

Modes of Transfer and Transfer Processes

Heat transfer data deals with the study of the rate at which exchange of heat takes place between hot (source) and cold (receiver) bodies.

Modes of Transfer

Heat may be transmitted from the source to the receiver by any of the three processes *viz.* conduction, convection and radiation or in combination of them.

1.1. CONDUCTION

It is the process of molecular transport of heat in a body due to temperature variation in the medium concerned. It is the only mode of transfer of heat practicable in solids. This process is particularly important with metals and accounts for their high thermal conductivity.

The rate of heat transfer by steady state conduction for unidirectional flow is given by Fourier's law as-

$$\frac{dq}{dt} = -KA \frac{dT}{dx} \qquad \dots (1.1)$$

$$\frac{dq}{dt} = Q - KA \frac{dT}{dx} = -KA \frac{dT}{L} = -\frac{\Delta T}{R} \qquad \dots (1.2)$$

q = heat transferred, K cal

i.e.

where,
$$q = \text{heat transferred}$$

 $t = \text{time, sec.}$

Q = heat transferred per unit time, K cal/sec (watt)

$$K =$$
thermal conductivity, $\frac{\text{watt}}{\text{m.sec}}$

$$dT$$
 = temperature gradient, °C/°K

dx (L) = thickness through which heat flows, m

$$R = \frac{L}{KA}$$
, thermal resistance or conductive conductance, $\frac{K}{\text{watt}}$

The negative sign in the above two expressions indicate transfer from higher temperature to lower temperature. However, in calculation, the numerical values are to be considered.

For unsteady state transfer, the fourier's law is

$$\frac{\partial T}{\partial t} = \frac{K}{\rho C p} \cdot \left(\frac{\partial^2 T}{\partial x^2}\right) \text{ for } x \text{-direction} \qquad \dots (1.3a)$$

$$= \alpha \, \frac{\partial^2 T}{\partial x^2} \qquad \dots (1.3b)$$

when all the tree direction are considered, expression (1.3b) becomes,

where,

$$a =$$
thermal diffusivity, $\frac{m^2}{sec.}$

The discussion on conduction will be primarily confined to steady state conditions.

1.1.1. Specific cases

(i) Conduction in composite wall

where, $R_1, R_2, R_3 \dots$ are the individual thermal resistances.

 $\Delta T_1, \Delta T_2, \dots$ are temperature drops for R_1, R_2, \dots respectively.

(*ii*) Conduction through tube or hollow cylinder:

$$Q = KA_{1n} \frac{\Delta T}{r_2 - r_1} ...(1.6)$$

where mean area = $\frac{A_2 - A_1}{1n^{A_2}}$

where $A_{1n} = \text{logrithmic mean area} = \frac{A_2 - A_1}{\ln \frac{A_2}{A_1}}$ $A_2 = \text{outer area, m}^2$

 $A_1 = \text{inner area, m}^2$ $r_2 = \text{outer radius, m}$ $r_1 = \text{inner radius, m}$

Also,
$$\frac{A_2 - A_1}{\ln \frac{A_2}{A_1}} = \frac{2\pi l (r_2 - r_1)}{\ln \frac{r_2}{r_1}} \qquad \dots (1.7)$$

So,

$$Q = 2\pi K l \frac{\Delta T}{\ln \frac{r_2}{r_1}}$$
...(1.8)

where, l = length, m

For thin-walled cylinder,

(iii) Radial conduction through hollow sphere:

$$Q = 4 \pi K \frac{\Delta T}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$
...(1.10)

which can also be expressed as

$$Q = KA_m \frac{\Delta T}{r_2 - r_1} \qquad \dots (1.11)$$

where A_m = Mean area of the spherical shell

 $\frac{Qr}{4 \pi K r^2 (T_1 - T_2)} = 1$

$$= \sqrt{A_1 A_2} = 4 \pi r_1 r_2$$

This is applied to the case of heat transfer by conduction through a stationary fluid surrounding a spherical particle or droplet of radius 'r'. When r_2 is very large $(r_2 \rightarrow \infty)$ and T_1 is the surface temperature of the drop.

$$Qr = 4 \pi K r^2 \Delta T \qquad \dots (1.12)$$

or *i.e*

Defining $rac{Q}{4 \, \pi \, r^2 \, (T_1 - T_2)}$ as *h*, a conductive heat transfer coefficient

$$\frac{hr}{K} = 1, i.e. \ \frac{hd}{K} = 2\left(\because r = \frac{d}{2}\right) \qquad \dots (1.13)$$

Here,
$$\frac{hd}{K}$$
 is dimensionless and is defined as Nusselt number

1.2. CONVECTION

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While the heat flow mechanism in solids is by conduction, the mechanism of transfer within the fluid is by convection. Here the transfer of occurs through the movement of the macroparticles of the fluid along the temperature gradient. Thus the concept of convection is linked to the movement of the medium of the itself. The fluid motion may be due to-

(i) The density difference as produced by temperature gradient. This type of heat transfer is called 'natural convection'.

(ii) an external agency such as a fluid motive force for the generation of circulating currents. This type of heat transfer is called 'Forced convection'.

Transfer of heat by convection is invariably accompanied by conduction. This is due to the fact that in case of a liquid or a gas in motion, individual particles which are at different temperatures, come in contact with each other resulting in conductive heat transfer between them.

In engineering practice, the transfer of heat by convection between a stream of liquid or gas and a solid surface is encountered. The transfer of heat between a solid surface and a fluid in contact is of importance for industrial heat exchangers. It is a case of simultaneous transfer of heat by conduction and convection. While the mode of transfer from solid surface to a static fluid will be essentially by conduction, the transfer to a fluid in motion will be by convection.

1.2.1. Film Coefficient

Whenever a fluid flows past a solid surface, a thin fluid film tends to adhere to the surface. In case of exchange of heat across tubes in industrial exchangers, two such films one on each side of the surface (inside and outside) of the tube will be formed. For transfer of heat, these two films will offer resistance to convective transfer while the tube wall will offer resistance to conductive transfer. The reciprocal of convective heat transfer resistance is called the film coefficient. The film offering higher resistance to convective transfer is called the 'controlling film'. The value of the overall heat transfer coefficient has a value nearer to the heat transfer coefficient of the controlling film.

...(1.14)

1.2.1.1. Calculation of film coefficient in natural (free) convection

In terms of dimensionless groups, the film coefficient can be expressed as-

$$Nu = f(Gr, Pr)$$

where, $Gr = \text{Grashof number}, \frac{\beta \Delta T g L^3}{v^2}$

$$Pr = Prandtl number, \frac{C_P \mu}{K}$$

Here,

- e, L = Characteristic length of the heat transfer surface (Length for flat surface and diameter for tubular surface), m
 - β = Coefficient of volume expansion, $\frac{1}{\circ \nu}$
 - v = Kinematic viscosity of fluid, m²/sec.

 μ = Viscosity of fluid, $\frac{\text{kg}}{\text{m.sec.}}$

The film coefficient for a few cases can be calculated with the help of the following empirical equations.

(*i*) Free convection from horizontal cylinders:

Churchill and Chu equation used for the present case over a wide range of Rayleigh number (Ra) is-

$$Nu^{0.5} = 0.60 + 0.387 \left[\frac{Ra}{\left[\left\{ 1 + 0.559 \,/\, \mathrm{Pr}^{9/16} \right\}^{\frac{16}{9}}} \right]^{1/6}} \dots (1.15)$$

This is used for the range $10^{-5} < Ra < 10^{12}$ where, Ra = Gr. Pr.

(*ii*) Free convection from spheres:

Yuge's equation is used for calculating heat transfer coefficient in terms of Nusselt number, which is given as –

$$Nu = 2.0 + 0.392 \, Gr^{0.25} \qquad \dots (1.16)$$

Equation (1.16) is applicable over the range $1 < Gr < 10^5$.

