightarrow \quad R_1 h_1 - R_2 h_2 - W \times 800 = 0$

$\Rightarrow \quad R_1 (h_1 - h_2) - W \times 800 = 0$

$\Rightarrow \quad R_1 (O_1 M) - W \times 800 = 0$

$\Rightarrow \quad R_1 [(400 + 600) \sin\alpha] - W \times 800 = 0$

$$\Rightarrow \quad W = \frac{0.75\,(1000 \sin 53.13)}{800}$$

$\Rightarrow \quad W = 0.75$ kN.

REVIEW EXERCISE

1. The resultant of two forces, one of which is double the other is 260 N. If the direction of the larger force is reversed and the other remains unaltered, the resultant reduces to 180 N. Determine the magnitude of forces and the angle between the forces.

[**Ans.** $F_1 = 100$ N, $F_2 = 200$ N, $\theta = 63.8961°$]

2. The magnitude of two forces is such that when acting at right angles produces a resultant force of $\sqrt{20}$ and when acting at 60° produces a resultant of $\sqrt{28}$. Find out the magnitude of two forces. [**Ans.** 2, 4 units]

3. A body is subjected to three forces as shown in Fig. 1.50. If possible determine the direction of the force F, so that the resultant is in x-direction, when

(i) $F = 5000$ N (ii) $F = 3000$ N [**Ans.** (i) 18.84° (ii) Nor possible]

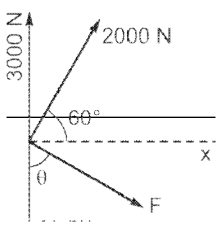


Fig. 1.50

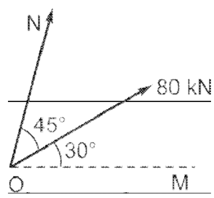


Fig. 1.51

4. Resolve 80 kN force into two components along OM and ON as shown in Fig. 1.51 (See above).

[**Ans.** 58.56 kN, 41.41 kN]

5. Find resultant of force system shown in Fig. 1.52. [**Ans.** 36.05, 86.31°]

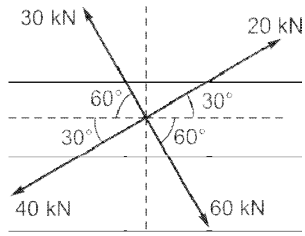


Fig. 1.52

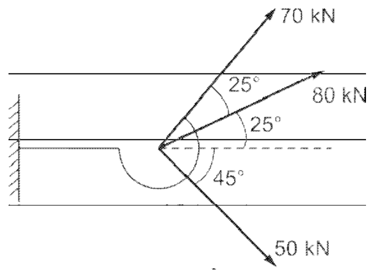


Fig. 1.53

6. Determine the resultant of the forces acting on a crane hook as shown in Fig. 1.53 (See above). [Ans. 161.48 N (71.18°) First quadrant]

7. A load is lifted in vertical direction with a force of 40 kN, applying by two cables passing over smooth pulleys *B* and *C*. Find tension T_1 if T_2 is to be minimum. What is the distance between two pulleys for this condition? [See in Fig. 1.54] [**Ans.** 32.77 kN, 12.77 m]

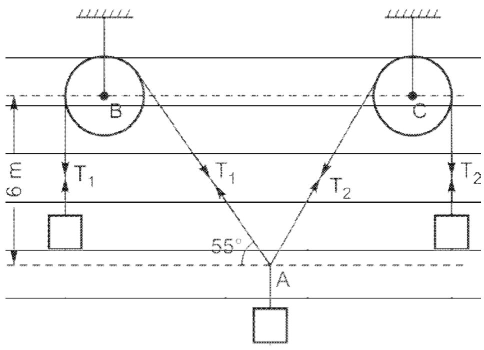


Fig. 1.54

8. In a jib crane, the jib and the tie rod are 5 m and 4 m long respectively. The height of crane post is 3 m and the tie rod remains horizontal. Determine the force produced in the jib and tie rod when a load of 2 kN is suspended at the crane head. [See in Fig. 1.55]

[**Ans.** 3.33 (compressive), 2.667 (tensile)]

