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Introductory Notes and Fluid Properties

I.0. Fluid Mechanics

Fluid mechanics is a physical science concerned with the behaviour of fluid at rest and in motion. It combines the two separate approaches—the empirical hydraulics and the classical hydrodynamics developed by the hydraulicians and the mathematicians respectively. Hydraulics is mainly concerned with the motion of water. It is an applied science consisting of an enormous amount of experimental data which have been accumulated over a period of many centuries. The hydraulicians relied heavily on the field observations and laboratory tests. The data thus obtained are usually reduced to empirical formulas. Barring a few exceptions, these formulas are generally presented in a form such that they are not dimensionally homogeneous. Their applicability is limited to flow conditions similar to those for which these formulas were derived.

On the other hand, hydrodynamics is essentially a mathematical science dealing with flow analysis based on the concept of an ideal fluid—a fictitious fluid in which both fluid viscosity and fluid compressibility are assumed absent. The mathematical solutions of flow problems involving an ideal fluid thus have limited applicability to the motion of real fluids, even to those with small viscosity like water and air.

It is possible to experience applications of fluid mechanics in daily life. Some of the examples are :

(*i*) The flight of birds in the air and the motion of fish in the water are governed by the laws of fluid mechanics.

(*ii*) The cricket ball bowler depends upon circulation principle to provide the ball with desired spin and flight.

(*iii*) The dentated golf ball is designed to traverse longer distance with a minimum effort exerted by a golf player.

(*iv*) The circulation of blood in veins and arteries follows the law of fluid resistance.

(*v*) The human heart is a fine example of a pump delicately designed by nature to work continuously non-stop for many decades.

(*vi*) The designs of aeroplanes and ships are based on the theory of fluid mechanics.

(*vii*) The oil and gas pipelines, the water supply systems are designed on the principles of fluid mechanics.

We live in an environment of air and of water to such an extent that almost every thing we do is related in someway to the laws of fluid mechanics.

I.1. Historical Development of Fluid Mechanics

The following paragraphs briefly deal with certain important contributions made by various distinguished investigators since the dawn of history. The entire period of development has been divided into three parts depending upon the extent and the type of contribution made during the period. These are : Ancient and Medieval Period, Eighteenth and Nineteenth century period and Twentieth century period which marks the advent of fluid mechanics.

I.1.1. Ancient and Medieval Period Developments. The important contributions to hydraulics made upto the end of the seventeenth century have been covered here. The main contributors who influenced the course of future developments may be regarded as Archimedes, Leonardo da Vinci, Torricelli, Descartes, Mariotte, Pascal, Huygens. Isaac Newton and the Bernoulli brothers—Jakob and Johann. The major contributions of some of these are described briefly as under :

The recorded history of hydraulics[1] begins with **Archimedes** (287—212 B.C.). The most significant contribution of this Greek genius is his discovery of the principle of buoyancy and floatation. The development in the field of hydraulics was delayed until the experimental and observational approach gained a prominent place in the study of mechanics. No important contribution was made by any individual till the second half of the 15th century.

It was the Italian genius, **Leonardo da Vinci** (1452—1519), who first advocated the experimental approach to understand the flow behaviour. His basic premise was that, "*When dealing with water, we must begin with experiment and try through it to discover the reason*". As a result of his observations he was first to sketch and comment upon many hydraulic phenomena such as, profiles of free jet, formation of eddies at abrupt expansions and in wakes, velocity distribution in a vortex, hydraulic jump etc. He was also first to propose streamlining of bodies and the centrifugal pump. Credit also goes to Vinci for being the first to quantitatively state the principle of Continuity.

Evangelista Torricelli (1608—1647) was also an Italian who generalized the analysis of trajectories of projectiles. His major contribution is the principle of efflux, which is now commonly written as $V = \sqrt{2gh}$. The discovery of barometer is also attributed to him.

Isaac Newton (1642—1727) was the first Englishman to have earned an eminent place amongst the scientists of his time. His approach was that of a mathematician while his predecessors investigated nature from philosophical point of view. In the "Principia" (1687) he enunciated concisely the three basic laws of motion now named after him. It is these basic laws which form the basis of analysis of problems of mechanics. He carried out extensive research on fluid resistance, and was first to report on the fact that in viscous flow the shear is proportional to the relative velocity of the adjacent zone. The statement of this fact is now known as the law of viscosity and bears his name. Newton was also first to introduce the use of coefficient of contraction in problems of efflux.

The direct and indirect contributions of two mathematicians of Basel, Switzerland, who happened to be brothers were also significant to the advancement of physical science. The eldest, **Jakob Bernoulli** (1654—1705), was a professor of physics at the University of Basel, who trained his younger brother **Johann Bernoulli** (1667—1748). Johann then worked with a French mathematician, taught mathematics in Holland and succeeded his brother as professor.

The four essential steps emerged as a result of evolution of mathematical physics in the 17th century are :

(*i*) the use of plotted curve to describe a phenomenon ;

(*ii*) the expression of the curve in equation form ;

(*iii*) the determination of its area and slope ; and

(*iv*) the application of these procedures to practical problems.

It is not known as to who accomplished the first step, but it was definitely Descartes who accomplished the second step, Leibnitz (1846—1916, Germany) the third and the Bernoullis the fourth.

Of the contemporary mathematicians, to whom the credit for laying the foundation of hydrodynamics goes, two belonged to the Bernoulli School at Basel. The first one was Daniel Bernoulli (son of Johann) ; the second one was his close friend Leonardo Euler and the third one was d' Alembert of France.

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1. History of hydraulics, by Hunter Rouse and Simon Ince, Dover publications, Inc., New York, 1963.

I.1.2. 18th and 19th Century Developments. Eighteenth and nineteenth centuries saw a tremendous advancement in the field of hydraulics and hydrodynamics. It was during this period that most of the experimental hydraulics was developed at the hands of the French, the Italian and the German engineers and hydraulicians. Noteworthy amongst them were Poleni, Pitot, Chezy, Borda, Venturi, Weber, Fourneyron, Belanger, Russel, Reech, Hagen, Poiseuille, Weisbach, Darcy, Bazin, Kutter, Manning and Froude. It was in the early 18th century when Daniel Bernoulli along with Leonhard Euler and d'Alembert founded the mathematical science of hydrodynamics. Others who significantly contributed to the development of hydrodynamics are Lagrange, Laplace, Navier, Stokes, Helmholtz, Kirchhoff, Boussinesq, Reynolds, Thomson (Lord Kelvin), Strutt (Lord Rayleigh) and Joukowsky. The important contributions of some of these distinguished investigators have been described in the following paragraphs.

Daniel Bernoulli (1700—1782) was a Swiss mathematician who published a treatise named "Hydrodynamica" dealing with various aspects of hydrostatics and hydraulics. He introduced for the first time the word 'Hydrodynamics' to encompass various topics of fluid statics and dynamics. He was first to use piezometer openings in the walls of conduits for pressure indication. His energy principle utilized only two terms, namely, the pressure and the velocity. Bernoulli's principle used for evaluating pressure seemed to indicate the constancy of pressure and velocity heads. He showed both experimentally and analytically that the pressure would become negative if the velocity increased sufficiently.

Leonhard Euler (1707—1783) was also a Swiss mathematician who worked mainly on hydrodynamics and hydraulic machinery. He was the first to explain the role of fluid pressure in fluid flow. Euler investigated the motion of a fluid under the action of an external force, rightly regarding the isotropic pressure as a function of space only ; formulated basic equations of motion (now known as Euler's equations of acceleration or motion). Assuming the fluid to be incompressible, flow to be steady and irrotational and utilizing the concept of a force potential he combined his three equations of acceleration to yield a single relationship involving pressure, velocity and elevation heads. This equation of Euler in its present familiar form is attributed to Bernoulli. Euler also contributed significantly to the hydrodynamics theory of centrifugal machinery. Analysing the performance of reaction turbines, he expressed for the first time the basic relationship by equating the torque to the change in the moment of momentum as the fluid is passed through the rotating part.

Jean le Rond d' Alembert (1717—1783), was a French mathematician first to introduce concepts of components of fluid velocity and acceleration and also the differential expression of continuity. Assuming similar condition in the rear of a body as in the front, the summation of the elementary pressures exerted on each part of the body surface led to the paradoxical result of a zero longitudinal force on the body. d'Alembert, however, left it on of future investigators to explain this anomaly between the theory and the reality. This is the paradox of zero resistance to steady non-uniform motion, known as *d' Alembert's paradox*.

Joseph Louis Lagrange (1736—1813) was a self-trained French mathematician who succeeded Euler as the world's leading mathematician. In the course of analysis of fluid motion, he introduced concepts of velocity potential and stream function which became of fundamental importance in describing the flow pattern. Lagrange was first to derive an equation for velocity of propagation of a wave of infinitesimal height in channel of finite depth ($V = \sqrt{gy}$).

Merquis Giovanni Poleni (1683—1761) is the only Italian who deserves a special mention for his contribution to the experimental hydraulics of the early 18th century. His major contributions were three-fold : (1) Based on measurements he obtained the coefficient of contraction for a sharp-edged orifice to be 0.62, a significant improvement over the value of $1/\sqrt{2}$ proposed by Newton. (2) He also experimented with short tubes (mouth-pieces) attached to the orifice and found that a maximum rate of discharge was obtained with an intermediate length of tube. (3) His last contribution was his treatment of the discharge through rectangular sharp- crested weirs. He considered the discharge as occurring in a series of horizontal elements, the velocity of each being assumed proportional to the square root of its depth below the liquid surface. The same approach was later used for deriving the discharge relationship for sharp-crested weirs, and hence the basic weir equation is often named after Poleni.

Henri de Pitot (1695—1771) was a French engineer whose principal contribution is the invention of a device (known after him as the Pitot-tube) for measuring fluid velocity. The original Pitot tube consisted of two parallel tubes mounted on a slender frame containing a scale and four petcocks ; one of the tubes being straight and the other bent through 90° at its lower end.

Antoine Chezy (1718—1798) was also a French engineer who contributed significantly to the understanding of the resistance in uniform open-channel flow. Credit goes to Chezy for not only presenting the first but also the most lasting resistance formula—later to be known by his name.

Jean Charles Borda (1733—1799) was a French military engineer who devoted himself to experiments in hydraulics and hydraulic machinery. His resistance studies verified the prevalent theory that the drag of an immersed body varied with the square of the relative velocity and showed that it would depend upon a still higher power if surface wave were produced. Borda was first to introduce the concept of elementary stream-tubes. He showed that not only the contraction of jet but also loss of energy must be taken into account in obtaining an expression for discharge. Making use of the momentum principle, he found that for the particular case of a re-entrant tube (*i.e.* the Borda mouth-piece) the coefficient of contraction has a value of 0.5.

Venturi's published work in Paris in 1797 reported his findings on various forms of mouthpieces fitted to the orifice. He demonstrated the effect of eddies formed at abrupt changes in section and, incidentally the change in discharge which would result from their elimination. Venturi observed that the replacement of the cylindrical tube with two conical sections essentially eliminated the eddies and increased the rate of flow, but it still produced the local reduction in pressure. This form of boundary is now used to measure flowrate in pipes and is rightly known as the venturimeter.

Giorgio Bidone (1781—1839) was an Italian hydraulician who is credited with having discovered the hydraulic jump. He was the first to study it systematically and to attempt its analysis.

Giuseppe Venturoli (1768—1816) also an Italian hydraulician, derived the elementary back water equation for rectangular channels. Through graphical integration, Venturoli succeeded in plotting various branches of the surface profile.

Claude Burdin (1790—1893), a French engineer, coined the word "turbine" and developed one with free efflux of water.

Benoit Fourneyron (1802—1867) improved upon Burdin's original device and developed a successful hydraulic turbine. More than hundred similar turbines were built by him for various parts of the world.

John Scott Russel (1808—1882), a Scottish engineer, was the first to study the problems of unsteady, non-uniform open-channel flow without discontinuity. He studied the effects of waves on the resistance of ships and proposed a reverse-curve form of bow which he believed would reduce the wave effect to a minimum.

A French contemporary of Russel, **Ferdinand Reech** (1805—1880) advocated the practicability of model tests and developed similitude principles based on Newton's laws of motion. Reech was first to express what is now known as the Froude criterion of similitude.

Gotthilf Heinrich Ludwig Hagen (1707—1884), a German hydraulic engineer made original contributions to the resistance of pipe flow. Based on extensive and accurate experiments on flow through small diameter tubes, he reasoned that the flow took place in series of cylindrical layers, the velocity of which varied linearly (an assumption which later proved wrong) from zero at the wall to a maximum at the centre. Hagen proposed an expression for the resistance to flow in small diameter tubes (*i.e.* laminar flow) based on the above assumption. He also carried out experiments on resistance of pipes in turbulent flow and correlated his measurements by means of a resistance equation.

Jean Louis Poiseuille (1799—1869), a French physician and not an engineer, was interested in experimental physiology. He conducted research on pumping power of the heart, the movement of blood in the veins and capillary vessels and the resistance to flow through tubes. He carried out accurate experiments on very small diameter tubes and presented an empirical relationship for the discharge in terms of the head loss, the tube diameter and the tube length. The resistance law for laminar flow was later named after Poiseuille rather than Hagen, and it still continues to be known by his name.

Julius Weisbach (1806—1871), a German hydraulician, wrote a treatise on hydraulics for engineering application which even now can be considered a textbook on hydraulics. Weisbach not only incorporated the best available experimental information, but in many cases supplemented it with the results of his own experiments. He advocated the use of the non-dimensional coefficient, and was first to express the resistance equation for pipes in the form $h = fLV^2/2gD$. Weisbach also modified the weir equation to include the velocity of approach and to eliminate the successive approximations involved in the determination of discharge he proposed instead and empirical equation of the type later adopted by Bazin.

Antoine Charles Bresse (1822—1883), a French engineer, accomplished integration of the equation of gradually varied open-channel flow and prepared tables of function, now known as Bresse's back water function, for general use. He also presented a correct formulation of the momentum characteristics of the hydraulic jump.

Henry Philibert Gaspard Darcy (1803—1858), also a French engineer, conducted studies on the flow of water in both pipes and permeable soils. His experiments included pipes of different sizes, materials and in various states of deterioration. His greatest contribution was his conclusive demonstration of the fact that the resistance depended upon the type and the condition of the boundary material. As a result of his studies on pipe flow, Darcy's name is commonly associated with that of Weisbach in designating the present-day resistance equation first formulated by Weisbach. On the basis of his filteration studies, Darcy concluded that the loss of head through a filter bed was proportional to the rate of flow rather than to its square root as was then generally believed.

Henri Emile Bazin (1829—1917), a French engineer and an associate of Darcy, conducted extensive experiments on open-channel resistance, propagation of waves and flow over weirs. Based upon his experiments in canals of various materials and shapes, Bazin proposed a formula of resistance in open-channel flow. He also carried out experiments for measurements of velocity distribution at various cross-sections with different linings. From these tests, he noted that the depth of point of maximum velocity varied with the relative width of cross-section, approaching zero as the width-depth ratio exceeded 5. Bazin's subsequent studies on discharge over vertical and inclined weirs included the precise determination of nappe profile and the distribution of velocity and pressure through the nappe. As a result of these studies he introduced a new dimensional term in the weir-discharge equation proposed by Weisbach.

Two Swiss engineers **Emile Oscar Ganguillet** (1818—1894) and **Wilhelm Rudolf Kutter** (1818—1888) are known for their contribution to the open-channel resistance. On the basis of several hundred experiments, they expressed the Chezy's coefficient C as a function of a roughness factor n, hydraulic radius R and the channel slope S.

Robert Manning (1816—1897), an Irish engineer, proposed in 1889 a relationship for open-channel flow of the form, $V = K\,R^{2/3}\,S^{1/2}$, which was in better agreement with the available data than the earlier ones. The present-day formula named after Manning was neither recommended nor ever devised in full by Manning himself. He also did not suggest use of Kutter's n—a coefficient now associated with the Manning's formula.

William Froude (1810—1879), an English engineer, developed and perfected the towing-tank techniques for testing of model ships. Froude believed that "*Experiments duly conducted on small scale model will give results truly indicating of the performance of the full size ships*". He considered the total resistance to be made up of the skin friction resistance and the resistance due to other factors such as waves. It is interesting to note that Froude's name has been inseparably associated with a law of similarity and a non dimensional number, which were neither originated nor even used by him.

Lousi Marie Henry Navier (1785—1836), a French engineer, derived through a purely mathematical analysis equations of motion for a viscous flow. His name is, therefore, most frequently associated with the present day equations for viscous flow. He however, did not identify the fluid viscosity as a variable affecting the flow but instead considered molecular spacing.

George Gabriel Stokes (1819—1903), a British mathematician, was the one whose name becomes finally associated with that of Navier in designating the equations of motion for a viscous fluid. His paper "On the Theories of Internal Friction of Fluids in motion" published in 1845 contained the derivations of what are now known as the Navier-Stokes equations. The general coefficient ε appearing in Navier's equations was replaced by the dynamic viscosity μ. Stokes also derived an expression for the terminal velocity of fall of spheres, which is now known as the Stokes' law.

Osborne Reynolds (1842—1912), a British engineer, was : (1) the first to demonstrate the phenomenon of cavitation and attribute the accompanying noise to the collapse of vapour bubbles, just as in a kettle beginning to boil ; (2) the first to correlate the length and time scales in the study of distorted models ; and (3) the first to introduce the viscosity into a parameter now bearing his name, demarcating the limit between laminar and turbulent flows. Reynolds demonstrated experimentally in 1888 that the velocity at which eddy motion (*i.e.* turbulent flow) began did indeed vary with the tube diameter and fluid characteristics in such a manner as to yield a fairly definite value of this parameter (now known as the Reynolds number). The most important contribution of Reynolds was his application and extension of Navier-Stokes equations to turbulent flow. His lasting contribution, however, was the derivation of the equations for motion—now known as the Reynolds equations for turbulent flow.

I.1.3. 20th Century Developments—Advent of Fluid Mechanics. Until the early 20th century the two distinctly divergent approaches namely, the experimental hydraulics and the theoretical hydrodynamics had developed to such an extent that the apparent gap between theory and the practical reality was bothering the genius amongst the hydraulicians and the hydrodynamists. The emergence of aeronautics at a rapid pace also hastened the activities of engineering talent towards bridging the gap between theory and the fact. This was brought about by a new concept of boundary layer originated by Prandtl of Germany.

Ludwig Prandtl (1875—1953) is regarded as the founder of the present-day fluid mechanics. He realized the need for better correlation between theory and experiment in problems of fluid flow. Prandtl conducted his first experiments on the flow of air while he was employed as a engineer in a large machinery firm. Later he joined as a faculty member of Polytechnic Institute, Hannover and continued his research on flow of air. Within three years he presented a paper containing his findings before the Third International Congress of Mathematicians in 1904. In the eight-page paper he introduced the concept of boundary layer according to which the motion of fluids of low internal resistance (*i.e.* low viscosity) can be divided into two interdependent zones : (*i*) very close to the fixed boundaries, there is a transition layer in which the fluid velocity changes from zero at the boundary to practically the same value as in the free-stream at the edge of this layer. It is in this layer that effects of viscosity are predominant, and (*ii*) away from the boundary lies the zone of free-stream across which there is hardly any change in velocity and within which viscous effects are negligible. The flow in this part can be analysed on the basis of potential flow theory. The layer in the vicinity of the fixed boundary and to which the viscous effects are confined was given the name of '*boundary layer*'.

The emergence of the theory of boundary layer has come to play a vital role not only in aeronautics but also in hydraulics and other related fields. This has enabled a much rapid development of the science of motion of fluids known as the fluid mechanics.