1.2.1.2. Calculation of film coefficient in forced convection (without phase change)

In terms of dimensionless groups, the film coefficient can be expressed as-

$$Nu =] f(Re, Pr)$$

The film coefficients for specific cases can be calculated with the help of empirical relations. A few specific cases are given below.

(*i*) Flow insider tubes:

Laminar flow

Sieder-Tate equation given as under is used-

$$Nu = 1.86 + [Re . Pr . D/L]^{0.33} \left(\frac{u}{\mu_w}\right)^{0.14} \qquad \dots (1.17)$$

where, $Re = \text{Reynolds number}, \frac{Dv\rho}{U}$

- D = Insider diameter of tube, m
- L = Length of tube, m
- μ = Fluid viscosity, kg/m. sec. (at bulk temperature)
- μ_w = Fluid viscosity, kg/m. sec. (at wall temperature)

Turbulent flow ($Re > 10^4$):

(a) Dittus-Boeltier equation is used, which is given as-

$$Nu = 0.023 \ (Re)^{0.8} \ (Pr)^{0.4 \text{ or } 0.3}$$
 ...(1.18)

The value of 0.4 is used in case of heating and 0.3 for cooling. A more generalised form of the above equation used for both cooling and heating is
$$-$$

$$Nu = 0.023 \ (Re)^{0.8} \ (Pr)^{0.33} \qquad \dots (1.19)$$

(b) Calburn's equation is also used for the calculation of heat transfer coefficient expressed in terms of dimensionless group as under-

$$St = 0.023 \ (Re)^{-0.2} \ (Pr)^{-0.67} \qquad \dots (1.20)$$

where,
$$St = \text{stanton number}, \frac{Nu}{(Re)(Pr)} = \frac{h}{C_p G}$$

G being mass velocity of fluid, $\frac{\text{kg}}{\text{hr m}^2}$. With the help of equation (1.20), a j_H -factor

defined as under is also used for the calculation of film coefficient-

$$j_H = 0.023 \ (Re)^{-0.2}$$
 ...(1.21)

where, $j_{H} = (St) (Pr)^{0.67}$

A plot is available in Fig. 24 (appendix) of Process Heat Transfer by D.Q. Kern to get the value of j_H as a function of Reynolds number for both laminar and turbulent flow conditions.

(*ii*) Flow in the annulus of a double pipe heat exchanger.

The above equations (1.17 to 1.21) used for flow insider tubes can be used with the following modifications.

Re in the equations will be replaced by $Re_a = \frac{Dev\rho}{\mu}$, De being the equivalent diameter

for the annulus.

(iii) Flow in the shell side of a shell-and-tube exchanger:

The equation for shell side film coefficient is given by Colburn's equation, where shell side Reynolds number (Re_s) is to be used in the equation.

$$Re_s = \frac{Dev\rho}{\mu} = \frac{DeG_s}{\mu}$$

where, De is the shell side equivalent diameter and G_s is the shell side fluid mass velocity to be calculated from the cross flow area (a_s) . (The calculation for De and a_s will be taken up in the next chapter)

(*iv*) Cross flow across a tube bank:

For gases,
$$Nu = 0.26 (Re)^{0.65} (Pr)^{0.33}$$
 ...(1.22)

$$Nu = 0.41 \ (Re)^{0.60} \ (Pr)^{0.33} \qquad \dots (1.23)$$

for staggered arrangement

The above two equations are applicable over the range $10^3 < Re < 10^5$.

For liquids, a factor $(Pr/Pr_w)^{0.25}$ is to be multiplied to the left hand side of the equations (1.22) and (1.23), where Pr and Pr_w are evaluated from properties corresponding to free stream and wall temperature respectively. Also in all the above cases, the fluid flows at right angle to the tube bank.

1.2.2. Convective heat transfer (with phase change) condensation

When a saturated vapour comes in contact with a cold surface, heat transfer takes place accompanied by a phase change, which is normally known as 'condensation' and the liquid thus formed is called 'condensate'. When the condensate liquid wets the surface resulting in the formation of a smooth film, the process is called 'film wise condensation'. If the liquid does not wet the condensing surface, droplets are formed which fall off and the phenomenon is called 'drop wise condensation'. In film-wise condensation, the surface gets covered by the condensate film which grows in thickness. Additional vapour is then required to condense into the liquid rather than form directly on the base surface. A temperature gradient comes to exist in the film and thus the film offers a thermal resistance to heat transfer. On the other hand, in dropwise condensation, a major portion of the condensing surface is directly exposed to the vapour. Resistance of the condensate film being absent, higher heat transfer rates are realized in case of dropwise condensation as compared to filmwise.

1.2.2.1. Film coefficient for condensation

The condensing film coefficient is influenced by the texture of the surface on which condensation occurs and also by whether the condensing surface is mounted vertically or horizontally. The thermal resistance of the condensing film on the cold tube wall offers controlling resistance while that of vapour to liquid for diffusional transfer is inappreciable. The final form of equation for the condensing coefficient may be obtained from dimensional

analysis where the average value of the condensing coefficient (h) is a function of the properties

of the vapour and condensate film viz. λ (latent heat of vapour), k_f , \mathbf{r}_f and \mathbf{m}_f ('f' refers to film); process parameter like D_0/L (outside diameter/length of the condensing surface), g and ΔT_f (the temperature difference between the film and the wall).

Based on Nusselt's theory of condensation, the expression for condensing film coefficients obtained are as under,

For horizontal condenser,
$$\overline{h}_{h} = 0.725 \left(\frac{K_{f}^{3} \rho_{f}^{2} \lambda_{g}}{\mu_{f} D_{0} \Delta T_{f}} \right)^{\frac{1}{4}}$$
 ...(1.24)

For vertical condenser,
$$\overline{h_v} = 0.943 \left(\frac{K_f^3 \rho_f^2 \lambda_g}{\mu_f L \Delta T_f} \right)^{\overline{4}}$$
 ...(1.25)

1.2.3. Temperature difference

Temperature difference is the driving force by which heat is transferred from a source to a receiver. Considering the transfer of heat from a hot fluid to a cold fluid flowing in the tube, a temperature difference exists along the tube length which varies from point to point and is called point temperature difference. In convective heat transfer calculation, however, the mean value of temperature difference (ΔT_m) is used. This can either be an arithmatic mean or a logarithmic mean and depends on the nature of flow of the two fluids (the one inside the tube and the other outside) *viz.* co-current or counter-current. The temperature variation along the length of a tube for the above two conditions are as under-



Distance along the length of the tube w.r.t. cold fluid inlet (Co-current flow)





The temperature difference at a particular end is called 'approach' and the difference between the inlet and the outlet (or vice versa) temperatures is called the 'range'. The cold and hot fluid ranges are $(T_{c_o} - T_{c_i})$ and $(T_{h_i} - T_{h_o})$ respectively. Based on the approaches, the overall mean temperature difference (ΔT_m) for heat transfer is calculated as under:

For parallel (co-current) flow:

