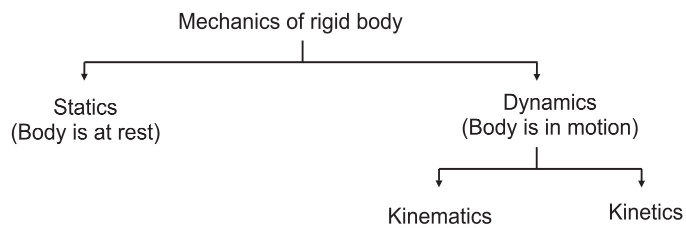
Chapter 1

Engineering Mechanics

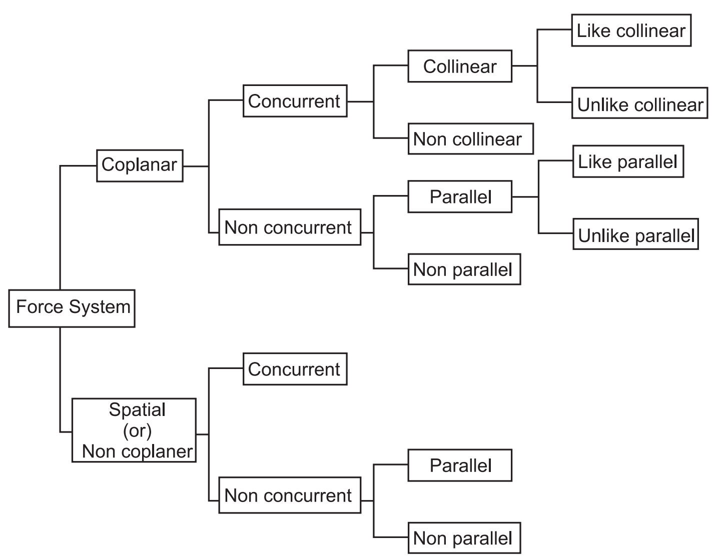
ENGINEERING MECHANICS

Engineering Mechanics is that branch of science,which deals the action of forces on the rigid bodies.

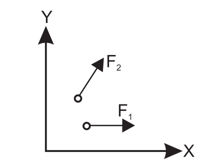
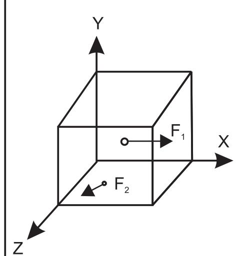
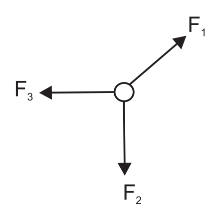
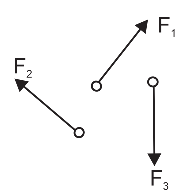
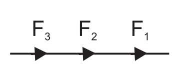
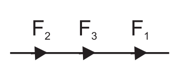
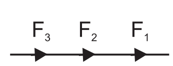
Branches of Mechanics

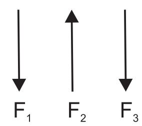
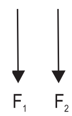
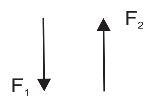
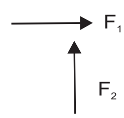
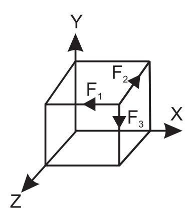
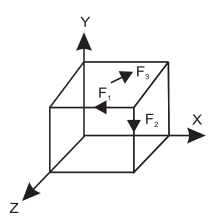
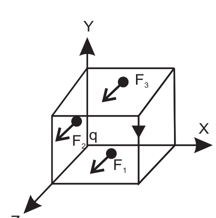


FORCE SYSTEM



Characteristics and Representation of Force System

Force System	Representation	Characteristics
1. Coplanar forces		Line of action of all forces lie on a single plane.
2. Non-coplanar forces		Line of action of all forces are not lying on a single plane.
3. Concurrent forces		Line of action of all forces pass through a single point.
4. Non-concurrent forces		Line of action of all forces do not pass through a single point.
5. Collinear forces		Line of action of all forces pass through a single line.
(a) Like collinear forces		Line of action of all forces pass through a single line in the same direction.
(b) Unlike collinear forces		Line of action of all forces pass through a single line in different direction.