As a result of his pioneering research on flow of low-viscosity fluids, Prandlt was invited as Professor and Director of Research Institute at the University of Göttingen. At Göttingen, he and his students further worked on the boundary layer theory, analyzed the phenomena of turbulence and drag and evolved dynamic principles of aerofoil behaviour. The first of his students to achieve recognition was **Paul Richard Heinrich Blasius** (1833—) of Berlin. He published in 1908 an analytical solution for velocity distribution and resistance of laminar boundary layer wherein Pandtl's qualitative theory was quantitatively verified by laboratory experiments.

Theoder von Karman (1881—1963) was perhaps the most illustrious alumnus of the Göttingen Institute where he worked under Prandtl. Karman was gifted with a combination of rare physical insight and mathematical ability. His primary contributions to fluid mechanics have been in the fields of form drag turbulence and surface resistance and the analogy between the sound and gravity waves. He investigated the problem of eddy formation behind circular cylinders and provided an analytical solution of what has since been known as the Karman vortex trail. Karman and Prandtl contributed to the analysis of velocity distribution and resistance to turbulent flow in pipes as well as long flat surfaces. The resulting logarithmic equations for resistance and velocity distributions are now known by their joint names.

Other notable investigators associated with the Göttingen Institute were : Walter Ludwig Christian **Schiller** (1882—) whose primary interest was in the field of pipe resistance. Walter Gustav Johannes **Tollmein** (1900—) distinguished himself by his analysis of flow stability and turbulent diffusion ; Hermann **Schlichting** (1907—) contributed greatly to the analysis of stability and boundary layer development. Carl **Wieselberger** (1887—1941) contributed significantly on the phenomena of drag ; Otty **Flachspart** (1898—1957) contributed on drag and particularly on wind pressures on buildings and Johann **Nikuradse** (1894—) contributed in the field of pipe resistance and is known for his famous experiments on artificially roughened pipes.

Credit goes to **Mortiz Weber** (1871—1951) of Berlin to put the general principles of similitude in their present form. It was he who specifically named the Frode and Reynolds numbers associated Cauchy's name with elastic similarity and introduced a capillarity parameter in his paper presented in 1919. The capillarity parameter **W** is subsequently named as the Weber number.

Geoffrey Ingram Taylor (1886—), a British physicist while employed as a meteorologist at the University of Cambridge, studied, eddy motion in the atmosphere. He published a series of papers dealing with the fundamental analysis of fluid turbulence by methods of statistics. Taylor presented his theory of diffusion by continuous movements and related the diffusive and dissipative characteristics of turbulent motion.

I.2. Systems of Measuring Physical Quantities

Any system of measurement is based on well defined units for certain basic quantities such as mass, length, time, temperature etc. The units for other quantities are derived from basis units by virtue of the relationship that exists among the quantities concerned. There is, however, an element of flexibility in making a choice for basic quantities. In the absolute system of units mass, length and time are considered the basic quantities and the corresponding units have been named as the basic or base units. The gravitational system of units, which has so far been used in engineering practice, considers force, length and time as basic quantities. While using the gravitational system of units, it is to be noted that the unit of force is not a universal constant, but varies from place to place owing to its dependence on the local acceleration due to gravity.

The metric (MKS) gravitational system of units was in engineering use in India for over past five decades. With growing acceptance of International System (SI) of units, the MKS units have been replaced by the SI units. In this book, SI units have been used alongwith the MKS to familiarize the reader with the new units.

I.2.1. Absolute (Coherent) and Gravitational Systems of Units. The basic or base units (mass or force, length and time) are inter-related to each other by Newton's second law of motion. This law states that a mass moving by virtue of an applied force will be accelerated and that the component of force in the direction of the acceleration is proportional to the product of the mass and the acceleration. Thus

$$F \propto m.a$$

$$= \frac{1}{C_n}\, m.a \qquad ...(1)$$

where F = force in the direction of acceleration,

m = mass

a = acceleration, and

C_n = constant of proportionality known as Newton's constant.

If we select the units such that one unit of force acting on one unit of mass produces one unit of acceleration, the proportionality constant C_n will be unity. For this case we can write Newton's second law in the conventional form, $F = m.a$, expressing that the force equals mass multiplied by acceleration. The proportionality constant C_n will be unity only when the absolute system of units is used, that is when the system of measuring units is *coherent*.

A coherent system of units may be defined as the one in which the product or quotient of any two unit quantities involved in the phenomenon is the unit of the resultant quantity. As examples of this statement, in any coherent system the quotient of unit length and unit time gives unit velocity and similarly the product of unit mass and unit acceleration gives unit force. Any absolute system of units is essentially a coherent system.

I.2.2. SI Units[2]. In the International System (SI) of units the base quantities are mass, length, time and the thermodynamic temperature and the corresponding base units are kilogram (kg), metre (m), second (s) and Kelvin (K) respectively. From these base units the derived unit of force is newton (N) which is the force required to accelerate 1 kg mass at the rate of 1 m/s^2, and is obtained from Eq. (1).

$$1\text{ N} = \frac{1}{C_n} \times 1\text{ kg (mass)} \times 1\text{ m/s}^2$$

from which the Newton's constant is

$$C_n = \frac{1\text{ kg (mass) m}}{\text{N.s}^2} \qquad ...(2)$$

I.2.3. MKS Gravitational Units. This system has the base quantities as force, length, time and thermodynamic temperature and the corresponding base units are kilogram (kg), metre (m), second (s) and Celsius (C). In the gravitational system, unit force is proportional to the product of unit mass and acceleration due to gravity. Unit force of one kilogram is defined as the force required to accelerate one kg mass at the rate of 9.81 m/s^2. Thus from Eq. (1), we obtain

$$1\text{ kg (force)} = \frac{1}{C_n} \times 1\text{ kg (mass)} \times 9.81\text{ m/s}^2.$$

$$C_n = 9.81\ \frac{\text{kg (mass) . m}}{\text{kg (force) . s}^2} \qquad ...(3)$$

Comparing Eqs. (2) and (3), 1 kg (force) = 9.81 newton (N) ...(4)

The derived unit of mass is called metric slug in analogy to slug—the British gravitational unit of mass. It is defined as the mass which will be accelerated at the rate of 1 m/s^2 when acted upon by a force of 1 kg. Equation (1) may be used to determine the Newton's constant.

2. "The International System of Units" Editors Chester H. Page and Paul Vigoureux, London, Her Majesty's Stationery Office 1973.

$$1 \text{ kg (force)} = \frac{1}{C_n} \times 1 \text{ metric slug} \times 1 \text{ m/s}^2.$$

$$C_n = \frac{\text{metric slug m}}{\text{kg (force) s}^2} \qquad ...(5)$$

Comparing Eqs. (3) and (5),

$$1 \text{ metric slug} = 9.81 \text{ kg (mass)} \qquad ...(6)$$

Table 1 shows SI and MKS (gravitational) units of various quantities used in fluid mechanics.

Table 1. SI and MKS (gravitational) units

Quantity	*System of units*			
	SI		*MKS (gravitational)*	
	Unit	*Symbol*	*Unit*	*Symbol*
Mass	kilogram	kg	metric-slug	m slug
Length	metre	m	metre	m
Time	second	s	second	s
Thermodynamic temperature	Kelvin	K	Celcius	C
Area	square metre	m^2	square metre	m^2
Volume	cubic metre	m^3	cubic metre	m^3
Velocity	metre per second	m/s	metre per second	m/s
Acceleration	metre per second square	m/s^2	metre per second squared	m/s^2
Force	newton	N	kilogram	kg
Moment of force	metre newton	Nm	metre kilogram	kgm
Pressure	newton per square metre (pascal)	N/m^2 (Pa)	kilogram per square metre	kg/m^2
Density	kilogram per cubic metre	kg/m^3	metric slug per cubic metre	m $slug/m^3$
Specific weight	newton per cubic metre	N/m^3	kilogram per cubic metre	kg/m^3
Dynamic viscosity	kilogram per metre second	kg/ms Pas	kilogram second per square metre	kgs/m^2
Surface tension	newton per metre	N/m	kilogram per metre	kg/m
Work, energy	joule	J = Nm	kilogram metre	kgm
Power	watt	W = J/s = Nm/s	kilogram per metre second	kg/ms

The conversion factors from MKS to SI, MKS to FPS and MKS to CGS units appear in Appendix E.

I.3. System

In the most general terms, a system may be defined as that region of space occupied by the quantity of fluid under consideration. The fluid contained within the system is separated from the surroundings by a boundary. The system may contain either a constant or a variable mass. Its boundaries may be fixed or deformable. It may be in motion or at rest with respect to a chosen co-ordinate system. The region outside the system's boundary is known as the surroundings. A system diagram shows the system's boundary and significant interactions between the system and its surroundings. If the surroundings of importance are forces then the system diagram reduces to the familiar '*free-body diagram*'.

I.3.1. Closed and Open Systems. A system is said to be a closed system if the same body of fluid remains within the system during a process. The process is then called a non-flow process. In such a process, work and heat may be transferred across the boundary but no fluid crosses the boundary. In Fig. 1 (*a*) dotted lines indicate the systems boundary and only the mass transfer has been considered.

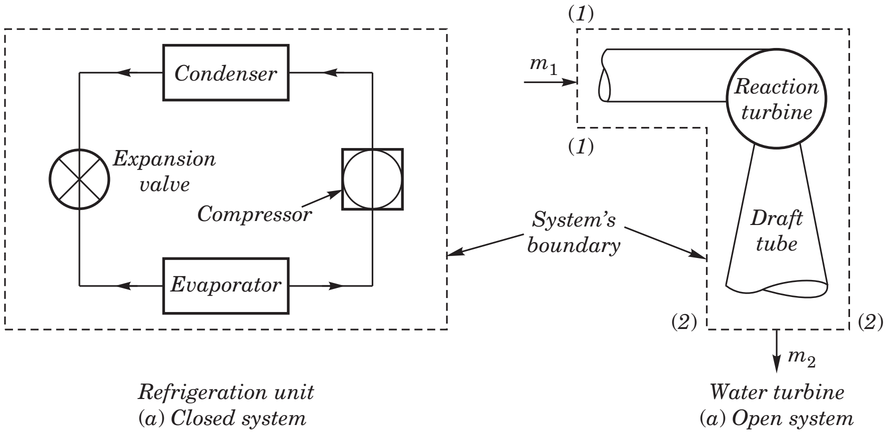


Fig. 1. Closed and open systems.

An open system is defined as one in which the fluid enters and/or leaves the system, and the process referred to as a flow process. In an open system fluid mass, momentum, energy and machinery all cross the boundary. Fig. 1 (*b*) shows such a system consisting of a water turbine. The mass rate of flow m_1 at (1)—(1) is the same as mass rate of flow m_2 at (2)—(2).

I.4. Control Volume

A control volume is an open-system which has its boundary fixed with respect to a fixed coordinate system. The control volume is thus, an arbitrary volume fixed in space and bounded by a closed surface which is known as the control surface. The fluid may enter and leave control volume by crossing the bounding surface (control surface) enabling transfer of mass, momentum and energy.

I.5. Free-body Diagram

As already pointed out, a free body diagram is a closed system in which the interactions between the system and its surroundings are the forces. The concept of a free-body diagram helps in cultivating a rational approach in respect of listing various forces which act on the system.

Consider the curved surface *AB* shown in Fig. 2 which closes an opening in the tank. The curved surface *AB* thus supports a liquid column contained in *ABCD*. The problem is pertaining to the statics of fluids, and the free-body of liquid *ABCD* must be in static equilibrium. The conditions of static equilibrium dictate that the algebraic sum of the force components in mutually perpendicular directions must be zero, and so also the algebraic sum of the moments of forces in the respective planes be zero, that is the forces must be co-planar.

The main problem is, therefore, of evaluating the forces acting on the free-body (magnitude, direction and their location). By the application of the above-stated principles of static equilibrium the unknown forces may be obtained. In Fig. 2, the forces acting on the free-body *ABCD* are :

(*i*) Weight *W*, of the liquid mass contained in *ABCD*.

(*ii*) Hydrostatic pressure force F_{H_2} exerted on the liquid by the tank wall.

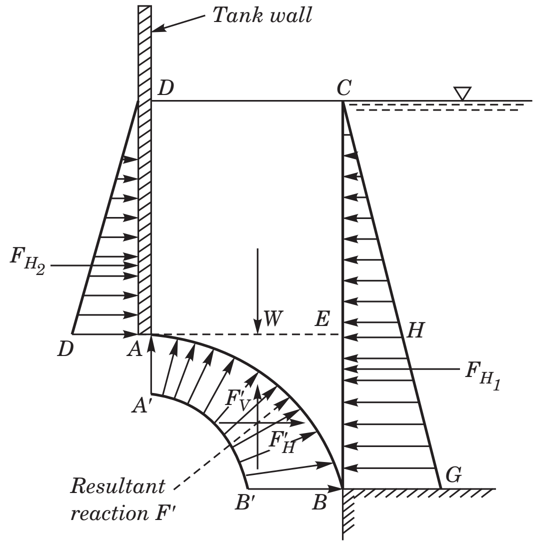


Fig. 2. Free-body diagram for *ABCD*.

(*iii*) Hydrostatic pressure F_{H_1} exerted on the free-body liquid by the surrounding liquid, and

(*iv*) The resultant reaction force F' (having components F_H' and F_V') exerted by the curved surface *AB* on the liquid.

The evaluation of hydrostatic forces is dealt with in Chapter II (refer articles 2.5 and 2.6).

PROPERTIES OF FLUID

1.0. Introduction

Matter can be distinguished by the physical form of its existence. These forms known as phases, are solid, liquid and gas.

Solid, Liquid and Gas

The liquid and gaseous phases are usually combined and given a common name of fluid, because of the common characteristics exhibited by liquids and gases. Solids differ from liquids and liquids from gases on account of their molecular structure (spacing of molecules and the ease with which they can move). The spacing of molecules is large in a gas, smaller in a liquid and extremely small in a solid. Very strong intermolecular attractive forces exist in solids which give them the property of rigidity. These forces are weaker in liquids and extremely small in gases.

Definition of a Fluid

The word fluid means a substance having particles which readily change their relative positions. A fluid may be defined as substance which deforms continuously under the action of shear stress, regardless of its magnitude.

Distinction between a Liquid and a Gas

A fluid may be either a liquid or a gas. The molecules of a liquid are very closely spaced as compared to those of a gas. While a liquid has a free surface and occupies a certain volume in a container, a gas does not possess a free surface and fills the entire space of the container regardless of its size. For all practical purposes, a liquid is incompressible while a gas is compressible and expands unless contained or enclosed in a container. In the words of Sir Oliver Lodge "*A solid has volume and shape, a liquid has volume but no shape, a gas has neither*." A vapour is a gas whose temperature and pressure are such that it is very near to the liquid phase, and hence the steam is considered as a vapour. A gas may be defined as a superheated vapour. Air is regarded as a gas on account of its state being normally very far from that of liquid air.

1.2. Density

The density of a substance is defined as the mass per unit volume and is denoted by the symbol ρ (Greek letter rho). It has the dimension $[ML^{-3}]$. The expansion or contraction of the substance results in a change in density. The density of liquids may be considered as constant while that of gases will be subjected to changes depending on the pressure and temperature. The fluid density at a point is defined by

$$\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V}$$

where Δm = mass contained in a small volume ΔV.

Specific Gravity. The specific gravity represents a numerical ratio of two densities, and water is commonly taken as a reference substance. Thus

$$\text{Specific gravity of a substance} = \frac{\text{Density of the substance}}{\text{Density of water}}.$$

It is also called the relative density.

1.3. Specific Weight

It is the weight of a given substance per unit volume, and is commonly denoted by symbol γ (Greek letter gamma). The specific weight represents the force exerted by gravity on a unit volume of fluid and, therefore, must have the units of force per unit volume. It is related to the density by the following expression

$$\gamma = \rho g.$$

The specific weight of fresh water under normal conditions is 1000 kg/m^3 in MKS units and 9.81×10^3 N/m^3 in SI units.

Specific Volume. It is the volume occupied by a unit mass of fluid. It is commonly applied to gases. The specific volume is reciprocal of the density, *i.e.* $v = 1/\rho$.

Example 1.1. *Calculate the mass density, specific weight and weight of 1 litre of petrol, if its specific gravity is 0.72.*

Solution. Volume of petrol

$$= 1 \text{ litre} = 1000 \text{ cu. cm.}$$
$$= 1 \times 10^{-3} \text{ m}^3$$

$$\text{Specific gravity} = \frac{\text{Mass density of petrol}}{\text{Mass density of water}}$$

$$0.72 = \frac{\text{Mass density of petrol}}{1000 \text{ (kg/m}^3)}$$

$\therefore$ Mass density of petrol, ρ

$$\rho = 0.72 \times 1000$$
$$= 720 \text{ kg/m}^3$$

Specific weight of petrol, γ

$$= \text{weight of 1 m}^3 \text{ of petrol}$$
$$= \rho g$$
$$= 720 \times 9.81$$
$$= \mathbf{7063.2\ N/m^3}$$

Weight of 1 litre of petrol = γV

$= 7063.2\ (\text{N/m}^3) \times 10^{-3}\ (\text{m}^3)$

$= \mathbf{7.0632\ N.}$

Example 1.2. *Ten litres of a liquid of specific gravity 1.3 is mixed with 6 litres of a liquid of specific gravity 0.8. If the bulk of the liquid shrinks by 1.5% on mixing, calculate the specific gravity, density, volume and weight of the mixture.*

Solution. Weight of 10 litres of liquid of specific gravity 1.3

$= 10 \times 10^{-3} \times 9810 \times 1.3\ \text{N} = 127.53\ \text{N}$

Weight of 6 litres of liquid of sp. gr. 0.8

$= 6 \times 10^{-3} \times 9810 \times 0.8 = 47.1\ \text{N}$

Total volume of liquids before mixing

$= 10 + 6 = 16$ litres

Upon mixing the bulk shrinks by 1.5%

$\therefore$ New total volume $= 0.985 \times 16 = 15.76$ litres

Weight of equal volume of water $= 15.76 \times 10^{-3} \times 9810 = 154.6\ \text{N}$

Weight of mixture $= 127.53 + 47.1 = 174.63\ \text{N}$

(*i*) Specific gravity of mixture $= \dfrac{\text{Wt. of mixture}}{\text{Wt. of equal vol. of water}} = \dfrac{174.63}{154.6} = \mathbf{1.128.}$

(*ii*) Density of mixture $= \dfrac{\text{Mass}}{\text{Volume}} = \dfrac{174.63/9.81}{15.76 \times 10^{-3}} = \mathbf{1128\ kg/m^3.}$

(*iii*) Volume of mixture $=$ **15.76 litre.**

(*iv*) Weight of mixture $=$ **174.63 N.**

Example 1.3. *If 5 m^3 of a certain oil weighs 40 kN, calculate the specific weight, mass density, specific volume and relative density of the oil.*

Solution. Specific weight $\gamma = \dfrac{\text{Weight}}{\text{Volume}} = \dfrac{40\ (\text{kN})}{5\ (\text{m}^3)} = 8\ \text{kN/m}^3$

$$\text{Mass density } \rho = \frac{\text{Mass}}{\text{Volume}} = \frac{W/g}{V} = \frac{\gamma}{g} = \frac{8 \times 10^3\ (\text{N/m}^3)}{9.81\ (\text{m/s}^2)} = 815.49\ \text{kg/m}^3$$

$$\text{Specific volume, } v = \frac{1}{\rho} = \frac{1}{815.49} = 1.23 \times 10^{-3}\ \text{m}^3/\text{kg}$$

$$\text{Relative density} = \frac{\text{density of oil}}{\text{density of water}} = \frac{815.49}{1000.00} = \mathbf{0.81549.}$$

1.4. Viscosity

Among all the fluid properties, viscosity is the most important and is recognised as the only single property which influences the fluid motion to a great extent. The viscosity is the property by virtue of which a fluid offers resistance to deformation under the influence of a shear force. For a given fluid, the rate of deformation is dependent upon the magnitude of shear force. The molecular friction or shear resistance within the fluid opposes such continuous deformation.