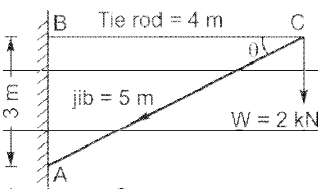


Fig. 1.55

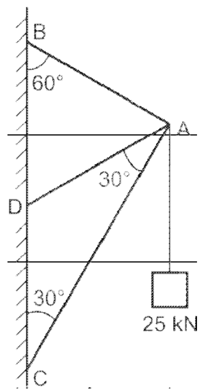


Fig. 1.56

9. The frictionless pulley A, shown if Fig. 1.56 (See above), is suspended by two bars *AB* and *AC* which are hinged at *B* and *C* to a vertical wall. The flexible cable *DA* is hinged at *D,* goes over the pulley and supports a load of 25 kN. Angle between the various members are shown in Fig. 1.56. Determine the forces in the bars *AB* and *AC*. Neglecting size of pulley and assume it as frictionless. [See in Fig. 1.56] [**Ans.** 0, 43.301 kN]

10. A force of 200 kN is acting at *A*; as shown in Fig. 1.57. Resolve the force along two members *AB* and *AC*. [**Ans.** 292.38 kN, 155.7 kN]

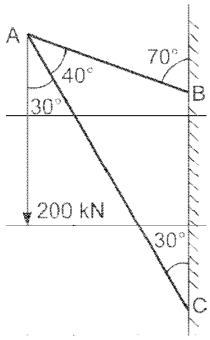


Fig. 1.57

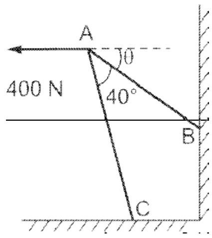


Fig. 1.58

11. Determine the angle θ for strut *AB* so that 400 N horizontal force has a component of 500 N acting from *A* to *C*. Also find component along member *AB*. [See above Fig. 1.58]

[**Ans.** 53.46°, 621.15 N]

12. Determine (*a*) the required tension in cable *BC*, if the resultant of three forces at *B* is to be vertical (*b*) the corresponding resultant to vertical. [See Fig. 1.59]

[**Ans.** 580 N, 300 N]

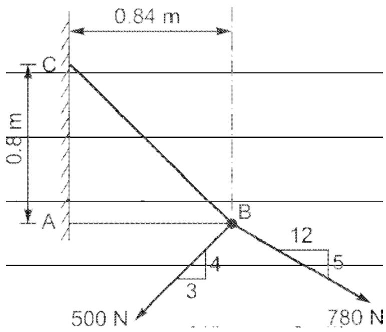


Fig. 1.59

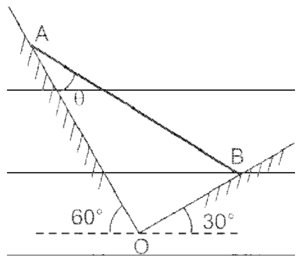


Fig. 1.60

13. A uniform bar *AB* of length *L* and weight *W* lies in a vertical plane with its ends resting on two smooth surfaces of *OA* and *OB*. Find angle θ for equilibrium of bar. [See above Fig. 1.60]

[**Ans.** 30°]

14. Two forces *P* and *Q* are applied to the corners *A* and *B* of a square plate as shown in Fig. 1.61. Find forces *P*, *Q*, and α, if resultant of two forces has a magnitude of 140 N, passing through *O* and making an angle of 30° with positive *x*-axis. [**Ans.** 70 N, 86.76 N, 53.79°]

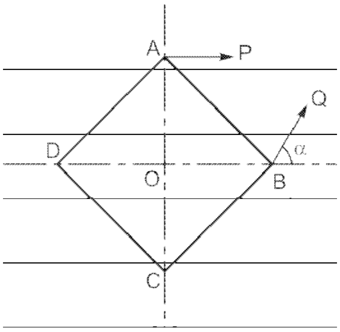


Fig. 1.61

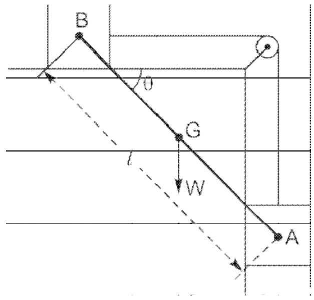


Fig. 1.62

15. A rod of length l and weight W is attached to two smooth collars A and B which can slides freely along the guides. The collars are connected by a string passing over a smooth pulley (see above Fig. 1.62). Express the tension in the string in terms of W and θ. $\left[\textbf{Ans. } T = \dfrac{W/2}{1 - \tan\theta}\right]$

