1. Engineering Mechanics

Statics deals with the effect of forces acting upon the bodies at rest and conditions of equilibrium of bodies. **Dynamics** deals with the effect of forces acting upon the bodies in motion. **Force** is any action which tends to change the state of rest of a body to which it is applied. Force would be defined completely when (*i*) its magnitude, (*ii*) point of application and (*iii*) its direction are known. Any quantity having both magnitude and direction is known as vector quantity. A force may be represented by an arrow-headed line called a 'vector' which gives 'magnitude', proportional to its length, its 'point of application' and its 'direction'. According to ***law of*** **parallelogram of forces**, if two forces represented by vector $\overline{OA}$ and $\overline{OB}$ are acting at point O and inclined to each other at angle θ, then their action is equivalent to action of one resultant force, represented by the vector $\overline{OC}$ which is the diagonal of the parallelogram formed as shown in Fig. 1.1 (*a*). It will be observed that it is also possible to find resultant by constructing triangle OBC which is called the **triangle of forces.** According to triangle law of forces, if two forces acting simultaneously on a particle, be represented in magnitude and direction by the two sides of a triangle, taken in order then their third side represents the resultant but direction will be in opposite order.

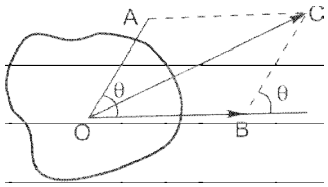


Fig. 1.1 (*a*)

According to **polygon of forces,** the force vectors may be added by drawing a polygon of forces. The line completing the polygon is the resultant (note that its arrow points in the opposite direction), and its angle to a reference direction may be found.

According to **equilibrium law**, two forces can be equilibrium only if they are equal in magnitude, opposite in direction, and collinear in action.

According to **law of superposition**, the action of a given system of forces on a rigid body will in no way be changed if another system of forces in equilibrium is added or subtracted from them.

Restriction to the free motion of a body in any direction is called *constraint*. Whenever a force acts on a support, an equal and opposite reaction is experienced from the support. A **free body diagram** is formed by showing all the forces acting on a body, and the body or its support being not shown.

If several forces, applied to a body at one point, all act in the same plane, then these can be replaced by a single resultant force.

For a body, acted upon by the several concurrent, coplanar forces, to be in equilibrium, these forces when geometrically added, must form a closed polygon.

The projection on the coordinate axes, of the resultant of a system of concurrent forces F_1, $F_2, F_3, \ldots, F_n$ acting in one plane are equal to the algebraic sums of the corresponding projections of the components. And when forces $F_1, F_2, F_3, \ldots, F_n$ are in equilibrium then ΣY_i and ΣX_i must be zero.

A system of coplanar forces will be in **equilibrium**, if the sum of the resolved parts of the forces of the system in any two perpendicular directions separately is zero.

The algebraic sum of their moments about any point in their plane is also zero.

According to **Lami's Theorem**, if three forces acting on a particle keep it in equilibrium then each is proportional to the sine of the angle between the other two.

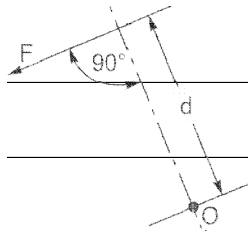


Fig. 1.1 (*b*)

The moment of force F about a point O at a perpendicular distance d from its line of action, is equal to Fd.

Resultant of several moments. If forces F_1, F_2, etc., act on a body at perpendicular distances d_1, d_2, etc., from a point O, the moments are, $M_1 = F_1d_1$, $M_2 = F_2d_2$, etc. about O.

The resultant moment is $M_r = M_1 + M_2 + ...$

Clockwise moment are reckoned positive and counterclockwise moments negative. If the moments 'balance' $M_r = 0$ and the system is in equilibrium.

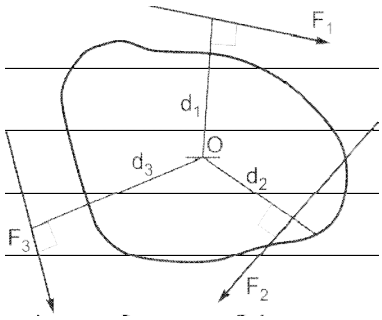


Fig. 1.1 (*c*)

According to **Varignon's Theorem of Moments,** the algebraic sum of moments of two forces about any point in their plane is equal to the moments of their resultant about that point.

According to **law of moments**, if a point remains in equilibrium under the effect of a number of coplanar forces, then the sum of clockwise moments must be equal to sum of anti-clockwise moments about any point in the same plane. For *equilibrium of a body* $\Sigma H = 0$, $\Sigma V = 0$ and $\Sigma M = 0$.

Couple. If two equal and opposite forces have parallel lines of action a distance a apart, the moment about any point O at distance d from one of the lines of action [Fig. 1.1 (*d*)] is

$$M = Fd - F(d - a) = Fa$$

This is *independent of d* and the resultant force is zero. Such a moment is called a 'couple'.

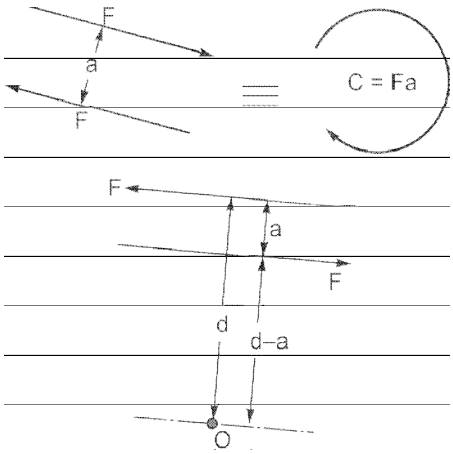


Fig. 1.1 (*d*)

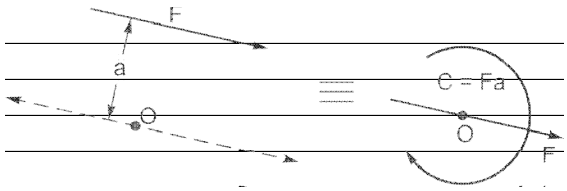


Fig. 1.1 (*e*)

Resolution of moment into a force and a couple. For a force F at a from point O; if equal and opposite forces are applied at O, then the result is a couple Fa and a net force F. [See above Fig. 1.1 (*e*)]

General condition for equilibrium of a body. Complete equilibrium exists when both the forces and the moments balance, *i.e.*, $F_r = 0$ and $M_r = 0$.

Properties of Couples

1. The algebraic sum of the moments of the two forces of a couple about any point in their plane is constant and is equal to the moment of the couple.

2. A number of coplanar couples acting on a rigid body are equivalent to a single couple whose moment in the algebraic sum of the moment of the couples.

3. Two couples acting in the same plane upon rigid body whose moments are equal in magnitude but opposite in sign balance each other.

4. The effect of a couple on a rigid body is unaltered if it be transferred to any plane parallel to its own, the arm remaining parallel to its original position.

Friction

Law of Static Friction

1. The force of friction always acts in a direction, opposite to that in which the body tends to move.

2. The magnitude of the force of friction, is always equal to the applied force.

3. The magnitude of the limiting friction, bears a constant ratio to the normal reaction between the body and the surface *i.e.*, F/R = a constant.

4. The force of friction depends upon the roughness of the surface.

5. The force of friction is independent of the area of contact between the bodies.

Laws of Dynamic Friction

1. The force of friction always acts in a direction opposite to that on which the body is moving.

2. The magnitude of the dynamic friction bears a constant ratio to the normal reaction between the surfaces. It is slightly less than that in case of static friction.

3. Friction force decreases at a slow rate with the increase of speed.

The stage when the body acted upon by an external force is just on the point of moving, is called **limiting equilibrium** of the body. The force of friction which is offered by the rough surface at the stage of limiting equilibrium, is called **limiting friction**. When a body starts moving, the force of friction offered by the surface is called the **dynamic friction.**

When a body is at the point of limiting equilibrium the force of friction is maximum. The angle which the resultant of the maximum force of friction and the normal reaction make with the normal reaction, is called the **angle of friction**. It is denoted by λ

$$\tan \lambda = \frac{\text{Maximum force of friction}}{\text{Normal reaction}}$$

The ratio of the limiting friction and the normal reaction is called **coefficient of friction.**

Angle of repose is the greatest angle at which a plane must be inclined to the horizontal, before the body lying on it just slides down. The angle of repose is always equal to angle of friction.

If the friction is not present, the reaction developed is normal to the surface. At the impending state *i.e.* when the motion just ensues, the maximum developed frictional force is

$$= \text{Coefficient of friction} \times \text{Normal reaction.}$$

Equilibrium of a Body on an Inclined Rough Plane

The force P acting horizontally when the body is about to move up is $W \tan(\alpha + \lambda)$ and when the body is about to move down, then $P = W \tan(\alpha - \lambda)$.