6. Parallel forces		Line of action of all forces are parallel to each other.
(a) Like parallel forces		Line of action of all forces are parallel to each other in the same direction.
(b) Unlike parallel forces		Line of action of all forces are parallel to each other in different direction.
7. Non-parallel forces		Line of action of all forces are not parallel to each other.
8. Non-coplanar concurrent forces		Line of action of all forces are not lying on a single plane but passing through a single point.
9. Non-coplanar non-concurrent forces		Line of action of all forces are not lying on a single plane and also not passing through a single point.
10. Non-coplanar parallel forces		Line of action of all forces are not lying on a single plane but parallel to each other.

Methods for Finding the Resultant of Coplanar Concurrent Force System

Methods for the resultant force

- Analytical method
  - Parallelogram law of forces (For two forces)
  - Method of resolution (For more an two forces)
- Graphical method
  - Triangle law of forces (For two forces)
  - Polygon law of forces (For more than two forces)
  - Funicular polygon

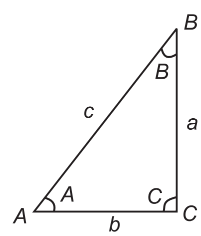
Resultant of Concurrent Coplanar Force System

➠ **Concurrent:** Line of action of forces meet at a point.

➠ **Coplanar:** All the forces lie in the same plane.

➤ **Resultant of Two Concurrent Forces**

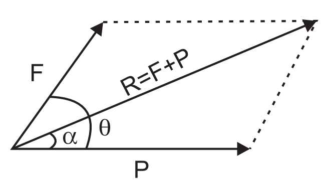
According to Sine law



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

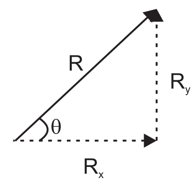
Using cosine law

$$R^2 = F^2 + P^2 - 2FP\cos\theta$$



Once the x and y components of resultant is given by

$$R = \sqrt{R_x^2 + R_y^2}$$



Its inclination is θ is given by

$$\theta = \tan^{-1}\frac{R_y}{R_x} \quad \left(R_x = R\cos\theta, R_y = R\sin\theta\right)$$

Truss

It is a rigid structure composed of number of straight members pin jointed to each other. It can sustain static or dynamic load without any relative motion to each other.

➤ **Types of Truss**

Trusses are classified into according to geometry:

1. Plane truss: It consists of coplanar system of members, which are essentially lies in a single plane.

Ex : Roof truss, Sides of bridges.

2. Space truss: It consists of three Dimensional system of members.

Ex : Electric power transmission tower.

3. Rigid truss: It means there is no deformation takes place due to internal strain in members.

Simple Truss

Simple truss is a rigid truss and removal of any of its members destroys its rigidity. If removing a member does not destroy rigidity the truss is over rigid.

➤ **Classification of Simple Truss**

1. Perfect truss stable truss : $m = 2j - 3$

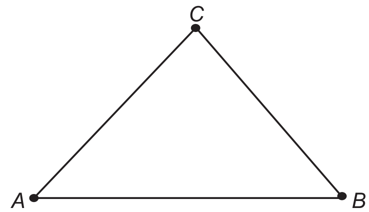
In a perfect truss the above relationship holds good. The truss does not collapse under loading.

Ex : Triangle truss *ABC*

No. of. joints $j = 3$.

No. of members $m = 3$.

For plane truss, $m = 2j - 3 = 2(3) - 3 = 3$



Perfect truss

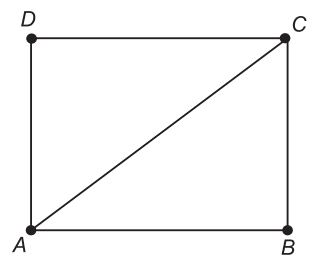
Simple space truss, $m = 3j - 6$

Simple plane truss, $m = 2j - 3$.

