1 Strength of Materials

1.1. The maximum value of Poisson's ratio for an elastic material is: (GATE, 1991)

(*a*) 0.25 (*b*) 0.5 (*c*) 0.75 (*d*) 1.0

Ans. The correct choice is (*b*)

1.2. A cantilever beam of tubular section consists of 2 materials copper as outer cylinder and steel as inner cylinder. It is subjected to a temperature rise of 20°C and $\alpha_{copper} > \alpha_{appear}$. The stress is developed in the tubes will be (GATE, 1991)

(*a*) compression on steel and tension in copper (*b*) tension in steel and compression in copper

(*c*) no stress in both (*d*) tension in both the materials.

Ans. The correct choice is (*b*)

1.3. In a linear elastic structural elements:

(*a*) stiffness is directly proportional to flexibility (*b*) stiffness is inversely proportional to flexibility

(*c*) stiffness is equal to flexibility (*d*) stiffness and flexibility are not related.

Ans. The correct choice is (*b*)

1.4. Pick up the incorrect statement from the following four statements: (GATE, 2000)

(*a*) on the plane which carries maximum normal stress, the shear stress is zero

(*b*) principal planes are mutually orthogonal

(*c*) on the plane which carries maximum shear stress, the normal stress is zero

(*d*) the principal stress axes and principal strain axes coincide for an isotropic material

Ans. The correct choice is (*c*)

1.5. The shear modulus (*G*) modules of elasticity (*E*) and the Poission's ratio (μ) of a material are related as: (GATE,, 2002)

(*a*) $G = \dfrac{E}{[2(1+\mu)]}$ (*b*) $E = \dfrac{G}{[2(1+\mu)]}$ (*c*) $G = \dfrac{E}{[2(1-\mu)]}$ (*d*) $G = \dfrac{E}{[2(\mu-1)]}$

Ans. The correct choice is (*a*)

1.6. For an isotropic material, the relationship between the Young's modulus (*E*), shear modulus (*G*) and Poission's ratio (μ) is given by: (GALE, 2007)

(*a*) $G = \dfrac{E}{(1+\mu)}$ (*b*) $G = \dfrac{E}{2(1+\mu)}$ (*c*) $G = \dfrac{E}{(1+2\mu)}$ (*d*) $G = \dfrac{E}{2(1+2\mu)}$

Ans. The correct choice is (*b*)

1.7. The number of independent elastic constants for a linear elastic isotropic and homogeneous material, is:

(*a*) 4 (*b*) 3 (*c*) 2 (*d*) 1 (GATE, 2007)

Ans. The correct choice is (*c*)

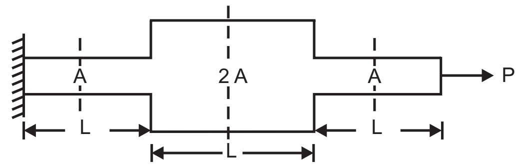
1.8. The Poission's ratio is defined as

(*a*) $\left|\dfrac{\text{axial stress}}{\text{lateral stress}}\right|$ (*b*) $\left|\dfrac{\text{lateral strain}}{\text{axial strain}}\right|$

(*c*) $\left|\dfrac{\text{lateral stress}}{\text{axial stress}}\right|$ (*d*) $\left|\dfrac{\text{axial strain}}{\text{lateral strain}}\right|$

Ans. The correct choice is (*b*).

1.9. The total elongation of the structural element fixed, at one end, free at the other end, and of varying cross-section shown in the figure when subjected to a force P at free end is given by:



(*a*) PL/AE (*b*) 3P/AE

(*c*) 2.5 PL/AE (*d*) 2P/LA

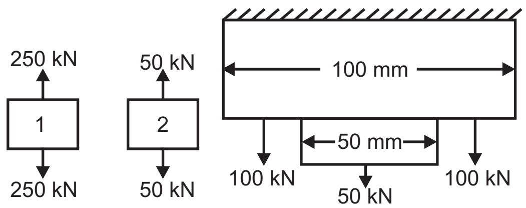
Sol. Applying the standard formula.

Elongation of the structural member

$$= \frac{PL}{AE} + \frac{PL}{2AE} + \frac{PL}{AE} = \frac{2PL + PL + 2PL}{2AE} = \frac{2.5PL}{AE}$$

Ans. The correct choice is (*c*).

1.10. A bar of varying square section is loaded symmetrically as shown in the figure, loads shown are placed on one of the axes of symmetry of cross section, ignoring self weight, the maximum tensile stress in N/mm^2 any where is: (GATE, 2007)



(*a*) 16.0 (*b*) 20.0 (*c*) 25.0 (*d*) 30.0

Sol..
$$\sigma_1 = \frac{250 \times 10^3}{100 \times 100} = 25\,\text{MPa}$$

$$\sigma_2 = \frac{50 \times 10^3}{50 \times 50} = 20\,\text{MPa}$$

$\therefore$ $= \sigma_{max} = \sigma_1 = 25$ MPa

Ans. The correct choice is (*c*).

1.11. A metal bar of length 100 mm is inserted between two rigid supports and its temperature is increased by 10°C. If the coefficient of thermal expansion is 12×10^{-6} per°C and the Young's modulus is 2×10^5MPa, the stress in the bar is. (GATE, 2007)

(*a*) Zero (*b*) 12 MPa (*c*) 24 MPa (*d*) 2400 MPa

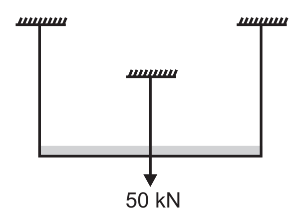
Sol. Given $\alpha = 12 \times 10^{-6}$ per°C; $t = 10$°C ; $E = 2 \times 10^5$ Ma

$\therefore$ Stress developed in the bar

$$\sigma = 12 \times 10^{-6} \times 10 \times (2 \times 10^5)$$
$$= 24 \text{ MPa}$$

Ans. The correct choice is (*c*).

1.12. A rigid bar is suspended by three rods made of the same material as shown in the figure. The area and length of the control rod are 3A and L, respectively while that of the two outer rods are $2A$ and $2L$ respectively. If a downward force of 50 kN is applied to the rigid bar, the forces in the central and each of the outer rods with be: (GATE, 2007)



(*a*) 16.67 kN each (*b*) 30 kN and 15 kN

(*c*) 30 kN and 10 kN (*d*) 21.4 kN and 14.3 kN

Sol. Resolving forces vertically,

$2\,P$ (out) $+ P$ (Central) = 50 kN ...(*i*)

Again δl(outer rod) . δl (Central rod)

$$\frac{\rho_{(o)}(2L)}{(2A)(E)} = \frac{\rho_{(c)}(L)}{(3A)(E)}$$

or $\rho_c = 3\,\rho_o$. By substituting the value of ρ_e in eqn. (*i*) ...(*ii*)

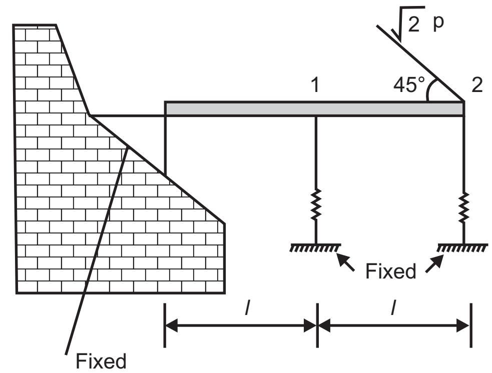
$$5\,\rho_{(o)} = 50 \quad \Rightarrow \quad \rho_o = 10 \text{ kN}$$

and $$\rho_{(c)} = 3 \times 10 = 30 \text{ kN}$$

Ans. The correct choice is (*c*)

Linked Answer Question 13 and 14:

A rigid beam is hinged at one end and supported on linear elastic spring (both having a stiffness of 'k') at prints '1' and '2' an inclined load acts at '2' as shown



1.13. Which of the following options represents the deflection δ_1 and δ_2 at points '1' and '2'?

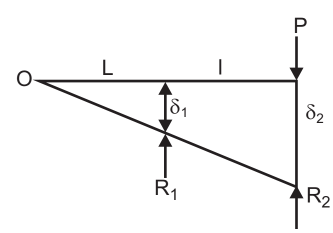
(*a*) $\delta_1 = \frac{2}{5}\left(\frac{2\rho}{K}\right)$ and $\delta_2 = \frac{4}{5}\left(\frac{2\rho}{K}\right)$ (*b*) $\delta_1 = \frac{2}{5}\left(\frac{\rho}{K}\right)$ and $\delta_2 = \frac{4}{5}\left(\frac{\rho}{K}\right)$

(*c*) $\delta_1 = \frac{2}{5}\left(\frac{\rho}{\sqrt{2}K}\right)$ and $\delta_3 = \frac{4}{5}\left(\frac{\rho}{K}\right)$ (*d*) $\delta_1 = \frac{2}{5}\left(\frac{\sqrt{2}\rho}{K}\right)$ and $\delta_2 = \frac{4}{5}\left(\frac{\sqrt{2}\rho}{K}\right)$

Sol. We know $\Sigma M_o = 0$

$\therefore$ $$R_1 \times 1 + R_2 2l = P \times 2l \qquad \ldots(i)$$

From similar triangles from in the figure



$$2\delta_1 = \delta_2$$

stiffness, $$K_1 = \frac{R_1}{\delta_1} \text{ and } k_2 = \frac{R_2}{\delta_2}$$

or $$R_1 = K\delta_1 \text{ and } R_2 = K2\delta_2$$

$$(\because\ K_1 = K_2 = K)$$

Substituting the values of R_1 and R_2 in eqn (*i*) we get

$$K\delta_1 l + K\delta_2 l = 2\rho l$$

or $$K\delta_1 + 4K\delta_1 = 2\rho \Rightarrow 5K\delta_1 = 2\rho \quad \text{or} \quad \delta_1 = \frac{2}{5}\left(\frac{\rho}{K}\right)$$

and $$\delta_2 = 2\delta_1 = 2 \times \frac{2}{5}\left(\frac{\rho}{K}\right) = \frac{4}{5}\left(\frac{\rho}{K}\right)$$

Ans. The correct choice is (*b*).

1.14. If the load P equals 100 kN, which of the following options represents forces R_1 and R_2 in the springs at points '1' and '2'? (GATE, 2011)

(*a*) $R_1 = 20$ kN ; $R_2 = 40$ kN (*b*) $R_1 = 50$ kN ; $R_2 = 50$ kN

(*c*) $R_1 = 30$ kN ; $R_2 = 60$ kN (*d*) $R_1 = 40$ kN ; $R_2 = 80$ kN

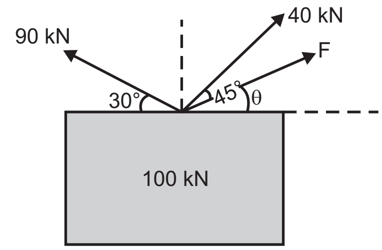
Sol. When $$\rho = 100 \text{ kN, than}$$

$$R_1 = k\delta_1 = \frac{2\rho}{5} = \frac{2 \times 100}{5} = 40 \text{ kN}$$

and $$R_2 = k\delta_2 = \frac{4\rho}{5} = \frac{4 \times 100}{5} = 80 \text{ kN}$$

Ans. The correct choice is (*d*).