The force P acting along the inclined plane, (Fig. 1.3) when the body tends to move up is

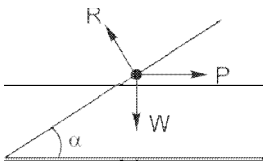


Fig. 1.2

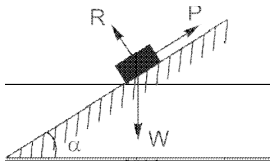


Fig. 1.3

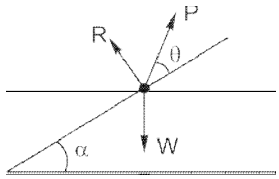


Fig. 1.4

$$P = \frac{W \times \sin(\alpha + \lambda)}{\cos \lambda}$$

And when the body tends to move down

then $$P = \frac{W \times \sin(\alpha - \lambda)}{\cos \lambda}$$

The force P acting at an angle θ to the inclined plane making angle α with horizontal and having angle of friction λ, when the body tends to move up is

$$P = \frac{W \times \sin(\alpha - \lambda)}{\cos(\theta + \lambda)}$$

And when body tends to move down,

then $$P = \frac{W \times \sin(\alpha - \lambda)}{\cos(\theta - \lambda)}$$

Analysis of Frame

A frame may be defined as a structure, made up of several bars, riveted or welded together. The bars are also known as the members of the frame. For calculation purposes, the joints are assumed to be hinged or pin-jointed

The frames are classified as:

1. Perfect Frame. 2. Imperfect Frame.

Perfect frame. A frame which satisfies the equation: $n = 2j - 3$

is known as perfect frame.

where n = No. of members, and j = No. of joints

The forces in a perfact frame are obtained by:

1. Analytical methods and
2. Graphical methods.

Imperfect frame. A frame which does not satisfy the equation. $n = 2j - 3$

is known imperfect frame. Imperfect frames are classified as:

1. Deficient Frame and 2. Redundant Frame.

A frame in which the number of members are less then $(2j - 3)$, is known as deficient frame.

A frame in which the number of members are more than $(2j - 3)$, is known as redundent frame.

Analytical methods for finding forces in the members of a frame are:

1. Method of sections and 2. Method of joints.

Method of section is particularly convenient, when the forces in a few members of a frame are required to be found out. In this method a section line is passed through the members in such a way that at one time the section line should not cut more than three members, in which the forces are unknown.

In the method of joints, each joint is treated separately as a free body in equilibrium. The unknown forces are determined from equilibrium equations *i.e.*,

$$\Sigma H = 0 \qquad \text{and} \qquad \Sigma V = 0$$

The joint should be selected in such a way that at any instant, the joint should not contain more than two members, in which the forces are unknown.

Centre of Gravity or Centroids of Various Figures and Bodies

The *centre of gravity* of a body is that point through which the resultant of the system of parallel forces formed by the weight of all the particles of the body passes, for all positions of the body.

Centroid is a point in the plane figure at which whole area of the figure is assumed to act.

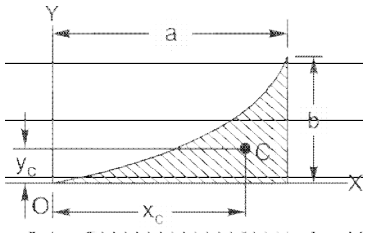


Fig. 1.5

(*i*) Semi-circle – $4r/3\pi$ above base on symmetrical radius (r).

(*ii*) Quadrant – $4r/3\pi$ from the centre.

(*iii*) Semi-circle arc – $2r/3\pi$ above base.

(*iv*) Quadrant arc – $2r/\pi$ from both sides.

(*v*) Parabola $x_c = 3a/4\ a$ and $y_c = 3b/10$ (Refer Fig. 1.5).

(*vi*) Hemisphere – $3r/8$.

(*vii*) Prism and cylinder – half the height.

(*viii*) Solid cone and pyramid – 1/4 height above base on axis.

(*ix*) Hollow cone – 1/3 height above base on axis.

(*x*) For right angle of base b and height h, (x, y) coordinates of $c \cdot g$. are $b/3$ and $h/3$.

(*xi*) For quarter ellipse in first quadrant with major axis 'a' and minor axis 'b', location of c.g. is at $x = 2a/5$ and $y = 3b/8$.

Centroids of composite plane figures and curves can be obtained by formulae

$$x_c = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} \quad \text{and} \quad y_c = \frac{A_1y_1 + A_2y_2}{A_1 + A_2}$$

where A_1, A_2 are areas of individual parts having their centroids at x_1, y_1, and x_2, y_2 respectively.

Moment of Inertia

Let m_1, m_2, ... be the masses of very small portions of a body of mass M and r_1, r_2, ... their distances from a fixed straight line, called the axis. Then $m_1r_1^2 + m_1r_2^2$ ... ..., *i.e.*, $\Sigma m_1r_1^2$ is called the moment of inertia of the body about the axis. The moment of inertia is also called the Second moment of area of the body.

If the moment of inertia be equal to Mk^2, then k is called the radius of gyration of the body about the axis.

Moment of Inertia of Geometrical Figures

M.I. of rectangle of width w and height h is $I_{xx} = wh^3/12$ and $I_{yy} = w^3h/12$

M.I. of triangle of base b and height h is $I_{xx} = bh^3/36$ and $I_{base} = bh^3/12$

M.I. of circle of diameter D is $I_{xx} = I_{yy} = \pi D^4/64$

M.I. by Routh's rule $\quad I = \dfrac{A \times S}{3}$ for rectangular section

$= \dfrac{A \times S}{4}$ for circular or elliptical section.

$$= \frac{V \times S}{5} \text{ for spherical body.}$$

where A = area, V = volume and S = sum of squares of remaining two semi-axes.

Parallel Axis Theorem

The moment of inertia of a lamina about any axis in the plane of the lamina equals the sum of the moments of inertia about a parallel centroidal axis in the plane of lamina together with the product of the area of the lamina and the square of the distance between the two axes.

Perpendicular Axis Theorem

If I_x and I_y be the moments of inertia of a lamina about mutually perpendicular axes OX and OY in the plane of the lamina and I_z be the moment of inertia of the lamina about axis normal to the lamina and passing through the point of intersection of the axes OX and OY, then $I_z = I_x + I_y$.

A **machine** is said to be reversible when the load gets lowered on removal of the effort. If the efficiency of a machine is less than 50% it is non-reversible or self-locking.

Flexible Strings

Flexible string is a string which offers no resistance on bending at any point and *catenary* is the curve in which a uniform string or chain hangs freely under gravity between two points which are not in the same vertical line. This curve is parabolic in nature when cable carries a uniformly distributed vertical load.

In the case of parabolic curve, if cable carries a uniformly distributed load of intensity w with respect to horizontal span l, the important formulae are

$$y = \frac{wx^2}{2H}$$

H = tension in cable at lowest point C

S = tension in cable at other point, such as

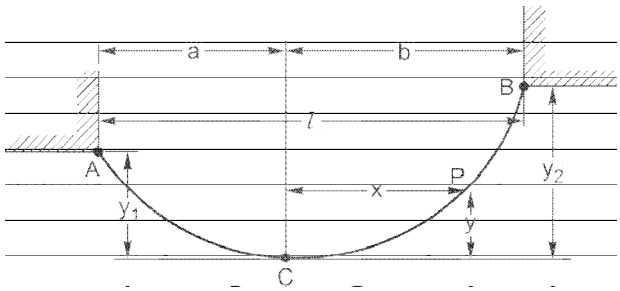


Fig. 1.6

$$P = \sqrt{H^2 + (wx)^2}$$

$$\therefore \quad S_A = \sqrt{H^2 + w^2a^2}, \; S_B = \sqrt{H^2 + w^2b^2}$$

$$y_1 = \frac{wa^2}{2H}, \; y^2 = \frac{wb^2}{2H}$$

$$a = \frac{l}{2} - \frac{hH}{wl}, \; h = y_2 - y_1, \; b = \frac{l}{2} + \frac{hH}{wl}$$

$$H = \frac{wl^2}{h^2}\left(y_2 - \frac{h}{2} \pm \sqrt{y_1 + y_2}\right)$$

If $\quad y_1 = y_2 = y, \; a = b = \frac{1}{2}$, then $H = \frac{wl^2}{8y}$

Projectiles

If an object is thrown in air with certain initial velocity and allowed to fall under influence of gravity, it will traverse along a certain path before falling to the ground, known as *trajectory* which has a parabolic shape.

If x and y are the horizontal and vertical distances covered during the time t,

then $$y = u \sin \alpha \times \frac{x}{u \cos \alpha} - \frac{1}{2} g \times \frac{x^2}{u^2 \cos^2 \alpha}$$

$$\therefore \quad y = x \tan \alpha - \frac{gx^2}{2u^2 \cos^2 \alpha}$$

where u = initial velocity

α = angle of projection *i.e.* angle between direction of projection and horizontal plane.