2. Imperfect truss (Unstable truss): $m = 2j - 3$

A truss which does not satisfy the above relationship is called imperfect truss. A truss which collapses when loaded is called unstable truss.

3. Redundant truss (Over rigid truss) $m > 2j - 3$



No. of joints $j = 4$

No. of members $m = 6$

$2j - 3 = 2(4) - 3 = 5$

$m > 2j - 3$

➠ One member is extra and can be removed.

4. Deficient truss

A truss which $m < 2j - 3$ is called deficient truss and will collapse under loading.

No. of members $m = 4$

No. of joints $j = 4$

$2j - 3 = 2 \times 4 - 3 = 5$

$4 < 6$

$m < 2j - 3$

➤ **Analysis of Simple Plane Truss**

The solution of simple truss consists of:

- Computation of supporting reactions that must exist to keep the truss in equilibrium.
- Computation of forces (in magnitude and direction) in each members of the truss so that every joint should be in equilibrium and the force acting at every joint should form system in equilibrium.

➤ **Assumption**

- The ends of the members are pin-connected (Hinged) and frictionless and cannot resist moments.
- The loads act only at the joints.
- Self weights of the members are neglected.
- The members of the truss are straight and their cross sections are uniform and are two force members.
- Truss is a perfect truss.
- Truss can be considered coplanar force system.
- Truss is statically determinate and the equations of external loads, members forces and support reactions can be completely solved using equation of equilibrium only.

➤ **Methods of Analysis**

There are two methods of analysis, namely graphical as analytical.

1. Methods of joints or method of resolution.
2. Method of sections.

1. Method of Joints

This method is very widely used to find faces in all members or most of the members of truss. A plane truss can be simply supported truss or a cantilever truss.

Steps :

(i) Check the stability of the truss by using the following equation.

For simply supported truss.

$$m = 2j - 3$$

For cantilever truss

$$m = 2j - 4$$

(ii) Draw free body diagram of entire truss using the basic principles.

(iii) Use three equation of equilibrium to find out the reaction at the supports. Determination of support reactions may not be necessary in case of cantilever truss.

2. Method of Section

This method is use full for calculating forces in some members of the truss and avoid laborious process of proceeding joint by joint reacting a joint on which the desired unknown force acts.

Steps:

(i) Check the stability of a truss.

(ii) Draw FBD and calculate reactions by applying three conditions of equilibrium.

(iii) Select a section, cutting maximum three members (maximum three unknown forces) including the members in which force is to be determined.

(iv) Draw FBD of any cut portion of truss and show forces in the cut members as towards the section (away from joints).

The FBD should include only external forces acting on that part and internal tensile forces in the cut members.

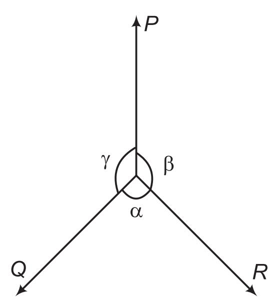
(v) Apply equations of equilibrium for coplaner forces system.

Important Facts

- A section can cut more than three members but unknown forces should not exceed three.
- In case of cantilever truss, draw FBD of part not containing the wall.

➤ **Lami's Theorem**

If a body is in equilibrium under three concurrent forces, the each force is proportional to the sine of the angle between other two.



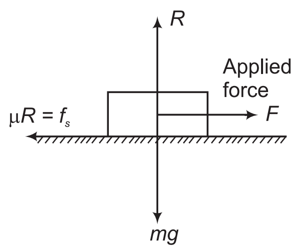
Three concurrent forces *P*, *Q* and *R*

Three concurrent forces *P*, *Q* and *R*

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

Friction Force

It is resistant force which acts in opposite direction at the surface in body which tend to move or its move.