1.15. A box of weight 100 kN shown in the figure is to be lifted without swinging. If all the forces are coplanar, the magnitude and direction (θ) of the force (*F*) with respect to X-axis should be: (GATE, 2014)



(*a*) $F = 56.389$ kN, $\theta = 28.28°$

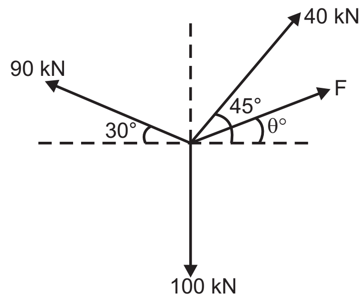
(*b*) $F = -56.389$ kN, $\theta = -28.26°$

(*c*) $F = 9.055$ kN, $\theta = 1.414°$

(*d*) $F = -9.055$ N, $\theta = -1.414°$

Sol. The coplaner diagram of the forces is shown in figure

Resolving the forces horizontally,



$-90 \cos 30° + 40 \cos 45° + F \cos \theta° = 0$

or $F \cos \theta° = 90 \cos 30° - 40 \cos 45°$

$= 77.942 - 28.284 = 49.658$ ...(*i*)

Resolving the forces vertically,

$90 \sin 30° + 40 \sin 45° + F \sin \theta = 100$

or $F \sin \theta = 100 - 90 \sin 30° - 40 \sin 45° = 100 - 45.000 - 28.285$

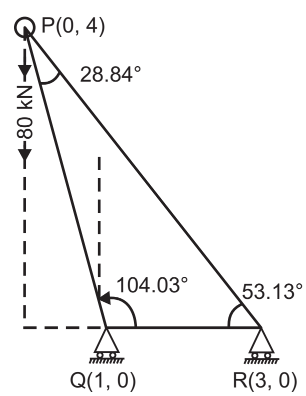
$= 26.715 \quad \therefore \tan \theta = \dfrac{26.715}{49.658} = 0.538 \quad \Rightarrow \quad \theta = 28° .28$

From equ (*i*) $F \cos 28.28 = 49.65$

$F = 56.38$ kN

Ans. The correct choice (*a*)

1.16. Mathematical idealization of a crane has three bars with their vertices arranged so shown in the figure with a load of 80 kN hanging vertically. The coordinates of the vertices are given in parentheses. The force in the member *QP*, F_{QR} will be: (GATE,2014)



(*a*) 30 kN Compressive

(*b*) 30 kN tensile

(*c*) 50 kN Compressive

(*d*) 50 kN tensile.

Sol. Taking moments of forces about *R* we get

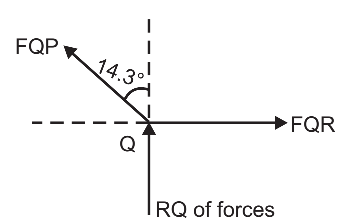
$R_Q \times 2 = 80 \times 3 \Rightarrow R_Q = 120$ kN

Resolving the forces vertically at joint Q,

$R_Q = FQ_P \cos 14.3$

or $FQ_P = \dfrac{R_\theta}{\cos 14.3} = \dfrac{120}{\cos 14.3°} = \dfrac{120}{0.9690}$

$= 123.84$ kN



Resolving the forces horizontally, atQ,

$FQP \sin 14.3° = FQ_R$ ...(*ii*)

$123.84 \sin 14.3° = FQR \quad \Rightarrow \quad FQR = 30.59$ kN (Compressive)

Ans. The correct choice is (*a*).

1.17. The symmetry of stress tensor at a point in the body under equilibrium is obtained from? (GATE, 2005)

(*a*) conservation of mass (*b*) Force equilibrium equations

(*c*) moment equilibrium equations (*d*) conservation of energy

Ans. The correct choice is (*c*)

1.18. Mohr's circle for the state of stress defined by $\begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix}$ MPa is a circle with (GATE, 2006)

(*a*) centre at (0, 0) and radius 30 *M*Pa (*b*) centre at (0, 0) and radius 60 *M*Pa

(*c*) center at (30, 0) and radius 30 *M*Pa (*d*) centre at (30, 0) and zero radius

Sol. Radius = $\tau_{max} = \sigma_1 - \sigma_2 =$ 30 – 30 = zero MPa

Ans. The correct choice is (*d*).

1.19. An axially loaded bar is subjected to a normal stress of 173 MPa. The shear stress in the bar: (GATE, 2007)

(*a*) 75 MPa (*b*) 86.5 MPa (*c*) 100 MPa (*d*) 122.3 MPa

Sol. We know that maximum shear stress in the bar

$$\text{Half the normal stress} = \frac{173}{2} = 8.65 \text{ mPa}$$

Ans. The correct choice is (*b*)

1.20. The major and minor principal stresses at a point are 3 MPa and –3 MPa respectively, the maximum shear stress at the point is: (GATE, 2010)

(*a*) 0 (*b*) 3 MPa (*c*) 6 MPa (*d*) 9 MPa

Sol. $\tau_{max} = \dfrac{\text{Major principal stress} - \text{minor principal stress}}{2}$

$$\tau_{max} = \frac{3-(-3)}{2} = 3 \text{ mPa}$$

Ans. The correct choice is (*b*)

1.21. If a small concrete cube is submerged deep in still water in such a way that the pressure was exerted on all faces of the cube is *P*, then the maximum shear stress developed inside the cube is: (GATE, 2012)

(*a*) *0* (*b*) $\dfrac{P}{2}$ (*c*) *P* (*d*) 2*P*

Sol. When a member is subjected to *a* hydrostatic pressure its Mohr circle is a point, the radius of Mohr circle *i.e.* maximum shear stress is zero.

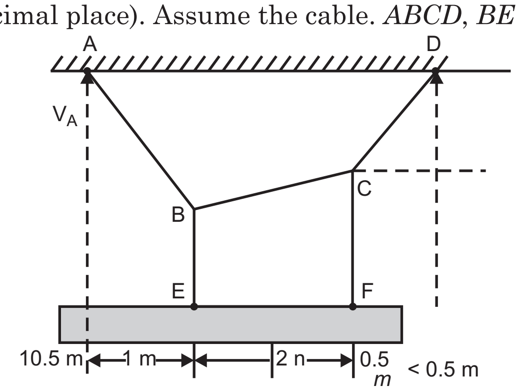
Ans. The correct choice is (*a*).

1.22. If principal stresses in a two-dimensional cases are (–) 10 MPa and 20 MPa respectively, then maximum shear stress at the print is: (GATE, 2004)

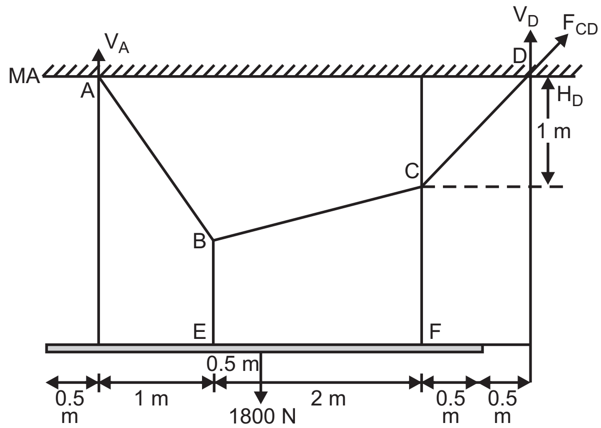
(*a*) 10 MPa (*b*) 15 MPa (*c*) 20 MPa (*d*) 30 MPa

Ans. The correct choice is (*b*).

1.23. A uniform beam weighing 1800 N is supported at *E* and *F* by cable ABCD. Determine the tension (in *N*) in segment *AB* of this cable (correct to 1 decimal place). Assume the cable. *ABCD*, *BE* and to be weightless: (GATE, 2013)



Sol. Taking moments of forces about *D*,



$$V_A \times 4 = 1800 \times 2.5 \quad \Rightarrow \quad V_A = \frac{1800 \times 2.5}{4} = 1125 \text{ N}$$

$$V_D = 1800 - 1125 = 675 \text{ N}$$

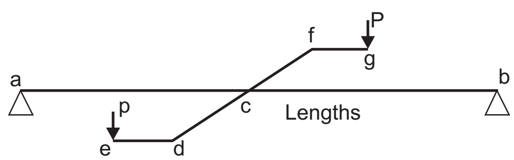
Taking moments of forces about C

$$V_A \times 3 - H_A \times 1 - 1800 \times 1.5 = 0$$

$$H_A = 1125 \times 3 - 2700 = 675 \text{ N}$$

Tension in $AB = \sqrt{1125^2 + 675^2} = 1311.96 \text{ N} = 1312.0 \text{ N}$ **Ans.**

1.24. A beam having a double cantilever attached at mid-span is shown in the figure. The nature of the force in '*ab*' is: (GATE, 1991)

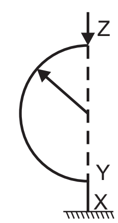


Lengths
$cd = cf$
$de = fg$
$ac = cb$

(*a*) bending and shear (*b*) bending, shear and torsion
(*c*) pure tension (*d*) torsion and shear

Ans. Two levers placed at *c* are balanced. The beam is subjected to S.F and *BM* only. The correct choice is (*a*).

1.25. A curved member with a straight vertical leg is carrying a vertical load *Z*, as shown in the figure the stress resultant in the *XY* segment are: Bending moment, shear force and axial force (GATE, 2003)



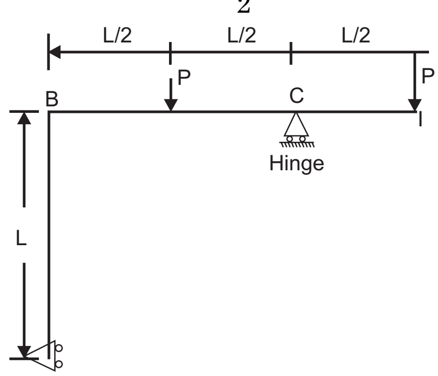
(*a*) bending short and axial force only (*b*) bending moment and shear force only
(*c*) axial force only (*d*) bending moment only

Sol. Since load passes axially through the segment *XY*, if experiences only axial stress.

Ans. The correct choice is (*c*).