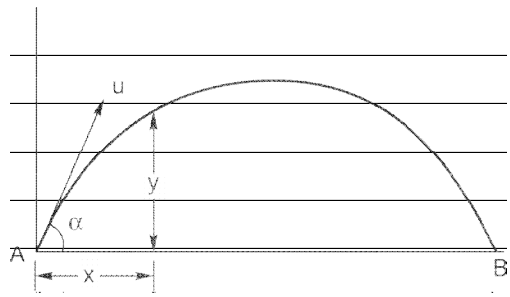


Fig. 1.7

Horizontal Range $R = \dfrac{u^2 \sin 2\alpha}{g}$

For a given value of u, R is maximum when $\sin 2\alpha$ is maximum, *i.e.*, $\sin 2\alpha = 1$ or $2\alpha = 90°$

$\therefore \quad \alpha = 45°$

$\therefore$ Maximum value of $R = u^2/g$

Time of flight $t = \dfrac{2u \sin^2 \alpha}{g}$

Maximum height $= \dfrac{u^2 \sin^2 \alpha}{2g}$

The velocity and direction of motion of a projectile at a given height h, above the point of projectile

$$v = \sqrt{u^2 - 2gh}$$

$$\tan \theta = \frac{\sqrt{u^2 \sin^2 \alpha - 2gh}}{u \cos \alpha}$$

Time of flight of a projectile up an inclined plane, $t = \dfrac{2u \sin^2 (\alpha - \beta)}{g \cos \beta}$

where α is angle of projection and β is angle of inclined plane.

Time of flight of a projectile down an inclined plane, $t = \dfrac{2u \sin (\alpha + \beta)}{g \cos \beta}$

The range of a projectile on an inclined plane, $R = \dfrac{2u^2 \sin (\alpha - \beta) \cos \alpha}{g \cos^2 \beta}$

For maximum range up an inclined plane, $\alpha = \pi/4 + \beta/2$.

For maximum range down an inclined plane, $\alpha = \pi/4 - \beta/2$.

Equations for Angular Motions

(*i*) $\omega = \omega_0 + \alpha t$, (*ii*) $\theta = \omega_0 t + 1/2\alpha t^2$ (*iii*) $\omega^2 = \omega_0^2 + 2\alpha\theta$

Also $\omega = d\theta/dt$, $\alpha = d^2\theta/dt^2$

where ω_0 = initial angular velocity in radians/sec., ω = final angular velocity in radians/sec.

t = time taken in seconds, θ = angle traversed in radians

α = angular acceleration in radians/sec^2.

A body performing rotary motion at uniform angular velocity ω is acted on by instantaneous acceleration towards the centre, known as centripetal acceleration whose value is $\omega^2 r$.

Momentum

The product of the mass of a body and its velocity is known as momentum of the body. Hence momentum = Mass × Velocity.

According to second law of motion,

External force = Rate of change of momentum

$$= \frac{\text{Change of momentum}}{\text{Time}}$$

$$= \frac{\text{Final momentum} - \text{Initial momentum}}{\text{Time}}$$

$$= \frac{m \times v - m \times u}{t} = m \times a$$

Lift motion. The tension (T) in the cables supporting a lift is given by

$$T = W\left(1 + \frac{f}{g}\right) \text{ ..... when lift is moving up}$$

$$= W\left(1 - \frac{f}{g}\right) \text{ ..... when lift is moving down}$$

where W = Weight carried by lift and f = Uniform acceleration of lift.

Angular momentum. The product of moment of inertiaa (I) and angular velocity (ω) of a rotating body, is known as angular momentum. Hence

Angular momentum = $I \times \omega$

According to the second law of motion for angular motion, external torque is directly proportional to the rate of change of angular momentum. Hence

Torque or external couple acting on body

T = Rate of change of angular momentum

$$= \frac{I \times \omega - I \times \omega_0}{t} = I \times \left(\frac{\omega - \omega_0}{t}\right)$$

$= I \times \alpha$ (α = Angular acceleration)

Kinetic energy of a body. If a body possesses both the motion of translation as well as the motion of rotation, then the kinetic energy of a body is equal to the K.E. (kinetic energy) due to translation plus K.E. due to rotation.

Let V = Liner velocity of the body, ω = Angular velocity,
m = Mass of the body, and I = Moment of inertia.

Then K.E. due to linear velocity $= mV^2/2$

K.E. due to angular velocity $= I\omega^2/2$

$\therefore$ Total K.E. $= mV^2/2 + I\omega^2/2$.

Virtual work. The work done by a force on a body due to small virtual displacement (*i.e.* imaginary displacement), is known as virtual work. The virtual work will be zero if the force is acting at right angles to the direction of virtual displacement. The virtual work is positive if the force and virtual displacement are in the same direction. But if the force and virtual displacement are in the opposite direction, then the virtual work is negative.

Principle of virtual work. It states that if a system of forces acting on a body or a system of bodies be in equilibrium and if the system is imagined to undergo a small displacement consistant with the geometrical conditions, then the algebraic sum of virtual work done by the forces of the system is zero.

Lifting machines. The machines which are used for lifting loads such as lever, screw jack, inclined plane etc. are known as lifting machines.

Input of machine. The product of effort applied and the distance moved by the effort is known as input of a machine.

Output of a machine. The product of weight lifted and the distance through which weight is lifted is known as output of a machine.

Efficiency of a machine. The ratio of output of the machine to the input of the machine is known as efficiency of the machine.

$$\therefore \qquad \eta = \frac{\text{Output}}{\text{Input}}.$$

Mechanical Advantage. The ratio of the weight lifted to the effort applied is know as mechanical advantage. It is represented by M.A.

$$\therefore \qquad \text{M.A.} = \frac{\text{Weight lifted}}{\text{Effort applied}} = \frac{W}{P}.$$

Velocity ratio. It is defined as the ratio of the distance moved by the effort to the distance moved by the weight. It is represented by V.R.

$$\therefore \qquad \text{V.R.} = \frac{\text{Distance moved by effort}}{\text{Distance moved by weight}}$$

Efficiency of a machine can also be expressed in terms of mechanical advantage and velocity ratio as

$$\eta = \frac{\text{M.A.}}{\text{V.R.}}$$

The efficiency can also be expressed in terms of ideal effort and actual effort as

$$\eta = \frac{\text{Ideal effort}}{\text{Actual effort}}.$$

Ideal effort is the effort applied to raise a load when the efficiency of the machine is expressed in %.

Reversible and lrreversible machine. A machine is said to be reversible if on the removal of the effort, the load falls down to the inital position (*i.e.* the load moves in the reverse direction). But if the load does not fall down to the inital position on the removal of the effort, then the machine is known as irreversible. The irreversible machine is also known as self-locking machine.

A machine will be irreversible (or self-locking) if its efficiency is less than 50%.

Law of a machine. The equation which gives the relationship between the effort and the load lifted, is known as law of the machine. The law of the machine is obtained by drawing

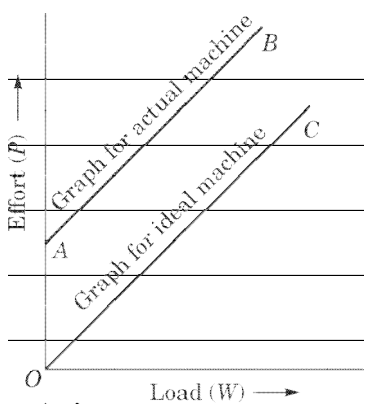


Fig. 1.8

a graph between the effort and the corresponding load lifted by the effort as shown in Fig. 1.8. The law of the machine is given mathematically as, $P = mW + C$

where P = Effort applied,

W = Load lifted,

m = Slope of the line AB and which is equal to a constant which is known as co-efficient of friction, and

C = Intercept of the line on Y-axis and is equal to effort required to overcome friction.

Maximum mechanical advantage and maximum efficiency are given by,

$$\text{Maximum M.A.} = \frac{1}{m} \quad \text{and} \quad \eta_{max} = \frac{1}{m \times (\text{V.R.})}.$$

Work and Power

Work = Force × distance

(Work by torque = $T\theta$)

Rotational Kinetic energy KE = $I\omega^2/2$

where I = moment of inertia of body

Change of kinetic energy = $m/2\ (v^2 - u^2)$

Potential energy PE = mgh

where g = acceleration due to gravity (9.81 ms^{-2}), h = height above a datum.

Strain energy SE = $Fx = kx^2/2$

where x = deflection, k = stiffness.

Conversion of potential energy to kinetic energy: $mgh = mv^2/2$

Therefore $v = \sqrt{2gh}$ or $h = v^2/2g$

Power

Power $P = \dfrac{W}{t} = \dfrac{Fx}{t} = \boldsymbol{Fv}$ ($Nms^{-1} = Js^{-1} = W$)

Rotational power P = torque × angular velocity = $T\omega = T\theta/t$

Also, if N = the number of revolutions per second $P = 2\pi NT$

where $2\pi N$ = angular velocity ω.