Friction force on a body

$$f_s^{\max} \propto R \quad \text{(or)} \quad f_s^{\max} = \mu R$$

$$\mu = \frac{f_s^{\max}}{R} = \frac{\text{Limiting friction}}{\text{Normal friction}}$$

μ = Co – efficient of static friction

Normal reaction force $R = mg$

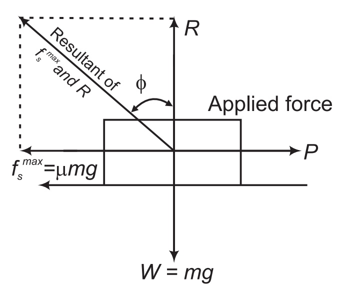
If $\mu\, mg > F$, the body will not move.

$\mu\, mg = F$ the body will tend to move.

$\mu\, mg < F$ the body will move.

➤ Angle of Friction

It is defined as the angle between normal reaction and resultant reaction when the body is in condition of just sliding.



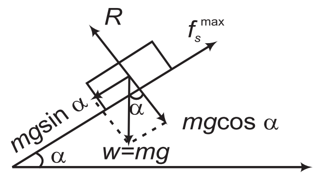
$$\tan\phi = \frac{f_s^{\max}}{R} = \frac{\mu\, mg}{R} = \frac{\mu\, mg}{mg} = \mu \qquad (\because R = mg)$$

$\phi = \tan^{-1}\mu$

μ = Co-efficient of friction

➤ Angle of Repose (α)

It is defined as angle of inclined plane with horizontal at which body is in condition of just sliding.



Angle of Repose (α)

$$f_s^{max} = mg \sin\alpha \qquad ...(1)$$

$$R = mg \cos\alpha \qquad ...(2)$$

Dividing eq.(1)by (2)

$$\frac{f_s^{max}}{R} = \frac{\mu mg}{mg} = \frac{mg \sin\alpha}{mg \cos\alpha}$$

$$\mu = \tan\alpha$$

$$\tan\phi = \tan\alpha$$

$$\phi = \alpha$$

Angle of friction is equal to angle of repose.

Important Facts

- μ_s-coefficient of static friction.
- μ_k-coefficient of kinematic friction.
- $\mu_s > \mu_k$.
- In motion μ_k is always acts as a constant value.
- If applied force is not able to start motion;frictional force will be equal to applied force.
- Unless and otherwise mentioned assume the body is in limiting equilibrium.

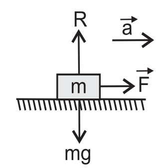
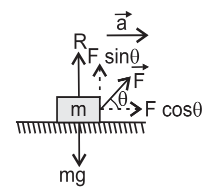
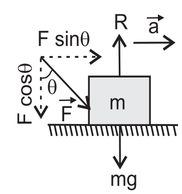
Factor Affecting the Coefficient of Friction

- The material of the meeting bodies.
- The roughness/smoothness of the meeting bodies.
- The temperature of the environment.

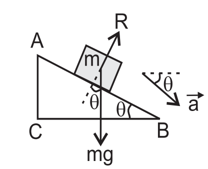
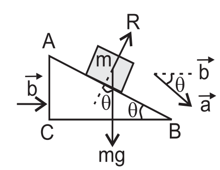
Efforts to Minimize it

- Use of proper lubrication, can minimize the friction.
- Proper polishing the surface can minimize it.

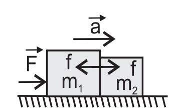
Motion of a Block on a Horizontal Smooth Surface

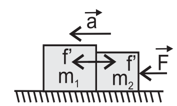
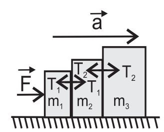
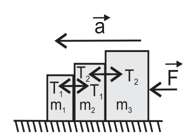
	Different Cases	Diagrams	Results
(a)	When subjected to a horizontal pull		(i) $R = mg$ (ii) $a = \frac{F}{m}$
(b)	When subjected to a pull acting at an angle θ to the horizontal		(i) $R = mg - F\sin\theta$ (ii) $a = \frac{F\cos\theta}{m}$
(c)	When subjected to a push acting at an angle θ to the horizontal		(i) $R = mg + F\sin\theta$ (ii) $a = \frac{F\cos\theta}{m}$