1.26. A frame *ABCD* is supported by a roller at *A* and is on a hinge at *C* as shown below:
The reaction at the roller end A is given by (GATE, 2000)

(*a*) P (*b*) $2P$ (*c*) $\frac{P}{2}$ (*d*) zero



Sol. The loads P on either side of the central support C are placed symmetrically on the frame. Hence, the reaction at horizontal hinge reaction is zero. It means there is no reaction at A.

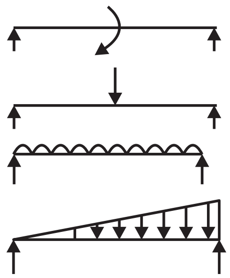
Ans. The correct choice is (*d*).

1.27. List-I shows different loads acting on a beam and list II shows different bending moment distributions. Match the load with the corresponding bending moment diagram.

(GATE, 2003)

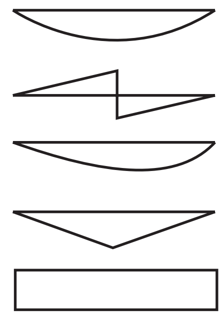
List I

A.
B.
C.
D.



List I

1.
2.
3.
4.
5.



Codes:

	A	**B**	**C**	**D**
(*a*)	4	2	1	3
(*b*)	5	4	1	3
(*c*)	2	5	3	1
(*d*)	2	4	1	3

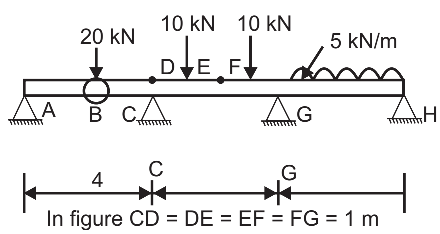
Ans. The correct choice is (*b*).

Data for Q. 28 and 29 are given below, solve the problems and choose the correct answers. A three-span continuous beam has an internal hinges at B. Section B is at mid span of AC, section E is at the mid span of CG. The 20 kN load is applied at section B where as 10 kN loads are applied at sections D and F as shown in the figure, span GH is subjected to uniformly distributed load of magnitude 5 kN/m, for the loading shown, shear force immediate to the right of section E is 9.84 kN upwards and the sagging moment at section E is 10.31 kNm.

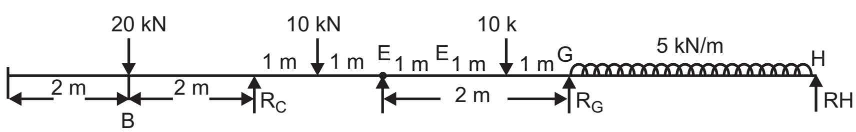
1.28. The magnitude of shear force immediate to left and immediate to the right of section B are respectively (GATE, 2004)

(*a*) 0 and 20 kN (*b*) 10 kN and 20 kN

(*c*) 20 kN and 0 (*d*) 9.84 kN and 10.16 kN



Sol.



Given: S.F. just right of section E = 9.84 kN ↑

Sagging moment at section E = 1031 kN.m

Let R_G and R_H be the reactions at G and H respectively

$R_H + R_G - 10\text{ kN} - (5 \times 4)\text{kN} = -9.54\text{ kN}$

or $\quad R_H + R_G = -9.84 + 30 = 20.16\text{ kN}$ ...(*i*)

Taking moments about E, we get

$R_H \times 6 + R_G \times 2 - 10 \times 1 - 5 \times 4 \times 4 = +10.31$

$$3R_H + R_G = -5.15 + 5 + 40 = 39.85 \quad \ldots(ii)$$

Solving eqn. (*i*) and (*ii*) we get

$$R_H = 9.85 \uparrow$$

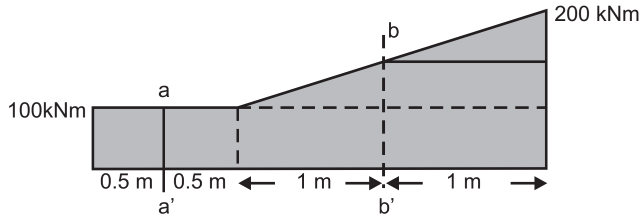
Ans. The correct choice (*b*).

1.29. The vertical reaction at support *H* is

(*a*) 15 kN upward (*b*) 9.84 kN upward (*c*) 15 kN downward (*d*) 9.84 kN downward

Ans. The correct choices (*b*)

1.30. The bending moment diagram for a beam is given below



The shear force at sections aa′ and bb′ respectively are of magnitude.

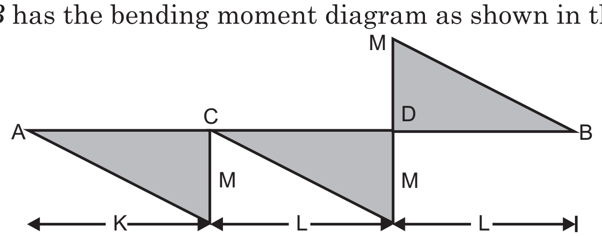
(*a*) 100 kN, 150 kN (*b*) zero, 100 kN (*c*) zero, 50 kN (*d*) 100 kN, 100 kN

Sol. We notice that bending moment left of section aa′ is uniform or constant, the shear force in the zone is zero, they by shear force at section aa′ is zero.

Again, the area of bending moment diagram from section bb′ up to right end is equal to the change in bending moment *i.e.*, 200 – 150 = 50

Ans. The correct choice is (*c*).

1.31. A simple supported beam *AB* has the bending moment diagram as shown in the following figure:



The beam is possibly under the action of following loads:

(*a*) couples of *M* at *C* and 2 m at *D*

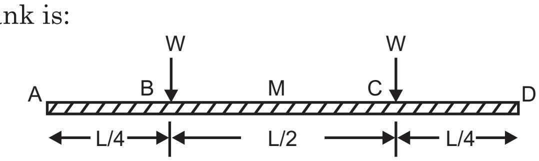
(*b*) couples of 2 *M* at *C* and *M* at *D*

(*c*) concentrated loads of *M*/*L* at *C* and 2 *M*/*L* at *D*

(*d*) concentrated load of *M*/*L* at *C* and couple of 2 *M* at *D*

Ans. The correct choice is (*a*).

1.32. Two people weighing *W* each are sitting on a plank of length *L* floating on water at *L*/4 from either end. Neglecting the weight of the plank, the bending moment at the centre of the plank is:



(*a*) $\frac{WL}{8}$ (*b*) $\frac{WL}{16}$

(*c*) $\frac{WL}{32}$ (*d*) zero

Sol. Let w = upward water pressure, per unit length

Resolving the vertical forces we get

$$2W = wL$$

or

$$W = \frac{wL}{2}$$

B.M. at $D = 0$

B.M at $C = W \times \frac{L}{4} \times \frac{L}{8} = \frac{WL^2}{32}$

B.M. at mid span $M = W - \frac{L}{2} \times \frac{L}{4} - W \times \frac{L}{4}$

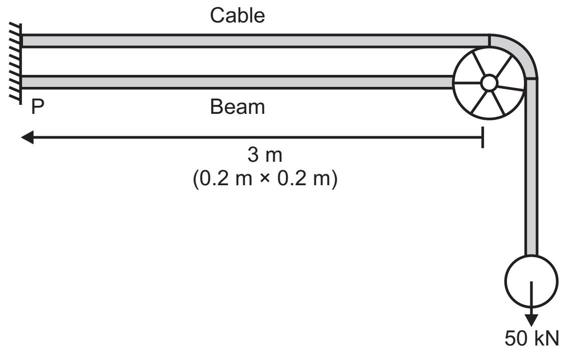
or $M = \frac{WL^2}{8} - \frac{WL}{2} \times \frac{L}{4} = \frac{WL^2}{8} - \frac{WL^2}{8} = 0$

Ans. The correct choice is (*d*).

1.33. The values of axial stress (σ) in KN/m^2, bending moment (*M*) in KN.m and shear force (*V*) in *kN* acting at point for the arrangement shown in the figure, are respectively.

(*a*) 1000, 75 and 25 (*b*) 1250, 150 and 50

(*c*) 1500, 225 and 75 (*d*) 1750, 300 and 100



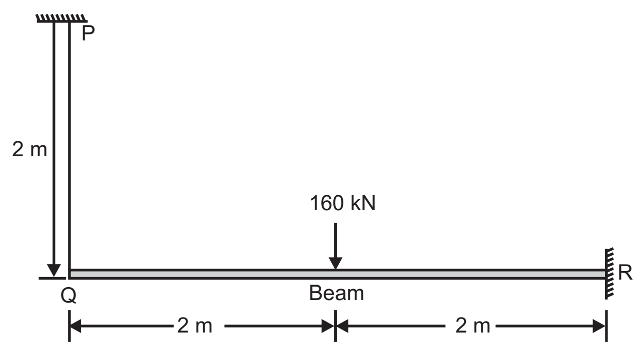
Sol. B.M. at $P = 50 \times 3 = 150$ KN.m

Shear force = 50 KN

Axial stress = $\frac{50}{0.2 \times 0.2} = \frac{5000}{4} = 1250$ KN/m^2

Ans. The correct choice is (*b*).

1.34. The axial load (in *kN*) in the member *PQ* for the arrangement/assembly shown in the figure given below, is (GATE, 2014)



Sol. Let V_θ = Vertical reaction at *Q* acting upward

Taking moments about *R* we get,

$$V_Q \times 4 = 160 \times 2 \Rightarrow V_Q = \frac{160 \times 2}{4} = 80\,\text{kN} \quad \textbf{Ans.}$$

1.35. The first moment of area about the axis of bending for a beam cross section is: (GATE, 2014)

(*a*) moment of inertia (*b*) section modules

(*c*) shape factor (*d*) polar moment of inertia

Ans. The correct choice is (*b*) since $Z = \frac{I}{Y_{max}}$

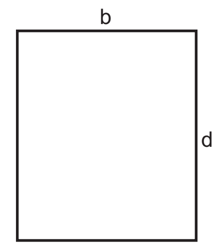
1.36. Polar moment of inertia (I_p) in Cm^4 of a rectangular section having width b = 2 cm and defame d = 60 m is ....
(GATE, 2014)

Ans. Polar moment of inertia

$$I_Z = I_x + I_y$$
$$= \frac{2\times 6^3}{12} + \frac{6\times 2^3}{12} = 36 + 4 = 40 \text{ cm}^4 \quad \textbf{Ans.}$$

1.37. For the section shown below, second moment of the area about an axis $d/4$ distance above the bottom of the area is: (GATE, 2006)

(*a*) $\frac{bd^3}{48}$ (*b*) $\frac{bd^3}{12}$ (*c*) $\frac{bd^3}{48}$ (*d*) $\frac{bd^3}{3}$

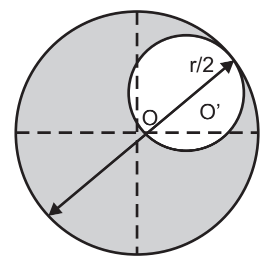


Sol.

$$I_{\frac{d}{4}} = \frac{bd^3}{12} + (bd)\left(\frac{d}{4}\right)^2$$
$$= \frac{bd^3}{12} + \frac{bd^3}{16} = \frac{4bd^3 + 3bd^3}{48}$$
$$= \frac{7bd^3}{48}$$

Ans. The correct choice is (*c*).