Let us consider a fluid contained in between two parallel plates as shown in Fig. 1.2, the bottom one being kept stationary while the top one moves at a constant speed U under influence of the applied shearing force F.

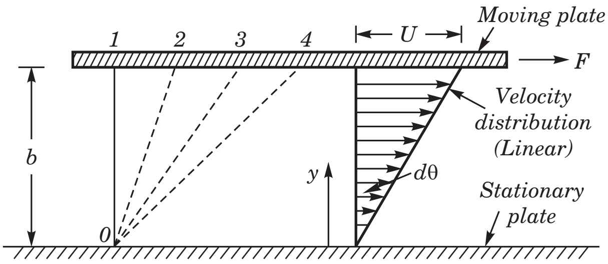


Fig. 1.2. Fluid motion between parallel plates, bottom plate stationary, top one moving.

Initially when the top plate is about to start moving all the fluid particles are at rest. The position of fluid particles lying along the vertical line 0—1 changes with time once the motion starts. The fluid particles sticking to the moving plate move with the same velocity U while those adhering to the bottom stationary plate are at rest. The velocity of the intermediate particles vary from 0 to U. If the gap separating the two plates is small, the velocity distribution will be linear (straight line) as shown, and the fluid particles originally lying on line 0—1 after a certain time, say $t = \Delta t$, will occupy the positions indicated by the line 0—2, and at times $t = 2\Delta t$ and $3\Delta t$ they will lie along the lines 0—3 and 0—4 respectively. The maximum deformation of fluid takes place at $y = b$, the magnitude of which in unit time is U, and the zero deformation occurs at $y = 0$, the bottom plate being stationary. The time rate of deformation is, therefore, equal to U/b. The rate of angular deformation $d\theta/dt$ is given by

$$b\,\frac{d\theta}{dt} = U \qquad \text{or} \qquad y\,\frac{d\theta}{dt} = u$$

$$\therefore \qquad \frac{d\theta}{dt} = \frac{U}{b} = \frac{u}{y}$$

where u is the velocity at a distance y from the stationary plate.

If A is the area of the moving surface and F is the force required to move the surface at a constant velocity, it has been established that the shear stress F/A is directly proportional to the time rate of deformation, thus,

$$\frac{F}{A} \propto \frac{d\theta}{dt}$$

or

$$\frac{F}{A} = \mu\,\frac{d\theta}{dt} = \mu\,\frac{U}{b} = \mu\,\frac{u}{y}$$

where μ is the proportionality constant and is called coefficient of viscosity. It is also known as the dynamic viscosity or simply viscosity. The shear stress which is usually denoted by the symbol τ (Greek letter tau) may be expressed as

$$\tau = \mu\,\frac{U}{b} = \mu\,\frac{d\theta}{dt} = \mu\,\frac{u}{y} \qquad \text{...(1.2)}$$

Equation (1.2) states that the rate of angular deformation is proportional to the shear stress.

Equation (1.2) in a differential form is expressed as

$$\tau = \mu\,\frac{du}{dy} \qquad \text{...(1.3)}$$

For a linear velocity distribution, du/dy, is a constant. But if the gap separating the parallel surfaces is large, the velocity distribution can no longer be assumed linear.

In case of a non-linear velocity distribution such as shown in Fig. 1.3, the rate of deformation is not constant, but changes from point to point. The shear stress, which depends on the rate of deformation will therefore not be constant throughout the fluid.

In Eq. (1.3), $\frac{du}{dy}$ represents the rate of shear deformation or rate of shear strain and is often called the velocity gradient. It relates the shear stress with the viscosity. Equation (1.3) is known as the *Newton's law of viscosity* or *Newton's law of fluid friction*. It may be noted here that when the fluid is at rest, no tangential forces exist and hence no fluid deformation takes place. In other words, as the velocity gradient, dy/dy is zero in a fluid at rest, no tangential or shearing force ever exists.

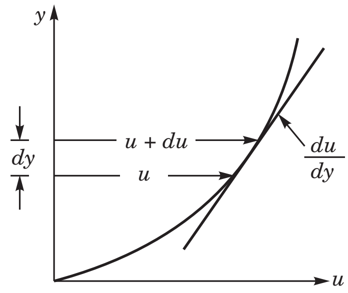


Fig. 1.3. Non-linear velocity distribution.

According to Newton's law of viscosity, for a given shear stress acting on a fluid element, the rate at which the fluid deforms is inversely proportional to the viscosity. This implies that for a constant shear stress the rate at which deformation takes place is larger for fluids of low viscosity. For solids, the resistance to shear deformation is due to modulus of elasticity whereas for fluids, the resistance to rate of shear deformation is on account of the viscosity. It is thus seen that the Hook's law for solids is analogous to the Newton's law of viscosity.

1.4.1. Newtonian and Non-Newtonian Fluids

A fluid which obeys Eq. (1.3) is known as a Newtonian fluid. Newtonian fluids have a certain constant viscosity, *i.e.*. the viscosity is independent of the shear stress. Many common fluids such as air, water, light oils and gasoline are Newtonian fluids under normal conditions. However, there are certain fluids which exhibit non-Newtonian characteristics—shear stress is not linearly dependent upon the velocity gradient. Non-Newtonian fluids, therefore, do not follow Newton's law of viscosity. Common examples of Non-Newtonian fluid are : human blood, lubricating oils, clay suspension in water, molten rubber, printer's ink, butter and sewage sludge. The following chart gives the classification of fluids :

Fluids

- *Newtonian*
- *Non-Newtonian*
  - Time-independent
    - (*i*) Dilatant
    - (*ii*) Bingham plastic
    - (*iii*) Pseudoplastic
  - Time-dependent
    - (*i*) Thixotropic
    - (*ii*) Rheopectic

A general relationship between shear stress and velocity gradient (rate of shear strain) for non-Newtonian fluid may be written as :

$$\tau = A \left(\frac{du}{dy}\right)^n + B \quad \text{...(1.4)}$$

where A and B are constants which depend upon the type of fluid and conditions imposed on the flow (shear stress).

The fluids which obey Eq. (1.4) are called *power-law fluids*. The additive constant B is zero for the fluids except Bingham plastic. Based on the value of power index n in Eq. (1.4), the non-Newtonian fluids are classified as :

(*a*) Dilatant, if $n > 1$, (example—quicksand, butter, printing inks)

(*b*) Bingham plastic, if $n = 1$, (*e.g.*, sewage sludge, drilling muds)

(*c*) Pseudoplastic, if $n < 1$, (*e.g.*, paper pulp, polymeric solutions such as rubbers, suspensions paints)

A Newtonian fluid is a special case of power law fluid having $n = 1$ and $B = 0$, and the constant A varying only with the type of fluid.

Time-independent fluids : In case of time-independent fluids, the rate of deformation or the velocity gradient depends only upon the shear stress, and is a single valued function of the latter. The viscosity of Newtonian fluids is independent of the shear stress, whereas in case of non-Newtonian fluids, the viscosity is a function of shear stress.

Time-dependent fluids : The rate of deformation and the viscosity depend upon both the shear stress and the duration of its application.

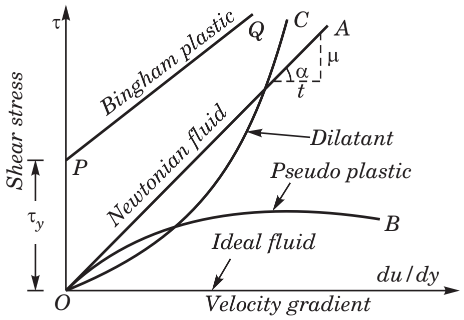


Fig. 1.4. Plot of τ *versus* $\dfrac{du}{dy}$.

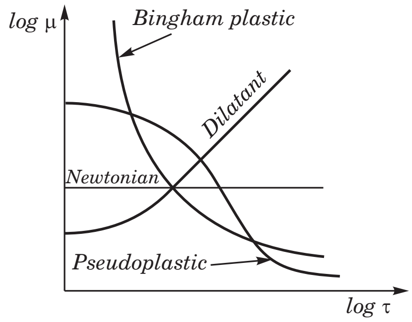


Fig. 1.5. Plot of log μ *versus* log τ.

Figs. 1.4 and 1.5 illustrate the shear or viscous characteristics of different fluids. Fig. 1.4 illustrates the shear stress-velocity gradient relationship for various type of fluids. The Newtonian fluids are characterised by linear relationship between the shear stress and the velocity gradient. They are represented by a straight line like *OA* passing through the origin and inclined at an angle α with the horizontal such that $\mu = \tan \alpha$.

The Bingham plastic fluids require a certain minimum shear stress τ_y known as the yield stress before they start flowing and exhibit a linear relationship between the shear stress and the velocity gradient as shown by the straight line *PQ*. The dilatant and pseudoplastic fluids are shown by curves marked *OC* and *OB* respectively.

Fig. 1.5 illustrates the viscosity μ and shear stress τ relationship for different fluids on log-log scale. For Newtonian fluids this relationship is a straight line parallel to τ-axis indicating that viscosity is independent of stress. For non-Newtonian fluids, the fact that the viscosity is a function of the shear stress can be noticed.

Thixotropic fluids are those which show an increase in apparent viscosity with time. Lipstic and certain paints and enamels exhibit thixotropic behaviour. The apparent viscosity may be defined as $\mu_{app} = \dfrac{\tau}{du/dy}$. Those fluids which show a decrease in the apparent viscosity with time are called *rheopectic*. Rheopectic fluids are much less common than thixotropic fluids. Gypsum suspensions in water and bentonite solutions are examples of rheopectic fluids. Thixotropy is an important property of paints and enamels. When subjected to high shear by the brush during application of paint, the apparent viscosity is reduced so that the paint covers the surface smoothly, and brush marks disappear subsequently.

1.4.2. Some Common Newtonian Fluids

The air and water both have relatively low viscosities, the viscosity of oils is much higher and the glycerine is the most viscous of the better known fluids (approximately one thousand times as viscous as water) having colourless appearance and is readily miscible with water, making it useful for laboratory purposes by providing glycerine-water solutions of any desired viscosity. At ordinary temperatures, a comparison of absolute viscosities of a few common fluids shows that water is about 50 times more viscous than air. As compared to water, castor oil is 1000 times more viscous, crude oil is 10 times more viscous and the gasoline is about $\frac{1}{3}$ times viscous than water.

Ideal Fluid. An ideal fluid is a conceptual fluid which is assumed non-viscous and incompressible. It is a concept that permits a fluid to possess nonexistential properties, like zero viscosity (inviscid fluid) and constant density implying zero compressibility. Such a concept was used by mathematicians for simplifying analysis of fluid motion. Truely speaking, there is no fluid that exists in nature that possesses zero viscosity and zero compressibility (*i.e.,* bulk modulus of elasticity having infinite value).

Real Fluid. All fluids that exist in nature are real fluids possessing properties like, viscosity, elasticity, surface tension and vapour pressure. Such fluids are viscous and compressible. Common examples are air, water, other gases and liquids.

1.4.3. Kinematic Viscosity

In the analysis of many fluid-flow problems, the dynamic viscosity divided by the density is commonly found to exist. This ratio of dynamic viscosity and mass density is called the kinematic viscosity and is denoted by the Greek letter ν (nu), thus

$$\text{Kinematic viscosity } \nu = \frac{\text{Dynamic viscosity } \mu}{\text{Mass density } \rho}$$

It is known as kinematic viscosity, because it can be defined dimensionally by only length and time dimensions, mass or force dimensions being not involved.

1.4.4. Dimensions of Dynamic and Kinematic Viscosities

The dimension of dynamic viscosity μ may be obtained by using Eq. (1.3)

$$\mu = \frac{\tau}{du / dy}$$

The fundamental dimensions are the mass (M), length (L) and time (T).

The dimensions of other quantities can be derived easily.

Writing the dimension of shear stress and velocity gradient, the dimension of the dynamic viscosity can be determined.

$$\text{Dimension of shear stress} = \frac{\text{Dimension of force}}{\text{Dimension of area}} = \left[\frac{MLT^{-2}}{L^2}\right] = ML^{-1}T^{-2}$$

Hence, dimension of [μ] $= \left[\dfrac{[MLT^{-2} / L^2]}{LT^{-1} / L}\right] = [ML^{-1}T^{-1}]$ ...(1/4 *a*)

dimension of μ in F-L-T system will be

$$[\mu] = \left[\frac{F / L^2}{LT^{-1} / L}\right] = [FL^{-2}T] \quad \text{...(1.4 } b\text{)}$$

the dimension of kinematic viscosity may be found as :

$$\text{Dimension of } [v] = \left[\frac{\text{Dimensions of } [\mu]}{\text{Dimensions of } [\rho]}\right] = \left[\frac{ML^{-1}\,T^{-1}}{ML^{-3}}\right] = [L^2\,T^{-1}] \quad \text{...(1.5)}$$

The unit of measurement of viscosity μ, may be obtained using Eq. (1.4 *b*).

In MKS system, the viscosity is expressed as kg s/m^2 ; in CGS system, it is expressed by dyne-s/cm^2, and is called a poise ; and FPS it is measured in lb-sec/ft^2. In SI units it is expressed as N s/m^2. The unit of viscosity may be converted from one system to another by using the above definitions and basic conversion factors.

(*i*) From FPS to CGS : $1 \frac{\text{1b} - s}{\text{ft}^2} = \frac{453.6 \times 981}{(30.48)^2} \frac{\text{dyne} - \text{s}}{\text{cm}^2} = 479 \frac{\text{dyne} - \text{s}}{\text{cm}^2} = 479$ poise.

(*ii*) From FPS to MKS : $1 \frac{\text{1b} - \text{s}}{\text{ft}^2} = \frac{453.6 / 1000}{(0.3048)^2} \frac{\text{kg} - \text{s}}{\text{m}^2} = 4.87 \frac{\text{kg} - \text{s}}{\text{m}^2}$

(*iii*) From MKS to SI : $1 \frac{\text{kg} - \text{s}}{\text{m}^2} = \frac{1000 \times 981}{(100)^2} \frac{\text{dyne} - \text{s}}{\text{cm}^2} = 98.1$ dyne-s/cm^2 = 98.1 poise.

$$1 \text{ kg-s/m}^2 = 9.81 \text{ N-s/m}^2 = 9.81 \text{ Pa.s}$$

$$98.1 \text{ poise} = 9.81 \text{ N-s/m}^2 \text{ ; } 1 \text{ poise} = \frac{1}{10} \text{ N-s/m}^2 = 0.1 \text{ N-s/m}^2 = 0.1 \text{ Pa.s}$$

The conversion factors of units are given in Appendix E.

Based on the dimensions of the kinematic viscosity $[L^2T^{-1}]$ the unit of ν in F.P.S. system is ft^2/sec, in C.G.S., it is cm^2/s and is called the stoke and in M.K.S. and SI units, it is expressed as m^2/s. These units can be converted from one system to the other using the same procedure as above :

$$1 \text{ ft}^2/\text{sec} = \frac{(30.48)^2}{1} \text{ cm}^2/\text{s} = (30.48)^2 \text{ stokes} = 930 \text{ stokes.}$$

$$1 \text{ stoke} = 1 \frac{\text{cm}^2}{\text{sec}} = \left(\frac{1}{100}\right) \text{m}^2/\text{s} = 10^{-4} \text{ m}^2/\text{s.}$$

1.4.5. Variation of Viscosity with Temperature

The variation of viscosity with changes in temperature may be understood by knowing the factors contributing to viscosity. The viscosity of a fluid depends upon its intermolecular structure. In gases, the molecules are widely spaced resulting in a negligible intermolecular cohesion, while in liquids the molecules being very close to each other, the cohesion is much larger. The viscosity of a fluid is due to intermolecular cohesion and transfer of molecular momentum in a direction normal to the flow. In liquids, this momentum transfer is small as compared to the force of cohesion between the molecules and, therefore, the viscosity is primarily dependent upon the magnitude of intermolecular cohesive force. With the increase of temperature, the cohesive force decreases rapidly resulting in the decrease of viscosity. Thus, the viscosity of liquids decrease with the increase of temperature.

Poiseuille developed a formula for determining the kinematic viscosity of water at any temperature T,

$$\nu = \frac{0.0179}{1 + 0.0337\,T + 0.000221\,T^2}$$

in which ν is the kinematic viscosity in cm^2/s and T is the temperature in degree centigrade.

In case of gases, the viscosity is mainly due to transfer of molecular momentum in the transverse direction brought about by the molecular agitation. The contribution to the viscosity by the intermolecular cohesive force being negligible due to large spacing of molecules. As the molecular agitation increases with the rise of temperature the viscosity of gases also increases with temperature rise.

The viscosity of a fluid may thus be considered to be composed of two parts :

1. that due to intermolecular cohesion, and
2. that due to transfer of molecular momentum, and thus

Viscosity of a flowing fluid = contribution from intermolecular cohesion + contribution from transfer of molecular momentum.

Table 1.1 gives viscosity of air at different temperatures. For water and other liquids see Tables 1.5 and 1.6. Table 1.7 deals with properties of the standard atmosphere.

Table 1.1. Density and Viscosity of Air at Atmospheric Pressure

Temperature degree C	*Density ρ* kg/m^3	*Dyn. viscosity,* μ $10^5\mu$, $N\text{-}s/m^2$
0	1.29	1.71
20	1.202	1.81
40	1.125	1.90
60	1.059	2.00
100	0.945	2.18
150	0.833	2.39
200	0.745	2.58
300	0.615	2.95
400	0.524	3.28
500	0.456	3.58

The viscosity of gases, like that of liquids, changes with temperature but is practically unaffected by pressure. The kinematic viscosity, depending as it does on density, varies with both temperature and pressure. The following equation, given by Holman, may be used for determining the value of μ at different temperatures :

$$\mu = 1.7150 \times 10^{-4} (1 + 0.00275\, T - 3.4 \times 10^{-7}\, T^2) \quad \text{(poise)}$$

in which μ is in poise and T in degrees centigrade. The viscosity in kg s/m^2 and Ns/m^2 are given by :

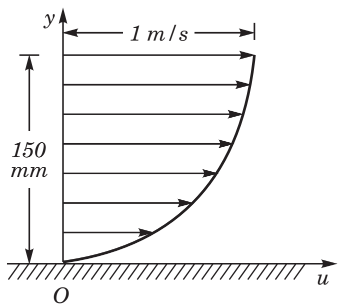
$$\mu = 1.7500 \times 10^{-6} (1 + 0.00265\, T - 3.4 \times 10^{-7}\, T^2) \qquad \text{(kg-s/m}^2$$

$$\mu = 1.715 \times 10^{-5} (1 + 0.00275\, T - 3.4 \times 10^{-7}\, T^2) \qquad \text{(N-s/m}^2\text{)}.$$

Example 1.4. *Calculate the velocity gradient at distances of 0, 100, 150 mm from the boundary if the velocity profile is a parabola with the vertex 150 mm from the boundary, where the velocity is 1 m/s. Also calculate the shear stresses at these points if the fluid has a viscosity of 0.804 N-s/m^2.*

Solution. Let the equation of the parabolic velocity profile be

$$u = Ay^2 + By + C \qquad ...(1)$$



where A, B and C are constants to be determined from the following boundary conditions :

(*i*) $u = 0$ at $y = 0$

(*ii*) $u = 1$ m/s, at $y = 0.15$ m

(*iii*) $du/dy = 0$ at the vertex, *i.e.*, $y = 0.15$ m

Boundary condition (*i*) gives $C = 0$ and from (*ii*), we obtain

$$1 = A(0.15)^2 + B(0.15) \qquad ...(2)$$

and from (*iii*), $$\frac{du}{dy} = 2Ay + B,$$

or $$0 = 2A(0.15) + B \qquad ...(3)$$

Solving Eqs. (2) and (3), $A = -44.4$ and $B = 13.33$

Eq. (1) for the velocity profile now becomes

$$u = -44.4y^2 + 13.33y.$$

The velocity gradients and the shear stresses the desired points may be obtained as below :

(a) At y = 0 mm, $\frac{du}{dy} = -2 \times 44.4 \times 0 + 13.33 = 13.33 \text{ sec}^{-1}$

Shear stress $\tau = \mu \frac{du}{dy} = 0.804\ (13.33) = 10.8 \text{ N/m}^2$

(b) At y = 100 mm, $\frac{du}{dy} = -2 \times (44.4) \times (0.1) + 13.33 = 4.45 \text{ sec}^{-1}$

$$\tau = 0.804\ (4.45)$$
$$= 3.575 \text{ N/m}^2$$

(c) At y = 150 mm, $\frac{du}{dy} = -2 \times (44.4)(0.15) \times 15 + 13.33 = 0$

$$\tau = \mu \frac{du}{dy} = 0.$$

Example 1.5. *Two horizontal plates are placed 12.5 mm apart, the space between them being filled with oil of viscosity 14 poise. Calculate the shear stress in the oil if the upper plate moves with a velocity of 2.5 m/s.*

Solution. 1 poise = 1 dyne-s/cm^2 = $\frac{1}{98.1}$ kg-s/m^2 = 0.1 N-s/m^2

Shear stress $\tau = \mu \frac{du}{dy}$

Relative velocity between the plates, du = 2.5 m/s.