Momentum and Impulse

Momentum of a body having a motion of translation = mv.

Momentum of rotating body = $I\omega$.

According to **law of conservation of momentum,** the total momentum of a system of bodies remains unaltered by mutual action between them.

Impulse is the change in momentum produced by the action of a force applied on a body within an infinitely short interval of time. Impulse = Force × Time.

It was observed by Newton that when two bodies impinge directly, their relative velocity after the impact is in a constant ratio to their relative velocity before impact and is in the opposite direction.

If two bodies of masses m and m' moving along the same line with velocity u and u' respectively collide, then after the direct impact if their velocities are v and v' respectively, then

$$v - v' = -e\,(u - u')$$

where, e is a coefficient called the **coefficient of restitution.**

S.H.M.

If a body moves in a straight line such that its acceleration is always directed towards a fixed point and is proportional to its distance from the fixed point, it is said to have **simple harmonic motion.**

The time taken by the body for one complete oscillation is called time period T which is equal to $2\pi/\omega$ (ω = angular velocity).

The acceleration of a particle moving with S.H.M. is equal to $\omega^2 y$ (y = distance from the mid point). Velocity of particle moving with S.H.M. is

$$v = \omega\sqrt{r^2 - y^2}, \; r = \text{ amplitude}$$

The time period of simple pendulum is given by $T = 2\pi\sqrt{l/g}$

where, l is the length of simple pendulum.

The motion of a particle from one extremity to the other constitutes half an oscillation and is called a **Beat.** If a pendulum executes one beat per second or whose time period is 2 seconds is called **second's pendulum.**

For a second's pendulum, $T = 2\pi\sqrt{l/g} = 2$

or $\sqrt{l/g} = 1/\pi \quad \therefore \quad l = \dfrac{g}{\pi^2} = 99.4$ cm.

A rigid body free to oscillate about a smooth horizontal axis passing through it is called a **compound pendulum.**

Time period of compound pendulum is $t = 2\pi\sqrt{\dfrac{k^2 + l^2}{gl}}$

where, l = length between point of suspension and centre of gravity and

k = radius of gyration.

In case of compound pendulum the centres of suspension and oscillation are interchangeable.

Centre of percussion is defined as that point on the body at which a blow may be struck so that the reaction at the point of suspension is zero.

D'Alembert's Principle

If a rigid body is acted upon by a system of forces, the system may be reduced to a single resultant force whose magnitude, direction and the line of action may be found out by the method of graphic statics.

i.e., $P = mf = 0.$

Motion of Lift

The tension in the cable supporting the lift of weight W when it is moving up with acceleration f is $T = W\left(1 + \dfrac{f}{g}\right)$ kg

And when it is moving down, then tension is $T = W\left(1 - \dfrac{f}{g}\right)$ kg

Motion of two bodies connected by a string

(1) For the system of two bodies connected by a string and passing over a smooth pulley (Fig. 1.9).

Acceleration, $f = \dfrac{(m_1 - m_2)g}{m_1 + m_2}$ metre/sec^2

and tension, $T = \dfrac{2m_1m_2}{m_1 + m_2}\, g$ Newtons

(2) The acceleration 'f' developed in a body of mass m_1 hanging freely and connected by a string to other body of mass W lying on a smooth horizontal plane (Fig. 1.10).

$$f = \frac{m_1 g}{m_1 + W} \text{ metre/sec}^2$$

(3) A body of mass m_2 is hanging freely and connected by a string passing over a pulley, to another body of mass W lying on a smooth plane, tension T in the string is

$$T = \frac{m_1 W}{m_1 + W} \times g \text{ Newton}$$

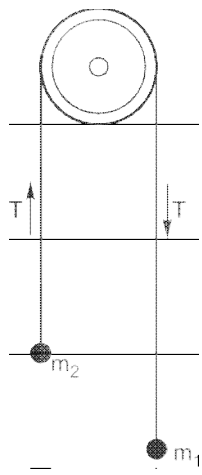


Fig. 1.9

(4) A body of mass m_1 is hanging freely and connected by a string passing over a pulley, to other body of mass W lying on a rough horizontal plane. The tension T in the string is

$$T = \frac{m_1 W(1 + \mu)}{m_1 + W} \times g \text{ Newtons}$$

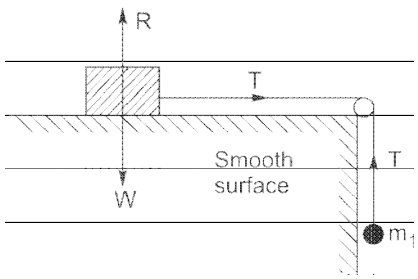


Fig. 1.10

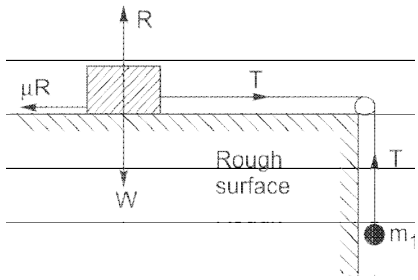


Fig. 1.11

(5) The body of mass m_1 is hanging freely and connected by a string passing over a pulley, to another body of mass m_2 lying on rough horizontal plane.

The acceleration of the system is $f = \dfrac{g(m_1 - \mu m_2)}{m_1 + m_2}$ metre/sec^2.

Pendulum

For simple pendulum, periodic time, $t_p = 2\pi\sqrt{L/g}$, and frequency $f = 1/g_p$.

Pendulum. *For conical pendulum*, periodic time, $t_p = 2\pi\sqrt{h/g}$,
and string tension $T = mL\omega^2$.

For compound pendulum, periodic time, $t_p = 2\pi\sqrt{\dfrac{(h^2 + k^2)}{gh}}$

$\dfrac{h^2 + k^2}{h} = L'$ = the length of the equivalent simple pendulum which is also equal to the distance to the centre of percussion.

Here k = radius of gyration about CG, h = distance from pivot to CG.

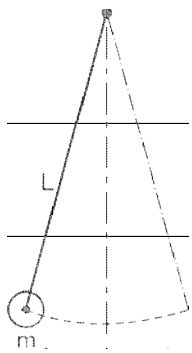


Fig. 1.11 (*a*)

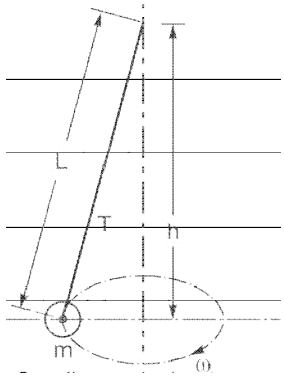


Fig. 1.11 (*b*)

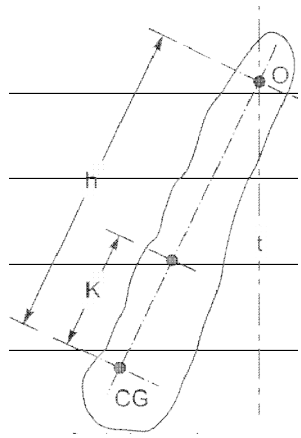


Fig. 1.11 (*c*)

Gravitation deals with the mutual attraction which exists between bodies. The magnitude of the force depends on the masses and the distance between them. For two masses m_1 and m_2 a distance d apart, the force is:

$$F = G\,\frac{m_1 m_2}{d^2}$$

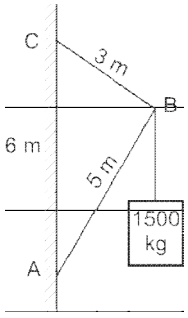
where, G is the 'gravitational constant' = 6.67×10^{-11} Nm^2 kg^{-2}

For a body m_2 on the earth's surface $m_1 = 5.97 \times 10^{24}$ kg (earth's mass), $d = 6.37 \times 10^6$ m (earth's radius). Then

$$F = \frac{6.67 \times 5.97}{6.37^2} \times 10\ m_2 = 9.81\ m_2 = gm_2.$$

SOLVED QUESTIONS

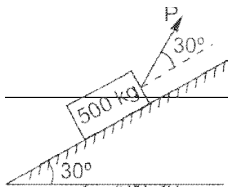
1. In given fig. is shown a hoisting apparatus in which the spar AB is 5 m long. It is free to turn in a vertical plane through A and B and is fastened by a cable BC 3 m long, to a point C, 6 m vertically above. A weight of 1500 kg is supported by a cable at B. Neglecting the weight of the spar and the cables, the force along AB will be



(*a*) 1500 kg (*b*) 1250 kg
(*c*) 1000 kg (*d*) 750 kg
(*e*) 2000 kg.

2. In the above problem, force along BC will be
(*a*) 1500 kg (*b*) 1250 kg
(*c*) 1000 kg (*d*) 750 kg
(*e*) 2000 kg.

3. A weight of 500 kg is held on a smooth plane, inclined at 30° to the horizontal by a force P acting 30° above the plane as shown in fig.