16. A uniform bar with end rollers has a mass of 50 kg and is supported by horizontal and vertical surfaces and by a wire *CD* as shown in Fig. 1.63. Determine the tension *T* in the wire and reaction against the rollers at *A* and *B*. [**Ans.** 580.2 N, 490.5, 212 N]

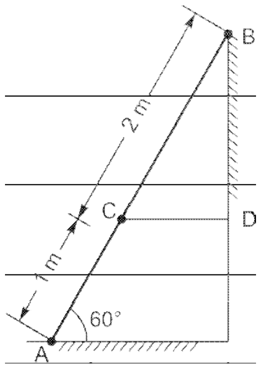


Fig. 1.63

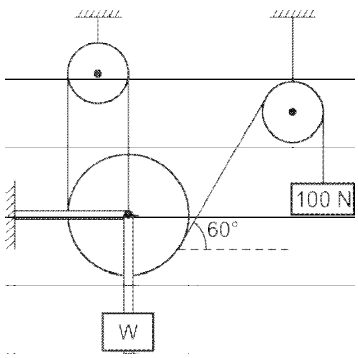


Fig. 1.64

17. The rope pulley arrangement shown in Fig. 1.64 (See above) is in equilibrium. Find the value of weight *W* . Neglect the weight of pulley and assume it as frictionless. [**Ans.** 286.60 N]

18. A uniform bar *AB* of weight 20 kN hinged at *A* is kept horizontal by supporting and setting a 50 kN weight with the help of a string tied at *B* and passing over a smooth peg *D* as shown in Fig. 1.65. Determine the tension in the string and reaction at support *A* and *D*. [**Ans.** 25 kN]

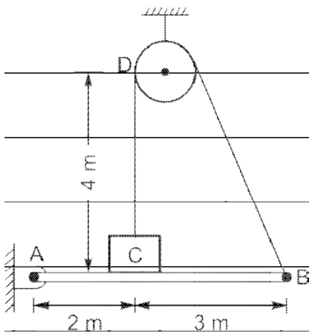


Fig. 1.65

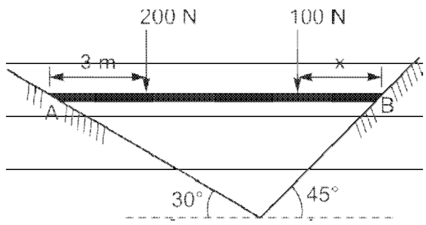


Fig. 1.66

19. A bar AB, 12 m long rests in horizontal position as shown in Fig. 1.66 (See above), on two smooth planes. Find distance x at which a load of 100 N is to be placed to keep the bar in equilibrium. Neglect weight of bar. [**Ans.** 4.82 m]

20. Two identical rollers each of weight 100 N are supported by an inclined plane and a wall as shown in Fig. 1.67, Find reactions at point of contact *A, B,* and *C.* [**Ans.** 86.6 N, 144.34 N, 115.47 N]

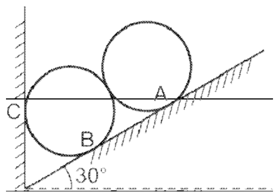


Fig. 1.67

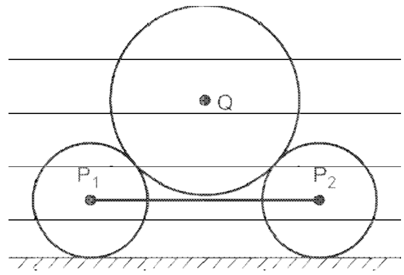


Fig. 1.68

21. Two smooth cylinders, each of weight 1000 N and radius 250 mm are connected at their centres by a string of length 800 mm, and rests upon a horizontal plane, supporting above a third cylinder of weight 2000 N and radius 250 mm as shown in Fig. 1.68 (See above). Find the tension in the string. [**Ans.** 1333.34 N]

22. Two steel cylinders are supported in a right angled wedge support as shown in Fig. 1.69. The diameter of the cylinders are 250 mm and 500 mm, their weights being 100 N and 400 N respectively. Determine the reaction R between the smaller cylinder and side OL. [**Ans.** 157 N]