Distance between the plates, dy = 1.25 cm = 0.0125 m

Viscosity of the oil, μ = 14 poise

$$= \frac{14}{98.1} = 0.143 \text{ kg-s/m}^2 = 1.4 \text{ N-s/m}^2$$

(since 1 Poise = 0.1 N-s/m^2)

Substituting in the formula,

$$\tau = 0.143 \times \frac{2.5}{0.0125} = 28.55 \text{ kg/m}^2 = 280.0 \text{ N/m}^2.$$

Example 1.6. *A rectangular plate 1.2 m × 0.4 m, weighing 970 N slides down a 45° inclined surface at a uniform velocity of 2.25 m/s. If the 2 mm gap between the plate and the inclined surface is filled with oil, determine its viscosity.*

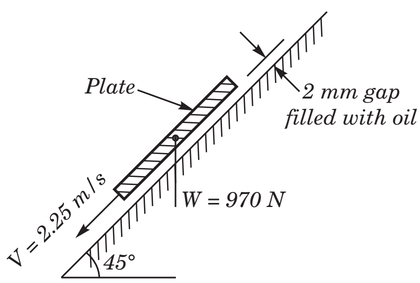
Solution. The sliding plate will attain the uniform velocity when the fore causing the motion (*i.e.* the component of the plates weight along the inclined surface) balances the fluid resistance offered by the oil filled in the gap.

$$W \sin 45° = \mu \frac{du}{dy} \,.\, A$$

$$= \mu \frac{V}{2/(10 \times 100)} \times 1.2 \times 0.4$$

or $$970 \sin 45° = \mu \frac{2.25 \times 10^3}{2} \times 1.2 \times 0.4$$

$$\therefore \quad \mu = \frac{970 \times 1/\sqrt{2}}{0.54 \times 10^3} = 2.54 \text{ Ns/m}^2 = \mathbf{25.4\ Poise.}$$

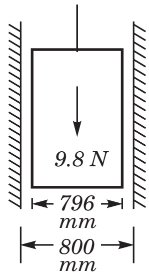


Example 1.7. *A piston 796 mm diameter and 200 mm long works in a cylinder of 800 mm diameter. If the annular space is filled with a lubricating oil of viscosity 5 cP, calculate the speed of descent of piston in vertical position. The weight of piston and the axial load are 9.8 N.*

Solution. Viscosity of oil

$$= 5 \text{ cP} = 5 \times 10^{-2} \text{ Poise}$$

$$= 0.5 \times 10^{-2} \text{ N-s/m}^2 = 5 \times 10^{-3} \text{ N-s/m}^2.$$



From Newton's law of viscosity, the shear stress

$$\tau = \mu \frac{du}{dy} \quad \text{...}(i)$$

Shear stress developed by the vertical descent of the piston

$$= \frac{\text{Shear force } (i.e. \text{ weight of piston etc.})}{\text{Piston area in contact with oil}}$$

$$= \frac{9.8}{\pi \times 0.796 \times 0.20} = 19.61 \text{ N/m}^2.$$

Velocity gradient when the piston attains a constant velocity of V (m/s) in the annular gap of 2 mm

$$\frac{du}{dy} = \frac{V}{2 \times 10^{-3}}.$$

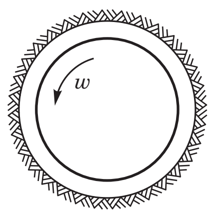
Substituting in the Newton's law, Eq. (i)

$$19.61 = 5.1 \times 10^{-3} \times \frac{V}{2 \times 10^{-3}}$$

from which, $V =$ **7.841 m/s.**

Example 1.8. *A cylinder of 150 mm radius rotates concentrically inside a fixed cylinder of 155 mm radius. Both cylinders are 300 mm long. Determine the viscosity of the liquid which fills the space between the cylinders if a torque of 0.98 N-m is required to maintain an angular velocity of 60 r.p.m.*

Solution. The torque is transmitted through the fluid layers to the outer cylinder.



Tangential velocity of the inner cylinder = $r\omega$

$$= r \frac{2\pi n}{60}$$

$$= 0.15 \times \frac{2\pi \times 60}{60} = 0.943 \text{ m/s}.$$

For the small space between the cylinders, the velocity profile may be assumed to be a straight line, then

$$\frac{du}{dy} = \frac{0.943}{\frac{(15.5 - 15.0)}{100}} = 188.6 \text{ per sec.}$$

Torque applied = Torque resisted

$$0.98 = \tau \times \text{Area} \times \text{Lever arm} = \tau \times (2\pi \times 0.15 \times 0.30) \times 0.15$$

$$\tau = \frac{0.98}{2\pi \times 0.045 \times 0.15} = 23.15 \text{ N/m}^2$$

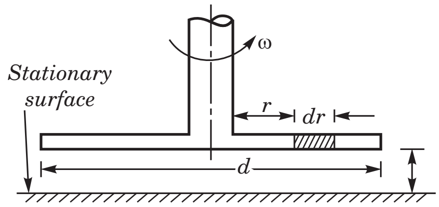
or

$$\mu = \frac{\rho}{du/dy} = \frac{23.15}{188.6} = 0.123 \text{ N-s/m}^2.$$

Example 1.9. *A circular disc of a diameter 'd' is slowly rotated in a liquid of large viscosity 'μ' at a small distance 'h' from a fixed surface. Derive an expression for torque 'T' necessary to maintain an angular velocity 'ω'.*

Solution. Consider an element of disc at a radius r and having a width dr.

Linear velocity at this radius = $r\omega$



Shear stress $\tau = \mu \dfrac{du}{dy}$

Torque = Shear stress × Area × r

$= \tau \times 2\pi r\, dr \times r$

$= \mu \dfrac{du}{dy} . 2\pi r^2\, dr.$

Assuming the gap h to be small so that the velocity distribution may be assumed linear.

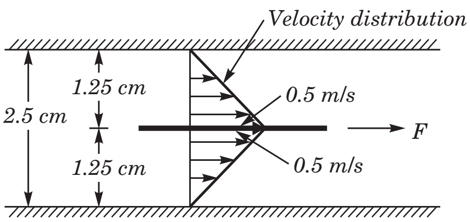
$$\frac{du}{dy} = \frac{r\omega}{h}$$

∴ Torque, dT, on the element

$$dT = \mu \frac{r\omega}{h} . 2\pi r^2\, dr = \frac{2\pi\mu\omega}{h} r^3 dr$$

Total torque,
$$T = \int_0^{d/2} \frac{2\pi\mu\omega}{h} r^3\, dr = \frac{2\pi\mu\omega}{h} \left| \frac{r^4}{4} \right|_9^{d/2} = \frac{\mu\pi d^4\omega}{32\, h}.$$

Example 1.10. *A space 25 mm wide between two large plane surfaces is filled with glycerine. What force is required to drag a very thin plate 0.75 sq metre in area between the surfaces at a speed of 0.5 m/s (i) if this plate remains equidistant from the two surfaces, (ii) if it is at a distance of 10 mm from one of the surfaces ? Take μ = 0.785 N-s/m².*



Solution. Total force required to drag the plate

= Sum of the forces on either side of the plate

∴ $F = F_1 + F_2.$

Case I. *When the plate is located midway between the surfaces.*

Since the space between the surfaces is small, the velocity distribution may be considered as straight line.

The shear force on the upper side of the plate

$F_1 = \tau_1$ × Area of the plate

$$= \mu \left(\frac{du}{dy}\right)_1 \times 0.75 = 0.785 \times \left(\frac{0.5}{2.5/2 \times 100}\right) \times 0.75 = 23.5 \text{ N}$$

The force on the bottom side of the plate

$$F_2 = \mu \left(\frac{du}{dy}\right)_2 \times \text{Area of plate} = 0.785 \left(\frac{0.5}{2.5/2 \times 100}\right) \times 0.75 = 23.5 \text{ N}.$$

∴ The total resistance force (which is equal to the force required to drive it) experienced by the plate = $F_1 + F_2$ = 47.0 N.

Case II. *When the plate is located at a distance of 10 mm from one of the surfaces.*

Force on the upper side of the plate

$$F_1 = \mu \left(\frac{du}{dy}\right) \times 0.75 = 0.785 \left(\frac{0.5}{1.5/100}\right) \times 5 = 19.60 \text{ N}.$$

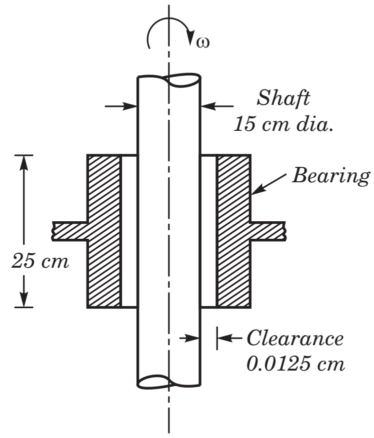
Force on the bottom side of the plate

$$F_2 = \mu \left(\frac{du}{dy}\right) \times 0.75 = 0.785 \left(\frac{0.5}{1/100}\right) \times 0.75 = 29.40 \text{ N}$$

Total force required = $F_1 + F_2$ = 49.0 N.

Example 1.11. *Lateral stability of a long shaft 150 mm in diameter is obtained by means of a 250 mm stationary bearing having an internal diameter of 150.25 mm. If the space between bearing and shaft is filled with a lubricant having a viscosity 0.245 N-s/m², what power will be required to overcome the viscous resistance when the shaft is rotated at a constant rate of 180 r.p.m. ?*

Solution. Circumferential velocity of the shaft



$$V = r\omega$$

$$= r \cdot \frac{2\pi n}{60} = \frac{15/2}{100} \times \frac{2\pi \times 180}{60} = 1.412 \text{ m/s}$$

Velocity gradient,

$$\left(\frac{du}{dy}\right) = \frac{1.412}{\frac{0.025}{2 \times 100}} = 1.13 \times 10^4 \text{ sec}^{-1}$$

Shear stress on the shaft

$$\tau = \mu \left(\frac{du}{dy}\right) = 0.245 \times 1.13 \times 10^4$$

$$= 2.77 \times 10^3 \text{ N/m}^2 = 2.77 \text{ kN/m}^2$$

Shear force on the shaft

$$F = \tau \cdot 2\pi r l = 2.77 \times 10^3 \times 2 \times \pi \times \frac{15.2}{2 \times 100} \times \frac{25}{100} = 327.5 \text{ N}$$

Torque to be overcome by the shaft = Shear force × Radius of shaft

$$T = F \cdot r$$

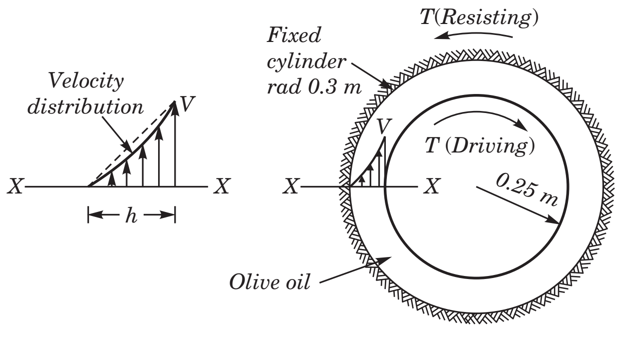
$$= 327.5 \times \frac{15}{2 \times 100} = 24.6 \text{ N-m.}$$

Power corresponding to this torque at a speed of n = 180 r.p.m.

$$P = \frac{2\pi n}{60} \times T = \frac{2\pi \times 180}{60} \times 24.6$$

$$= 463.0 \text{ N-m/s} = 463.0 \text{ W} = 0.463 \text{ kW.}$$

Example 1.12. *A cylinder 0.25 m in radius and 2 m length rotates coaxially inside a fixed cylinder of the same length and 0.30 m radius. Olive oil of viscosity 4.9 × 10⁻² Ns/m² fills the space between the cylinders. A torque 4.9 N-m is applied to the inner cylinder. After constant velocity is attained, calculate the velocity gradient at the cylinder walls, the resulting r.p.m., and the power dissipated by fluid resistance ignoring end effects.*



Solution. The surface area of the outer cylinder is larger than that of the inner one, since the former has a larger radius. Accordingly the shear force and the velocity gradient at the outer cylinder will be less than the respective quantities on the inner one. The velocity profile through the fluid will be non-linear as indicated in the figure, since the gap between the inner and outer cylinders is comparatively larger.

The torque of 4.9 Nm is transmitted from inner cylinder to the outer one through fluid friction (viscous effect). Let r be the radial distance of any fluid layer.

Then $$4.9 = \tau \times (2\pi r l) \times r = \tau .2\pi r \times 2 \times r$$

$$= \tau .4\pi r^2 = \mu \frac{du}{dy} . 4\pi r^2$$

$$= 4.9 \times 10^{-2} \times 4\pi r^2 \frac{du}{dy}$$

$$\therefore \quad \frac{du}{dy} = \frac{100}{4\pi r^2} = \frac{7.95}{r^2}.$$

The velocity gradients at the inner at outer cylinders are :

$$\left(\frac{du}{dy}\right)_i = \frac{7.95}{(0.25)^2} = 127.2 \text{ sec}^{-1}$$

and $$\left(\frac{du}{dy}\right)_0 = \frac{7.95}{(0.30)^2} = 88.3 \text{ sec}^{-1}.$$

Substituting $(-dr)$ for dy in the equation for du/dy since velocity decreases as r increases. Integrating,

$$\int_0^V du = -7.95 \int_{0.30}^{0.25} \frac{dr}{r^2}$$

$\therefore$ Velocity of inner cylinder,

$$V = 7.95 \left[\frac{1}{r}\right]_{0.30}^{0.25} = 5.30 \text{ m/s}$$

speed of inner cylinder, $$\omega = \frac{V}{r} = \frac{5.3}{0.25} = 21.2 \text{ rad/sec.}$$

$$n = \frac{60\omega}{2\pi} = \frac{60 \times 21.2}{2\pi} = \mathbf{202.4 \ r.p.m.}$$

Assuming the velocity profile to be linear for an approximate calculation

$$V = 127.1 \times 0.05 = 6.35 \text{ m/s}$$

and $$n = 242.5 \text{ r.p.m.}$$

Since this result differs from the former by nearly 20%, the approximation is not satisfactory in this case.

The power dissipated in fluid friction

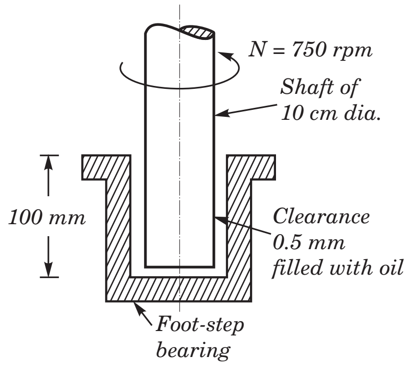
$$= \frac{2\pi n T}{60} = \frac{2\pi \times 202.4 \times 4.9}{60} = 104.0 \text{ Nm/s} = \mathbf{104.0 \ W.}$$

Example 1.13. *The lower end of a vertical shaft of diameter 10 cm rests in a foot step bearing (length 100 mm). The clearance between the lower end of the shaft and the bearing surface is 0.5 mm. If the shaft has to run at 750 rpm, find the torque required to keep the shaft in motion. Find also the power required. Take dynamic viscosity as 1.5 poise.* (*RGPV, 2013 June*)

Solution. Torque required to keep the shaft in motion

= shear force on shaft × its radius

Shear stress, $\tau = \mu \frac{d\mu}{dy}$



Referring to the figure, the foot-step bearing and the shaft is shown. The plan of the shaft and the bearing surface shows the velocity distribution in the annular clearance.

The shear stress on the shaft's outer surface is given by the Newton's law of viscosity, and will be calculated with the data given :

$$\mu = 1.5 \text{ poise} = 1.5 \times 0.1 \text{ (N.s/m}^2\text{)}$$
$$(10 \text{ poise} = 1 \text{ Ns/m}^2)$$

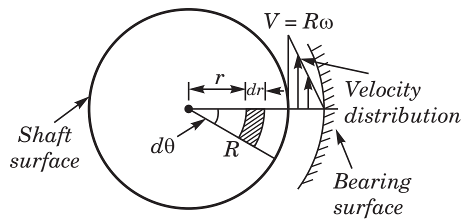
Speed of rotation of shaft, $N = 750$ rpm

Velocity of rotation, $V = R\omega$

$$= \frac{5}{100} \times \frac{2\pi \times 750}{60} \text{ (m/s)}$$
$$= 3.928 \text{ m/s}$$

The bearing surface being stationary, the velocity is zero, and

$$du = V - O \quad = 3.928 \text{ m/s}$$
$$dy = \text{clearance} = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$$

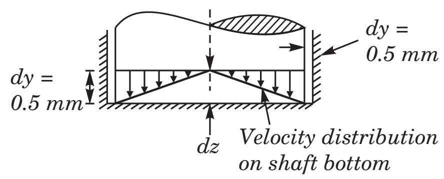


Shear stress, $\tau = 0.15 \times \dfrac{3.928}{0.5 \times 10^{-3}}$

$$= 1178.57 \text{ N/m}^2$$

Shear force on the shaft surface of area $2\pi Rl$

$$F = \tau.\ 2\pi Rl = 1178.57 \times 2\pi \times (10 \times 10^{-2}) \times (5 \times 10^{-2})$$
$$= 18.689 \text{ N}$$



Torque exerted on the shaft

$$= F.R = 11.689 \times (5 \times 10^{-2})$$
$$= \mathbf{0.9344 \text{ Nm}}$$

$$\text{Power required} = \text{Torque} \times \omega = 0.9344 \times \frac{2\pi \times 750}{60}$$
$$= 73.35 \text{ Nm/s} = \mathbf{73.35 \text{ W}} \qquad ...(i)$$

To this, there is also the contribution of the shear force exerted on the shaft bottom.