1.38. A disc of radius 'r' has a hole of radius $\frac{'r'}{2}$ cut-out as shown. The centroid of the remaining disc (shaded portion) at a radial distance from the centre 'O' is: (GATE, 2011)



(*a*) $\frac{r}{2}$ (*b*) $\frac{r}{3}$ (*c*) $\frac{r}{6}$ (*d*) $\frac{r}{8}$

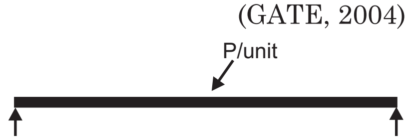
Sol. Let us assume that Y-axis passes through the origin of the big circle.

$$\bar{x} = \frac{A_1x_1 - A_2x_2}{A_1 - A_2} = \frac{\pi R^2(o) - \pi\left(\frac{R}{2}\right)^2\left(\frac{R}{2}\right)}{\pi R^2 - \pi\left(\frac{R}{2}\right)^2}$$

$$= \frac{-\pi\frac{R^3}{8}}{\pi R^2 - \pi\frac{R^2}{4}} = \frac{-\frac{R}{8}}{\frac{3}{4}} = -\frac{R}{8}\times\frac{4}{3} = -\frac{R}{6}$$

Ans. The centre oil of the shaded portion of the disc is at $\frac{R}{6}$ distance from the origin on left.
The correct choice is (*c*).

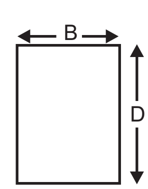
1.39. A homogeneous, simply supported prismatic beam of width B, depth D and span L is subjected to a concentrated load of magnitude P. The load can be placed anywhere along the span of the beam. The maximum flexural stress developed in beam is: (GATE, 2004)



(*a*) $\frac{2PL}{3BD^2}$ (*b*) $\frac{3}{4}\frac{PL}{BD^2}$

(*c*) $\frac{4}{3}\frac{PL}{BD^2}$ (*d*) $\frac{3}{2}\frac{PL}{BD^2}$

Sol.



Maximum $BM = \frac{PL}{4}$

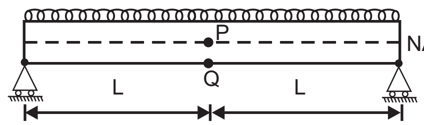
Maximum section modulus $= \left(\frac{BD^2}{6}\right) = \frac{BD^2}{6}$

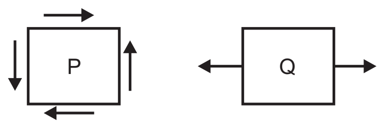
∴ Maximum flexural stress

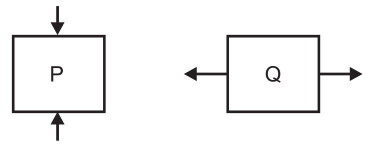
$$= \frac{M}{Z} = \frac{\left(\frac{PL}{4}\right)}{\left(\frac{BD^2}{6}\right)} = \frac{3PL}{2BD^2}$$

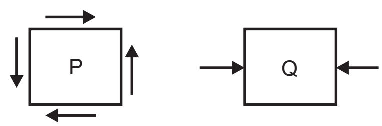
Ans. The correct choice (a).

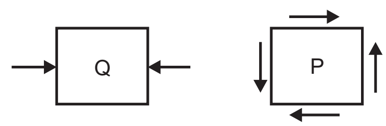
1.40. Consider a simply supported beam with a uniformly distributed load having a neutral axis (NA) as shown. For points P (on the neutral axis) and Q (at the bottom of the beam), the state of stress is best represented by which of the following pairs? (GATE, 2011)



(a) 

(b) 

(c) 

(d) 

Ans. The correct choice is (a).

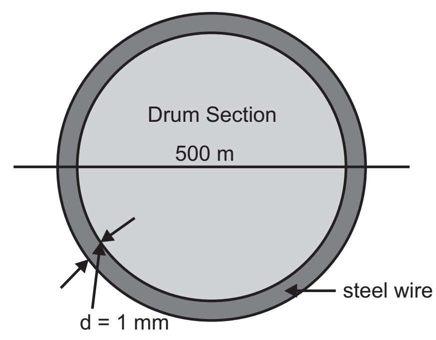
1.41. The plane section remains "plane" the assumption in bending theory implies: (GATE, 2013)

(a) strain profile is linear (b) stress profile is linear

(c) both strain and stress profiles are linear (d) Shear deformations are neglected

Ans. The correct choice is (a).

1.42. The maximum bending stress induced in a steel wire of modulus of elasticity 200 kN/mm^2 and diameter 1 mm when wound on a drum of diameter 1 m is approximately equal to :



(a) 50 N/mm^2 (b) 100 N/mm^2

(c) 200 N/mm^2 (d) 400 N/mm^2

Sol. Given: $E = 200$ kN/mm^2

$R = 500$ mm^2

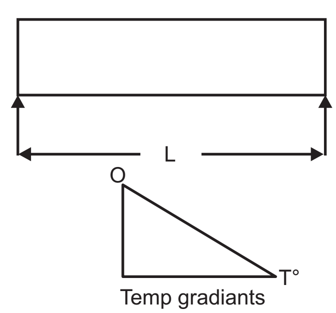
$Y = 0.5$ mm

∴ Bending stress $f = \frac{E}{R}.Y$

$$= \frac{200 \times 10^3}{500} \times (0.5) = 200 \text{ MPa}$$

The correct choice (c).

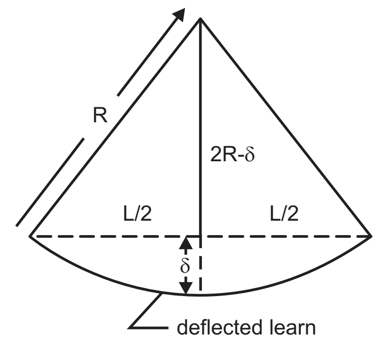
1.43. A simply supported beam of uniform rectangular cross section of width b and depth h is subjected to a linear temperature gradient, 0° at the top and T° at the bottom, as shown in the figure. The coefficient of linear expansion of the beam material is α. The resulting vertical deflection at the mid- span of the beam is:



(a) $\frac{\alpha T h^2}{8L}$ upward (b) $\frac{\alpha T L^2}{8h}$ upward

(c) $\frac{\alpha T h^2}{8L}$ downward (d) $\frac{\alpha T L^2}{8h}$ downward

Sol. Due to temperature variation, the beam warps. Average variation of temperature from centroid axis to the extreme fiber at the bottom = $T/2$



The extreme strain at the bottom = $\alpha.\frac{T}{2}$

Applying the bending equation, we get

$$R = \frac{E.Y}{f} = \frac{Y}{\text{Strain}} = \frac{h/2}{\alpha.\frac{T}{2}} = \frac{h}{\alpha T}$$

From the simple geometry of a circle

$$\frac{L}{2}\times\frac{L}{2} = (2R-\delta)\delta$$

$$\frac{L^2}{4} = 2R\delta - \delta^2$$

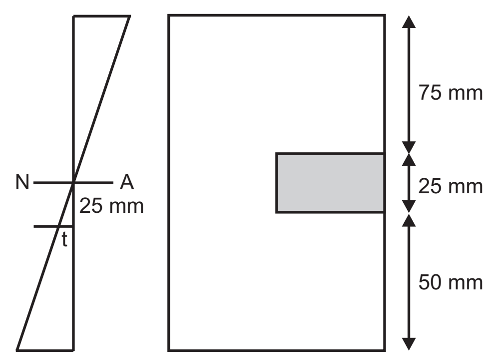
or $$\delta = \frac{L^2}{8R} \text{ neglecting } \delta^2\text{, being too small.}$$

Substituting the value of R we get

$$\delta = \frac{L^2}{8\times\frac{h}{\alpha T}} = \frac{L^2\times\alpha T}{8h}, \text{ downward}$$

Ans. The correct choice is (*d*).

1.44. A beam with the cross-section given below is subjected to a positive bending moment (causing compression at the top) of 16 kNm acting around the horizontal axis. The tensile force acting on the hatched area of his cross-section is : (GATE, 2006)



(*a*) zero (*b*) 5.9 kN (*c*) 8.9 kN (*d*) 17.8 kN

Sol. By applying the bending equation we get

$$f = \frac{M}{I}Y = \frac{16\times10^6}{\left(\frac{50\times150^3}{12}\right)}\times 25$$

$$= 28.44 \text{ MPa}$$

The tensile force f on the hatched area

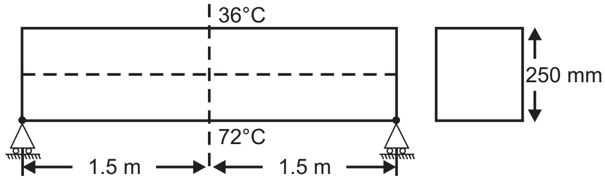
= average tensile stress below neutral axis area × hatched area

$$= \left(\frac{0+28.44}{2}\right)25\times50 = 17.8 \text{ kN}$$

Ans. The correct choice is (*d*).

1.45. The beam of an over all depth 250 mm (shown below) is used in a building subjected to two different thermal environments. The temperatures at the top and bottom surfaces of the beam are 36°C and 72°C respectively.

Considering coefficient of thermal expansion (α) 1.5×10^{-5} per °C, the vertical deflection of the beam (in mm) at its mid-span due to temperature gradients is – (GATE, 2014)



Sol. Here, $\Delta T = 72° - 36° = 36°C.$

Since the bottom temperature is more than that of the top surface, the beam deflects downward.

By using the standard formula

$$\delta = \frac{\alpha.(\Delta T)\times L^2}{8h} = \frac{1.5\times10^{-5}\times36\times3000^2}{8\times250}$$

$$= 2.43 \text{ mm downward}$$

Ans. The answer choice is 2.43 mm.

1.46. For a given shear force across a symmetrical I-section, the intensity of shear is maximum at (GATE, 1994)

(*a*) Extreme fibers (*b*) centroid of the section

(*c*) at the junction of the flange and the web on the web

(*d*) A the junction of the flange and web, but not on the flange

Ans. The correct choice is (*b*).