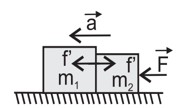
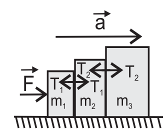
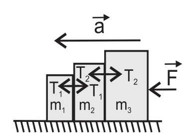
Motion of Bodies on a Smooth Inclined Plane

	Different Cases	Diagrams	Results
(a)	When smooth inclined plane is stationary		(i) $R = mg\cos\theta$ (ii) $a = g\sin\theta$
(b)	When the smooth inclined plane is moving horizontal with an acceleration b		(i) $R = m(g\cos\theta + b\sin\theta)$ (ii) $a = (g\sin\theta - b\cos\theta)$

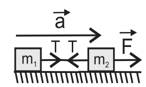
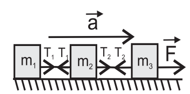
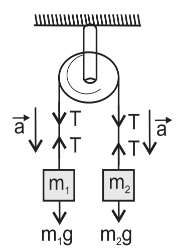
Motion of Bodies in Contact (Force of Contact)

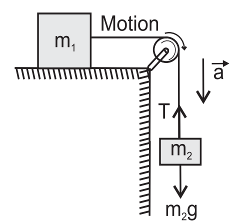
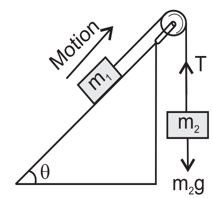
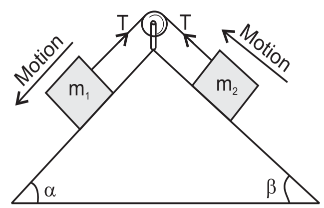
	Different Cases	Diagrams	Results
(a)	When two bodies are kept in contact and force is applied on the body of mass m_1		(i) $a = \dfrac{F}{m_1 + m_2}$ (ii) $f = \dfrac{m_1 F}{m_1 + m_2}$

(b)	When two bodies are kept in contact and force is applied on the body of mass m_2		(i) $a = \frac{F}{m_1 + m_2}$ (ii) $f^{'} = \frac{m_2 F}{m_1 + m_1}$
(c)	When three bodies are kept in contact and force is applied on the body of mass m_1		(i) $a = \frac{F}{m_1 + m_2 + m_1}$ (ii) $T_1 = \frac{(m_2 + m_3)\ F}{(m_1 + m_2 + m_3)}$ (iii) $T_2 = \frac{m_3 F}{(m_1 + m_2 + m_3)}$
(d)	When three bodies are kept in contact and force is applied on the body mass m_3		(i) $a = \frac{F}{m_1 + m_2 + m_3}$ (ii) $T_1 = \frac{m_1 F}{(m_1 + m_2 + m_3)}$ (iii) $F_2 = \frac{(m_1 + m_2)F}{(m_1 + m_2 + m_3)}$

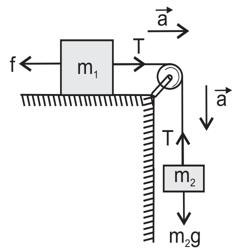
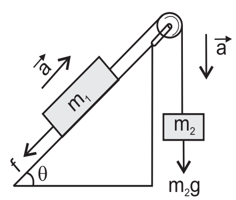
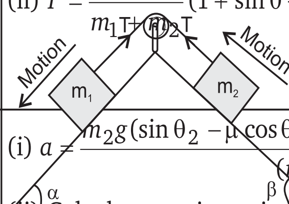
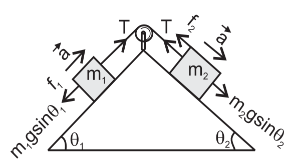
(b)	When two bodies are kept in contact and force is applied on the body of mass m_2		(i) $a = \frac{F}{m_1 + m_2}$ (ii) $f' = \frac{m_2F}{m_1 + m_1}$
(c)	When three bodies are kept in contact and force is applied on the body of mass m_1		(i) $a = \frac{F}{m_1 + m_2 + m_1}$ (ii) $T_1 = \frac{(m_2 + m_3)\,F}{(m_1 + m_2 + m_3)}$ (iii) $T_2 = \frac{m_3F}{(m_1 + m_2 + m_3)}$
(d)	When three bodies are kept in contact and force is applied on the body mass m_3		(i) $a = \frac{F}{m_1 + m_2 + m_3}$ (ii) $T_1 = \frac{m_1F}{(m_1 + m_2 + m_3)}$ (iii) $F_2 = \frac{(m_1 + m_2)F}{(m_1 + m_2 + m_3)}$