For counter-current flow:

$$\Delta T_{1} = T_{h_{o}} - T_{c_{i}}; \Delta T_{2} = T_{h_{i}} - T_{c_{o}}$$

$$\Delta T_{m} = \Delta T_{\ln} = \frac{\Delta T_{1} - \Delta T_{2}}{\ln \frac{\Delta T_{1}}{\Delta T_{2}}} \qquad \dots (1.27)$$

when $\frac{\Delta T_1}{\Delta T_2} < 2.0$, ΔT_m is calculated as the arithmatic mean temperature difference as below–

$$\Delta T_m = \frac{\Delta T_1 + \Delta T_2}{2} \qquad \dots (1.28)$$

1.2.4. Overall heat transfer coefficient

As has been mentioned earlier, convective heat transfer is accompanied by conductive heat transfer too. While calculating the overall heat transfer coefficient, it may be required to compute the total thermal resistance offered to both the above-mentioned modes of transfer in case of a particular heat transfer process. If the transfer of heat between two fluids flowing in the tube and the annulus of a set of concentric pipe is considered then the resistance encountered are the pipe fluid film resistance, the pipe wall resistance and the annulus fluid film resistance. Thus, the equation for overall heat transfer incorporating these resistance is-

$$Q = \frac{\Delta T_m}{\Sigma R} = \frac{\Delta T_m}{R_i + R_w + R_o} \qquad \dots (1.29)$$

where,

$$R_i$$
 = thermal resistance of the fluid film at the inside surface of the inner tube,
 $\frac{1}{w}$, K/w

 $R_o =$ thermal resistance of the fluid film at the outer surface of the inner tube, $\frac{1}{h_c A_c}, K/w$

$$\overline{a_0 A_0}, \overline{A_0}$$

 A_i = inside surface of the inner tube, m²

 A_o = outside surface of the inner tube, m²

 h_i = insider fluid film coefficient, $\frac{W}{m^2 b}$ h_o = outside fluid film coefficient, $\frac{W}{W^2 k}$

 R_w = thermal resistance offered by the tube (inner) wall, $\frac{\ln r_0 / r_i}{2\pi L K_w}$, $\frac{K}{w} r_o$ and r_i are the outer and inner radii of the inner tube respective

L = Length of the tube, m

 K_w = thermal conductivity of the tube material, $\frac{w}{m-k}$

Defining U as overall heat transfer coefficient, the heat transfer equation can be written as-

$$Q = U_i A_i \Delta T_m = U_o A_o \Delta T_m \qquad \dots (1.30)$$

 U_i = overall heat transfer coefficient based on inside diameter of the tube, $\frac{W}{m^2 b}$ where,

 U_o = overall heat transfer coefficient based on outside diameter of the tube, $\frac{W}{m^2 k}$ Putting for Q from (1.29) in (1.30) and simplifying,

$$U_{i} = \frac{1}{\frac{1}{h_{i}} + A_{i} \frac{\ln r_{o} / r_{i}}{2\pi L K_{w}} + \frac{A_{i}}{A_{o}} \cdot \frac{1}{h_{o}}}$$
$$= \frac{1}{\frac{1}{\frac{1}{h_{i}} + \frac{r_{i}}{K_{w}} \ln \frac{r_{o}}{r_{i}} + \frac{r_{i}}{r_{o}} \cdot \frac{1}{h_{o}}}} \dots (1.31)$$

which can also be written as-

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{r_i}{K_w} \ln \frac{r_o}{r_1} + \frac{1}{h_o} \left(\frac{r_i}{r_o} \right) \qquad \dots (1.32)$$

Similarly in terms of U_{o} , the above equation will be-

$$\frac{1}{U_o} = \frac{1}{h_i} \left(\frac{r_i}{r_o} \right) + \frac{r_o}{K_w} \cdot \ln \frac{r_o}{r_i} + \frac{1}{h_o} \qquad \dots (1.33)$$

1.2.5. Fouling (dirt) factor and its effect on overall heat transfer coefficient

When a heat transfer equipment is in service for sometime, foreign materials deposit both on the outside and inside of the pipe surface adding two more resistances to heat transfer. These additional resistances decrease the value of overall heat transfer coefficient and the required amount of heat is no longer transferred by the original heating surface. To overcome this eventuality, these additional resistances are to be taken into consideration in the calculation of total resistance and thereafter fixing the area of heat transfer surface. Let,

 h_{d_i} = heat transfer coefficient of the dirt film on the inside of the inner tube

 h_{d_o} = heat transfer coefficient of the dirt film on the outside of the inner tube

The thermal resistance due to inside and outside dirt films are-

$$R_{d_i} = rac{1}{A_i \ h_{d_i}}$$
 and $R_{d_o} = rac{1}{A_o \ h_{d_o}}$

With dirt factors, the rate of heat transfer is-

$$Q = \frac{\Delta T_m}{R_i + R_{d_i} + R_{d_o} + R_o}$$

= $\frac{\Delta T_m}{\frac{1}{A_i h_i} + \frac{1}{A_i h_{d_i}} + \frac{\ln r_o / r_i}{2\pi L K_w} + \frac{1}{A_o h_{d_o}} + \frac{1}{A_o h_o}}$...(1.34)

which can be written in terms of U_i as-

$$U_{i} = \frac{1}{\frac{1}{h_{i}} + \frac{1}{h_{d_{i}}} + \frac{r_{i}}{K_{w}} \ln \frac{r_{o}}{r_{i}} + \frac{r_{i}}{r_{o}} \left(\frac{1}{h_{d_{o}}} + \frac{1}{h_{o}}\right)} \qquad \dots (1.35)$$

Similarly in terms of U_o , equation (1.34) becomes,

$$U_{o} = \frac{1}{\frac{r_{o}}{r_{i}} \left(\frac{1}{h_{i_{o}}} + \frac{1}{h_{d_{i}}}\right) + \frac{r_{o}}{K_{w}} \ln \frac{r_{o}}{r_{i}} + \frac{1}{h_{d_{o}}} + \frac{1}{h_{o}}} \qquad \dots (1.36)$$

1.2.6. Variable overall heat transfer coefficient

When overall heat transfer varies with temperature, the heat transfer equipment may be looked upon as a number of small units in series and that the coefficient varies linearly with temperature in each of these sections. The heat transfer equation becomes,

$$\begin{split} Q &= q_1 + q_2 + q_3 + \dots = A_1 \frac{U_2 \Delta T_1 - U_2 \Delta T_2}{\ln \frac{U_2 \Delta T_1}{U_1 \Delta T_2}} + A_2 \frac{U_3 \Delta T_2 - U_2 \Delta T_3}{\ln \frac{U_3 \Delta T_2}{U_2 \Delta T_3}} \\ &+ A_3 \frac{U_4 \Delta T_3 - U_3 \Delta T_4}{\ln \frac{U_4 \Delta T_3}{U_3 \Delta T_4}} \quad \dots (1.37) \end{split}$$

where, q_1, q_2, q_3 ... are the heat transferred in respective sections U_1, U_2, U_3, U_4 ... are the local overall heat transfer coefficients at the end of the sections considered $\Delta T_1, \Delta T_2, \Delta T_3, \Delta T_4$... are temperature approaches at corresponding ends of the sections considered.

1.3. RADIATION

All substances at temperature above absolute zero emit radiation. This radiation which is the result of temperature only is called thermal radiation. This is given off from a hot body in the form of radiant energy which moves through space in straight lines or beams and substances only in the sight of a radiant body can intercept radiation from that body. The total radiation thus intercepted will distribute into reflected, absorbed and emitted radiations which depends on reflectivity (ρ) , absorptivity (α) and transmissivity (ε) of the material respectively. The relation between these properties is–

$$\alpha + \alpha + \varepsilon = 1.0 \qquad \dots (1.38)$$

Part of the radiation that is absorbed by the intercepting body is transformed quantitatively to heat.

1.3.1. Black body

When the total intercepted radiation is absorbed by a body with the result that there is no reflection or emission, such a body is called a 'black body'. For a black body

$$\alpha = 1$$
 and $\rho = \varepsilon = 0$

A black body has also the maximum attainable emissive power at any given temperature and is the reference to which all other radiators are referred.