To evaluate this, it is necessary to assume the velocity distribution at the bottom end of the shaft, as shown in the bottom-most sketch. The velocity at the shaft axis zero and it varies radially, there being a triangular velocity distribution with maximum velocity equal to $R\omega$. Consider the sectorial sketch in the plan, an element of shaft at a radius r, having thickness dr, the sector angle being $d\theta$.

Shear force on this element of area $rd\theta.dr$ is (dy being 0.5 mm)

$$dF = \tau \times \text{area of element} = \mu \frac{r\omega}{0.5 \times 10^{-3}} \times rd\theta\, dr$$

Torque exerted on this elementary area situated at radius r

$$dT = dF.r = \frac{\mu\omega r^2\, d\theta dr}{0.5 \times 10^{-3}}.r$$
$$= 2\mu\omega \times 10^3\, r^3\, d\theta dr$$

To get the total torque exerted on this shaft bottom, we integrate the expression for dT

$$T = \int_0^{2\pi}\int_0^R dT = 2\mu\omega \times 10^3 \int_0^{2\pi}\int_0^R r^3 dr\, d\theta$$

$$= 2 \times 10^3\ \mu\omega \int_0^{2\pi} \left| \frac{r^4}{4} \right|_0^R d\theta = 2 \times 10^3\ \mu\omega \int_0^{2\pi} \frac{R^4}{4}\, d\theta$$

$$= 2 \times 10^3\ \mu\omega \left| 0 \right|_0^{2\pi} \frac{R^4}{4}$$

$$= \frac{2\mu\omega R^4}{4} .\ 2\pi = \pi R^4 \mu\omega$$

Power reqd. by the shaft bottom = $T.\omega$

$$= \pi R^4 \mu \omega^2$$

$$= \pi \times (5 \times 10^{-2})^4 \times 0.15 \times \left(\frac{2\pi \times 750}{60}\right)^2 \text{ Nm/s}$$

$$= \pi \times 625 \times 10^{-8} \times 0.15\ (25\ \pi)^2 \text{ Nm/s}$$

$$= 1962.5 \times 10^{-8} \times 0.15 \times 6162.25 \text{ Nm/s}$$

$= 0.01814$ Nm/s = **0.1814 W** ...(*ii*)

$\therefore$ Power required to keep the shaft in motion

= 73.35 + 0.1814 = **73.5314 W**

The contribution of the torque and therefore of power required by the shaft bottom to the total power

$$= \frac{0.01814}{73.5314} = 0.0002466 \text{ or } \mathbf{0.0247\%}$$

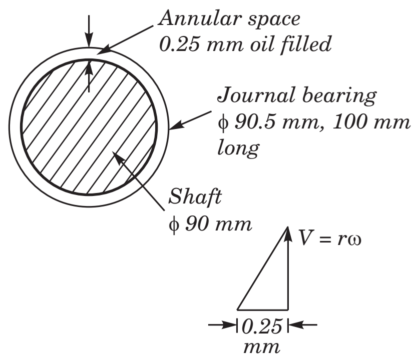
and is negligible.

Example 1.14. *A 90 mm diameter shaft rotates at 1200 rpm in a 100 mm long journal bearing of 90.5 mm internal diameter. The annular space in the bearing is filled with oil having a dynamic viscosity of 0.12 Pa.s. Estimate the power dissipated.*

Solution. Torque exerted on the rotating shaft due to viscous resistance offered to it by the journal bearing :

T = Shear force exerted on the shaft × radius of shaft

$$= \left(\mu \frac{du}{dy}\right) A \times r = \mu \frac{du}{dy} .\ \pi d\ L\ .\ r$$



Assuming linear variation of velocity in the annular space, as shown, the velocity gradient ($du = v = rw$, dy = 0.25 mm)

$$\frac{du}{dy} = \frac{V}{0.25 \times 10^{-3}} = \frac{(2\pi r\ N / 60)}{0.25 \times 10^{-3}}$$

$$= 2\pi \left(\frac{90 \times 10^{-3}}{2}\right) \times \frac{1200}{60} = 36\pi \times 10^2 \text{ m/s/m}$$

$\therefore$ $T = 0.12$ (N.s/m^2) $(36\pi \times 10^2)$ $(\pi \times 90 \times 10^{-3} \times 100 \times 10^{-3}) \times 45 \times 10^{-3}$

= 1.728 N.m

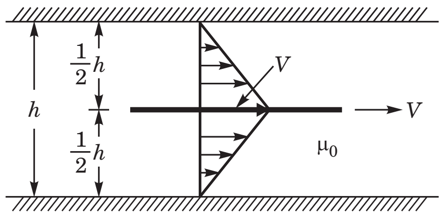
Power dissipated in overcoming viscous friction

$$P = \frac{2\pi\ NT}{60} = \frac{2\pi \times 1200 \times 1.728}{60}$$

= 217.06 N.m/s = **217.06 W**.

Example 1.15. *A thin plate of large area is placed midway in a gap of height h filled with oil of viscosity* μ_0 *and the plate is pulled at a constant velocity V. If a lighter oil of viscosity* μ_1 *is then substituted in the gap, it is found that for the same velocity V, the drag force will be the same as before if the plate is located unsymmetrically in the gap but parallel to the walls. Find* μ_1 *in terms of* μ_0 *and the distance from the nearer wall to the plate.*

Solution. Case (*i*) *When the liquid of viscosity* μ_0 *fills the gap*. Since the plate is placed midway in the gap, the velocity profile on both the sides of the plate will be symmetrical, thus



$$\frac{du}{dy} = \frac{V}{h/2} = \frac{2V}{h}.$$

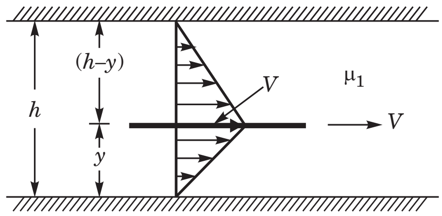
The shear force on the upper and the bottom side of the plate will be same, and hence the drag force on the plate

$$F_1 = \left[\mu_0\left(\frac{du}{dy}\right)_u + \mu_0\left(\frac{du}{dy}\right)_b\right]A = \left[\mu_0\frac{2V}{h} + \mu_0\frac{2V}{h}\right]A = \frac{4V\mu_0 A}{h}$$

where A is the area of plate.

Case (*ii*) *When the liquid of viscosity* μ_1 *fills the gap*. Let the plate be placed at a distance of y from the bottom wall as shown.

The velocity gradients for the upper and bottom sides of the plate are :



$$\left(\frac{du}{dy}\right)_u = \frac{V}{(h-y)}$$

and
$$\left(\frac{du}{dy}\right)_b = \frac{V}{y}.$$

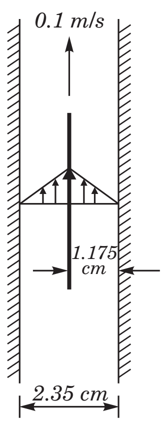
Now, the drag force on the plate

$$F_2 = \left[\mu_1\left(\frac{du}{dy}\right)_u + \mu_1\left(\frac{du}{dy}\right)_b\right]A = \left[\mu_1\frac{V}{(h-y)} + \mu_1\frac{V}{y}\right]A = \frac{\mu_1 VhA}{y(h-y)}.$$

But since the drag forces F_1 and F_2 are equal, we have

$$\frac{4V\mu_0 A}{h} = \frac{\mu_1 VhA}{y(h-y)} \qquad \text{or} \qquad \mu_1 = 4\mu_0\frac{y}{h}\left[1-\frac{y}{h}\right]$$

Example 1.16. *A vertical gap 23.5 mm wide of infinite extent contains oil of specific gravity 0.95 and viscosity 2.45 N-s/m*^{2}*. A metal plate 1.5 m × 1.5 m × 1.5 mm weighing 49 N is to be lifted through the gap at a constant speed of 0.1 m/s. Estimate the force required.*



Solution. Let the plate be placed midway in the gap. The velocity gradient

$$\frac{du}{dy} = \frac{0.1}{1.10 \times 100} = 9.09\ \text{sec}^{-1}.$$

Viscous resistance to be overcome by the plate

= Shear force on the plate

= Sum of the shear force acting on each face

$$= 2 \, . \, \mu \frac{du}{dy} \, . \, A$$
$$= 2 \times 2.45 \times 9.09 \times (1.5 \times 1.5)$$
$$= 100.1 \text{ N}$$

Total force required

= Immersed weight of the plate + Viscous resistance

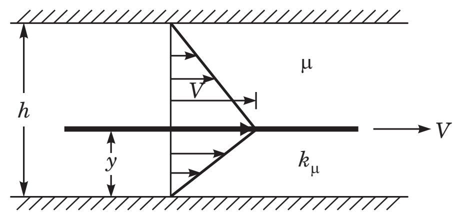
$$= (49.0 - 0.95 \times 9810 \times 1.5 \times 1.5 \times 0.0015) + 100.1$$
$$= 118.0 \text{ N.}$$

Example 1.17. *Through a very narrow gap of height h, a thin plate of a very large extent is being pulled at constant velocity V. On one side of the plate is oil of viscosity* μ *and on the other side oil of viscosity* $K\mu$. *Calculate the position of the plate so that drag force on it will be a minimum.*

Solution. Let the thin plate be placed at a distance y from one of the surfaces as shown. The drag force per unit area of the plate

F = Sum of shear forces per unit area on both the faces of plate

$$= \mu \left(\frac{du}{dy}\right)_u + K\mu \left(\frac{du}{dy}\right)_b$$



where the subscripts u and b refer to the upper and bottom sides of the plate, thus

$$F = \mu \frac{V}{(h-y)} + K\mu \frac{V}{y}$$

For the drag force to be minimum,

$$\frac{dF}{dy} = 0$$

$$\frac{dF}{dy} = \mu V \frac{1}{(h-y)^2} - K\mu \frac{V}{y^2} = 0$$

or $$\frac{\mu}{K\mu} = \frac{h^2 + y^2 - 2hy}{y^2} = \frac{h^2}{y^2} + 1 - 2\frac{h}{y}$$

or $$\frac{h^2}{y^2} - 2\frac{h}{y} + \left(1 - \frac{\mu}{K\mu}\right) = 0.$$

Solving the quadratic equation for h/y

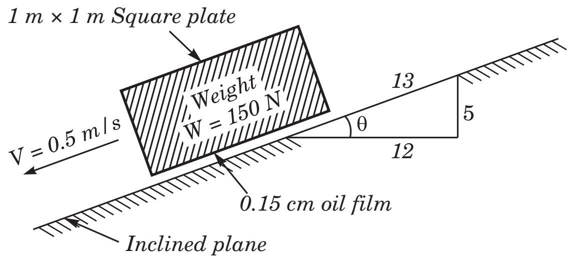
$$\frac{h}{y} = \frac{2 \pm \sqrt{4 - 4\left(\frac{\mu}{K\mu}\right)}}{2} = 1 \pm \sqrt{\frac{\mu}{K\mu}}$$

Since h/y cannot be less than unity, using the plus sign

$$\frac{h}{y} = 1 + \sqrt{\frac{\mu}{K\mu}} \qquad \text{or} \qquad y = \frac{h}{1 + \sqrt{\frac{\mu}{K\mu}}}$$

Example 1.18. *Calculate the approximate viscosity of the oil for the following case* :

Solution. When the constant velocity of 0.5 m/s is attained, the viscous resistance to the motion is equal to the component of the weight of plate along the slope.



Component of weight along the slope

$$= W \sin \theta = 150 \times \frac{5}{13} = 56.6 \text{ N}$$

Viscous resistance $\quad = \mu \cdot \dfrac{du}{dy} \cdot A = \mu \cdot \dfrac{0.5}{(0.15/100)} (1 \times 1) = 333.5 \, \mu$

Hence $\quad 333.5 \, \mu = 56.6$

$$\therefore \quad \mu = \frac{56.6}{333.5} = 0.0173 \times 9.81 = 0.17 \text{ N-s/m}^2 = 0.17 \text{ Pa.s.}$$

1.5. Surface Tension

The molecules of a liquid are held together by a force of attraction known as *cohesion*, the magnitude of which is very small, yet it enables the liquid to withstand a small tensile stress. The liquid molecules exert an attractive force upon all other molecules with which it comes into contact. The force of attraction between the molecules of two different liquids which do not mix or between the liquid molecules and molecules of solid boundary containing the liquid, is known as *adhesion*.

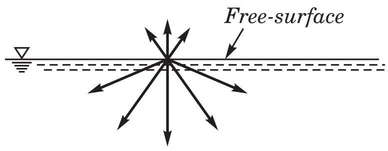


Fig. 1.6. Intermolecular force near a free surface.

At surface of contact between a gas and a liquid (like air and water) or between two different immiscible (liquids that do not mix with each other) liquids, molecular attraction introduces a force which causes the interface (*i.e.* the contact surface) to behave like a membrane under tension. Within the body of a liquid a molecule is attracted equally in all directions by the other molecules surrounding it, but at the liquid-air interface or at the contact surface between two immiscible liquids the upward and downward attractions are unbalanced, giving rise to the phenomenon of surface tension. The liquid-air interface behaves as if it were an elastic membrane under tension. This surface tension is the same everywhere on the surface, and acts in the plane of the surface normal to any line in the surface. As shown in Fig. 1.6, a liquid molecule at the free surface will be exerted upon by a smaller force from the free surface side, giving rise to a resultant downward force acting at right angles to the free surface. This imbalance of molecular force gives rise to surface tension. It is a force which exists on the surface of a liquid when it is in contact with another fluid or a solid boundary. Its magnitude depends upon the relative magnitude of cohesive and adhesive forces. Surface tension is a force in the liquid surface and acts normal to a line of unit length drawn imaginarily on the surface. Thus, it is a line force. It represents surface energy per unit area (unit surface energy), and is denoted by Greek letter σ (sigma). It has dimensions $[FL^{-1}]$ or $[MT^{-2}]$ and is expressed in kg/m in MKS units and N/m in SI units.

A small needle gently placed on the liquid surface will not sink and will be supported by the surface tension. This is made possible by the localised curvature and depression caused by the needle.

1.5.1. Effect of temperature

As the surface tension depends directly upon the intermolecular cohesion, and since cohesion is known to decrease with temperature rise, the surface tension decreases with the rise of temperature. Its value for water-air contact (free-surface of water) reduces from 0.00745 kg/m (0.0731 N/m) at 17.8°C to 0.00596 kg/m (0.0585 N/m) at 100°C.

1.5.2. Capillary action

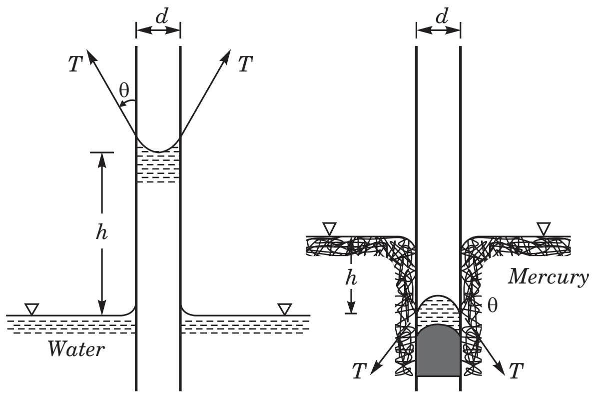


Fig. 1.7. Capillary action in a glass tube.

The molecules of a solid surface attract liquid molecules with a greater force than that which exist between the liquid molecules (except mercury). Because of this, most of the liquids completely wet the surface. The extent of wetting will depend upon the relative magnitude of the forces of adhesion and cohesion. If the adhesive force is greater than that of cohesion, the liquid tends to spread out and wet the surface. If the cohesive force is greater than the adhesive force, a small drop of the liquid placed on the solid surface will remain in the drop form *e.g.* a small mercury drop retains its almost spherical shape while resting on a solid surface. Mercury does not wet the surface because of its greater cohesion. If water drops are placed on a solid surface, they will completely spread out over the surface and will wet it. This is due to the fact that the adhesion between the molecules of water and the solid surface is greater than the cohesion between the water molecules.

The free surfaces of mercury and water will, therefore, behave differently at places where they come in contact with a solid surface. When a glass tube is dipped vertically into water, the water rises in the tube. If the glass tube is placed in mercury, the surface of mercury inside the tube will be lower than the outside level. Fig. 1.7 exhibits, what is known as the capillary action. The rise of water in the tube is called the capillary rise and the fall of mercury is termed as the capillary depression. The phenomenon of rise and fall of liquid in a capillary tube is known as *capillarity*. Its magnitude depends upon the diameter of tube, the specific weight of the liquid and its surface tension, and may be obtained by the following analysis. If the angle of contact between the liquid and the solid surface is θ, the water in the glass tube will continue to rise until the vertical component of the surface tension force ($T \cos \theta$) which acts over the wetted length (circumference of the tube) at the free surface equals the weight of the water column. Thus

$$T \cos \theta = \frac{\pi d^2}{4} hy$$

where $T = \sigma \pi d$. Substituting this value of T, the capillary rise h is given by

$$h = \frac{(4 \cos \theta)\sigma}{\gamma d} \quad ...(1.6)$$

For pure water and clean glass surface, θ is almost equal to zero, but under actual conditions the water is neither pure nor the glass is clean. For water in contact with glass and air, Gibson has obtained the value of θ as 25° 32′, and that of σ = 0.0075 kg/m (0.0735 N/m). If h and d are expressed in mm. Eq. (1.6) reduces to

$$h = \frac{27.07}{d} \text{ mm} \quad ...(1.7)$$

A similar analysis for glass tube placed in mercury shows that the mercury is depressed by an amount h given by Eq. (1.6). For mercury Gibson has obtained θ = 128° 52′ and the specific gravity and the surface tension may be taken as 13.55 and 0.53 kg/m (5.2 N/m) respectively. With h and d both expressed in mm, Eq. (1.6) for glass-air contact becomes

$$h = \frac{9.6}{d} \text{ mm} \qquad ...(1.8)$$

Glass tubes are commonly used for measuring pressure of flow, and in order that they give correct pressure observations, it is necessary that the rise of water or any other liquid should not be influenced by the capillary action. To ensure this, the diameter of tube should be large enough so that the capillary rise is negligible.

Table 1.2 gives the values of h for glass tubes of different diameter for water and mercury as calculated from the above equation.

Table 1.2

	Capillary rise or depression, h in mm					
Tube diameter, d in mm	2	5	10	15	20	25
1. Water	13.4	5.4	2.7	1.8	1.3	1.1
2. Mercury	4.8	1.9	1.0	0.6	0.5	0.4

From this table it is obvious that if a smaller tube is used for pressure measurement, the height of liquid in the tube which indicates pressure of flow will not represent the correct pressure, as it includes the capillary rise. For this reason, the diameter of tube should never be less than 1 cm.