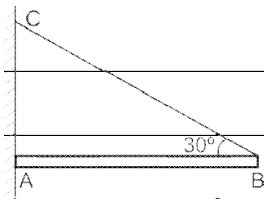
The reaction of plane on the weight will be

(*a*) 500 kg (*b*) 250 kg
(*c*) 476 kg (*d*) 288 kg
(*e*) none of the above.

4. In above problem, the force P should be

(*a*) 500 kg (*b*) 250 kg
(*c*) 476 kg (*d*) 288 kg
(*e*) none of the above.

5. A uniform bar AB of weight 100 kg is hinged at A to a vertical wall and held in horizontal position by a cord BC (refer given figure). The tension in the cord BC will be

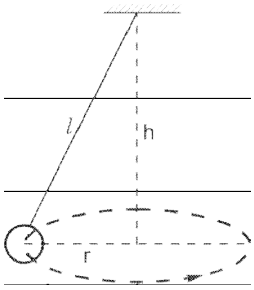


(*a*) 100 kg (*b*) 50 kg
(*c*) 200 kg (*d*) 150 kg
(*e*) unpredictable.

6. In above problem, the reaction on the bar of the hinge at A will be

(*a*) 100 kg (*b*) 50 kg
(*c*) 200 kg (*d*) 50 kg
(*e*) unpredictable.

7. A conical pendulum consisting of a weight W suspended from a cord is made to rotate in a horizontal circle about a vertical axis with a constant angular velocity of ω rad/sec. Tension in cord is equal to



(*a*) $\frac{W}{g} \times l\omega^2$ (*b*) $\frac{W}{g} \times \frac{l}{\omega^2}$
(*c*) $\frac{2W}{g} l\omega^2$ (*d*) $\frac{W}{2g} \frac{l}{\omega^2}$
(*e*) none of the above.

8. A freight car weighing 50,000 kg is moving with a velocity of one m/sec when it strikes a bumping post. If the draw bar spring on the car takes all of the compression, and the deflection is not to be more than 10 cm, then scale of spring should be approximately equal to

(*a*) 50×10^4 kg/cm (*b*) 100×10^4 kg/cm
(*c*) 25×10^4 kg/cm (*d*) 250×10^4 kg/cm
(*e*) not possible to determine.

9. A body weighing 1000 kg falls 8 cm and strikes a 500 kg/cm spring. The deformation of spring will be

(*a*) 8 cm (*b*) 4 cm
(*c*) 16 cm (*d*) 2 cm
(*e*) not possible to determine.

10. An elevator weighing 1000 kg attains an upward velocity of 4 m/sec in two sec with uniform acceleration. The tension in the supporting cables will be

(*a*) 1000 kg (*b*) 800 kg
(*c*) 1200 kg (*d*) 2000 kg
(*e*) not possible to determine.

11. If in the above problem, the tension be reduced so that the elevator comes to rest in a distance of 2 m, then tension in the cable will be

(*a*) 1000 kg (*b*) 500 kg
(*c*) 0 kg (*d*) 590 kg
(*e*) not possible to determine.

12. A 13 m ladder is placed against a smooth vertical wall with its lower end 5 m from the wall. What should be the coefficient of friction between ladder and floor so that it remains in equilibrium

(*a*) 0.1 (*b*) 0.15
(*c*) 0.2 (*d*) 0.21
(*e*) 0.22.

13. A circular disc rolls down an inclined plane. The fraction of its total energy associated with its rotation is

(*a*) 1/2 (*b*) 1/3
(*c*) 1/4 (*d*) 2/3
(*e*) none of the above.

14. The kinetic energy of a body rotating with an angular speed ω depends on
(*a*) ω only (*b*) ω^2 only
(*c*) its mass only
(*d*) the distribution of mass and angular speed
(*e*) all of the above.

15. The velocity of the satellite in an orbit close to earth's surface depends on
(*a*) radius of the orbit only
(*b*) acceleration due to gravity only
(*c*) product of radius and acceleration due to gravity
(*d*) product of radius and gravitational constant
(*e*) none of the above.

16. A thief stole a box full of jewellery of W kg and while carrying it on his head jumped down from third storey of the building. Before he reached the ground, he experienced a load of
(*a*) zero (*b*) infinite
(*c*) less than W (*d*) greater that W
(*e*) $W/2$.

17. The escape velocity of a body on earth
(*a*) increases with the increase of its mass
(*b*) decreases with the increase of its mass
(*c*) remains unchanged with variation of mass
(*d*) varies as the square of the change in mass
(*e*) varies as the square root of change in mass.

18. The length of a second's pendulum on the surface of earth is 1 metre. The length of second's pendulum on the surface of moon, where g is 1/6th of the value of g on the surface of earth, is
(*a*) 1/6 metre (*b*) 6 metres
(*c*) 1/36 metre (*d*) 36 metres
(*e*) none of the above.

19. The energy of a damped oscillator
(*a*) decreases linearly with time
(*b*) increases linearly with time
(*c*) decreases exponentially with time
(*d*) increases exponentially with time
(*e*) remains constant with time.

20. A tunnel is dug through the earth from one end to the opposite end along a diameter and a particle is dropped at one end of the tunnel. The particle will
(*a*) come out of the other end
(*b*) execute simple harmonic motion about the centre of the earth
(*c*) immediately come to rest at the centre
(*d*) stay at the point where it is dropped
(*e*) unpredictable.

21. A small metal ball is tied to a light string and is suspended inside a lift. The ball is set to oscillations. The period of oscillations is maximum when the lift is
(*a*) at rest
(*b*) moving downward at a constant speed
(*c*) moving upward at constant speed
(*d*) moving downward with acceleration
(*e*) moving upward with acceleration.

22. Period of simple harmonic vertical oscillation of a loaded light spring
(*a*) is independent of mass attached to the spring
(*b*) increases with increase in mass attached to the spring
(*c*) decreases with increase in the mass attached to the spring
(*d*) increases with decrease in mass attached to the spring
(*e*) none of the above.

23. A boy is swinging on a swing. If another boy sits along with him without disturbing his motion, then the time period of swing will
(*a*) increase (*b*) decrease
(*c*) be doubled (*d*) remain the same
(*e*) is halved.

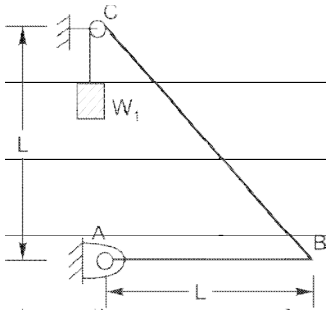
24. A uniform, heavy rod AB of length L and weight W is hinged at A and tied to a weight W_1 by a string at B.

The massless string passes over a frictionless pulley (of negligible dimension) at C as shown in the fig. beow.

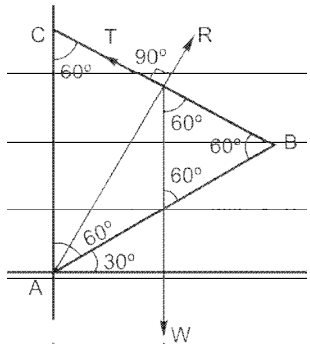
If the rod is in equilibrium at horizontal configuration, then

(*a*) $W_1 = W$ (*b*) $W_1 = W/2$

(*c*) $W_1 = \sqrt{2}\ W$ (*d*) $W_1 = W/\sqrt{2}$.



25. A uniform boom *AB* (see given figure below) pinned at *A* is held by the cable *BC* in the position shown.



If the tension in the cable is 200 kgf, then the weight of the boom and the reaction of the pin at *A* on the boom are respectively

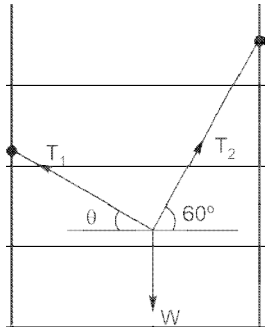
(*a*) 300 kgf; $100\sqrt{3}$ kgf, 30°

(*b*) 400 kgf; $100\sqrt{3}$ kgf, 60°

(*c*) 300 kgf; $200\sqrt{3}$ kgf, 30°

(*d*) 400 kgf; $200\sqrt{3}$ kgf, 60°.

26. A weight *W* is supported by two cables as shown in the given fig. below. The tension in the cable making angle θ will be the minimum when the value of θ is



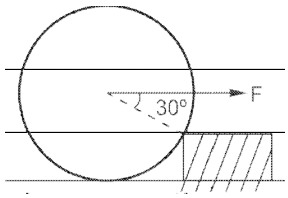
(*a*) 0° (*b*) 30°

(*c*) 45° (*d*) 60°.