1.3.2. Emissivity

The ratio of the total emissive power of a body (E) to that of a black body (E_B) is called emissivity $\varepsilon.$

Thus,
$$\varepsilon = \frac{E}{E_p}$$
 ...(1.39)

1.3.3. Kirchhgoff's law

This law states that at temperature equilibrium, the ratio of the total radiating power of a body to its absorptivity is a constant and depends upon the temperature of the body. Considering any two bodies in temperature equilibrium with common surroundings, according to the Kirchhoff's law-

$$\frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} \qquad \dots (1.40)$$

where, E_1 and E_2 are total radiating powers of the two bodies and a_1 and a_2 are absorptivity of the bodies respectively. This law is applicable to both monochromatic and total radiation.

If the first body referred to in equation (1.40) above is a black body, then $\alpha_1 = 1$ and

 $E_1 = E_B = \frac{E_2}{\alpha_2}$ where E_b refers to total radiating power of a black body.

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So,

$$\alpha_2 = \frac{E_2}{E_B}$$

But by definition of emissivity (equation 1.39),

$$\varepsilon_2 = \frac{E_2}{E_B} = \alpha_2$$

Thus for a body at temperature equilibrium with surroundings, its emissivity and absorptivity are equal.

1.3.4. Radiant heat transfer

Expression for radiant heat transfer is given as-

$$Q = sF(T_1^{\prime 4} - T_2^{\prime 4}) \qquad \dots (1.41)$$

where,

$$\sigma$$
 = Stefan-Boltzmann constant (4.92 × 10⁻⁸ Kcal/hr. m² K⁴)

F = view factor

 T_1^1 and T_2^1 are absolute temperature of source and receiver respectively

When heat source and sink (receiver) are connected by refractory walls view factor is

replaced by interchange factor (\overline{F}) . The interchange factor is a function of surface emissivities. It also depends on the nature of the surfaces and their orientation. A few common cases are given below.

(i) Two large grey parallel surfaces

$$\overline{F}_{1-2} = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} \qquad \dots (1.42)$$

where, \overline{F}_{1-2} is overall interchange factor ε_1 , ε_2 are surface emissivities.

(ii) One grey surface completely surrounded by the other (spheres)

$$\overline{F}_{1-2} = \frac{1}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2} - 1\left(\frac{1}{\epsilon_2} - 1\right)} \qquad \dots (1.43)$$

where, A_1 and A_2 are the areas of the surfaces.

(*iii*) One grey surface completely surrounded by the other (cylinders)

$$\overline{F}_{1-2} = \frac{1}{\frac{1}{\epsilon_1} + \left(\frac{1}{\epsilon_2} - 1\right)\frac{r_1}{r_2}} \qquad \dots (1.44)$$

where, r_1 and r_2 are the radii.

For grey surfaces in general the following approximate equation is used to calculate the overall interchange factor:

$$\overline{F}_{1-2} = \frac{1}{\left(\frac{1}{F_{1-2}}\right) + \left[\left(\frac{1}{\varepsilon_1}\right) - 1\right] + \frac{A_1}{A_2}\left[\left(\frac{1}{\varepsilon_1}\right) - 1\right]} \qquad \dots (1.45)$$

If no refractory is present, F is used in place of (\overline{F}) as in equation (1.34).

An expression for radiant heat transfer coefficient is obtained from equations 1.41 as,

$$h_r = \frac{Q}{A(T_1 - T_2)} = \frac{4.920 \times 10^{-8} F(T_1'^4 - T_2'^4)}{T_1 - T_2} \qquad \dots (1.46)$$

In some radiation calculation (*e.g.* steam line exposed to atmosphere) air is assumed to be non absorbing and non-reflecting. This assumption is essentially correct for gases such as oxygen, nitrogen, hydrogen and chlorine. Other gases, however, such as carbon monoxide,

carbon dioxide, sulphur dioxide, ammonia, organic gases and water vapour, exhibit considerable ability to absorb radiant energy. Hence while dealing with heat transfer in furnaces or other equipments where absorbing gases are present, it is necessary to consider radiation from gases as well as ordinary surface radiation.

1.4. BOILING HEAT TRANSFER

Heat transfer to boiling liquid is a necessary step in evaporation where a solution is concentrated by boiling off the solvent. The concentrated solution is generally the desired product which is used as such or is processed further in a crystallizer to get crystalline product. The boiling of liquid can take place in a vessel equipped with heating surface in form of plate or tubes to supply the required heat. Alternately the liquid may flow through heated tubes under either natural or forced convection with the heat being transferred to the fluid through the walls of the tubes.

1.4.1. Pool boiling of a saturated liquid

When the heating surface is submerged below the free surface of liquid, the process is called 'pool boiling'. If the liquid is maintained at saturation temperature the process is 'pool boiling of a saturated liquid'. In this case, the boiling point of the liquid corresponds to the pressure existing in the equipment.

Mechanism: When a pool of liquid is heated by a submerged heating surface, the heat flux will depend on the temperature difference between the heating surface (T_w) and that of the boiling liquid (T). For low rates of heat addition, vapour will be formed at the free surface. With the increase of heat flux, bubble formed at the heater surface get detached, rise through the liquid and also grow in size while in transit. The bubbles finally disengage themselves from the liquid surface and accummulates in the vapour space from where it is removed through a vapour outlet immediately. The leaving vapour is in equilibrium with the boiling liquid.

If the temperature drop $\Delta T(T_w - T)$ is increased by steps and the corresponding heat flux (q/A) is measured, a boiling curve as under is obtained on a logarithmic plot. The boiling curve thus obtained has clearly four regimes. In the first regime AB, the q/A versus DT is a linear one with a slope of 1.25. Heat is primarily transferred by free convection. In the second regime BC, the relation is also straight but has a slope between 2 and 3. Bubbles start forming at the heating surface and move up through the liquid to increase the velocity of circulation currents and coefficient of heat transfer

becomes more than that in case of undisturbed natural convection (*i.e.* regime AB). Point C is called critical temperature drop and the corresponding heat flux is called 'Peak flux'. The segment BC marks the onset of 'Nucleate boiling'.

In the third regime CD, with further increase of DT, the flux decreases and reaches a minimum value at D, which is called the "Leiden frost point". In this region, there is vigorous generation of bubbles and a tendency for their coalescence also on the heating surface in form of a layer of insulating vapour. From this layer, miniature explosions



Fig. 1.2. Boiling curve for water 100°C.

send jet of vapour away from the heating surface into the bulk of the liquid. This type of mechanism is termed as "Transition boiling". In this regime, increase of temperature drop increases the thickness of vapour layer and reduces the number of explosions thereby reducing both the heat flux and the transfer coefficient.

As the Leiden frost point is approached, the heating surface is blanketed with a quiscent vapour film through which heat transfer takes place by conduction and radiation. Random and vigorous release of jets of vapour of transition boiling is substituted by the slow and orderly formation of bubbles at the interface of liquid and hot vapour film. The bulk of the resistance to transfer of heat is confined to the vapour blanket over the heating surface. With further increase in temperature drop, the heat flux rises and the boiling action in this region (DE) is called 'film boiling'.

Variation of heat transfer coefficient with temperature drop is plotted on a logarithmic coordinate ad under-



Fig. 1.3. Boiling heat transfer coefficient vs DT for water boiling at 1 atm.

Film boiling is generally not desired in process equipment because of slow heat transfer rate corresponding to a large temperature drop. In general, the high rate of heat transfer, realised in boiling is due to turbulence in liquid created by the dynamic action of the generated bubbles from the heat transfer surface.

1.4.2. Evaporation

Evaporation is a case of a boiling heat transfer where a solution consisting of a nonvolatile solute and a volatile solvent is concentrated. It is conducted by vapourising a portion of the solvent to produce a concentrated liquor. Normally in evaporation, the thick liquor is the valuable product and the vapour is condensed and discarded. However, the reverse is true in case of production of mineral-free water from mineral-bearing water for boiler and drinking water from sea water where the condensed vapour from the evaporation process becomes the desired product.