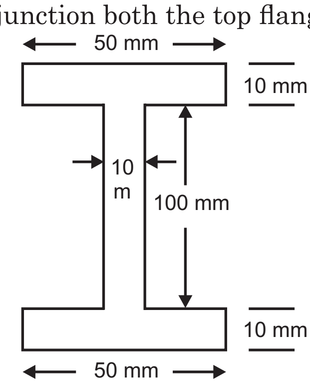
1.47. In a section, shear centre is a point through which, if the resultant load passes, the section will not be subjected to any.

(*a*) bending (*b*) tension (*c*) compression (*d*) torsion

Ans. The correct choice is (*d*),

Remember torsion is eliminated by applying load through shear centre.

1.48. A symmetric I-section (with width of each flange = 50 mm, thickness of each flange = 10 mm, depth of web = 100 mm and thickness of web = 10 mm) of steel is subjected to a shear force of 100 kN. Find the magnitude of the shear stress (kN/mm^2) in the wcb at its junction both the top flange – (GATE, 2013)



Sol. $$I = \frac{50\times10^3}{12} - \frac{40\times100^3}{12} = 3.87\times10^6 \text{mm}^4$$

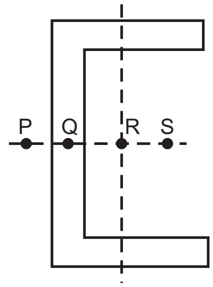
Shear stress in the web at the junction of flange and web

$$\tau = \frac{V.A.Y}{Ib} = (100\times10^3)(50\times10)(\frac{100}{2}+5)$$

$$= \frac{(100\times10^3)(500)(55)}{(3.87\times10^6)(10)} = 71\text{MPa}$$ **Ans.**

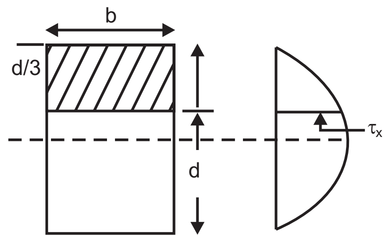
1.49. The possible location of shear centre of the channel section, shown below, is:

(*a*) *P* (*b*) *Q* (*c*) *R* (*d*) *S*



Ans. The correct choice is (*a*).

1.50. If a beam of rectangular cross section is subjected to a vertical shear force V, the shear force carried by the upper one third of the cross section is



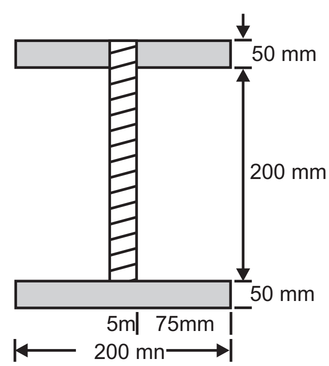
(*a*) zero (*b*) $\frac{7V}{27}$ (*c*) $\frac{8V}{27}$ (*d*) $\frac{V}{3}$

So.
$$\tau_x = \frac{V\,A\bar{Y}}{Ib} = \frac{V\left(b\times\frac{d}{3}\right)\left(\frac{d}{2}-\frac{d}{6}\right)}{\left(\frac{bd^3}{12}\right)\times b} = \frac{(V.b.d^2)/9}{\frac{b^2d^3}{12}} = \frac{Vbd^2}{9}\times\frac{12}{b^2d^3} = \frac{4V}{3bd}$$

Force on the hatch used area $= \frac{2}{3}\tau_x = \frac{2}{3}\left(\frac{4V}{3bd}\right)\left(b.\frac{d}{3}\right) = \frac{8V}{27}$

Ans. The correct choice is (*c*).

1.51. *I*-section of a beam is formed by gluing wooden planks as shown in the figure below. If this beam transmits a constant vertical shear force of 3000 N, the glue at any of the four joints will be subjected to a shear force *C* in kN per metre length) of:



(*a*) 3.0 (*b*) 4.0 (*c*) 8.0 (*d*) 10.7

Sol. Shear at the joint/unit length $= \frac{VA\bar{Y}}{I}$

Here
$$I = \frac{200\times300^3}{12} - \frac{150\times200^3}{12}$$
$$= 350\times10^6\ \text{mm}^4$$

$$\therefore \quad \tau_b = \frac{(3000)(50\times75)(125)}{350\times10^6} = 4\ \text{N/mm} = 4\ \text{kN/m}$$

Ans. The correct choice is (*b*).

1.52. The shear stress at the neutral axis in a beam of triangular section with a base of 40 mm and/height 20 mm, subjected to a shear force of 3 kN, is (GATE, 2007)

(*a*) 3 MPa (*b*) 6 MPa (*c*) 10 MPa (*d*) 20 MPa

Sol. We know that

$$\tau_{NA} = \frac{4}{3}[\tau_{\max}] = \frac{4}{3}\left[\frac{F}{\frac{1}{2}bh}\right]$$

$$= \frac{4}{3}\left[\frac{3\times10^3}{\frac{1}{2}\times40\times20}\right] = 10\,\text{MPa}$$

Ans. The correct choice is (*c*).

1.53. The point within the cross-sectional plan of a beam through which the resultant of the external forces on the beam has to pass through, to ensure pure bending without twisting of the cross section of the beam is called:

(*a*) moment centre (*b*) centroid (*c*) shear centre (*d*) elastic centre

Ans. The correct choice is (*c*).

1.54. A long shaft of diameter 'd' is subjected to twisting moment T at its ends. The maximum normal stress acting at its cross section is equal to (GATE, 2006)

(*a*) zero (*b*) $\frac{16T}{\pi d^3}$ (*c*) $\frac{32T}{\pi d^3}$ (*d*) $\frac{64T}{\pi d^3}$

Ans. Since a member subjected to torsion the normal stresses developed in the cross-section in always zero, the correct choice is (*a*).

1.55. A circular solid shaft of span L = 5 m is fixed at one end and free at the other end. A twisting moment T = 100 kNm is applied at the free end. The torsional rigidity GI per unit angular twist is 50000 kNm^2/rad. Following statements are made for this shaft:

1. The maximum rotation is 0.01 rad (GATE, 2004)
2. The torsional strain energy is 1 kNm

With reference to the above statements, which of the following applies?

(*a*) Both statements are true
(*b*) Statement 1 is true but 2 is false
(*c*) Statements 2 is true but 1 is false
(*d*) Both the statements are false

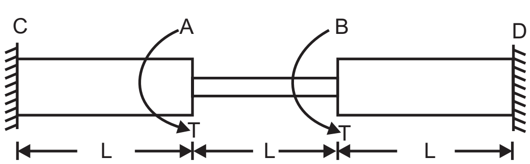
Sol. We known $\tau_{max} = \frac{TL}{GJ} = \frac{(100\times10^6)(5000)}{5000\times1000^3} = 0.01$ rad

and Torsional strain energy $u = \frac{1}{2}T = \frac{1}{2}(100\times10^6)(0.01) = 0.5\,\text{kNm}$

i.e., statement 1 is true and statement 2 is false

Ans. The correct choice is (*a*)

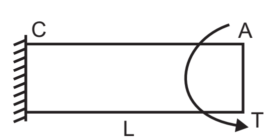
1.56. A circular shaft shown in the figure is subjected to torsion T at two points A and B. The torsional rigidity of portions CA and BD in GJ and that portion $AB = GJ_2$. The rotations of shaft at points A and B are θ_1 and θ_2. The rotation θ_1 is



(*a*) $\frac{TL}{GJ_1 + GJ_2}$ (*b*) $\frac{TL}{GJ_1}$

(*c*) $\frac{TL}{GJ_2}$ (*d*) $\frac{TL}{GJ_1 - GJ_2}$

Sol. Part AB simply rotates without any angle of twist. Considering FBD of portion CA,

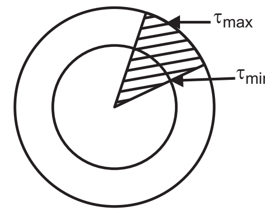


Angle of twist at $A = \frac{TL}{GJ_1}$

Ans. The correct choice (*b*)

1.57. The maximum and minimum shear stresses in a hollow circular shaft of outer diameter 20 mm and thickness 23 m, subjected to a torque of 92.7 Nm will be: (GATE, 2007)

(*a*) 59 MPa and 47.2 MPa (*b*) 100 MPa and 80 MPa
(*c*) 118 MPa and 160 MPa (*d*) 200 MPa and 160 MPa



Sol. We know $\dfrac{T}{J} = \dfrac{\tau}{R}$

τ_{max} occurs at the outer surface of the shaft.

$$\tau_{max} = \left(\frac{T}{J}\right) = \frac{\pi}{32}\frac{92.7\times10^3}{(20^4-16^4)}\left(\frac{20}{2}\right) = 100\,\text{mPa}$$

τ_{min} occurs inside the shaft

$$\frac{\tau}{J}(R_{min}) = \frac{\pi}{32}\frac{92.7\times10^3}{(20^4-16^4)}\left(\frac{16}{2}\right) = 80\text{ mPa}$$

Ans. The correct choice is (*b*).

1.58. The maximum shear stress in a solid shaft of circular cross-section having diameter *d* subjected to a torque *T* is τ. If the torque is increased by four times and diameter of the shaft is increased by two times the maximum stress in the shaft will be: (GATE, 2006)

(*a*) 2τ (*b*) τ (*c*) τ/2 (*d*) τ/4

Sol. We are given that torque is increased by 4 times and diametre is increased by 2 times

$$\frac{\tau_B}{\tau} = \frac{(d)^3(4T)}{(2d)^3(T)} = \frac{d^3\times4T}{8d^3\times T} = \frac{1}{2}$$

or $$\tau_B = \frac{\tau}{2}$$

Ans. The correct choice is (*c*).

1.59. A hollow circular shaft has an outer diameter of 100 mm and a wall thickness of 25 mm. The allowable shear stress in the shaft is 125 MPa. The maximum torque the shaft can transmit, is (GATE, 2009)

(*a*) 46 kNm (*b*) 24.5 kNm (*c*) 23 kNm (*d*) 11.5 kNm

Sol. Torque $T = \tau z P$

$$= 125\times\frac{(100^4-50^4)\times2}{32\times100} = 23\text{ kNm}$$

Ans. The correct choice is (*a*)

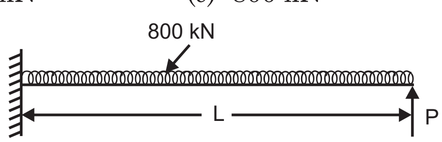
1.60. A solid circular shaft of diameter *d* and length *L* is fixed at one end and free at the other end. The shear modulus of the material is *G*, the angle of twist at the free end, is : (GATE, 2010)

(*a*) $\dfrac{16TL}{\pi d^4G}$ (*b*) $\dfrac{32TL}{\pi d^4G}$ (*c*) $\dfrac{64TL}{xd^4G}$ (*d*) $\dfrac{128TL}{\pi d^4G}$

Ans. The correct choice is (*b*).