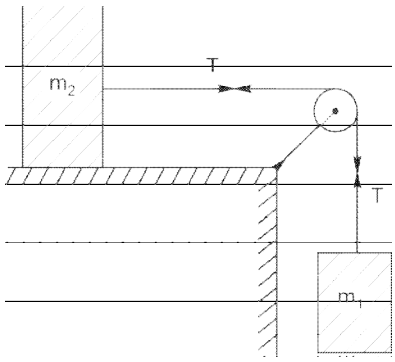
27. A roller of weight *W* is rolled over the wooden block shown in the given fig. (below). The pull *F* required to just cause the said motion is

(*a*) *W*/2 (*b*) *W*

(*c*) $\sqrt{3}\ W$ (*d*) 2*W*.



28. In the given figure (below), two bodies of masses m_1 and m_2 are connected by a light inextensible string passing over a smooth pulley. Mass m_2 lies on a smooth horizontal plane. When mass m_1 moves downwards, the acceleration of the two bodies is equal to



(*a*) $\dfrac{m_1 g}{m_1 + m_2}$ m/s^2 (*b*) $\dfrac{m_2 g}{m_1 - m_2}$ m/s^2

(*c*) $\dfrac{m_2 g}{m_1 + m_2}$ m/s^2 (*d*) $\dfrac{m_1 g}{m_1 - m_2}$ m/s^2.

29. A ball is projected vertically upward with a certain velocity. It takes 40 seconds for its upwards journey. The time taken for its downward journey is

(*a*) 10 s (*b*) 20 s
(*c*) 30 s (*d*) 40 s.

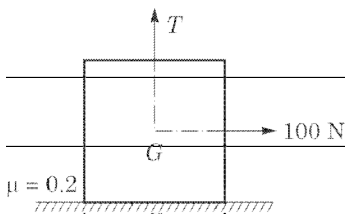
30. A spring of stiffness 1000 N/m is stretched initially by 10 cm from the undeformed position. The work required to stretch it by another 10 cm is

(*a*) 5 Nm (*b*) 7 Nm
(*c*) 10 Nm (*d*) 15 Nm.

31. A straight rod of length $L(t)$, hinged at one end and freely extensible at the other end, rotates through an angle $\theta(t)$ about the hinge. At time t, $L(t) = 1$ m, $\dot{L}(t) = 1$ m/s, $\theta(t) = \pi/4$ rad and $\dot{\theta}(t) = 1$ rad/s. The magnitude of the velocity at the other end of the rod is

(*a*) 1 m/s (*b*) 2 m/s
(*c*) $\sqrt{3}$ m/s (*d*) $\sqrt{2}$ m/s

32. A block weighing 981 N is resting on a horizontal surface. Coefficient of friction between the block and horizontal surface is $\mu = 0.2$. A vertical cable attached to the block provides partial support as shown. A man can pull horizontally with a force of 100 N. What will be the tension, T (in N) in the cable if the man is just able to move the block to the right?



(*a*) 176.2 (*b*) 196.0
(*c*) 481.0 (*d*) 981.0

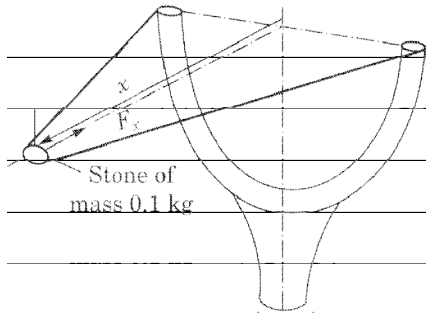
33. There are two points P and Q on a planar rigid body. The relative velocity between the two points

(*a*) should always be along PQ
(*b*) can be oriented along any direction
(*c*) should always be perpendicular to PQ
(*d*) should be along QP when the body undergoes pure translation

34. A band brake having band-width of 80 mm, drum diameter of 250 mm, coefficient of friction of 0.25 and angle of wrap of 270° is required to exert a friction torque of 1000 Nm. The maximum tension (in kN) developed in the band is

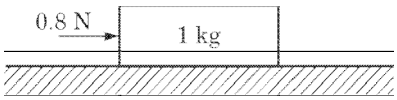
(*a*) 1.88 (*b*) 3.56
(*c*) 6.12 (*d*) 11.56

35. A stone with mass of 0.1 kg is catapulted as shown in the fig. The total F_x (in N) exerted by the rubber band as a function of distance x (in m) is given by $F_x = 300\,x^2$. If the stone is displaced by 0.1 m from the un-stretched position ($x = 0$) of the rubber band, the energy stored in the rubber band is



(*a*) 0.01 J (*b*) 0.1 J
(*c*) 1 J (*d*) 10 J

36. A 1 kg block is resting on a surface with coefficent of friction $\mu = 0.1$. A force of 0.8 N is applied to the block as shown in the fig. The friction force is



(*a*) 0 (*b*) 0.8 N
(*c*) 0.98 N (*d*) 1.2 N

37. If a body is moving with a uniform acceleration "a", then the distance travelled by the body in the "n"th second is given by (where u is the initial velocity).

(*a*) $\frac{u+a}{2}(1-2n)$ (*b*) $\frac{u+a}{2}(2n-1)$

(*c*) $u + \frac{a}{2}(2n-1)$ (*d*) $u + \frac{a}{2}(2n+1)$.

38. The angle of projection for which the horizontal range and maximum height of a projectile are equal, is

(*a*) 45° (*b*) $\tan^{-1} 4$

(*c*) $\tan^{-1} 1/2$ (*d*) $\tan^{-1} 2$.

39. Two masses, 1 kg and 4 kg, are moving with equal kinetic energy. The ratio of magnitudes of their momentum is

(*a*) 0.25 (*b*) 0.50

(*c*) 1.00 (*d*) 2.00.

40. A truck weighing 981 kN and travelling at 2 m/s impacts with a buffer which compresses 1 mm per 10 kN. The maximum compression of the spring is

(*a*) 0.2 m (*b*) 2 m

(*c*) 0.1 m (*d*) 1 m

41. If the kinetic energy of the body becomes four times its initial value then the new momentum will be

(*a*) three times its initial value

(*b*) four times its initial value

(*c*) twice the initial value

(*d*) unchanged.

42. A man throws bricks to a height of 10 m where they reach with a speed of 10 m/s. If he throws the bricks such that they just reach that height, what percentage of energy will he save ?

(*a*) 100% (*b*) 50%

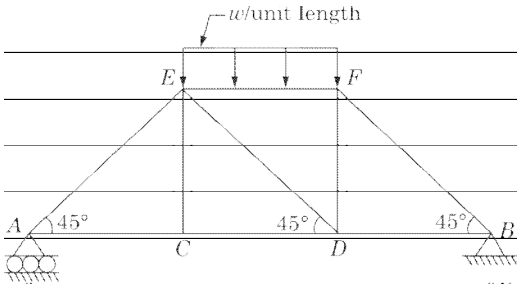
(*c*) 60% (*d*) 83.3%.

43. A circular disc rolls without slip on an inclined plane. The ratio of its rotational kinetic energy to the total energy is

(*a*) 1/4 (*b*) 1/3

(*c*) 1/2 (*d*) 2/3.

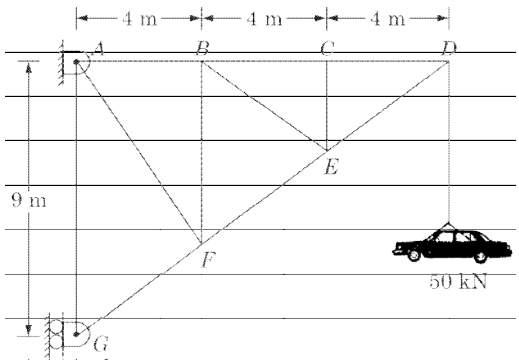
44. A truss consists of horizontal and vertical members having length l each. For the uniformly distributed load w per unit length on the member EF of the truss, the force in the member CD is



(*a*) $wl/2$ (*b*) $wl/4$

(*c*) zero (*d*) wl.

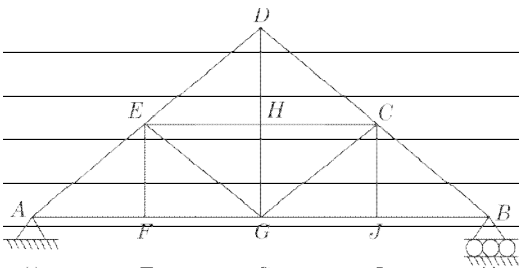
45. The crane structure shown in figure supports a load of 50 kN. The force in member AB is



(*a*) 66.6 kN (*b*) 50 kN

(*c*) 33.3 kN (*d*) 25 kN.

46. Determine the type of truss shown in the fig. below.



(*a*) perfect (*b*) deficient

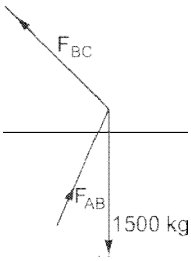
(*c*) redundant

(*d*) none of the above.