Amount of heat transferred in case of evaporation is given by-

$$Q = UA \ \Delta T \qquad \dots (1.47)$$

where,

A = Heat transfer area

U = Overall heat transfer coefficient

 ΔT = Difference in temperature between the heating medium and the boiling liquid.

If feed to the evaporator is at the boiling point corresponding to the absolute pressure in the vapour space, all heat from the heating medium is transferred to the heating surface and available for evaporation. So capacity is proportional to the heat supplied. In case of cold feed (feed below its boiling point), the value is reduced. If feed is above its boiling point, a portion of the feed evaporates spontaneously, and is called 'flash evaporation'.

1.4.2.2. Boiling point rise and Dühring's rule

Vapour pressure of most of the aqueous solutions is less than that of water at the same temperature. As a result, the boiling point of solution is higher than that of water. This increase in boiling point over that of water is known as Boiling Rise (B.P.R.) of the solution. The value of B.P.R. is small for dilute solutions and organic colloids while it is high for solutions of inorganic salts.

For finding B.P.R., Dühring's rule is used. According to this rule, boiling point of a solution is a linear function of boiling point of pure water of same pressure. If boiling point of

solution is plotted against boiling point of water at the same pressure, a straight line is obtained for each concentration which is called a "Duhring's line".

Boiling point rise of common solutions can be predicted with the heep of Fig. 11.19 of Chemical Engineer's Handbook (J.H. Perry -4th. edition).

The effect of B.P.R. is to be considered if the value is appreciable Rise in boiling point of solution results in decreasing the available temperature difference



Boiling Point of water, °C.

Fig. 1.4. Dühring's lines for sodium hydroxide

and for a given heat load, the heat transfer area required, increases.

1.4.2.3. Hydrostatic head effect

The depth of the liquid (or liquid head) above the tube bundle adds to the pressure at the heating surface and this raises the boiling temperature of the liquid. This is called 'hydrostatic heat effect' and the elevation in boiling point due to this is calculated as under-

$$\Delta t_h = 0.87 \ \frac{v_R T_R}{\lambda} \cdot \Delta p \qquad \dots (1.48)$$

where Δt_h = Hydrostatic elevation of the boiling point, °C

- T_R = Solution boiling temperature, °k
- V_{R} = Specific volume of water vapour at T_{R} , m³/kg.
- λ = latent heat of vapourisation corresponding to the saturation pressure, K cal/ $$\rm kg$$
- Δp = hydrostatic head, m(equal to one half of the liquid level above the heating surface)

Equation (1.48) has been plotted as a nomograph in Fig. 1.5 below for a direct prediction of $\Delta th.$

*Reference: Chem. Engg. World, XXI, No. 2. 1986. p. 43-44.



Fig. 1.5.* Prediction of Hydrostatic Elevation of Boiling Point.

1.4.2.4. Overall boiling heat transfer coefficient

The overall heat transfer coefficient in most of the evaporators strongly depends on DT. Hence the value of coefficient is generally specified for the service in particular, which is based on practical data. For forced circulation evaporator with no boiling, the heat transfer coefficients can be calculated in a similar manner as is done for the shell-and-tube heat exchanger. The steam film coefficient is calculated from the Nusselt equation and the liquid film coefficient from Dittus-Boeltier equation (equation 1.18).

Economy and Capacity. Economy is defined as the amount evaporated per unit amount of steam used whereas the capacity (or, evaporative capacity) of one or more effects in series is directly proportional to:

(a) the difference between the condensing temperature of the steam supplied and the temperature of the boiling solution in the effect (in case of multiple effect it is the last one).

(b) the overall coefficient of heat transfer from steam to solution.

When the above two factors remain constant, the capacity of one is the same as a combination of more than one effects. Thus the economy depends on the number of effects whereas the capacity is independent of the same.

1.4.2.5. Use of vacuum in evaporation System

The purpose of employing a vacuum to an evaporation system is generally two-fold-

(*i*) to increase the available temperature difference.

(ii) to protect the heat sensitive liquids that would otherwise be damaged by high temperature.

An evaporator either single or multiple effect is not necessarily operated under vacuum. The use of vacuum on evaporator is the result of the almost universal tendency to supply evaporators with low pressure steam with saturation temperature between 105°C to 120°C. In order to obtain satisfactory working temperature difference especially for multiple effects, it has thus been necessary to use vacuum. If the desired temperature difference can otherwise be obtained, an evaporator may work under pressure above atmospheric. As vacuum increases, boiling point decreases and available temperature difference increases. But as boiling point decreases, heat transfer coefficient decreases. Optimum vacuum depends on the type of the evaporator, the liquid characteristics and the cost of producing vacuum. In general, a vacuum of 65-66 of mercury (referred to a 76 cm mercury barometer) is a fairly acceptable value.

1.4.2.6. Multiple effect evaporation

Since latent heat necessary to evaporate one kilogram of water at a temperature T_L (solution boiling temperature) is very nearly equal to the latent heat liberated in condensing one kilogram of steam at T_S (steam saturation temperature), an evaporator will evaporate approximately one kilogram of water for every kilogram of steam condensed in the evaporator. If the vapour formed from this is used as a heating medium in a second evaporator, more amount of evaporation is possible without using additional amount of steam in the second one. This is the principle of multiple effect evaporation. Thus in an *N*-effect evaporation system, there will be approximately *N*-kilograms of water evaporated per kilogram of steam supplied to the first effect. A term 'steam economy' used in such system is defined as the amount of evaporation obtained per unit amount of steam used. Thus the reason for operating an evaporation system in multiple effects is to secure increased steam economy.

1.4.2.7. Method of feeding tom multiple effect evaporation system

Feeding a single effect evaporator calls for little special comment. It is well to have the feed introduced at such a point that the natural convection currents mix it with the liquid in the evaporator as completely as possible. The commonest method of feeding a multiple effect evaporator system is a forward feeding. This signifies the feeding of dilute solution to the first effect and getting the thick liquor from the last effect. This is illustrated in the Figure below:



F Feed, S = Steam, C = Condensate

 E_1, E_2, E_3 -evaporation

 P_1 and P_2 -intermediate product (Thick liquor) steam

 P_3 -final product stream



Backward feed signifies introducing the dilute liquor to the last effect, then advancing it from effect to effect by pump and withdrawing the thick liquor from the first effect. While the flow of the liqour and vapour are parallel in case of forward feed arrangement, it is counter current in case of backward feed. When it is parallel feed, dilute liquor is introduced to individual effect. Mixed feed uses all other arrangements. A mixed feed arrangement is given below-



Fig. 1.7. A double-effect backward feed evaporation system.



 P_1 – Concentrated product stream P_2 , P_3 – intermediate product stream

Fig. 1.8. A triple effect mixed feed evaporation system.

While parallel and mixed feeds are not very common, the usual industrial practice is to go for either a forward or a backward feeding.

The advantages claimed of forward feeding are-

(i) It is a simple arrangement of equipment with simple control and there is only one pump whose suction is under vacuum.

(*ii*) In case of heat sensitive material, no damage is done to the concentrated product as it is removed from the last effect with minimum boiling temperature.

As against the above, this arrangement has the following disadvantages.

 $(i)\ {\rm In}\ {\rm case}\ {\rm of}\ {\rm cold}\ {\rm feed},$ there is under heat load on the first effect resulting in high steam consumption.

(*ii*) As concentrated product is taken from the effect with lowest temperature, the value of overall heat transfer coefficient will be low due to high value of liquor viscosity.

(*iii*) There will be a wide variation in the temperature difference available in the effects with a very low value in the last effect especially with systems exhibiting boiling point rise. Thus there is wide variation in the values of overall heat transfer coefficients for the effects.

For a cold feed, backward feed is generally preferred. The advantages of backward feed are-

(i) Better steam economy is achieved as comparatively less amount of liquor is heated in the first effect.