1.61. A cantilever beam of span '*L*' is subjected to a downward load 800 kN uniformly distributed over its length and *a* concentrated upward load *P* at its free end. For vertical displacement to be zero at the free end, the value of *P* is: (GATE, 1992)

(*a*) 300 kN (*b*) 500 kN (*c*) 800 kN (*d*) 1000 kN



Sol. Deflection of the free end downward = $\dfrac{wL^4}{8EI}$ ...(*i*)

Deflection of the free end upward = $\dfrac{PL^3}{3EI}$ ...(*ii*)

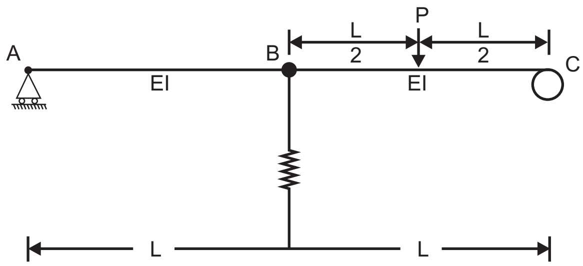
Equating (*i*) and (*ii*) we get

$$\frac{PL^3}{3EI} = \frac{WL^4}{8EI}$$

or $$P = \frac{3}{8}(wL)\ \frac{3}{8}\times 800 = 300\,\text{kN}$$

Ans. The correct choice is (*a*).

1.62. Two elastic rods *AB* and *DC* are hinged at *B*. The joint *A* is a hinge one, joint *C* is over a roller and the joint *B* is supported on a spring having its stiffness as *k*. (GATE, 1990)



A load *P* acts at mid point of the rod *BC*. The downward deflection of joint *B* is:

(*a*) P/k (*b*) $2P/k$

(*c*) $P/2k$ (*d*) 0

Sol. Apparently load on spring = $P/2$

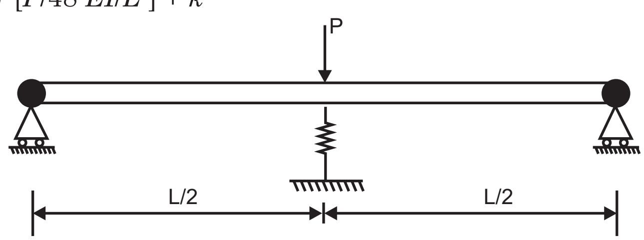
But, stiffness of spring $k = \dfrac{\text{load}}{\text{deflection}} = \dfrac{P/2}{\delta} \Rightarrow \delta = \dfrac{P}{2k}$

Ans. The correct choice is (*c*).

1.63. A simply supported beam of span length *L* and flexural stiffness *EI* has another spring support at the centre span of stiffness *k* as shown in figure. The central deflection of the beam due to a central load (concentrated) of *P* would be

(*a*) $[PL^3/48\ EI] + P/k$ (*b*) $[P/48\ EI/L^3] - k$

(*c*) $[PL^3/48\ EI] - P/k$ (*d*) $[P/48\ EI/L^3] + k$



Sol. The central deflection without the spring

$$\delta = \frac{PL^3}{48EI}$$

Deflection counter balanced by spring = P/K

∴ The resultant deflection at mid-span

$$\delta = \frac{PL^3}{48EI} - P/K$$

Ans. The correct choice is (*c*).

1.64. A cantilever beam of span *l* subjected to a uniform ly distributed load '*w*' per unit length resting on a rigid prop at the tip of the cantilever. The magnitude of the reaction at the prop is:

(*a*) $\dfrac{1}{8}Wl$ (b) $\dfrac{2}{8}Wl$ (*c*) $\dfrac{3}{8}Wl$ (*d*) $\dfrac{4}{8}Wl$

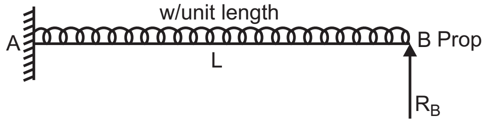
Sol. Equating the standard equations

$$\frac{Wl^4}{8EI} = \frac{Rl^3}{3EI}$$

$\therefore$ Prop reaction $R = \frac{We^4}{8EI} \times \frac{3EI}{l^3} = \frac{3}{8}Wl$

Ans. The correct choice is (*c*).

1.65. A propped cantilever beam of span *L*, is loaded with uniformly distributed load of intensity w/unit length, all through the span, bending moment at the fixed end is: (GATE, 1995)



(*a*) $\frac{WL^2}{8}$ (*b*) $\frac{WL^2}{2}$ (*c*) $\frac{WL^2}{12}$ (*d*) $\frac{WL^2}{24}$

Sol. Let R_B = prop reaction

The deflection end *B* due to *u. d.l* = $\frac{wl^4}{8EI}$ downward

The deflection end $B = \frac{R_B l^3}{3E}$ upward due to reaction

$$\therefore \quad \frac{R_B.l^3}{3El} = \frac{wl^4}{8El} \Rightarrow R_B = \frac{3wl}{8}$$

The B.M. at end

$$= R_B \times l - wl \times l/2$$

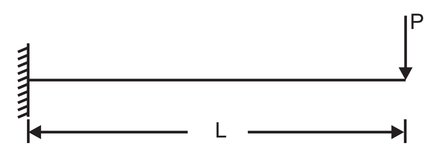
$$= \frac{3wl^2}{8} - \frac{wl^2}{2} = \frac{3wl^2 - 4wl^2}{8} = -\frac{wl^2}{8} \text{acting anticlockwise}$$

Ans. The correct choice is (*a*)

1.66. A cantilever beam of span '*L*' is loaded with a concentrated load '*P*' at the free end Deflection of the beam at the free end is: (GATE, 1997)

(*a*) $\frac{PL^3}{48EI}$ (*b*) $\frac{5PL^3}{384EI}$ (*c*) $\frac{PL^3}{3EI}$ (*d*) $\frac{PL^3}{6EI}$

Sol. The maximum deflection of the cantilever at the free end

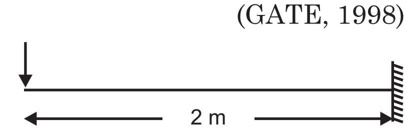


$$= \frac{PL^3}{3EI}$$

Ans. The correct choice is (*c*).

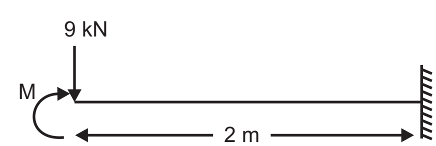
1.67. A cantilever beam is shown in the figure, the moment to be applied at free end for zero vertical deflection at the point, is: (GATE, 1998)

(*a*) 9 kN.m clockwise (*b*) 9 kN.m anti-clockwise
(*c*) 12 kN.m clockwise (*d*) 12 kN.m anti-clockwise



Sol. The deflection of free end due to load 9 kN

$$= \frac{WL^3}{3EI} = \frac{9 \times (2)^3}{3EI}$$



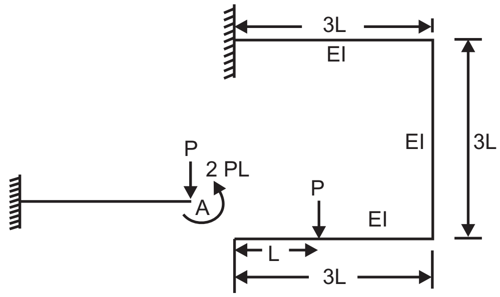
The deflection due to *M* at free end, $= \frac{Ml^3}{2EI} = \frac{M^4}{2EI}$

$$\therefore \quad \frac{4M}{2EI} = \frac{9 \times 8}{3EI} \Rightarrow M = \frac{9 \times 8}{6EI} = 12 \text{ kNm clockwise}$$

Ans. The correct choice is (*c*).

1.68. For the structure shown below, the vertical deflection at print A is given by:

(*a*) $\dfrac{PL^3}{\delta EI}$ (*b*) $\dfrac{2PL^3}{81EI}$ (*c*) zero (*d*) $\dfrac{PL^3}{72EI}$ (GATE, 2000)



Sol. Consider the free body diagram of the top horizontal member

$$M = P \times 2L = 2PL$$

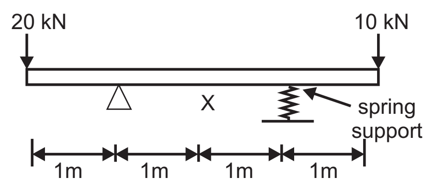
Deflection at $\quad A = \dfrac{PL^3}{3EI}(-)\dfrac{ML^2}{2EI}$

$$= \frac{P(3L)^3}{3EI} - \frac{2PL \times (3L)^2}{2EI} = \frac{27PL^3}{3EI} - \frac{18PL^3}{2EI}$$

$$= 9PL^3 - 9PL^3 = 0$$

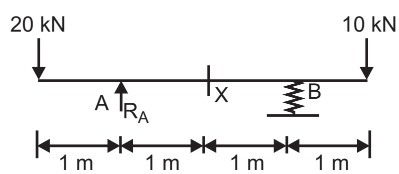
Ans. The correct choice is (*c*).

1.69. The bending moment (in kNm units) at the mid-span location X in the beam with overhangs shown below is equal:



(*a*) 0 (*b*) –10 (*c*) –15 (*d*) –20

Sol. Taking moments about B,



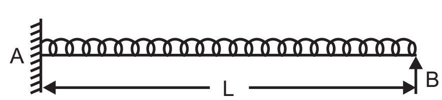
$$R_A \times 2 - 20 \times 3 + 10 \times 1 = 0$$

$$R_A = 25 \text{ kN}$$

The BM at mid section $x = R_A \times 1 - 20 \times 2 = 25 - 40 = -15$ kNm.

Ans. The correct choice is (*c*).

1.70. In the propped cantilever beam carrying a uniformly distributed load of 'w' N/m, shown in the following figure, the reaction at the support 'B', is;



(*a*) $\dfrac{5}{8}wL$ (*b*) $\dfrac{3}{8}wL$ (*c*) $\dfrac{1}{2}wL$ (*d*) $\dfrac{3}{4}wL$

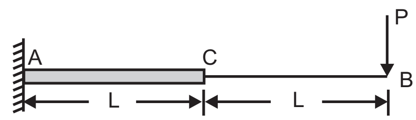
Sol. Using the standard formula, we get

$$\frac{WL^4}{8EI} = \frac{R_B L^3}{3EI}$$

or $$R_B = \frac{wL^4 \times 3EI}{8EI \times L^3} = \frac{3}{8}wL$$

Ans. The correct choice is (*b*).