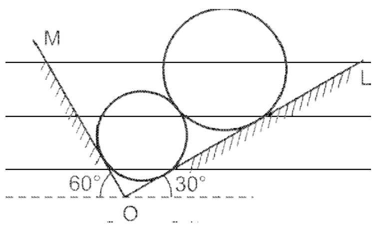


Fig. 1.69

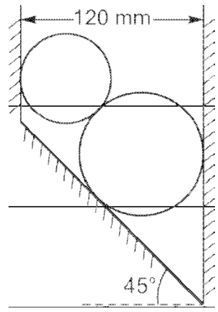


Fig. 1.70

23. Two cylinder of diameters 100 mm and 50 mm, weighing 200 N and 50 N respectively are placed in a trough as shown in Fig. 1.70 (See above). Neglecting friction, find the reaction at contact surface 1, 2, 3 and 4. [**Ans.** 37.5, 62.5, 287.5, 353.5]

24. Determine the moment of 400 N force acting at B about point A as shown in Fig. 1.71. [**Ans.** 78564 N mm CW]

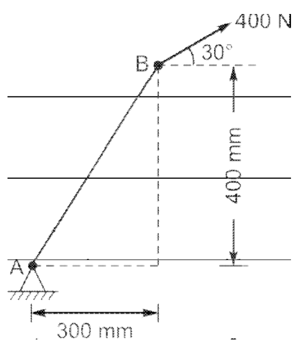


Fig. 1.71

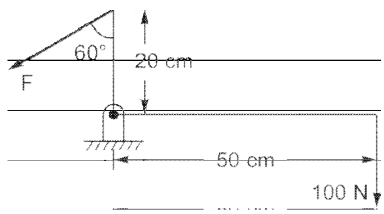


Fig. 1.72

25. A bell crank lever is subjected to the force system shown in Fig. 1.72 (See above). If the lever is in equilibrium, find the magnitude of force *F*. [**Ans.** 288.67 N]

26. The system of forces acting on a bell crank is shown in Fig. 1.73. Determine the resultant.

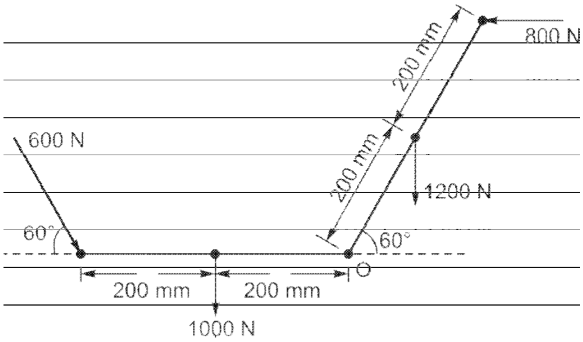


Fig. 1.73

[**Ans.** 2765.2 N, 79.58 (3rd quadrant), 207.74 mm leftward from *O*]

27. An equilateral triangular plate *ABC* of sides 200 mm is acted upon by four forces as shown in Fig. 1.74. Determine the resultant of the force system and its *x* interception.

[**Ans.** 57.35 N, 6.711° (IVth quadrant), 98.56 mm leftward from *A*]

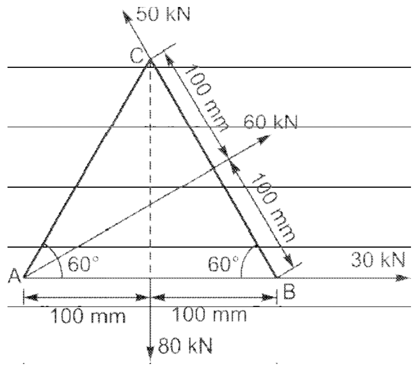


Fig. 1.74

28. Determine the resultant of the forces shown in Fig. 1.75.

[**Ans.** 200 N, 60° (4th quadrant), 5.768 m below *A*]

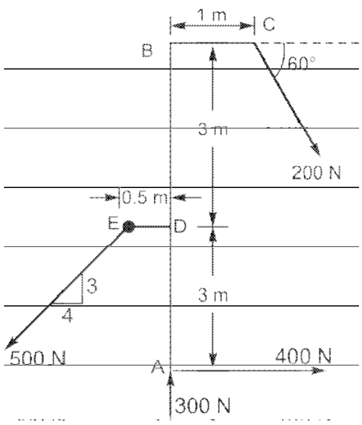


Fig. 1.75