Liquid drop, jet and soap bubble :

In case of a liquid drop or inside a jet, the action of the surface tension is to increase the pressure inside, in relation to the outside pressure. In a liquid drop of diameter d, if Δp is the difference of pressure between the inside and outside of the drop, then using Fig. 1.8 (a).

$$\Delta p \,.\, \pi d^2/4 = \sigma \pi d$$

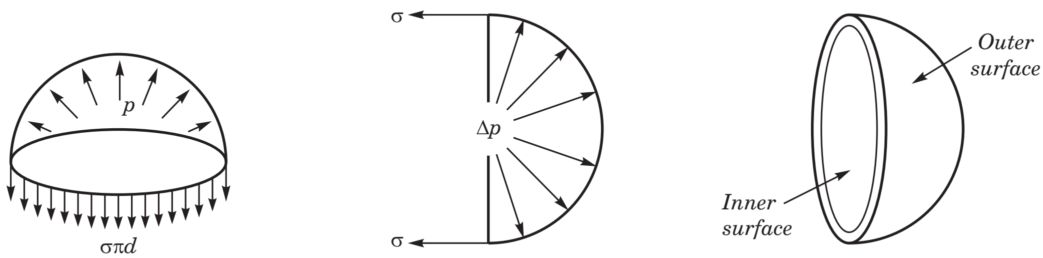
$$\Delta p = \frac{4\sigma}{d} \qquad ...(1.9)$$

and in case of a liquid jet of diameter d and of unit length, we have from Fig. 1.8 (b).

$$2\sigma = \Delta p \,.\, d$$

$$\Delta p = \frac{2\sigma}{d} \qquad ...(1.10)$$

A soap bubble in air has two surfaces in contact with air, one inside and the other outside, Fig. 1.8 (c). The forces that act on the hemispherical section are same as those for the drop, but the surface tension force is twice as great. The pressure difference is given by



(a) Forces on hemispherical section of liquid drop (b) Process on half-cylindrical section of liquid jet (c) Two surfaces of a soap bubble

Fig. 1.8

$$\Delta p \,.\, \frac{\pi d^2}{4} = 2\pi d\sigma$$

or
$$\Delta p = \frac{8\sigma}{d} \qquad \text{...(1.11)}$$

A soap solution has a high value of σ, which causes a soap bubble to be larger in diameter for small pressure of blowing.

Example 1.19. *A soap bubble 25 mm in diameter has inside pressure of 20.0 N/m² above atmosphere. Calculate the tension in the soap film.*

Solution. Using Eq. (1.11), $\Delta p = \dfrac{8\sigma}{d}$

∴ Surface tension in soap film $\sigma = \dfrac{\Delta p \,.\, d}{8}$

$$= \frac{20}{8}\,(25/1000) = \mathbf{0.0625\ N/m.}$$

Example 1.20. *If the surface tension of water in contact with air is 0.075 N/m, what correction need be applied toward capillary rise in the manometric reading in tube of 3 mm diameter.*

Solution. Assuming the manometer tube made of glass, for water-air-glass contact gibson has determined the angle of contact θ as 25° 32′, and surface tension σ = 0.0075 kg/m. The capillary rise is given by Eq. (1.6) as

$$h = \frac{(4\cos\theta)\,\sigma}{\gamma d}$$

$$= \frac{4\cos 25.533° \times 0.075}{9810 \times (3/1000)} = 0.009198 \text{ m} = \mathbf{9.198\ mm}$$

However for pure water and clean glass, θ = 0°, and the capillary rise,

$$h = \frac{4\sigma}{\gamma d}$$

$$= \frac{4 \times 0.075}{9810 \times (3/1000)} = 0.01019 \text{ m} = 10.19 \text{ mm}$$

Note. From the above two values of h, it can be seen that the effect of impure water and unclean glass is reflected in decrease in the value of capillary rise.

Example 1.21. *Calculate the capillary effect in millimetres in a glass tube of 4 mm diameter, when immersed in (i) water and (ii) mercury. The temperature of the liquid is 20°C and the values of surface tension of water and mercury at 20°C in contact with air are 0.0736 N/m and 0.51 N/m respectively. The angle of contact for water is zero and that for mercury 130°.*

Solution. The capillary effect is given by Eq. (1.6),

$$h = \frac{(4\cos\theta)\,\sigma}{\gamma d}$$

For water : σ = 0.0736 N/m γ at 20°C = 9790 N/m³

θ = 0°, $d = 4 \times 10^{-3}$ m

$$h = \frac{4\cos 0° \times 0.0736}{9790 \times 4 \times 10^{-3}} = 7.51 \times 10^{-3} \text{ m}$$

= 7.51 mm (rise of water)

For mercury : σ = 0.51 N/m γ = 13.6 × 9790 = 133 kN/m³

θ = 130°, $d = 4 \times 10^{-3}$ m

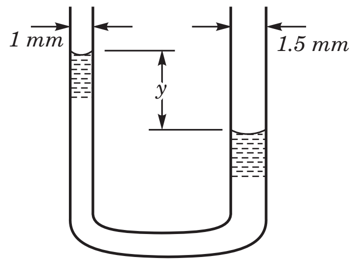
$$\therefore \qquad h = \frac{4 \times \cos 130° \times 4.51}{133 \times 10^3 \times 4 \times 10^{-3}} = -2.46 \times 10^{-3} \text{ m}$$

$$= -2.46 \text{ mm (depression).}$$

Example 1.22. *A U-tube is made of two capillaries of diameter 1.0 mm 1.5 mm respectively. The tube is kept vertically and partially filled with water of surface tension 0.0736 N/m and zero contact angle. Calculate the difference in the levels of miniscii caused by the capillarity.*

Solution. Capillary rise in a circular tube is given by Eq. (1.6),

$$h = \frac{4\sigma \cos \theta}{\gamma d}$$



According to the data given

$$\theta = 0°, \ \sigma = 0.0736 \text{ N/m}$$

(*i*) Capillary rise in 1.0 mm tube

$$h_1 = \frac{4 \times 0.0736 \times 1}{9810 \times 1.0 \times 10^{-2}} = 0.030 \text{ m}$$

$$= 30 \text{ mm}$$

(*ii*) Capillary rise in 1.5 mm tube

$$h_2 = \frac{4 \times 0.0736 \times 1}{9810 \times 1.5 \times 10^{-2}} = 0.020 \text{ m}$$

$$= 20 \text{ mm.}$$

$\therefore$ The difference in levels of water in the two limbs caused by the surface tension effect

$$y = h_1 - h_2 = 30 - 20 = \mathbf{10 \text{ mm.}}$$

Example 1.23. *The diameters of the two glass limbs of a differential U-tube manometer were found to be 5 mm and 6 mm respectively. In an experiment the differential pressure readings of 50, 100, 250, 400 and 500 mm were indicated by the manometer. Determine the percentage error caused by the capillary effect. Take surface tension of water as 0.0736 N/m and the angle of contact as zero.*

Solution. Capillary rise in

(*i*) **5 mm tube**

$$h_1 = \frac{4\sigma \cos \theta}{\gamma d_1}$$

$$= \frac{4 \times 0.0736 \times 1}{9810 \times 5 \times 10^{-3}} = 0.006$$

$$= 6 \text{ mm}$$

(*ii*) **6 mm tube**

$$h_2 = \frac{4\sigma \cos \theta}{\gamma d_2} = \frac{4 \times 0.0736 \times 1}{9810 \times 6 \times 10^{-3}}$$

$$= 0.005 = 5 \text{ mm}$$

$\therefore$ Capillary effect $= h_1 - h_2$

$$= 6 - 5 = 1 \text{ mm.}$$

Percentage Errors :

(*i*) When pressure difference is 50 mm, % error $= \pm \frac{1}{50} \times 100 = \pm 2\%$

(*ii*) When pressure difference is 100 mm, % error $= \pm \frac{1}{100} \times 100 = \pm 1\%$

(*iii*) When pressure difference is 250 mm, % error $= \pm \frac{1}{250} \times 100 = \pm 0.4\%$

(*iv*) When pressure difference is 400 mm, % error $= \pm \frac{1}{400} \times 100 = \pm 0.25\%$

(*v*) When pressure difference is 500 mm, % error $= \pm \frac{1}{500} \times 100 = \pm 0.20\%$.

Example 1.24. *Determine the absolute pressure and the gauge pressure that would exist within :*

(*i*) *a spherical droplet of water 5 mm in diameter*

(*ii*) *a jet of water 5 mm in diameter.*

Surface tension of water at the prevalent temperature is 0.0736 N/m and the barometer reading stands at 750 mm of mercury. Take specific gravity of mercury as 13.55 and specific weight of water as 9810 N/m³.

Solution. Case (*i*) For a spherical droplet of water, Eq. (1.9) gives differential pressure within the drop as compared to the outside atmosphere as

$$\Delta p = 4\sigma/d = 4 \times 0.0736/5 \times 10^{-3} = 58.86 \text{ N/m}^2$$

This being the pressure measured above the local atmospheric one, it represents the gauge pressure.

Local atmospheric pressure = 13.55 × 750 mm water = 9810 × 13.55 × 750 × 10^{-3} N/m²
= 99.67 kN/m² = 99.67 kPa.

Absolute pressure inside the droplet
= Gauge pressure + Local atmospheric pressure
= 58.86 + 99.67 × 10^3 = 99.73 kPa

Case (*ii*) For the liquid jet, the differential pressure is

$$\Delta p = \frac{2\sigma}{d} = \frac{2 \times 0.0736}{5 \times 10^{-3}} = 29.43 \text{ N/m}^2 = 29.43 \text{ Pa}$$

Absolute pressure inside the liquid jet
= 29.43 + 99.67 × 10^3 N/m² = **99.7 kN/m².**

Example 1.25. *In measuring the unit surface energy of a mineral oil (sp. gr. = 0.85) by the bubble method, a tube having an internal diameter of 1.5 mm is immersed to a depth of 12.5 mm in the oil. Air is forced through the tube forming a bubble at the lower end. What magnitude of unit surface energy will be indicated by a maximum bubble pressure intensity of 147.15 Pa ?*

Solution. Specific weight of the mineral oil

$$\gamma = 0.85 \times 9810 = 8338.5 \text{ N/m}^3$$

Pressure at a depth of 12.5 mm = $\gamma h = 8338.5 \times \frac{12.5}{1000} = 104.2 \text{ N/m}^2 = 104.2 \text{ Pa}$

Pressure difference between inside and outside of the bubble

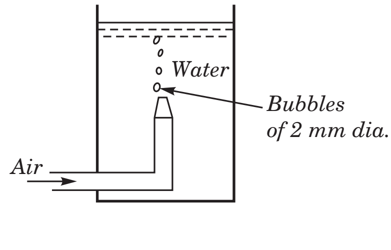
$$\Delta p = 147.15 - 104.2 = 42.95 \text{ Pa}$$

Using Eq. (1.9), the unit surface energy

$$\sigma = \frac{\Delta p \, . \, d}{4} = 4 \times \frac{1.5}{10^3} = 16.1 \text{ N/m.}$$

Example 1.26. *Air is introduced through a nozzle into a tank of water to form a stream of bubbles. If the bubbles are intended to have a diameter of 2 mm, calculate by how much the pressure of the air at the nozzle must exceed that of the surrounding water. Assume that surface tension is 71.6 mN/m.*

Solution. The pressure at the tip of the nozzle must be the same as the excess pressure inside the air bubble. The difference in pressure between inside and outside of the bubble is given by Eq. (1.9),



$$\Delta p = \frac{4\sigma}{d} = \frac{4 \times 71.6 \times 10^{-3}}{2 \times 10^{-3}}$$

$$= \mathbf{143.23 \text{ N/m}^2 = 143.23 \text{ Pa.}}$$

1.6. Vapour Pressure

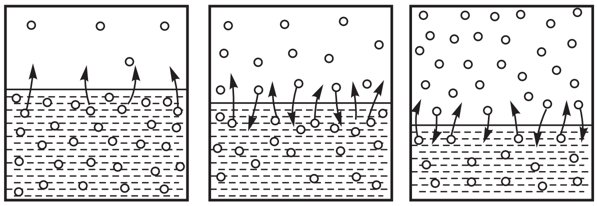


Fig. 1.9

All liquids possess a tendency to vaporise when exposed to air or gaseous atmosphere. The vaporisation takes place due to liquid molecules escaping from the free surface. The rate at which this vaporisation occurs depends upon the molecular energy of the liquid (which is dependent upon the nature of liquid and its temperature) and the condition of the atmosphere adjoining it. Consider a liquid contained in a sealed container, Fig. 1.9. Let a constant temperature be maintained within the container. Some of the liquid molecules have sufficient energy to break away from the liquid surface and enter the air space in the vapour state, see Fig. 1.9 (*i*). After a certain time, the air will contain enough liquid molecules to exert a partial pressure of air on some of the molecules forcing them to rejoin the liquid surface, Fig 1.9 (*ii*). Eventually, when equilibrium condition is established, the rate at which the molecules are leaving the liquid surface will be the same as the rate of return of molecules. In this condition the air above the liquid is saturated with liquid vapour molecules and the pressure exerted on the liquid surface is called the vapour pressure. A liquid with a high vapour pressure evaporates readily and is known as a volatile liquid. The boiling of a liquid is closely related to its vapour pressure. When the pressure impressed on the liquid surface is brought slightly below the vapour pressure limit, the liquid starts boiling. This means that the boiling can be achieved either by raising the temperature of the liquid so that its vapour pressure rises or by lowering the pressure of the overlying air below the vapour pressure of the liquid.

The vapour pressure depends upon the molecular activity which is a function of temperature, as such it depends upon the temperature and increases with it. Table 1.3 gives the vapour pressure of some important liquids, while Table 1.4 shows the vapour pressure of water at different temperatures.

Table 1.3. Vapour Pressure at 20°C

Liquid	*N/m² (Pa)*
Water	2.345×10^3
Kerosine	3.310×10^3
Benzene	10.000×10^3
Petrol	30.400×10^3
Mercury	0.160

Table 1.4. Vapour Pressure of Water

T(°C)	*N/m²(Pa)*	*m of water*
0	0.610×10^3	0.063
10	1.230×10^3	0.125
15	1.62×10^3	0.165
20	2.345×10^3	0.239
30	4.27×10^3	0.437
40	7.400×10^3	0.762
50	12.36×10^3	1.275
60	20.0×10^3	2.075
80	47.5×10^3	4.960
100	101.5×10^3	10.790

The low vapour pressure of mercury, as evidenced from Table 1.3 (along with its high density) makes it very suitable for use in barometers and other pressure measuring devices. The vapour pressure of mercury is so low that there is an almost perfect vacuum above the mercury column in a barometer.

The problem of cavitation, encountered in hydraulic structures like spillways and sluice gates, and hydromachinery such as turbines and pumps, is a direct result of local pressures being equivalent to or less than the vapour pressure of the liquid. When such a situation develops, vapour bubbles or cavities are formed in the flow. The unsteady nature of these bubbles and their ultimate collapse is responsible for the high pressure which often leads to vibrations, noise, pitting and erosion of metal parts of machines and concrete surfaces of hydraulic structures.

Tables 1.5 and 1.6 show the properties of some common fluids and that of water respectively. Table 1.7 deals with the properties of standard atmosphere.

Table 1.5. Properties of some common fluids at 20°C and atmospheric pressure

	Density, ρ	*Specific wt.*, γ	*Dyn. viscosity* $10^5\ \mu$	*Kin. viscosity* $10^7\ v$	*Surface tension** σ	*Vapour pressure*, p_v	*Bulk Modulus of Elasticity,* E_v
	kg/m^3	N/m^3	$Ns/m^2(Pa.s)$	m^2/s	N/m	$N/m^2(Pa)$	$N/m^2(Pa)$
Air	1.235	12.12	1.853	150.0	—	—	—
Benzene	888.0	8650.0	65.3	7.43	0.0255	1.0×10^4	1.035×10^9
Carbon tetrachloride	159.40	15.64×10^3	96.5	6.04	0.0265	1.305×10^4	1.105×10^9
Castor oil	960.0	9410.0	8.04×10^4	1.0×10^4	0.0392	—	1.44×10^9
Ethyl alcohol	788.9	7730.0	119.7	15.0	0.0216	5.78×10^3	1.206×10^9
Glycerine	1270.0	12.45×10^3	9.04×10^4	6.3×10^3	0.0490	1.37×10^4	4.35×10^9
Kerosine	800.0	7850.0	188.0	22.9	0.0235	—	—
Mercury	13.53×10^3	13.28×10^4	155.0	1.16	0.5100	0.173	2.62×10^{10}
Water	998.0	8.9×10^3	100.0	10.0	0.0735	2.39×10^3	2.11×10^9

*In contact with air.

Table 1.6. Properties of water at different temperatures

Temperature T degree C	*Density*, ρ	*Specific wt.*, γ	*Dyn. viscosity* $10^5\ \mu$	*Kin. viscosity* $10^7 v$	*Surface tension** $10^3\sigma$	*Vapour pressure* p_v	*Bulk modulus of Elasticity* $10^{-6} E_v$
	kg/m^3	N/m^3	$N\text{-}s/m^2$ $(Pa.s)$	m^2/s	N/m	N/m^2 (abs) (Pa)	N/m^2 (Pa)
0	1000.0	9810.0	179.3	17.93	75.7	611.0	2016.0
5	1000.0	9810.0	155.5	15.30	74.9	883.0	
10	1000.0	9810.0	131.2	13.20	74.2	1229.0	
15	999.0	9800.0	113.8	11.50	73.6	1618.0	
20	990.0	9780.0	100.1	10.00	72.9	2340.0	2110.0
25	997.0	9770.0	90.0	9.02	72.2	3217.0	
30	996.0	9760.0	79.9	8.09	61.2	4270.0	

35	994.0	9750.0	71.9	7.30	70.5	5585.0	
40	992.0	9730.0	65.1	6.62	69.6	7300.0	
45	990.0	9710.0	59.3	6.04	68.7	9510.0	
50	988.0	9690.0	55.0	5.53	68.0	12.36×10^3	2286.0
55	985.0	9660.0	50.7	5.21	67.1	16.03×10^3	
60	983.0	9640.0	46.6	4.82	66.0	20.45×10^9	
65	981.1	9620.0	43.6	4.50	65.3	24.90×10^3	
70	978.1	9600.0	40.7	4.17	64.4	31.05×10^4	
75	975.2	9560.0	38.0	3.90	63.6	38.55×10^3	
80	971.6	9520.0	35.6	3.68	62.7	47.60×10^3	
85	968.3	9500.0	33.5	3.46	61.8	57.30×10^3	
90	964.0	9450.0	31.5	3.28	60.9	69.20×10^3	
95	961.0	9425.0	29.9	3.01	59.9	84.75×10^3	2138.0
100	959.0	9400.0	28.4	2.90	59.0	101.30×10^3	

*In contact with air.

Table 1.7. Properties of standard atmosphere

Altitude Z *km*	*Temperature* *T* *degree C*	*Pressure absolute* *N/m²* (*Pa*) $10^{-3}\,p$	*Density* *kg/m³* ρ	*Kin. viscosity* *m²/s* $10^5\,\nu$	*Velocity of sound* *m/s* *C*
0	15	101.2	1.225	1.45	341.0
0.3	13.3	97.6	1.193	1.49	339.2
0.6	11.1	94.6	1.156	1.52	338.0
0.9	9.1	91.2	1.125	1.56	337.4
1.2	7.2	87.7	1.090	1.59	337.3
1.5	5.2	83.2	1.085	1.64	335.0
3.0	– 4.7	70.6	0.910	1.85	329.0
4.5	– 14.6	68.2	0.775	2.10	322.5
6.0	– 23.6	47.5	0.667	2.42	316.0
7.5	– 34.4	38.7	0.555	2.70	310.0
9.0	– 44.3	31.4	0.460	—	303.4
10.5	– 54.2	25.0	0.382	—	297.0
11.0	– 55.3	22.5	0.363	—	296.0
11.3	– 55.3	21.6	0.347	—	296.0

1.7. Properties of Gases

Gases are highly compressible fluids and are characterised by change in density. The change in density is achieved by both change in pressure and change in temperature. The absolute pressure p, the specific volume v and the absolute temperature T are related by the equation of state. For a perfect gas, the equation of state per unit weight is

$$pv = RT \qquad \text{...(1.12)}$$

where R = a gas constant, the value of which depends upon the particular gas.