1.71. Consider the beam *AB* shown in the figure below, Part *AC* of beam is rigid while part *CB* has the flexural rigidity *EI*. Identify the correct combination of deflection at end *B* and bending moment at end *A* (GATE, 2007)



(*a*) $\frac{PL^3}{3EI}$, $2PL$ (*b*) $\frac{PL^3}{3EI}, PL$

(*c*) $\frac{8PL^3}{3EI}, 2PL$ (*d*) $\frac{8PL^3}{3EI}, PL$

Sol. Apparently, $M_A = P(2L) = 2PL$

Since portion *AC* is rigid, it does not undergo any deflection, only the proton *CB* deflects as a cantilever due to loading

$\therefore$ $$\delta_B = \frac{PL^3}{3EI}$$

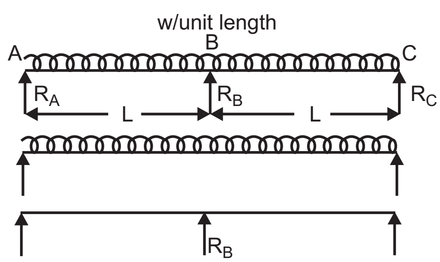
Ans. The correct choice is (*a*).

Statement for linked answer Q. 72 and 73.

A two span continuous beam having equal spans, each of length *L* is subjected to a uniformly distributed load 'ω' per unit length. The beam has constant flexural rigidity

1.72. The reaction at the middle support is:

(*a*) wL (*b*) $\frac{5wL}{2}$ (*c*) $\frac{5wL}{4}$ (*d*) $\frac{5wL}{8}$



Sol. Treating AC as a beam. The deflection at mid span of $AC = \frac{5w(2L)^4}{384EI}$ downward ...(*i*)

The deflection at mid span due to reaction R_B up ward = $\frac{R_B(2L)^3}{48EI}$ ...(*ii*)

Equating (*i*) and (*ii*) we get

$$\frac{5w(2L)^4}{384EI} = \frac{R_B(2L)^3}{48EI}$$

$$R_B = \frac{480}{384} \times wL = \frac{5}{4}wL$$

Ans. The correct choice is (*c*).

1.73. The bending moment at the middle support is:

(*a*) $\frac{WL^2}{4}$ (*b*) $\frac{WL^2}{8}$ (*c*) $\frac{WL^2}{12}$ (*d*) $\frac{WL^2}{16}$

Sol. Resolving the forces vertically,

$$R_A + R_B + R_C = w(2L)$$

But, $R_A = R_C$ due to symmetry,

$$2R_A + \frac{5}{4}wL = 2wL$$

$$R_A = wL\left(2 - \frac{5}{4}\right) \Rightarrow R_A = \frac{3}{8}wL$$

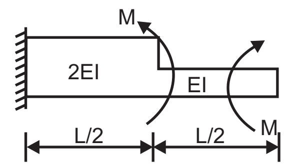
B.M. at mid support

$$M_B = R_A.L - W\,.\,L\frac{L}{2}$$

$$M_B = \frac{3}{8}wL^2 - \frac{wL^2}{2} = \frac{wL^2}{8}$$

Ans. The correct choice is (*b*).

1.74. The stepped cantilever is subjected to moment M as shown in the figure below. The vertical deflection at the free end (neglecting the self weight), is: (GATE, 2008)



(*a*) $\frac{ML^2}{8EI}$ (*b*) $\frac{ML^2}{4EI}$ (*c*) $\frac{ML^2}{2EI}$ (*d*) zero

Sol. Apply a dummy load F at free end where deflection is required.

B.M. from $x = 0$ to $x = L/2$ $\quad x = 0$ to $L/2$,

$$M_x = \text{Fx} - \text{M}$$

$$\therefore \quad \frac{\delta M_x}{\delta F} = x$$

B.M. from $\quad x = L/2$ to L,

$$M_x = F_x - 2m$$

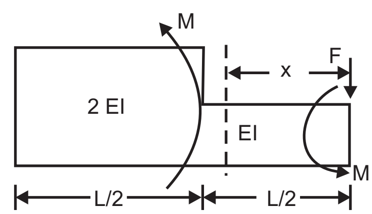
or $$\frac{\partial M_x}{\partial F} = x$$

Deflection $$\delta = \int_o^{L/2} \frac{(F_x - M)xdx}{EI} + \int_{L/2}^{L} \frac{(F_x - 2M)xdx}{2EI} \quad \ldots(i)$$

The general equation by Castigation, is

$$\delta = \int \frac{ma}{EI}\frac{\delta mx}{\delta_F}$$

But $\quad F = 0°$, a dummy



$$\delta = \int_o^{L/2} \frac{-mx}{EI}dn + \int_{L/2}^{L} \frac{-2Mx}{2EI}dx$$

$$\delta = \left[\frac{-Mx^2}{2EI}\right]_o^{L/2} + \left[\frac{-2Mx^2}{2EI}\right]_{L/2}^{L}$$

On simplification, we get

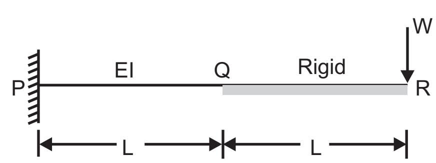
$$\delta = -\frac{ML^2}{2EI}$$

Since the sign is, negative, the deflection is upward,

Ans. The correct choice is (*c*).

1.75. In the cantilever beam PQR shown in figure, the segment PQ has flexural EI and segment QR has infinite flexural rigidity.

The deflection and slope of the beam at Q are respectively



(*a*) $\frac{5ML^3}{6EI}$ and $\frac{3ML^2}{2EI}$ (*b*) $\frac{WL^3}{2EI}$ and $\frac{WL^2}{2EI}$ (*c*) $\frac{ML^3}{2EI}$ and $\frac{WL^2}{EI}$ (*d*) $\frac{WL^3}{3EI}$ and $\frac{3WL^2}{2EI}$

Ans. The correct choice is (*a*).

1.76. The deflection of the beam at 'R' is

(*a*) $\frac{8WL^3}{EI}$ (*b*) $\frac{5WL^3}{6EI}$ (*c*) $\frac{7WL^3}{3EI}$ (*d*) $\frac{8WL^3}{6EI}$

Sol. The free body diagram of the beam is shown in figure below by removing the rigid part of the beam

The slope at Q

$$\theta_Q = \frac{ML^2}{EI} + \frac{WL^2}{2EI}$$

$$= \frac{WL^2}{EI} + \frac{WL^2}{2EI} = \frac{3}{2}WL^2$$

Deflection at Q

$$Y_Q = \frac{WL^2}{2EI} + \frac{WL^3}{3EI} = \frac{(WL)L^2}{2EI} + \frac{WL^3}{3EI} = \frac{WL^3}{EI}\left(\frac{1}{2}+\frac{1}{3}\right) = \frac{5WL^3}{6EI}$$

Deflection at free end R

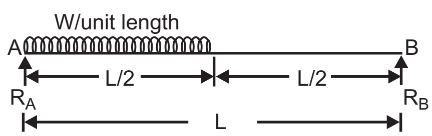
$$Y_R = Y_Q + \theta_Q(L)$$

$$= \frac{5WL^3}{6EI} + \frac{3WL^2}{2EI}(L) = \frac{WL^3}{EI}\left(\frac{5}{6}+\frac{3}{2}\right) = \frac{WL^3}{EI}\left(\frac{5+9}{6}\right) = \frac{7WL^3}{3EI}$$

Ans. The correct choice is (*c*).

1.77. A simply supported beam is subjected to a uniformly distributed load of intensity 'ω' per unit length. On half of the span from one end. The length of span and flexural stiffness are denoted by L and EI, respectively. The deflection at mid-span of the beam is

(*a*) $\frac{5}{6144}\frac{wL^4}{EI}$ (*b*) $\frac{5}{768}\frac{wL^4}{EI}$ (*c*) $\frac{5}{384EI}$ (*d*) $\frac{5}{192}\frac{wL^4}{EI}$



(GATE, 2012)

Sol. By using Mac Auley's double integration method,
integration method,

$$R_B \times L = w \times \frac{L}{2}\times\frac{L}{4} \Rightarrow R_B = \frac{WL}{8}$$

$$EI\frac{d^2y}{dx^2} = R_B.\,x - \frac{w\left(x-\frac{L}{2}\right)\left(x-\frac{L}{2}\right)}{2}$$

$$EI\frac{dy}{dx} = R_B.\frac{x^2}{2} - \frac{w\left(x-\frac{L}{2}\right)^3}{2\times 3} + C_1$$

$$EI\,.\,y = R_B.\frac{x^3}{2\times 3} - \frac{w\left(x-\frac{L}{2}\right)^4}{2\times 3\times 4} + C_1 + C_2$$

Now, consider a section at $x = 0$, $y = 0$

$\therefore$ $0 = 0 - 0 + C_Z = 0$

At $x = L, \quad y = 0$

$\therefore$ $$0 = \left(\frac{wL}{8}\right)\times\frac{L^3}{6}\frac{\left(w\frac{L}{2}\right)^4}{24} + C_1x$$

$$= \frac{wL^4}{48} - \frac{wL^4}{384} + C_1 \times L$$

$$C_1 = \frac{wL^4}{48} - \frac{wL^4}{384} = \frac{7}{384} wL^3$$

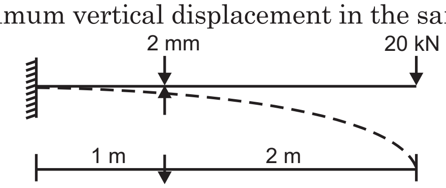
At $$x = \frac{L}{2}$$

$$EIy = \frac{wL}{8}\left(\frac{L^3}{48}\right) - \left(\frac{7}{384} wL^3\right) \times \frac{L}{2}$$

$$= wL^4\left[\frac{1}{384} - \frac{7}{768}\right] = -\frac{5wL^4}{768EI}$$

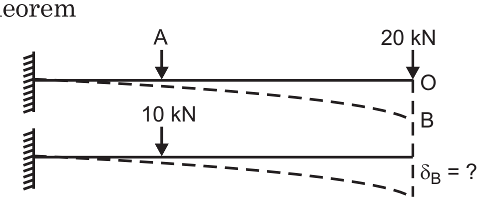
Ans. The correct choice is (*b*).