(ii) As concentrated solution is at higher temperature thereby the viscosity is reduced and thus the overall heat transfer coefficient is less affected as is the case with forward feed arrangement.

(iii) There is less variation of temperature differences in the effects and hence in the values of the overall heat transfer coefficient.

The drawbacks for a backward feed arrangement are-

(*i*) For each of the effects, pumps are required for transfer of the liquor.

(ii) Since the concentrated product is taken out from the first effect, which has the highest liquor temperature in the system, it is likely to affect the product quality in case of heat sensitive materials.

Mixed feed arrangement (Fig. 1.8) is especially used when a solution is to be concentrated through a wide range of densities and where the thick liquor is very viscous. This has the advantage of forward feed as one pump is used and of backward feed where the thick liquor is withdrawn from an effect with high temperature.

1.4.2.8. Capacity and economy in evaporators

While economy is defined as the amount of evaporation per unit amount of steam used, capacity is the total evaporation obtained per hour. The use of multiple effect though increases steam economy, has little effect on capacity, since latent heat values are nearly constant over the ranges of pressure ordinarily involved. Capacity is also measured by the total heat transferred in the effects. The economy of multiple effect evaporation depends on heat balance and not on the rate of heat transfer. The capacity (evaporative capacity) of one or more effects in series is directly proportional to:

(i) the difference between the condensing temperature of the stem supplied and the temperature of the boiling solution in the effect (in case of multiple effect it is the last one).

(ii) the overall coefficient of heat transfer from the steam to solution.

When the above two factors remain constant, the capacity of one is the same as a combination of more than one effects. Further, it has been found that, if a solution has elevation in boiling point, the capacity of a double effect evaporator is less than half the capacity of two single effects each of which is operating between two same temperature differences.

Thus the economy depends on the number of effects where as the capacity is independent of the same.

1.4.2.9. Calculation for evaporation process

1.4.2.9.1. Single effect evaporation

(a) Material and Energy balance

For single effect evaporation, the material balance is-F = E + P

where, F = Feed, kg/hr

E = Evaporation, kg/hr

P = Product, kg/hr.

The solute balance can be written as

$$F x_f = P x_p \qquad \dots (1.50)$$

where, $x_f = \text{mass fraction of solution in feed}$ $x_p = \text{mass fraction of solute in product.}$

The solute and heat content of various streams are presented as under-

The energy balance can be written as-

$$Fh_F + SH_S = EH_E + Eh_c + Ph_p \qquad \dots (1.51)$$

or,
$$Ph_F + Sl_s = EH_E + Ph_p \qquad \dots (1.52)$$

(Since $\lambda = H_c - h_c$)

(Since, $\lambda_s = H_s - h_c$)

In the above equation, the enthalpy of vapour (H_E) is obtained from the steam table at a pressure corresponding to that in the evaporator. Values of feed and product enthalpy depend on solute concentration and stream temperature

(*b*) Heating area:

For an evaporator, the heat balance equation is-

$$Q = UA \Delta T$$

which can be written as,



S-stea, kg/hr

 h_f – feed enthalpy, kcal/kg

 h_p – product enthalpy, kcal/kg

 H_E – total heat of vapour, kcal/kg

 $H_{\rm s}$ – total heat of steam, kcal/kg

 h_c – sensible heat of condensate, kcal/kg

Fig. 1.9. Solute and heat content of various streams of a single effect evaporator.

...(1.47)

18

$$Q = UA \ \Delta T = Sl_s \qquad \dots (1.53)$$

Thus the heat transfer area is,

$$A = \frac{S\lambda_s}{U\,\Delta T} \qquad \dots (1.54)$$

1.4.2.9.2. Multiple effect evaporation

For multiple effect evaporator calculations, the following assumptions are generally made:

(i) Equal heat transfer in each of the effects

(ii) Equal heat transfer area

(*iii*) Equal evaporation/equal pressure drop in each effect.

However, in actual practice all the three assumptions cannot hold good. With equal heat load, trial and error calculations are made to get almost equal heat transfer area (with \pm 10% variation) in each of the effects. The third assumptions is taken into account only for the distribution of temperature and/or to get the intermediate product concentration (for getting the B.P.R. in case of solutions exhibiting the same). The actual evaporation (or the pressure drop) in each of the effects are obtained by subsequent calculations.

Forward feed arrangement



 HE_1, HE_2, HE_3 – total heat of vapours hE_1, hE_2 – sensible heat of vapours.

Fig. 1.10. Solute and heat content of various streams of a triple effect forward feed evaporation system.

(a) Distribution of temperature difference and calculation of enthalpy of various streams: Let q_1 , q_2 and q_3 refer to heat fluxes; U_1 , U_2 and U_3 refer to overall heat transfer coefficients, A_1 , A_2 and A_3 refer to heat transfer are and DT_1 , DT_2 and DT_3 refer to temperature drop in the first, second and third effect respectively (refer to: Fig. 1.10). The heat transfer equation can be written as–

$$q_1 = q_2 = q_3$$
 ...(1.55)

$$U_1 A_1 \Delta T_1 = U_2 A_2 \Delta T_2 = U_3 A_3 \Delta T_3 \qquad \dots (1.56)$$

with equal area in each of the effects,

$$U_1 \, \Delta T_1 = U_2 \, \Delta T_2 = U_3 \, \Delta T_3 \qquad \dots (1.57)$$

$$\Delta T_2 = \frac{U_1}{U_2} \ \Delta T_1 \qquad \dots (1.58)$$

which gives,

or

$$\Delta T_3 = \frac{U_1}{U_3} \ \Delta T_1 \qquad \dots (1.59)$$

Overall DT for the evaporation system,

$$\Delta T = \Delta T_1 + \Delta T_2 + \Delta T_3 \qquad \qquad \dots (1.60)$$

$$= \Delta T_1 \left(1 + \frac{U_1}{U_2} + \frac{U_1}{U_3} \right) \qquad \dots (1.61)$$

Overall $\Delta T = T_S - T_{L3} (= T_{E3})$ in case there is no B.P.R. ...(1.62)

$$= (T_S - T_{E_3} - \Sigma BPR) \text{ in case of solution with B.P.R.} \qquad \dots (1.63)$$

where, $SBPR = (BPR)_1 + (BPR)_2 + (BPR)_3$ and BPR_1 , BPR_2 and BPR_3 refer to the *B.P.R.* values in the first, second and third effects respectively.

Hence, from steam pressure (to the steam chest of the first effect) and calendria pressure (in terms of vacuum) in the third effect, overall temperature drop is calculated. With U_1 , U_2 and U_3 values and by use of equations (1.58), (1.59) and (1.61) the temperature chops in the three effects are calculated. With the help of these drops and boiling point rises, if any, the temperature distribution in the effects are as under:

	First effect	Second effect	Third effect
Steam	T_S	T_{E1}	T_{E3}
Liquor	T_{L1}	T_{L2}	T_{L3}
Vapour	T_{E1}	T_{E2}	T_{E3}
ΔT	$T_S - T_{L1}$	$T_{E1} - T_{L2}$	$T_{E2} - T_{L3}$
BPR	$T_{L1} - T_{E1}$	$T_{L2} - T_{E2}$	$T_{L3} - T_{E3}$

From the above temperature distribution, the enthalpy of various streams can be obtained.