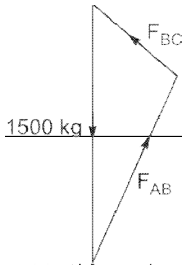
SOLUTIONS

1 and 2. Ans. (*b*) and (*d*) Fig. (*a*) shows the free body diagram for all the forces and Fig. (*b*) shows the force triangle for these.

It will be noted that force triangle is similar to $\Delta\, ABC$ shown in Fig. in the Prob. 1.



Free Body Diag.
Fig. (*a*)



Force Triangle
Fig. (*b*)

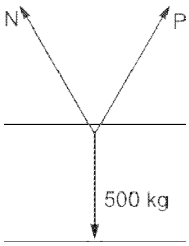
$\therefore$ From similar triangles

$$\frac{F_{AB}}{5} = \frac{F_{BC}}{3} = \frac{1500}{6}$$

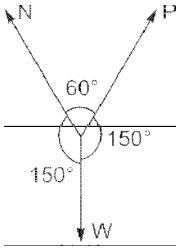
or $$F_{AB} = \frac{1500 \times 5}{6} = 1250 \text{ kg}$$

and $$F_{BC} = \frac{1500 \times 3}{6} = 750 \text{ kg}$$

3 and 4. Ans. (*d*) and (*d*) Fig. (*a*) below shows the force diagram and (*b*) shows the free body diagram for the forces.



Free Body Diag.
Fig. (*a*)



Force Diag.
Fig. (*b*)

By Lami's theorem,

$$\frac{P}{\sin 150°} = \frac{N}{\sin 150°} = \frac{500}{\sin 60°}$$

$$P = 500 \times \frac{\sin 150°}{\sin 60°} = 5000 \times \frac{\sin 30°}{\sin 60°}$$

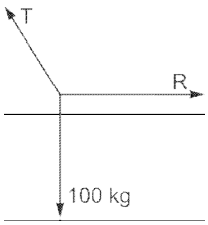
$$= \frac{500 \times 1/2}{\sqrt{3}/2} = \frac{500}{\sqrt{3}} = \frac{500 \times \sqrt{3}}{3}$$

$$= \frac{500 \times 1.732}{3} = \frac{866}{3} = 288 \text{ kg.}$$

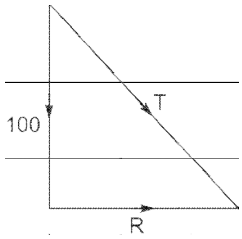
Similarly $N = 288$ kg.

5 and 6. Ans. (*a*) and (*a*) Weight of bar *AB* can be assumed to act at the mid point. Since the bar is in equilibrium under the action of three external forces, the lines of action of these forces must be concurrent. Fig. (*a*) below shows the free body diagram of the bar and Fig. (*b*) below shows the corresponding force triangle, which is equilateral triangle.

$\therefore \qquad R = T = 100$ kg.



Free Body Diag.
Fig. (*a*)



Force Δ
Fig. (*b*)

7. (*a*) As the forces in the vertical direction must be balanced.

$\therefore \qquad T \cos \theta = W$

Forces in the direction normal to the circular path of rotation are balanced such that

$$T \sin \theta = \frac{W}{g}\, a_n = \frac{W}{g} \,.\, \omega^2 r$$

$$= \frac{W}{g}\, \omega^2 \,.\, l \sin \theta$$

$$T = \frac{W}{g}\, \omega^2 l$$

8. (*a*) Work of spring = K.E. of car

$$= \left(\frac{\text{Resistance of spring in kg/cm}}{2}\right) \times (\text{Deflection of spring})^2$$

or $$\frac{1}{2}\frac{W}{g} v^2 = \frac{R}{2} \times h^2$$

or $$\frac{50{,}000}{9.81} \times (100)^2 = R \times 10^2$$

or $R = \dfrac{500 \times 10^4}{9.81} \cong 50 \times 10^4$ kg/cm.

9. (*a*) Weight × Distance of free fall + Weight × Displament of spring (h)

= Work of spring

= Average force of spring × h^2

$$1000 \times 8 + 1000 \times h = \frac{500}{2} \times h^2$$

or $32 + 4h = h^2$

or $h^2 - 4h - 32 = 0$

or $(h - 8)(h + 4) = 0$

or $h = 8$ cm or $= -4$ cm

10. (*c*) Velocity = Acceleration × Time

or 4 m/sec = $a \times 2$

and $a = 2$ m/sec^2

Tension in cable

$$= \frac{W}{g}(g + a) = \frac{1000}{9.81}(9.81 + 2)$$

$$= \frac{11{,}810}{9.81} \cong 1200 \text{ kg.}$$

11. (*d*) (Final velocity)^2

= (Initial velocity)^2 + $2a$ × Distance

or $0 = 4^2 + 2a \times 2$ or $a = -4$ m/sec

$\therefore$ Tension in cable $= \dfrac{W}{g}(g + a)$

$$= \frac{1000}{9.81}(9.81 - 4)\ \frac{5810}{9.81} = 590 \text{ kg.}$$

12. (*c*) Vertical height of wall where the ladder rests is

$$\sqrt{13^2 - 5^2} = 12 \text{ m.}$$

At vertical wall, only horizontal normal force (S) acts, and at ground, normal reaction R and frictional force F act.

$\therefore$ $F = S$

and R = Weight of ladder (W)

Taking moments about point on ground where ladder rests,

$W \times 2.5 = S \times 12 = F \times 12$

or $F = \dfrac{2.5}{12} W \cong 0.21\ W$

$$\mu = \frac{F}{R} = \frac{F}{W} \cong 0.2.$$

13. (*b*) When the disc rolls down an inclined plane, apart from possessing translatory motion, it rotates about an axis passing through its centre of gravity and perpendicular to its plane.

$\therefore$ K.E. of rotation of the disc about this axis

$$= \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}Mr^2\right)\omega^2$$

$$= 1/4Mr^2w^2 = 1/4Mv^2.$$

Here

I = M.I. of the disc about the said axis,

M = mass of the disc,

r = radius of the disc

v = linear velocity of the disc,

ω = angular velocity of the disc.

K.E. of translation $= 1/2\ Mv^2$

Total energy of the disc

$= 1/4\ Mv^2 + 1/2\ Mv^2 = 3/4Mv^2$

$\therefore$ Fraction of total energy associated with rotation

$$= \frac{\frac{1}{2}Mv^2}{\frac{3}{4}Mv^2} = \frac{1}{3}.$$

14. (*d*) As K.E. of rotation = $\frac{1}{2}I\omega^2$, it depends upon I (moment of inertia) and ω. Further, I depends upon the *distribution of mass*, apart from its dependence upon the mass and the position and direction of the axis of rotation.

15. (*c*) The velocity of satellite in an orbit close to the surface of the earth is given by

$$V_e = \sqrt{gR}$$

16. (*a*) It is due to the state of weightlessness during the time of free fall.

17. (*c*) The escape velocity depends upon the planet.

18. (*a*) The length of the second's pendulum(T = 2 seconds) is given by

$$L = \frac{g}{\pi^2}. \quad \left(\because\ 2 = 2\pi\sqrt{\frac{L}{g}} \text{ or } 1 = \pi^2\frac{L}{g}\right)$$

Since the value of g on moon is 1/6th of its value on earth, the length of the

second's pendulum on the surface of moon is 1/6th of that on earth.

19. (*c*) A damped oscillator is one whose amplitude goes on decreasing. The energy of a harmonic oscillator is directly proportional to the square of its amplitude. Since the amplitude decreases exponentially, the energy will also decrease exponentially with time.

20. (*b*) When the body is dropped from one end of the tunnel, it will be attracted towards the centre of the earth due to the gravitational force. Under the influence of this force, its velocity goes on increasing till it reaches the centre. At the centre its velocity is maximum. This maximum velocity will take the body away from the centre towards the other end. But as it moves away from the centre, its velocity goes on decreasing as it is being attracted towards the centre of the earth. On reaching the other end, the velocity becomes zero and the body is again attracted towards the centre with increasing velocity. Thus the body executes S.H.M. about the centre of earth.

21. (*d*) When the lift is moving downwards, the apparent weight of the bob decreases and as also the effective value of g. Since $T = 2\pi\sqrt{l/g}$, with decrease in the effective value of *g*, *T* also increases (*i.e.,* it is maximum when lift is moving down with acceleration).

22. (*b*) The time period of a loaded light spring is given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Obviously, as *m* increases, *T* also increases.

23. (*d*) The swing is like a simple pendulum whose time period is given by $T = 2\pi\sqrt{\frac{l}{g}}$, which obviously is independent of the mass on the swing. Thus when another boys sits on the swing without disturbing its motion, the mass on the swing increases, but the time period remains unchanged.