For air the value of the gas constant R is 287 N-m/kg-K or (J/kg-K) in SI. Equation (1.12) may also be written as

$$p = \rho RT \qquad ...(1.13)$$

which may be used to compute the density if the gas constant R is known. The absolute temperature (T) is measured in °K (Kelvin), and is related to temperature in Celcius by the equation

$$T = 273 + T_c, \text{ where } T_c = \text{Temperature in °C.}$$

Another fundamental equation for a perfect gas is

$$pv^n = p_1 v_1^{\,n} = p_2 v_2^{\,n} = \text{constant} \qquad ...(1.14)$$

in which n may have any value from zero to infinity depending upon the process to which the gas is subjected. By combining Eqs. (1.12), (1.13) and (1.14) the following useful relationship can be obtained :

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(n-1)/n} = \left(\frac{v_1}{v_2}\right)^{n-1} = \left(\frac{\rho_1}{\rho_2}\right)^{n-1} \qquad ...(1.15)$$

Example 1.27. *One cubic metre of air at 40°C and pressure 0.105 × 10⁶ N/m² is compressed adiabatically to 0.5 m³. What are the temperature and pressure of the gas ? If the process had been isothermal, what would be the temperature and pressure* ?

Solution. The gas constant for air, R = 287 N-m/kg °K

From the equation of state, $pv = RT$

Substituting the given data,

$$0.105 \times 10^6\, v_1 = 287(273 + 40), \text{ from which } v_1 = 1.17 \text{ m}^3\text{/kg}$$

For adiabatic process, pv^k = constant

$$p_2(0.5)^{1.4} = 0.105 \times 10^6(1.17)^{1.4}$$

$$p_2 = 0.277 \times 10^6 \text{ N/m}^2.$$

Also, $0.277 \times 10^6(1.17 \times 0.5) = 287(273 + T_2)$

$$T_2 = 292\text{°K}.$$

If the process is isothermal, T_2 = 40°C.

$$pv = \text{constant}$$

and

$$p = \frac{0.105 \times 10^6 \times 1}{0.5} = 52.5 \times 10^3 \text{ N/m}^2.$$

1.7.1. Isothermal Process

The compression and expansion of a gas may take place according to various laws of thermodynamics. If the temperature is kept constant, the process is known as *isothermal* and the value of n is unity. Eq. (1.13) is then written as : p/ρ = constant.

1.7.2. Isentropic Adiabatic Process

If the process is such that no heat is added to or withdrawn from the gas (the case of zero heat transfer), it is said to be an *adiabatic process*. An isentropic process is the one in which there is no friction and hence is a reversible process. An isentropic adiabatic process is accompanied by a decided change in temperature. The value of the exponent n in Eq. (1.14) is then denoted by k, the ratio of specific heats at constant pressure and constant volume. The ratio k is called the adiabatic constant for the gas. Eq. (1.14) then changes to :

$$p/\rho^k = \text{constant}.$$

For actual gases the exponent k is nearly constant over a wide range of states. If the process is adiabatic but not frictionless, it is described by an equation like Eq. (1.14) with an exponent slightly different from k. The exponent is smaller than k for expansion and is larger than k for compression.

1.8. Incompressible and compressible Fluids

Fluid mechanics deals with both incompressible and compressible fluids, that is, with fluids of constant and variable densities. Although there is no such thing in reality as an incompressible fluid, this term is applied where the change in density with pressure is so small as to be negligible. This is usually the case with all liquids. Gases, too, may be considered incompressible when the pressure variation is small compared with the absolute pressure.

In liquid pipelines at times of sudden or rapid valve closure, a high pressure wave is generated giving rise to the phenomenon of *water hammer*. In water hammer problems, it is, therefore, necessary to consider the compressibility of liquid.

The flow of air in a ventilating system is a case where a gas may be considered incompressible, because the pressure variation is so small that the change in density is of no importance. But in case of a gas or steam flowing through a pipe line at high velocity, the drop in pressure may be so great that the resulting change of density cannot be ignored. For an aeroplane flying at or less than 400 kmph, the air may be considered to be of constant density. But as an object moving through the air approaches the velocity of sound, which is around 1150 kmph, the pressure and density adjacent to the body become significantly different from those of the air at some distance away. Under such circumstances, the air must be treated as a compressible fluid.

1.8.1. Compressibility

The compressibility is the measure of change in volume (or density) when a substance is subjected to pressure. The reciprocal of coefficient of compressibility β is known as the *bulk modulus of elasticity*. Thus

Coefficient of compressibility = Percentage change in volume for a given change in pressure.

or $$\beta = 1/E_v$$

A fluid may be compressed by the application of pressure, thereby reducing its volume and giving rise to a volumetric strain. Such a compressed fluid will expand to its original volume when the applied pressure is withdrawn. This property of compressibility of a fluid is expressed by the bulk modulus of elasticity. If by applying a pressure dp, the decrease in the fluid volume is dV, then the bulk modulus of elasticity is defined as

$$E_v = -\frac{dp}{dV/V} \quad \text{...(1.16)}$$

where V = original fluid volume. The negative sign indicates a decrease in volume with the increase in pressure. Since most of the liquids have a comparatively high value of bulk modulus of elasticity, the compressibility is very close to zero and hence the liquids are considered practically incompressible under ordinary conditions. The bulk modulus of elasticity of fluid is not a constant but increases with increasing pressure. Tables 1.5 and 1.6 indicate E_v-values for some common fluids and water respectively.

As the density is equal to the mass divided by volume, we have

$$\rho = \frac{m}{V}$$

Since the mass m of a certain volume V is constant, differentiating ρ,

$$d\rho = d\left(\frac{m}{V}\right) = -\frac{mdV}{V^2} = -\rho\frac{dV}{V}$$

$$-\frac{dV}{V} = \frac{d\rho}{\rho} \quad \text{...(1.17)}$$

From Eqs. (1.16) and (1.17),

$$E_v = \rho\frac{dp}{d\rho} \quad \text{...(1.18)}$$

(*i*) For an isothermal process, p/ρ = constant.

$$\therefore \qquad \frac{dp}{d\rho} = \text{constant} = \frac{p}{\rho}$$

and the bulk modulus $E_v = p$ ...(1.19)

(*ii*) If the process is isentropic,

$$\frac{p}{\rho^k} = \text{constant.}$$

Differentiating, $dp = \text{constant}.\ k\rho^{k-1}\, d\rho = k\rho^{k-1}\, d\rho\, \dfrac{p}{\rho^k} = k\, \dfrac{d\rho}{\rho} p$

$$\therefore \qquad \frac{dp}{d\rho} = k\left(\frac{p}{\rho}\right)$$

and the bulk modulus, $E_v = \rho k \left(\dfrac{p}{\rho}\right) = kp$ ...(1.20)

The velocity of sound through a fluid medium is expressed by

$$C = \sqrt{\frac{dp}{d\rho}} \qquad \text{...(1.21)}$$

Small pressure disturbances travel through the fluid medium at a velocity which depends upon bulk modulus and the density of the fluid. Using Eqs. (1.18) and (1.21),

$$C = \sqrt{\frac{E_v}{\rho}} \qquad \text{...(1.22)}$$

in which C is the sonic velocity or the velocity of sound in the fluid medium.

The values of the bulk modulus E_v for air and water at standard atmospheric conditions are:

Fluid	*Value of* E_v (N/m²)
1. Air (isothermal process)	100×10^3
2. Air (isentropic process)	140×10^3
3. Water	2.11×10^9

The bulk modulus is also designated by E.

In the present age of high speed flight, the compressibility of fluid has assumed paramount importance. The Mach number M, which is the ratio of the velocity of flow V (or the velocity of a body) to the velocity of sound in the fluid medium ($M = V/C$) is a measure of the compressibility effects. If $M < 1$, the compressibility effects are of little importance, and when the Mach number exceeds unity ($M > 1$), the compressibility of the fluid affects the flow phenomena to an appreciable extent. Thus the Mach number offers a criterion to decide whether or not the fluid be assumed incompressible in the flow analysis. In case of an incompressible fluid, $E = \infty$ and so also is the sonic velocity $C = \infty$ resulting in a zero Mach number. It has been found through experience that if $M \leq 0.2$, the effects of compressibility are negligible. For air at room temperature, $M = 0.2$ corresponds to $V = 70$ m/s.

The Mach number of flow is also used to describe the flow as subsonic if $M < 1$, supersonic if $M > 1$, and hypersonic if $M > 5$.

Example 1.28. *The volume of a liquid is reduced by 1.2% by increasing the pressure from 0.40 MPa to 12.3 MPa. Estimate the modulus of elasticity of the liquid.*

Solution. The modulus of elasicity is given by

$$E_v = -\frac{dp}{dV/V} = -\frac{(12.3 - 0.40)}{\left(-\dfrac{1.2}{100}V\right)/V} = \mathbf{991.7\ MPa.}$$

Example 1.29. *A liquid with a volume of 0.2 m^3 at 300 kPa is subjected to a pressure of 3000 kPa and its volume is found to decrease by 0.2%. Calculate the bulk modulus of eleasticity of the liquid.*

Solution. Bulk modulus of elasticity of a fluid is defined by Eq. (1.16),

$$E_v = -\frac{dp}{dV/V}$$

Pressure increase, $dp = 3000 - 300 = 2700$ kPa

Resulting dcrease in volume, $dV/V = 0.2\% = -\dfrac{0.2}{100} = \dfrac{(-2)}{1000}$

∴ Bulk modulus of the liquid

$$E_V = -\frac{2700}{(-2/1000)} = \mathbf{13.5 \times 10^5\ kPa.}$$

Example 1.30. *A cylinder contains 0.75 m^3 of gas at 20°C and 2.5 bar pressure. After compression, the volume gets reduced to 0.15 m^3. Determine final pressure and bulk modulus of compressed gas if compression takes place under :*

(*i*) *Isothermal conditions*

(*ii*) *Adiabatic conditions* (n = *1.4*) (*RGPV, June 2013*)

Solution. (*i*) **Isothermal conditions :**

Equation of state, for a perfect gas Eq. (1.12) gives

$pv = RT$, which for isothermal conditions, T is constunt, hence = constant

or $p_1v_1 = p_2v_2$, where v_1 and v_2 are volumes per unit weight pressure after compression, *i.e.,*

$$p_2 = p_1 \frac{v_1}{v_2} = 2.5 \times \frac{0.75}{0.15} = \mathbf{12.5\ bar}$$

∴ Final pressure is 12.5 bar.

Bulk modulus of compressed gas, E_v is given by Eq. (1.16) :

$$E_v = -\frac{dp}{dv/v} = -\frac{p_2 - p_1}{(v_2 - v_1)/v_1} = -\frac{12.5 - 0.75}{(0.15 - 0.75)/0.75} = 12.5 \text{ bar}$$

$$1 \text{ bar} = 10^5 \text{ N/m}^2 = 10^5 \text{ Pascal}$$

$$E_v = \mathbf{12.5 \times 10^5\ N/m^2}$$

(*ii*) **Adiabatic compression :**

For a perfect gas, Eq. (1.14) gives another relationship

$$pv^n = \text{constant}$$

or

$$p_1{v_1}^n = p_2{v_2}^n, \quad \therefore \quad p_2 = p_1\left(\frac{v_1}{v_2}\right)^n$$

$$= 2.5\left(\frac{0.75}{0.15}\right)^{1.4} = \mathbf{23.79\ bar}$$

∴ Final pressure after compression = 23.79 bar

Bulk modulus of compressed gas

$$E_v = -\frac{dp}{dv/v} = -\frac{23.79 - 2.5}{(0.15 - 0.75)/0.75} = 26.6125 \text{ bar}$$

$$= \mathbf{26.61 \times 10^5\ N/m^2}$$

Example 1.31. *Determine the velocity of sound at 20°C and 101.2 kN/m^2 in (i) air, (ii) water, and (iii) mercury. Use table 1.5 for* ρ *and* E_v.

Solution. The velocity of sound in a fluid medium is given by

$$C = \sqrt{E_v/\rho}\ .$$

(*i*) The thermodynamic process involving air may be considered as adiabatic and frictionless (isentropic) for which, $E_v = kp$. The velocity of sound then becomes

$$C = \sqrt{kp/\rho}$$

Using the equation of state $p = \rho RT$, the above equation may be written

$$C = \sqrt{kRT}$$

in which R has the dimensions $(L^2/T^2\theta)$.

For air, $k = 1.4$, $R = 287$ J/kg °K.

Making substitutions, $\quad C = \sqrt{1.4 \times 287 \times (273 + 20)} = 343.0$ m/s.

(*ii*) From table 1.5, for water,

$$\rho = 998.0 \text{ kg/m}^3$$
$$E_v = 2.11 \times 10^9 \text{ N/m}^2$$
$$C = \sqrt{E_v/\rho} = 1470 \text{ m/s.}$$

(*iii*) From table 1.5, for, mercury,

$$\rho = 13530 \text{ kg/m}^3$$
$$E_v = 2.62 \times 10^{10} \text{ N/m}^2$$
$$C = \sqrt{E_v/\rho} = 1392 \text{ m/s.}$$

Example 1.32. *For determining the depth of sea at a place, a charge was exploded at 100 m below the sea water surface. The first reflected wave was recorded after 2.5 seconds at the surface. Calculate the depth assuming the sea has a flat bottom. Average value of bulk modulus of elasticity of sea water is* 1.96×10^9 *N/m*^{2} *and its specific weight is* 10×10^3 *N/m*^{3}.

Solution. $E_v = 1.96 \times 10^9$ N/m^2, $\rho = 10^4/9.81 = 1020$ kg/m^3.

Velocity of sound in sea water $= \sqrt{E_v/\rho} = 1385$ m/s

Let the depth of sea below the sea water surface be d m. The distance travelled by the reflected sound wave

$$= (d - 100) + d = 2d - 100$$

Time taken by the first reflected wave to reach the surface is

$$2.5 = \frac{2d - 100}{1385}$$

from which $d =$ **1781.25 m.**

Example 1.33. *At standard atmospheric conditions, determine the increase in pressure necessary to cause* :

(*i*) *1% reduction in the volume of water,*

(*ii*) *1% reduction in the volume of air subjected to isentropic compression,*

(*iii*) *1% reduction in the volume of air when subjected to isothermal compression.*

Take the volume of the bulk modulus of elasticity the same as given in Ex. 1.22.

Solution. From Eq. (1.16), $E_v = -\,dp/dV/V$

Reduction of volume by 1% results in, $-\,dV/V = \dfrac{1}{100} = 0.01$

(*i*) Increase in pressure of water to produce 1% reduction in its volume

$$dp = 1.96 \times 10^9 \times 0.01 = 1.96 \times 10^7 \text{ N/m}^2$$
$$= 19.6 \text{ MN/m}^2 = 19.6 \text{ MPa.}$$

(*ii*) For air subjected to isentropic process, the bulk modulus is given by Eq. (1.20) :

$$E_v = kp = 1.4 \times 101.3 \times 10^3 = 1.42 \times 10^5 \text{ N/m}^2 = 142 \text{ kPa.}$$

$\therefore$ Increase in pressure, $dp = 142 \times 10^3 \times 0.01 = 1.42 \times 10^3 \text{ N/m}^2 = 1.42$ kPa.

(*iii*) For air under isothermal process, Eq. (1.19) gives the bulk modulus,

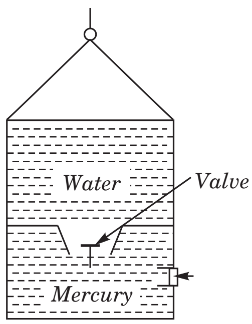
$$E_v = p = 101.3 \text{ kN/m}^2$$

$\therefore$ Increase in pressure, $dp = 101.3 \times 10^3 \times 0.01 = 1.01 \times 10^3 \text{ N/m}^2 = 1.01$ kPa.

From the values, it is evident that the increase in pressure of water to cause 1% reduction in its volume is extremely large, being about 15,000 times greater, as compared to that required for air for similar reduction in volume. Liquids are, therefore, considered incompressible, unless subjected to sudden and large pressure changes.

Example 1.34. *To measure the depth of sea, a device employed takes the form of a solid steel vessel with a partition having a recess and a valve. The upper portion of the vessel is filled with water and the lower one with mercury. When the vessel is lowered, the seawater penetrates into the lower part of the vessel through a small orifice and forces the mercury up through the valve. The compressibility of water reduces its volume. Determine the depth of the sea if the upper part of the vessel contains 600 g of mercury when the device reaches the bottom. The volume of water in the upper part of the vessel is 1000 cm³. Assume the specific weight of water constant and equal to 1050 kg/m³. Take Bulk modules of elasticity of water $E_v = 2.13 \times 10^4$ kg/cm². Neglect the compressibility of mercury.*

Solution. $$E_v = -\frac{\Delta p}{\Delta V/V} = \frac{\Delta p}{(-\Delta V/V)}$$



$V = 1000$ cm³ ; weight = volume × density

$$600 \text{ g} = \Delta V \times (13.6 \times 1 \text{ gm/cm}^3)$$

where ΔV = change in volume of water

or $$\Delta V = \frac{600}{13.6} = 44.12 \text{ cm}^3$$

$$\left(-\frac{\Delta V}{V}\right) = \frac{44.12}{1000} = 0.04412$$

Making substitutions,

$$2.13 \times 10^4 = \frac{\Delta p}{0.04412}$$

or $$\Delta p = 938.67 \text{ kg/cm}^2$$

From the pressure-depth relationship,

$$h = \frac{\Delta p}{\gamma_{sea}} = \frac{938.67}{(1/100)^2 \times 1050} = \mathbf{8939.74 \text{ m}}$$

$\therefore$ Depth of the sea = **8939.74 m**

OBJECTIVE TYPE QUESTIONS

Select the correct answer(s)

1. A fluid is defined as a substance which—
(*a*) takes the shape and volume of the container into which it is poured
(*b*) is highly compressible
(*c*) has a constant shear stress throughout
(*d*) deforms continuously under the action of a shear stress.

2. An ideal fluid is one which—

(*i*) is compressible (*ii*) is non-viscous and incompressible

(*iii*) has low density (*iv*) is elastic and viscous.

3. Fluid continuum is a concept in which—

(*a*) fluid is non-homogeneous (*b*) fluid density is very low

(*c*) fluid particles are very closely spaced

(*d*) fluid particles are widely scattered in space.

4. Viscous deformations in fluid flow are—

(*i*) inversely proportional to the dynamic viscosity

(*ii*) directly proportional to the dynamic viscosity

(*iii*) independent of shear stress and kinematic viscosity

(*iv*) dependent on the pressure.

5. The Newton's law of viscosity is a relationship between—

(*a*) shear stress and pressure (*b*) viscosity and temperature of fluid

(*c*) shear stress and velocity gradient (*d*) pressure and viscosity

6. A Newtonian fluid is one which—

(*i*) has a specific weight of 1 newton/m^3

(*ii*) has a linear relationship between the shear stress and the resulting rate of deformation (*i.e.* velocity gradient)

(*iii*) is non-viscous and incompressible

(*iv*) has a high viscosity.

7. Viscosity of liquids—

(*a*) decreases with decrease in fluid temperature

(*b*) increases with decrease in fluid temperature

(*c*) does not change with fluid temperature

(*d*) is dependent of pressure.

8. Viscosity of gases—

(*i*) decreases with decrease in fluid temperature

(*ii*) increases with decrease in fluid temperature

(*iii*) does not change with fluid temperature

(*iv*) is dependent on pressure.

9. The dimensions of dynamic viscosity μ are—

(*a*) $ML^{-1}T^{-2}$ (*b*) $ML^{-1}T^{-1}$

(*c*) MLT^{-2} (*d*) $ML^{2}T^{-1}$

10. In case of solid mechanics, the law similar to Newton's law of viscosity is—

(*i*) Hooke's law (*ii*) Newton's second law of motion

(*iii*) Archemede's principle (*iv*) Newton's first law

11. The following numerical values represent the magnitude of kinematic viscosity (in m^2/s) of water at different temperatures—

(*a*) 6.62×10^{-7} (*b*) 10.0×10^{-7} (*c*) 17.93×10^{-7}.