1.78. For the cantilever beam of span 3 m(shown below), a concentrated load of 20 kN applied at the free end causes a vertical displacement of 2 mm at a section located at a distance of 1 m from the fixed end. If a concentrated vertically downward load of 10 kN is applied at the section located at distance of 1 m from the fixed end (with no other load on the beam) the maximum vertical displacement in the same beam (in mm) is–



(GATE, 2014)

Sol. By using Maxwell's Reciprocal Theorem



$$(20\text{ kN})\delta_B = (10\text{ kN})\ (2\text{ mm})$$

or $$\delta_B = \frac{(10\,\text{kN})(2\,\text{mm})}{20\,\text{kN}} = 1\text{ mm.}$$

Ans. 1 mm

1.79. A thin-walled cylindrical pressure vessel having a radius of 0.5 m and wall thickness of 25 mm is subjected to an internal pressure of 700 kPa. The hoop stress developed is:

(*a*) 14 MPa (*b*) 1.4 MPa
(*c*) 0.14 MPa (*d*) 0.014 MPa (GATE, 2008)

Sol. We know that hoop stress

$$\sigma_n = \frac{PD}{2It}, \text{ where } P = \text{internal pressure, } D = 0.5 \times 2 = 1\text{ m}$$

$$t = \text{thickness} = 25\text{ mm}$$

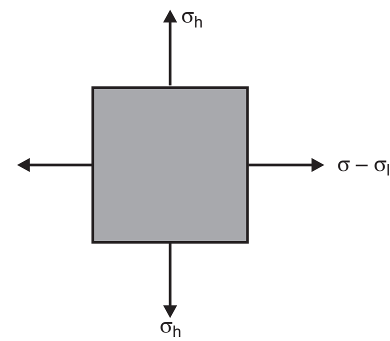
$\therefore$ $$\sigma_n = \frac{700 \times 1000 \times 1000}{2 \times 25} = 14\text{ MPa}$$

Ans. The correct choice is (*a*).

1.80. A thin-walled long cylindrical tank of inside radius r is subjected simultaneously to internal gas pressure P and axial compressive force F at its ends. In order to produce pure shear state of stress in the wall of the cylinder, F should be equal to: (GATE, 2008)

(*a*) $P\pi r^2$ (*b*) $2P\pi r^2$
(c) $3P\pi r^2$ (*d*) $4P\pi r^2$

Sol. An element in a thin cylinder to have shear stress



$$\sigma_n = \sigma - \sigma_l$$

$$\frac{PD}{2t} = \frac{F}{2\pi rt} - \frac{PD}{4t}$$

$$\frac{F}{2\pi rt} = \frac{3PD}{4t}$$

$$F = \frac{3P(2r)}{4t}(2\pi rt) = 3P\pi r^2$$

Ans. The correct choice is (*c*).

1.81. The kern area (core) of a solid circular section column of diameter, *D*, is a con-centric circle of diameter '*d*' equal to: (GATE, 1992)

(*a*) $D/8$ (*b*) $D/6$ (*c*) $D/4$ (*d*) $D/2$

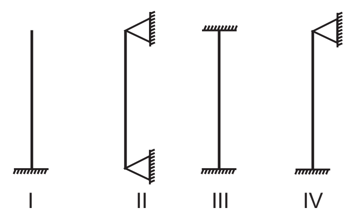
Ans. The correct choice is (*b*) $D/6$

1.82. When a column is fixed at both ends corresponding Euler's criterion load is:

(*a*) $\frac{\pi^2 EI}{L^2}$ (*b*) $\frac{2\pi^2 EI}{L^2}$ (*c*) $\frac{3\pi^2 EI}{L^2}$ (*d*) $\frac{4\pi^2 EI}{L^2}$

Ans. The correct is (*d*).

1.83. Four columns of the same material and having identical geometric properties are supported in different ways as shown below: (GATE, 2000)



It is required to order there four beams in the increasing order of their respective first buckling boards.

(*a*) I, II, III (*b*) III, IV, II, I (*c*) II, I, IV, III (*d*) I, II, IV, III

Ans. The correct choice is (*b*)

1.84. A long structural column (length = *L*) with both ends hinged is acted upon by an axial compressive load *P*. The differential equation governing the bending of column, is given by $EI\frac{d^2y}{dx^2} = -Py$ where *y* is the structural lateral deflection and *EI* is the flexural rigidity the first critical load on column responsible for its buckling is given by: (GATE, 2003)

(*a*) $\frac{\pi^2 EI}{L^2}$ (*b*) $\frac{\sqrt{2}\,\pi^2 EI}{L^2}$ (*c*) $\frac{2\pi^2 EI}{L^2}$ (*d*) $\frac{4\pi^2 EI}{L^2}$

Ans. The correct choice is (*a*).

1.85. The effective length of a column of length *L* fixed against rotation and translation at one end is (GATE, 2010)

(*a*) 0.5 L (*b*) 0.7 L (*c*) 1.414 L (*d*) 2 L

Ans. The correct choice is (*d*).

1.86. The ratio of the theoretical critical buckling load for a column with fixed ends to that of another column with the same dimensions and material, but with pinned ends, is equal to (GATE, 2012)

(*a*) 0.5 (*b*) 1.0 (*c*) 2.0 (*d*) 4.0

Ans. The correct choice is (*d*).

1.87. Two steel columns *P*(length *L* and yield strength f_y = 250 MPa) and *Q* (length 2*L* and yield strength f_y = 500 MPa) have the same cross-sections and end conditions. The ratio of buckling load of columns *P* to that of column *Q* is (GATE, 2013)

(*a*) 0.5 (*b*) 1.0 (*c*) 2.0 (*d*) 4.0

Sol. According to Euler's Theory

$$P_e = \frac{\pi EI}{L^2} \Rightarrow P_e \; \alpha \; \frac{1}{L^2}$$

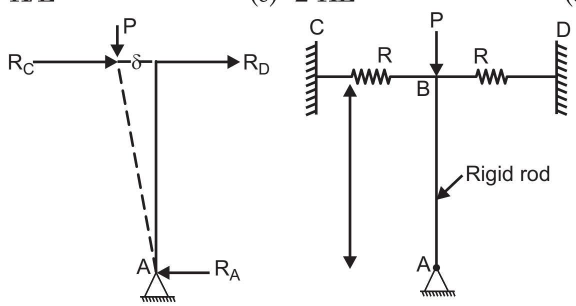
$$P_Q = \frac{\pi EI}{(2L)^2} \Rightarrow P_Q \; \alpha \; \frac{1}{(2L)^2}$$

$\therefore$ The ratio $\dfrac{P_e}{P_Q} = \dfrac{1/L^2}{1/(2L)^2} = \dfrac{1}{\cancel{L^2}} \times \dfrac{4\cancel{L^2}}{1} = 4$

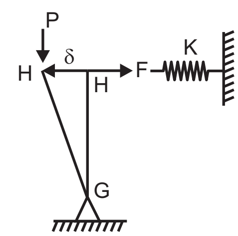
Ans. The correct choice is (*d*).

1.88. A rigid rod *AB* of length *L* is hinged at *A* and is maintained in its vertical position by two springs with spring constants k attached at end *B*. The system is under stable equilibrium under the action of load *P* when $P < P_{cr}$. The system will be is unstable equilibrium when *P* attains a value greater than: (GATE, 1990)

(*a*) KL (*b*) K/L (*c*) 2 KL (*d*) 4 KL



Sol. Let us assume that the end *B* sway left by δ



Apparently, $R_A = R_C = R = k\delta$, the reactions in springs

$$\sum A = 0$$

$$(R_c + R_D)L = P\delta$$

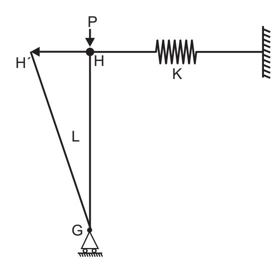
or $(K_\delta + K_\delta)L = P\delta$

$$2k\delta L = P\delta \Rightarrow P = 2\text{KL}$$

Ans. The correct choice is (*c*).

1.89. A rigid bar *GH* of length *L* is supported by a hinge and a spring of stiffness *k* as shown in the figure below. The buckling load P_{er} for the bar will be

(*a*) 0.5 KL (*b*) 0.8 KL (*c*) 1.0 KL (*d*) 1.2 KL



Sol. At the time of bucking, the end *H* moves to *H'*

For spring stiffness $K = \dfrac{F}{\delta} \Rightarrow F = K\delta$

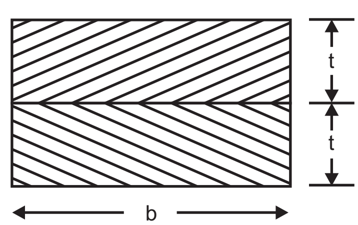
Taking moments about *G*

$$P \times \delta = F \times L \Rightarrow P = \frac{FL}{\delta} \Rightarrow P = \frac{K\delta L}{\delta}$$

$\therefore$ $P = KL$

Ans. The correct choice is (*c*)

1.90. Cross-section of a column consisting of two steel strips each of thickness *t* and width *b* is shown in the figure below. The critical loads of the column with perfect bond and without bond between the strips are *P* and P_o respectively. The ratio P/P_o is



(*a*) 2 (*b*) 4 (*c*) 6 (*d*) 8

Sol. According to the Euler's theory of crippling load

$$P = \frac{\pi^2 EI}{L^2} \Rightarrow P \ \alpha \ I$$

$$\frac{\text{The crippling load of bonded column}}{\text{The crippling load of loose column}} = \frac{I \text{ bonded}}{I \text{ loose}} = \frac{\left[\dfrac{b(2t)^3}{12}\right]}{2\left[\dfrac{lb\,t^3}{12}\right]}$$

$$= \frac{b\,8t^3}{12} \times \frac{12}{2b\,t^3} = 4$$

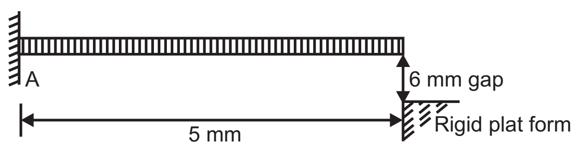
Ans. The correct choice is (*b*).

1.91. The fixed end moment of a uniform beam of span L and fixed at the ends, subjected to a central point load P, is (GATE, 1994)

(*a*) $\frac{PL}{2}$ (*b*) $\frac{PL}{8}$ (*c*) $\frac{P}{8}$ (*d*) $\frac{P}{16}$

Ans. The correct choice is (*b*).

1.92. For the linear elastic beam shown in the figure, the flexural rigidity, EI is 781250 kN.m^2. When w = 10 kN/m, the vertical reaction at A is 50 kN. The value of R_A for w = 100 kN, is: (GATE, 2004)



(*a*) 500 kN (*b*) 425 kN (*c*) 250 kN (*d*) 75 kN

Sol. When the applied u.d.l. w = 10 kN/m, the free end deflects

$$\delta = \frac{wl^4}{8EI} = \frac{10 \times 5^4}{8 \times 781250} = 1 \text{ mm}$$

As the gap between free end and rigid platform is more than 1 mm, no reaction develops at the prop

When w = 100 kN/m, the free end deflection

$= 1 \times 10 = 10$ mm greater than to gap = 6 mm

The reaction developed at the point is equal to affective vertical deflection

i.e., $$\frac{R_B \times l^3}{3EI} = 10 - 6 = 4$$

or $$R_B = \frac{4 \times 3EI}{5^3} = \frac{4 \times 3 \times 781250}{625} = 15 \text{ kN}$$

The reaction at $$A = \frac{15}{10} \times 50 = 75 \text{ kN}$$

The correct choice is (*d*).