(b) Material and Energy balance:

Material balance equations are-

For the first effect, overall balance,	$F = P_1 + E_1$	(1.64)
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 $Fx_F = P_1 x_{p_1}$ Solute balance,

For the second effect, overall balance,

$$P_1 = P_2 + E_2 \qquad \dots (1.66)$$

...(1.65)

$$P_1 x_{p_1} = P_2 x_{p_2} \qquad \dots (1.67)$$

For the third effect,

Solute balance,

Overall balance,

 $P_2 = P_3 + E_3$ Overall balance, ...(1.68) $P_2 x_{p_2} = P_3 x_{p_3}$...(1.69) Solute balance,

For the evaporation system,

$$F = E + P_3$$
 ...(1.70)

Solute balance,

$$Fx_F = P_3 x_{p_3}$$
 ...(1.71)

 Vapour balance,
 $E = E_1 + E_2 + E_3$
 ...(1.72)

$$E = E_1 + E_2 + E_3 \qquad \dots (1.72)$$

From equation (1.71), P_3 can be calculated and putting its value in equation (1.70). Ecan be obtained. In the equations (1.64) to (1.72),

E = total evaporation from the system

 E_1, E_2, E_3 = evaporation from the first, second and third effects respectively

P_3 = final product

 $x_{p_1}, x_{p_2}, x_{p_3} = \text{mass fraction of solute in } P_1 \text{ and } P_2 \text{ (intermediate product streams) and}$ P_3 (final product) respectively.

Energy balance equations are-

For the first effect,

$$Fh_F + S\lambda_S = E_1 H_{E_1} + P_1 h_{p_1} \qquad \dots (1.73)$$

For the second effect,

$$P_1 h_{p_1} + E_1 \lambda_{E_1} = E_2 H_{E_2} + P_2 h_{p_2} \qquad \dots (1.74)$$

For the third effect,

$$P_2 h_{p_2} + E_2 \lambda_{E_2} = E_3 H_{E_3} + P_3 h_{p_3} \qquad \dots (1.75)$$

where.

 H_{E_1}, H_{E_2} and H_{E_3} are total heat of vapour E_1, E_2 and E_3 respectively.

 $\lambda_{E_1}, \lambda_{E_2}$ and λ_{E_3} are the latent heat of E_1, E_2 and E_3 .

 λ_{S} is latent heat of steam $h_{p_{1}}, h_{p_{2}}$ and $h_{p_{3}}$ are enthalpy of P_{1} and P_{2} and P_{3} streams.

With the help of equations (1.64) and (1.72),

$$P_1 = F - E_1 = F - (E - E_2 - E_3) \qquad \dots (1.76)$$

Using for P_1 from (1.76) and P_2 from (1.63), the equations (1.74) and (1.75) can be solved for the two unknown E_2 and E_3 . (In equation (1.74) $-E_1$ is substituted as $E - E_2 - E_3$). Thereafter, with the help of (1.72), equation (1.73) can be solved for steam consumption (s).

(*C*) Heating area:

For the first effect,
$$A_1 = \frac{S\lambda_S}{U_1 \Delta T_1}$$
 ...(1.77)

For the second effect,
$$A_2 = \frac{E_1 \lambda_{E_1}}{U_2 \Delta T_2}$$
 ...(1.78)
For the third effect $A_2 = \frac{E_2 \lambda_{E_2}}{U_2 \Delta T_2}$ (1.79)

For the third effect,
$$A_3 = \frac{E_2 \kappa_{E_2}}{U_3 \Delta T_3}$$
 ...(1.79)

If $A_1 \neq A_1 A_2 \neq A_3$ and the deviation is more than 10%, a second trial with now temperature differences is taken up. The temperature differences are recalculated as follows:

$$\Delta T_1^1 = \Delta T_1 \times \frac{A_1}{A_{av}} \qquad \dots (1.80)$$

where, $A_{av} = \frac{A_1 + A_2 + A_3}{3}$

$$\Delta T_2^1 = \Delta T_2 \times \frac{A_2}{A_{av}} \qquad \dots (1.81)$$

$$\Delta T_3^1 = \Delta T_3 \times \frac{A_3}{A_{av}} \qquad \dots (1.82)$$

$$\frac{3}{av}$$
 ...(1.82)

i.e.

$$\Delta T_3^1 = \Delta T - \Delta T_1^1 - \Delta T_2^1 \qquad \dots (1.82A)$$

The energy balance calculations are to be repeated with the values of enthalpies of liquor and vapour streams corresponding to the new values of boiling temperatures obtained as a result of the modified values of the temperature difference in the effects.

Backward feed arrangement



Fig. 1.11. Solute and heat content of various streams of a triple effect backward feed evaporation system.

While the temperature difference and calculation of enthalpy as per boiling temperatures prevailing in the effects follow the same procedure as in the case of a forward feed arrangement, the material and energy balance equations are modified as given below.

Material balance equations:

For the first effect,			
overall balance, $P_2 = P_1 + E_1$	(1.83)		
solute balance, $P_2 x_{p_2} = P_1 x_{p_1}$	(1.84)		
For the second effect,			
overall balance, $P_3 = P_2 + E_2$	(1.85)		
solute balance, $P_3 x_{p_3} = P_2 x_{p_2}$	(1.86)		
For the third effect,			
overall balance, $F = P_3 + E_3$	(1.87)		
solute balance, $Fx_F = P_3 x_{p_3}$	(1.88)		
For the evaporation system,			
overall balance, $F = P_1 + E$	(1.89)		
solute balance, $Fx_F = P_1 x_{p_1}$	(1.90)		
vapour balance, $E = E_1 + E_2 + E_3$	(1.91)		

From equation (1.90), P_1 can be calculated and with its value, ${\it E}$ can be obtained from (1.89).

Energy balance equations:

For the first effect,

$$P_2 h_{p_2} + S\lambda_S = E_1 H_{E_1} + P_1 h_{p_1} \qquad \dots (1.92)$$

For the second effect,

$$P_3h_{p_3} + E_1\lambda_{E_1} = E_2H_{E_2} + P_2h_{p_2} \qquad \dots (1.93)$$

For the third effect,

$$Fh_{F} + E_{2}\lambda_{E_{2}} = E_{3}H_{E_{2}} + P_{3}h_{p_{2}} \qquad \dots (1.94)$$

With the enthalpy values of the streams, the procedure for calculation of evaporation from the effects, steam requirement and heating area is similar to that followed in case of forward feed arrangement.

In case of a mixed feed arrangement, the material and energy balance equations can be modified as per the actual flow of the liquor streams and the above procedure can be followed for the calculation of heating area.

1.5. CRYSTALLIZATION

Crystallization is the formation of solid particles within a homogeneous phase. It is applied to chemical processes essentially as a method of separation of a substance in a solid form of a well-defined structure (called a crystal) from its homogeneous solution. Crystallization is important in the preparation of a pure product, since a crystal usually separates out as a substance of definite composition from a solution of varying composition.

The rate of crystallization depends on two distinct phenomena *viz*. (*i*) the rate of formation of new crystals or nucleation, either in a clear solution or in a solution containing solids,

(*ii*) the rate of precipitation of crystals already present known as crystal growth.

A crystal is a highly organised array of a atoms, molecules or ions arranged in three dimensional space lattices. In commercial crystallization processes, unhindered crystal growth seldom occurs. The crystallization process is governed by definite laws and is strongly affected by the degree of supersaturation.

1.5.1. ∆L laws

It has been shown that all geometrically similar crystals of the same material suspended in the same solution grow at the same rate, if the growth is measured as the increase in length of geometrically corresponding distances on all the crystals. If ΔL is the linear increase of one crystal, it is at the same time equalt to the corresponding dimension of each of the other crystals and is independent of the initial size of any of the original crystals, provided that all crystals in the suspension are treated exactly alike.