24. (*d*) If *T* be tension in string *BC* and since it passes over smooth pulley *C*, $T = W_1$. Reaction at *B* is *W*/2,

$$\therefore \quad W/2 = T\cos 45° = W_1 \times 1/\sqrt{2}$$

or $$W_1 = \frac{W\sqrt{2}}{2} = \frac{W}{\sqrt{2}}.$$

25. (*d*) $$\frac{W}{\sin 90°} = \frac{T}{\sin(90+60)} = \frac{R}{\sin(90+30)}$$

$$W = \frac{200 \times 2}{1} = \frac{R \times 2}{\sqrt{3}}$$

$\therefore$ $W = 400$ kgf and $R = 200\sqrt{3}$ kgf

and angle *R* makes with horizontal = 60°.

26. (*b*) T_1 should be minimum

$$\frac{T_1}{\sin 150°} = \frac{T_2}{\sin(90+\theta)}$$

$$= \frac{W}{\sin\{180-(60+\theta)\}} = \frac{W}{\sin(90+30-\theta)}$$

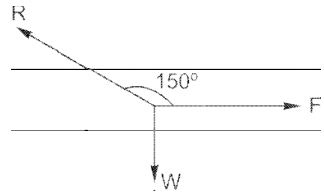
Since $T_1 \propto \frac{1}{\sin(90+\theta)}$, for T_1 to be least θ should be minimum

Also $T_1 \propto \frac{W}{\sin(90+30-\theta)}$. Again for min. value of T_1, θ should be 30°.

27. (*c*) $R\cos 60° = W$, or $R = W/1/2 = 2W$

Also $R\cos 30° = F$,

and $F = 2W \times \sqrt{3}/2\, W$.



28. (*a*) $m_1 - T = m_1 \times a$

Also $m_2 a = T - m_2$

From these equations, $a = \frac{m_1 g}{m_1 + m_2}$

29. (*d*) Time in upward journey is same as in downward journey.

30. (*d*) Initial stretch of spring is 10 cm

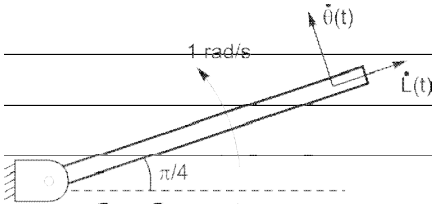
$\therefore$ Force in spring = 1000 × 0.1 = 100 N

To further stretch it by 10 cm, new force will be 200 N

$\therefore$ Work to stretch by 10 cm

$$= \frac{100+200}{2} \times 0.1 \text{ m} = 15 \text{ Nm.}$$

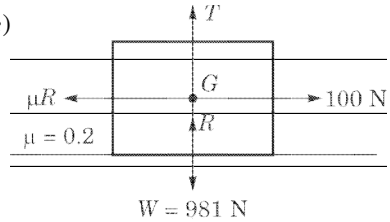
31. (*d*) Two components of velocity encountered *viz*. $L(t)$ and ωl are shown below



$\dot{L}(t) = 1$ m/s, and $\dot{\theta}(t) = \omega l = 1$ m/s

$$\therefore \quad v = \sqrt{1^2 + 1^2} = \sqrt{2} \text{ m/s}$$

32. (*c*)



Let normal reaction between body and surface = R.

$\therefore$ Net normal force on block

$$R = W - T = 981 - T$$

Frictional force, $F = \mu R$

Under equilibrium, *i.e.* when man is just able to move the block

$$\mu R = 100 \quad \mu (981 - T) = 100$$

$$\Rightarrow \quad 0.2 (981 - T) = 100$$

$$T = -\frac{100}{0.2} + 981 = 981 - 500$$

$$= 481 \text{ N}$$

33. (*c*) Relative motion between two points P and Q on a planar rigid body is $\perp$ to PQ.

34. (*d*) $\theta = 270° = 270 \times \pi/180 = 4.712$ radians

Friction torque, $F_t = 1000$ N-m

Now $\frac{T_1}{T_2} = e^{\mu\theta}$(ratio of tensions in band)

or $T_1 = e^{\mu\theta} . T_2$

$$= e^{0.25 \times 4.712} . T_2 = 3.25 T_2$$

Now $F_t = (T_1 - T_2) R$

$$\therefore \quad 1000 = \left(T_1 - \frac{T_1}{3.25}\right) 0.125$$

T_1 = Max. tension in belt

= 11.56 kN

35. (*b*) Energy stored in the band

= Work done by the stone

$$= \int_0^{0.1} F_x \, dx = \int_0^{0.1} 300 \times x^2 dx$$

$$= 300 \times \frac{x^3}{3} = 100 \times 0.1^3 = 0.1 \text{ J}$$

36. (*b*) Limiting friction force between block and the surface is 0.98 N. But applied force is 0.8 N which is less than the limiting friction force. Therefore friction force is restricted to 0.8 N.

37. (*c*) $s_n = un + 1/2\, an^2$

$s_{n-1} = u(n-1) + 1/2\, a(n-1)^2$

$s_{n\text{th}} = s_n - s_{n-1}$

$= u_n + 1/2\, an^2 - [u(n-1) + 1/2\, a(n-1)^2]$

$= u - a/2 + an = u + a/2\,(2n - 1)$.

38. (*b*) Horizontal range = $u^2/g \sin 2\alpha$

$$\text{Maximum height} = \frac{u^2 \sin^2 \alpha}{2g}$$

$$\frac{u^2}{g} \sin 2\alpha = \frac{u^2 \sin^2 \alpha}{2g}$$

$$2 \sin \alpha \cos \alpha = 1/2 \sin^2 \alpha$$

$$\tan \alpha = 4, \alpha = \tan^{-1} 4.$$

39. (*b*) $1/2 \times 1 \times u^2 = 1/2 \times 4 \times v^2$

$$u = 2v$$

Ratio of momentum

$$= \frac{m_1 u}{m_2 u} = \frac{1 \times 2v}{4 \times v} = \frac{1}{2} = 0.5$$

40. (*a*) $1/2\, mv^2 = 1/2\, k\delta^2$

$$1/2 \times \frac{981}{9.81} \times 2^2 = 1/2 \times \frac{10 \times \delta^2}{1 \times 10^{-3}}$$

$$\delta = 0.2 \text{ m}$$

41. (*c*) $E_2 = 1/2\, mv_1^2$, $E_2 = 1/2\, mv_2^2$

$E_1 = 4E_2$

$1/2\, mv_2^2 = 4 \times 1/2\, mv_1^2$ and $v_2 = 2v_1$

$$\frac{M_2}{M_1} = \frac{2mv_2}{2mv_1} = 2$$

42. (*d*) $E_1 = 1/2\ m v_2^2 + mgh$

$= 50mg + 10mg = 60mg$

$E_2 = mgh = 10mg$

$E_1 - E_2 = 50mg$

Percentage saving $= \dfrac{(E_1 - E_2)}{E_1} = 83.3\%$.

43. (*b*) Translational kinetic energy,

$E_1 = 1/2\ mv^2 = 1/2\ mr^2w^2$

Rotational kinetic energy,

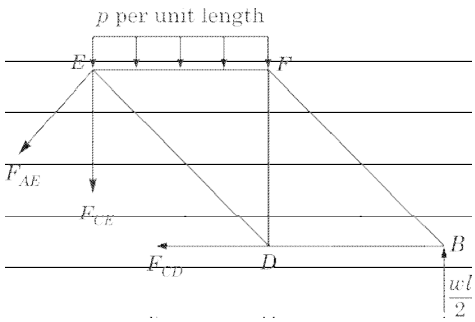
$E_2 = 1/2Iw^2 = 1/2 \times 1/2\ mr^2w^2$

$= 1/4\ mr^2\ w^2$

Total kinetic energy, $E = 3/4\ mr^2w^2$

$\therefore\ \dfrac{E_2}{E} = 1/4 \times 4/3 = 1/3.$

44. (*a*) Consider a section cutting members *AE*, *EC* and *CD*.

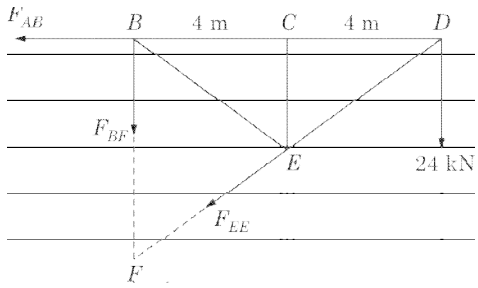


$R_A = R_B = wl/2$, Taking moments about point E; $\Sigma M_E = 0$

$wl \times l/2 - wl/2 \times 2l + F_{CD} \times l = 0$

$F_{CD} = wl/2$.

45. (*a*) Pass a section cutting members *AB*, *BF* and *FE*. Consider right hand side portion of truss.



$BF = 8 \times \dfrac{9}{12} = 6$ m

$\Sigma M_F = 0$; $F_{AB} \times 6 = 50 \times 8$

$F_{AB} = 66.6$ kN.

46. (*a*) $j = 9, n = 15$

$2j - 3 = 2 \times 9 - 3 = 15$

Since $n = (2j - 3)$, it is a perfect truss.