If the temperatures involved are 0, 20 and 40 degree C, identify the above values with the corresponding temperature.

12. Match the kinematic viscosity of each liquid choosing the correct value from column *B*—

Col. (*A*) Name of liquid	*Col.* (*B*) Kinematic viscosity in m^2/s
(*i*) Water	(*a*) 150×10^{-7}
(*ii*) Glycerine	(*b*) 10×10^{-7}
(*iii*) Air	(*c*) 6.3×10^{-4}.

13. The following table shows the dynamic viscosity of air in N s/m^2 at atmospheric pressure at different temperatures—

(*i*) 2×10^{-5} (*ii*) 1.90×10^{-5}
(*iii*) 1.81×10^{-5} (*iv*) 1.715×10^{-5}

If the temperatures involved are 0, 20, 40 and 60 degree C, identify each viscosity value with its temperature.

14. MKS unit of 1 kg force is equal to how many newtons—

(*a*) 0.981 (*b*) 98.1 (*c*) 9.81 (*d*) 981.

15. To convert the MKS unit of dynamic viscosity (kg s/m^2) into poise, the multiplying factor is—

(*i*) 89.1 (*ii*) 981
(*iii*) 98.1 (*iv*) 9.81.

16. 1 poise is equal to

(*a*) 1 dyne s/cm^2 (*b*) 98.1 dyne s/cm^2
(*c*) 1 dyne s/m^2 (*d*) 1 kg s/m^2
(*e*) 1 kg s/cm^2.

17. To convert the MKS unit of viscosity (kg s/m^2) into its SI equivalent (N s/m^2) multiply by—

(*i*) 98.1 (*ii*) 981
(*iii*) 9.81 (*iv*) 0.981.

18. One stoke is equal to—

(*a*) 1 cm^2/s (*b*) 1 m^2/s
(*c*) 1 ft^2/s (*d*) 1 mm^2/s.

19. The FPS unit of kinematic viscosity, ft^2/s, is equal to how many stokes—

(*i*) 93 (*ii*) 930
(*iii*) 9.30 (*iv*) 9300.

20. Surface tension is a phenomenon due to—

(*a*) cohesion only (*b*) viscous force
(*c*) adhesion between liquid and solid molecules
(*d*) difference in magnitude between the forces due to adhesion and cohesion.

21. Weight of liquid that rises in a capillary tube is supported by—

(*i*) the friction between the tube wall and the liquid
(*ii*) the atmospheric pressure
(*iii*) the vertical component of force due to surface tension
(*iv*) the curvature of the miniscus.

22. The capillary depression in mercury is on account of—

(*a*) adhesion being greater than cohesion
(*b*) surface tension being larger than the viscosity
(*c*) cohesion being greater than the adhesion
(*d*) vapour pressure being small.

23. The capillary rise or depression in a small diameter tube is—

(*i*) directly proportional to the diameter

(*ii*) inversely proportional to the surface tension

(*iii*) directly proportional to the surface tension

(*iv*) inversely proportional to the diameter.

24. The following are the values of surface tension in N/m for certain liquids :

(*a*) 0.5100 (*b*) 0.0735 (*c*) 0.0235

If the liquids involved are water, kerosene and mercury, identify the above values with the corresponding liquid.

25. The pressure within a soap bubble is—

(*i*) the same as that of the surrounding atmosphere

(*ii*) greater than the external pressure

(*iii*) less than the external pressure

(*iv*) equal to the vapour pressure.

26. The following expressions give the pressure difference between inside and outside of a bubble, a liquid drop and a liquid jet (not in that order, necessarily) :

(*a*) $\frac{2\sigma}{d}$ (*b*) $\frac{8\sigma}{d}$ (*c*) $\frac{4\sigma}{d}$.

Identify each of thee above values with the corresponding item.

27. An incompressible flow is one in which—

(*a*) the temperature of fluid remains constant

(*b*) the density does not change with pressure

(*c*) the fluid is non-viscous

(*d*) the fluid compressibility is non-zero.

28. A measure of the effect of compressibility in fluid flow is the magnitude of a dimensionless parameter known as—

(*i*) Reynolds number (*ii*) Mach number

(*iii*) Weber number (*iv*) Froude number

(*v*) Strouhl number.

29. Gas-flows can be treated as incompressible when the Mach number is less than—

(*a*) 0.5 (*b*) 1.0

(*c*) 0.2 (*d*) 0.1

(*e*) 0.05.

30. For air flow at room temperature to be incompressible, the fluid velocity must not exceed—

(*i*) 100 m/s (*ii*) 70 m/s

(*iii*) 50 m/s (*iv*) 25 m/s.

31. Density, pressure and temperature in a gas flow are related by the—

(*i*) First law of thermodynamics (*ii*) Newton's law of viscosity

(*iii*) Equation of state (*iv*) Equation of motion.

32. A perfect gas is the one—

(*a*) which is incompressible and viscous

(*b*) which obeys the equations of state

(*c*) which follows the Newton's law of gravity

(*d*) which exists in isothermal flows only.

33. A fluid-flow process is isothermal only when—

(*i*) the fluid pressure does not change

(*ii*) the density change is small

(*iii*) the fluid temperature remains constant

(*iv*) there is no heat transfer.

34. An adiabatic fluid-flow is one in which—

(*a*) the fluid temperature does not vary

(*b*) the heat is neither added to nor withdrawn from the gas (the case of zero heat transfer)

(*c*) the pressure remains constant

(*d*) the heat transfer has a non-zero value.

35. An isentropic adiabatic process of fluid-flow is one in which—

(*i*) the heat-transfer is non-zero

(*ii*) there is no friction and the process is reversible

(*iii*) there is no change in the temperature

(*iv*) the process is irreversible.

36. The following table shows the vapour pressure of certain liquids at 20 degree C—

(*a*) 3100 (*b*) 0.0163

(*c*) 239 (*d*) 337.

If the liquids happen to be water, kerosene, mercury and petrol, identify these with their respective values.

37. Mercury is used in barometers on account of—

(*i*) its high density (*ii*) negligible capillarity effect

(*iii*) very low vapour pressure (*iv*) its low compressibility.

38. Spherical shape of droplets of mercury is due to—

(*a*) high density (*b*) high surface tension

(*c*) high adhesion (*d*) low vapour pressure.

39. Arrange the "evaporability" of the following liquids in the decreasing order of magnitude—

(*a*) Kerosene (*b*) Petrol

(*c*) Mercury (*d*) Water.

(**Note.** Evaporability is directly proportional to the vapour pressure).

40. Arrange the following fluids in the decreasing order of their compressibility—

(*a*) water (*b*) air at 1 atm. pressure

(*c*) a gas at 5 atm. pressure (*d*) air at 0.5 atm. pressure.

(**Note.** Compressibility is the reciprocal of the bulk modulus of elasticity).

41. Capillary rise of water in a glass tube depends primarily on its diameter and the angle of contact. Arrange the following sizes of glass tube in increasing order of capillary rise of water.

(*a*) 5 mm (*b*) 2 mm

(*c*) 10 mm (*d*) 1 mm

42. Glass tubes of the same diameter are dipped vertically in vessels containing different liquids. Arrange the following liquids such that the capillary effect in the respective tubes is in the decreasing order of magnitude—

(*a*) kerosene (*b*) water (*c*) mercury.

(**Note.** The capillary effect for the same tube but different liquids is proportional to σ/γ.)

43. Vapour pressure of a liquid is due do—
(*i*) the pressure of flow
(*ii*) the molecules of liquid which hang over the free-surface
(*iii*) the pressure of air above the free surface
(*iv*) the existence of free-surface.

44. The unit of dynamic viscosity of a fluid is
(*a*) m^2/s (*b*) $N.s/m^2$
(*c*) $Pa.s/m^2$ (*d*) $kg.s^2/m^2$.

ANSWERS TO OBJECTIVE TYPE QUESTIONS

1. (*e*)	**2.** (*ii*)	**3.** (*c*)	**4.** (*i*)	**5.** (*c*)
6. (*ii*)	**7.** (*b*)	**8.** (*i*)	**9.** (*b*)	**10.** (*a*)
11. (*a*) : 0°C, (*b*) : 20°C, (*c*) : 40°C		**12.** (*i*) (*b*), (*ii*) (*c*), (*iii*) (*a*),		
13. (*i*) 60°C, (*ii*) 40°C, (*iii*) 20°, (*iv*) 0°C			**14.** (*c*)	**15.** (*iii*)
16. (*i*)	**17.** (*iii*)	**18.** (*a*)	**19.** (*ii*)	**20.** (*d*)
21. (*iii*)	**22.** (*c*)	**23.** (*iii*), (*iv*)		
24. (*a*) mercury, (*b*) water, (*c*) kerosene			**25.** (*ii*)	
26. (*a*) jet, (*b*) bubble, (*c*) drop		**27.** (*b*)	**28.** (*ii*)	**29.** (*c*)
30. (*ii*)	**31.** (*iii*)	**32.** (*b*)	**33.** (*iii*)	**34.** (*b*)
35. (*ii*)	**36.** (*a*) petrol, (*b*) mercury, (*c*) water, (*d*) kerosene			
37. (*iii*), (*i*)	**38.** (*b*)	**39.** (*b*), (*a*), (*d*), (*c*),	**40.** (*d*), (*b*), (*c*), (*a*)	
41. (*c*), (*a*), (*b*), (*d*)	**42.** (*b*), (*c*), (*a*)	**43.** (*ii*)	**44.** (*b*).	

REVIEW QUESTIONS

1. Distinguish between solids, liquids and gases.
2. Is there any analogy of Hooke's law in fluids ? If so, state the parallel law in fluids.
3. Explain why the following statements are right or wrong :
(*i*) Ideal fluid can sustain a shearing stress when in motion.
(*ii*) Fluids cannot sustain shearing stress when at rest.
(*iii*) Molecular viscosity may be expressed in gm sec^{-1} cm^{-1} or dyne sec^{-2}.
4. Enunciate Newton's law of viscosity and distinguish between Newtonian and non-Newtonian fluids.
5. Explain why petrol evaporates more readily than water at ordinary temperature ?
6. Explain how certain insects are able to walk on the surface of water ?
7. Under what conditions is the miniscus between two liquids in a class tube : (*i*) concave upwards and (*ii*) concave downwards ?
8. Comment on the role of capillary action in (*i*) an oil lamp and (*ii*) a fountain pen.
9. Why is it necessary in winter to use a *lighter* oil for automobiles than in summer ? To what property does the term *light* refer ?
10. Will the viscous resistance to the flow of honey be greater or lesser than the viscous resistance to the flow of water ?
11. Is the pressure intensity, (*a*) within a soap bubble (*b*) within a drop of liquid, probably greater than, equal to or less than that of the surrounding medium ?
12. Explain what do you mean by capillarity ?

13. Cite examples where surface tension effects play a prominent role.

14. Answer briefly the following questions : (*i*) define a fluid from mechanics point of view, (*ii*) can a fluid sustain (*a*) tension (*b*) volumetric compression (*c*) shear ? In each case state the relationship between the applied stress and the corresponding shear developed.

15. To what type of flow is the Newton's law of viscosity applicable ? Can it be used to determine shear stress in turbulent flow ?

16. Classify the fluids on the basis of existence of interface (Free-surface).

17. What are the characteristic fluid properties to which the following phenomena are attributable, *viz* :

(*i*) rise of sap in a trees, (*ii*) spherical shape of a drop of liquid,
(*iii*) cavitation, (*iv*) flow of a jet of oil in an unbroken stream,
(*v*) water hammer ?

Express the quantities involved in the metric system and also in terms of fundamental units.

18. Explain the property of fluids on the basis of molecular motion.

19. Define Newtonians fluids.

PROBLEMS

1.1. If the equation of velocity profile is $u = 3y^{2/3}$ (u in cm/s, y in cm), what is the velocity gradient at the boundary and at 10 cm from it ?

1.2. A plate 2.5×10^{-5} m distant from a fixed plate moves at 0.60 m/s and requires a force of 1.96 N/m^2 to maintain this speed. Determine the fluid viscosity of the substance that fills the space between the plates.

1.3. A piston 12 cm dia and 15 cm long moves down in a 12.04 cm dia cylinder. The oil filling the annular space has a viscosity of 8.0×10^{-2} Ns/m^2 and the weight of the piston is 9.81 N. Find the speed with which the piston slides down.

1.4. If two coaxial cylinders 10 cm and 9.75 cm in dia and 25 cm high have a certain liquid filled in between, find the viscosity of the liquid which produces a torque of 0.98 Nm upon the inner cylinder when the outer one rotates at the rate of 90 rpm.

1.5. If the velocity distribution over a plate is given by $u = \frac{2}{3} y - y^2$

in which u is the velocity in m/s at a distance y metres above the plate, determine the shear stress at $y = 0$ and $y = 0.15$ m. Take $\mu = 0.863$ Ns/m^2.

1.6. Derive the dimensions of :

(*i*) dynamic viscosity, and (*ii*) kinematic viscosity

and hence obtain their units in (*i*) MKS, (*iii*) FPS and (*iii*) SI systems.

1.7. A piece of pipe 0.5 m long weighing 9.81 N and having internal diameter of 5.25 cm is slipped over a vertical shaft 5.0 cm in diameter and allowed to fall. Calculate the approximate velocity attained by the pipe if a film of oil of viscosity 0.196 Ns/m^2 is maintained between pipe and shaft.

1.8. A piece of pipe of 5.25 cm internal diameter and 15 cm long slides down a vertical shaft of 5.0 cm diameter at a constant speed of 0.1 m/s. A vertical force 14.7 N is required to pull the pipe back up the shaft at the same constant speed. Calculate the approximate viscosity of oil which fills the small gap between the pipe and shaft.

1.9. A Newtonian fluid is filled in the clearance between a shaft and concentric sleeve. When a force of 490 N is applied to the sleeve parallel to the shaft the sleeve attains a speed of 70 cm/sec. If 2450 N force is applied, what speed will the sleeve attain ? The temperature of the sleeve remains constant.

1.10. A very large thin plate is centred in a gap of width 6 cm with different oils of unknown viscosities above and below, the viscosity of one being twice that of the other. When the plate is pulled at a velocity of 30 cm/sec, the resulting force on one square metre of plate due to viscous shear on both sides is 29.4 N. Assuming viscous flow and neglecting all end effects, calculate viscosities of the oils.

1.11. Two coaxial cylinders with the gap in between completely filled with a viscous fluid, one cylinder rotating while the other one remains stationary, show that the shear stress on the inner cylinder is always greater than that on the outer one. Indicate the velocity profiles for (*a*) inner cylinder rotating while outer one is stationary, and (*b*) outer cylinder rotating while the inner one is kept stationary.

1.12. A rotating viscometer consists of a disc pivoted above a stationary boundary, the fluid to be tested filling the very small space between the parallel surfaces. Means are available to measure the driving torque and the rotational speed of the disc. Determine by integration of the expression for shear stress over the lower surface of the disc., the torque-speed ratio which would be obtained for a liquid having a viscosity of 0.15 Ns/m^2 with a disc having 20 cm diameter and a boundary spacing of 0.19 cm.

1.13. (*a*) On a plot of shear stress *versus* velocity gradient represent a Newtonian and non-Newtonian fluid.

(*b*) Two large parallel flat plates are placed 1.25 cm apart. A 0.25 cm thick plate of 0.2 m^2 area is being towed in glycerine filled between the above plates with a constant force of 9.81 N. Calculate the towing speed of the plate when it is held equidistant from the two parallel plates. Take μ = 0.01 cm (mass) per cm sec.

1.14. A shaft of diameter 74.9 mm rotates in a bearing of diameter 75 mm and of length 75 mm. The annular space between the shaft and the bearing is filled with oil having a coefficient of viscosity of 0.2 stokes and the specific gravity 0.94. Determine the power in overcoming viscous resistance in this bearing at 1400 rpm.

1.15. Calculate the maximum capillary rise of water (20°C) to be expected in a vertical glass tube 1 mm in diameter. The surface tension at 20°C is 0.0718 N/m.

1.16. Derive an equation for theoretical capillary rise between vertical parallel plates.

1.17. Calculate the maximum rise of water (20°C) to be expected between two vertical, clean glass plates spaced 1 mm apart.

1.18. What force is necessary to lift a thin wire ring 2.5 cm in diameter from a water surface at 20°C ? Neglect the weight of ring.

1.19. A soap bubble 5 cm in diameter contains a pressure (in excess of atmosphere) of 20.07 N/m^2. Calculate the tension in the soap film.

1.20. Determine the velocity of sound in air at (*i*) 20°C and 101.2 kN/m^2, and (*ii*) 267°C and 706.0 kN/m^2 and find out the ratio of the two velocities.

1.21. Calculate the dynamic viscosity of standard air using table 1.7. Comment upon the effect of pressure on the dynamic viscosity.

1.22. Compute kinematic viscosity of air at atmospheric pressure using table 1.1. What conclusions can be drawn regarding the effect of temperature on the viscosity (both dynamic and kinematic) ?

1.23. A cubical block weighing 196.2 N and having a 20 cm edge is allowed to slide down on an inclined plane surface making an angle of 20° with the horizontal on which there is a thin film of oil having a viscosity of 2.16×10^{-3} Ns/m^2. What terminal velocity will be attained by the block, if the film thickness is estimated to be 0.025 mm ?

1.24. In a stream of glycerine in motion at a certain point the velocity gradient is 0.25 per second. The mass density of fluid is 1268 kg per cubic metre and the kinematic viscosity is 6.30 × 10^{-4} m²/s. Calculate the shear stress at the point.

1.25. Calculate the capillary effect in millimeter in a glass tube of 4 mm diameter when immersed in (*i*) water and (*ii*) in mercury. The temperature of the liquids is 20°C, and the value of surface tension of water and mercury at 20°C in contact with air respectively 0.07357 N/m and 0.490 N/m. The contact angle for $\theta = 0°$, and for mercury $\theta = 130° \, 24'$.

ANSWERS

1.1. ∞, 0.9275 s^{-1}.
1.2. 81.8 × 10^{-6} Ns/m^2.
1.3. 0.434 m/s.
1.4. 0.7. Ns/m^2.
1.5. 0.574 N/m^2, 0.316 N/m^2.
1.6. (*i*) FTL^{-2}, kgs/m^2, lbs/ft^2, Ns/m^2. (*ii*) L^2T^{-1}, m^2/s, ft^2/s, m^2/s,
1.7. 1.515 m/s.
1.8. 3.72 Ns/m^2
1.9. 3.5 m/s.
1.10. 0.98 and 1.96 Ns/m^2
1.12. 2/3978.
1.13. 1.275 m/s.
1.14. 199.75 kW
1.15. 2.932 cm.
1.16. $h = \dfrac{2\sigma \cos\theta}{r_w d}$
1.17. 1.466 cm.
1.18. 11.28 × 10^{-2} N.
1.19. 0.1293 N/m.
1.20. 343 m/s, 465.5 m/s, 1.354.
1.21. 1.18 × 10^{-6} msl/m-s (1.775 × 10^{-5} kg/n-s), 1.50 × 10^{-6} msl/ms (1.50 × 10^{-6} kg/m-s).
1.22. 13.3 × 10^{-6} m^2/s, 7.85 × 10^{-6} m^2/s.
1.23. 19.4 m/s.
1.24. 19.94 × 10^{-2} N/m^2.
1.25. 7.5 mm in water, and 2.47 mm in mercury.