1.5.2. Nuclear formation

Crystallization from a solution takes place in two steps—(a) formation and (b) growth. Crystal formation is also called 'nucleation'. Both nucleation and growth require that the solution should be super-saturated. The usual solubility relation is an equilibrium between saturated solution and crystals at least of moderate size. As particle size reduces, solubility increases. Actually chance encounters between high energy solute molecules lead to formation of nucleii large enough to survive. The magnitude of the smallest nucleus that is just enough to exist in equilibrium with the super saturated solution in which it is born in called critical size Nucleii smaller than critical immediately dissolve. A critical nucleus contains in the order of ten to several hundred molecules. The larger the super saturation, the smaller the critical size and the more rapid is the nucleation measured in the number of nucleii per unit time per unit volume. A quantitative explanation to this is given by Miers super solubility curve.

1.5.3. Rate of crystal growth

In commercial crystallization it is highly desirable to have the product not only of uniform size, but of a particular uniform size. If follows that if too many million nucleii are started in a crystallization process, there may not be material enough to grow them upto the desired size before the solution has been brought upto saturation. It is, therefore, desirable in the commercial process to control the rate of nuclear formation and limit the number of nucleii that start to grow down to the number which can be grown to the desired crystal size within the limit of the amount of material handled.

Once the nucleus has formed and started to grow the laws for the rate of growth are not clearly understood. The controlling factor may be the rate of diffusion of solute at the interface. If this is true, the rate of growth of all the crystals would have been the same. In practice it is

found that, different faces of the same crystal grow at different rates. Moreover, if diffusion would have been the controlling factor, then with the increase of viscosity, the rate should decrease. But in some cases, the rate has been found to be independent of viscosity. Consequently it has been postulated that some process goes on at the surface of the crystal in orienting to the crystal lattice the solid molecules that come out of solution which in itself has the effect or resistance. Apparently in many cases, the resistance of orientation at the actual interface is large enough to overcome completely any effect of the rate of diffusion of the solute to the inner face. The impurities in crystal also influence the crystal growth.

1.5.4. Calculation of crystallization process

The process of crystallization may more correctly be viewed as a case of combined heat and mass transfer operations. The material and energy balance equations are employed for the crystallization process to make various

calculation relating to yield of product, area of heat transfer and sizing of the crystallizer.

Material balance

The material flow diagram for a crystallization process is given below:

F, M and C represent the amount of feed, mother liquor and crystal respectively.

 x_F , x_M and x_c are the solid mass fractions in F, M and C respectively.

E represents evaporation which is solid-free.

Overall material balance is:

$$F = M + C + E \qquad \dots (1.95)$$

Solid balance is, $Fx_F = Mx_M + Cx_C$...(1.96) For negligible evaporation (*i.e.* E = 0), from the above two equations, we have,

$$\frac{F}{C} = \frac{x_C - x_M}{x_F - x_M} \qquad ...(1.97)$$

The value of x_M is obtained from solubility data.

Enthalpy balance

For enthalpy balance, heat of crystallization or the latent heat evolved during crystal formation, is necessary. Heat of crystallization is related to heats of solution and dilution and is a function of concentration and temperature. The information for this is available from enthalpy concentration charts.

The heat removed in a crystallization process consists of two parts-

(*a*) the sensible heat, and

(b) the latent heat

Thus,

$$Q_C = FC_{PF} dT + C\lambda_C \qquad \dots (1.98)$$

where, Q_C = Total heat removed in the crystallizer, kcal/hr

F = Feed rate, kg/hr

 C_{PF} = Specific heat of feed solution, kcal/kg°C

dT = Cooling range, °C

C = Amount of crystals formed, kg/hr.

 λ_{C} = Heat of crystallization kcal/kg



Fig. 1.12. Material flow diagram for the crystallization process.

Calculation of Heat Transfer Area:

The overall heat transfer coefficient for a crystallizer is-

$$Q = UA \left(\Delta T\right)_{\ln}$$

...(1.99)

where,
$$U = \text{overall heat transfer coefficient for the crystallizer, kcal/hr. m2°C$$

 $A = \text{Heat transfer area, m}^2$

 $\left(\Delta T\right)_{\rm ln}$ = Log mean temperature difference, °C

$$=\frac{\Delta T_1 - \Delta T_2}{\ln \frac{\Delta T_1}{\Delta T_2}}$$

 ΔT_1 and ΔT_2 are calculated from the inlet and outlet cooling water, feed and crystallization temperatures.

From the heat transfer area calculated and the effective heat transfer are available per running meter, the length of a crystallizer of the Swenson-Walker type is computed.

List of Symbols Used

Specific heat at constant pressure	—
Diameter of tube	L
Diameter of sphere	L
Mass velocity of fluid	$ML^{-2}\theta^{-1}$
Acceleration due to gravity	$L \mathrm{q}^{-2}$
Heat transfer coefficient for convection	$ML^{-2}\theta^{-1}$
Heat transfer coefficient in boiling (Fig. 1.3)	$ML^{-2}\theta^{-1}$
Average value of condensing film coefficient for horizontal condenser	$ML^{-2}\theta^{-1}$
Average value of condensing film coefficient for vertical condenser	$ML^{-2}\theta^{-1}$
Heat transfer coefficient for radiation	$ML^{-2}\theta^{-1}$
'j-factor' for heat transfer	_
Thermal conductivity	$ML^{-2}\theta^{-1}$
Thermal conductivity of tube material	$ML^{-2}\theta^{-1}$
Length of the path for the heat conduction, length of tube of	
characteristic length	L
Inside radius	L
Outside radius	L
Temperature	T
Absolute temperature	T
Temperature of condensing vapour or steam	T
Temperature difference for heat transfer	T
Temperature difference across condensate film	T
Overall heat transfer coefficient	$ML^{-2}\theta^{-1}$
Design value of overall heat transfer coefficient considering dirt factor	$ML^{-2}\theta^{-1}$
Linear velocity	$L \mathrm{q}^{-1}$
Distance measured along the path of heat flow of <i>x</i> -direction	L
	Specific heat at constant pressure Diameter of tube Diameter of sphere Mass velocity of fluid Acceleration due to gravity Heat transfer coefficient for convection Heat transfer coefficient for convection Heat transfer coefficient in boiling (Fig. 1.3) Average value of condensing film coefficient for horizontal condenser Average value of condensing film coefficient for vertical condenser Heat transfer coefficient for radiation 'j-factor' for heat transfer Thermal conductivity Thermal conductivity of tube material Length of the path for the heat conduction, length of tube of characteristic length Inside radius Outside radius Temperature Absolute temperature Temperature of condensing vapour or steam Temperature difference for heat transfer Temperature difference for heat transfer Temperature difference for heat transfer Temperature difference across condensate film Overall heat transfer coefficient Design value of overall heat transfer coefficient considering dirt factor Linear velocity Distance measured along the path of heat flow of <i>x</i> -direction

26	FUNDAMENTALS OF HEAT AN	FUNDAMENTALS OF HEAT AND MASS TRANSFER	
x_{m}	Thickness of tube wall	L	
y	Distance measured along the path of heat flow in y-direction	L	
z	Distance measured along the path of heat flow in <i>z</i> -direction	L	
α	Thermal diffusivity	$L^2 heta^{-1}$	
β	Coefficient of cubical expansion	T^1	
θ	Time	θ	
ρ	Density	Ml^{-3}	
v	Kinematic viscosity	$L^2 heta^{-1}$	
λ	Latent heat	T	
μ	Viscosity	$ML^{-1} \theta^{-1}$	
G_r	Grashof number, $\frac{\beta g \Delta T L^3 \rho^2}{\mu^2}$	_	
Nu	Nusselt number, $\frac{hD}{k}, \frac{hd}{k}$	_	
Pr	Prandtl number, $\frac{C_P \mu}{k}, \frac{\nu}{\alpha}$	_	
Re	Reynolds number, $\frac{D\nu\rho}{\mu}$, $\frac{Du\rho}{\mu}$		
St	Stanton number, $\frac{h}{C_P G}$		
Suffi	xes:		
f	Film		
i	Inside		

L

0

Liquor

Outside