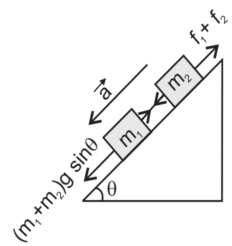
Motion of Connected Bodies

	Different Cases	Diagrams	Results
(a)	When two bodies are connected by a string and placed on a smooth horizontal surface		(i) $a = \frac{F}{m_1 + m_2}$ (ii) $F = \frac{m_1 F}{m_1 + m_2}$
(b)	When three bodies are connected through strings as shown in fig. and placed on a smooth horizontal surface.		(i) $a = \frac{F}{(m_1 + m_2 + m_3)}$ (ii) $T_1 = \frac{m_1 F}{(m_1 + m_2 + m_3)}$ (iii) $T_2 = \frac{(m_1 + m_2) F}{(m_1 + m_2 + m_3)}$
(c)	When two bodies of masses m_1 and m_2 are attached at the ends of a string passing over a pulley as shown in the figure (neglecting the mass of the pulley). If in the above system mass (m) of the pulley is taken into account then		(i) $a = \frac{(m_1 - m_2)\, g}{(m_1 + m_2)}$ (ii) $T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right) g$ (iii) $a = \frac{(m_1 - m_2)\, g}{\left(m_1 + m_2 + \frac{m}{2}\right)}$

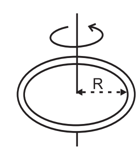
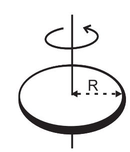
(d)	When two bodies of masses m_1 and m_2 are attached at the ends of a string passing over a pulley in such a way that mass m_1 rests on a smooth horizontal table and mass m_2 is hanging vertically		(i) $a = \frac{m_2 g}{(m_1 + m_2)}$ (ii) $T = \frac{m_1 - m_2\ g}{(m_1 + m_2)}$
(e)	If in the above case mass m_1 is placed on a smooth inclined plane making an angle θ with horizontal as shown in figure then		(i) $a = \frac{(m_2 - m_1 \sin\theta)g}{m_1 + m_2}$ (ii) $T = \frac{m_1 mg(1 + \sin\theta)}{m_1 = m_2}$ (iii) If the system remains in equilibrium, then $m_1 g = \sin\theta = m_2 g$
(f)	In case (d) masses m_1 and m_2 are placed on inclined plane making angle α and β with the horizontal respectively then.		(i) $a = \frac{g\ (m_1 \sin\alpha - m_2 \sin\beta)}{(m_1 + m_2)}$ (ii) $f = \frac{m_1 m_2}{m_1 + m_2}(\sin\alpha + \sin\beta)g$

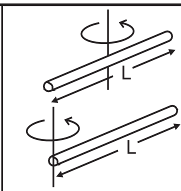
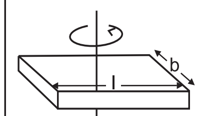
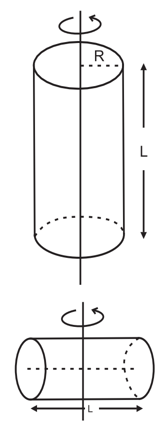
Motion of Connected Bodies on Rough Surfaces

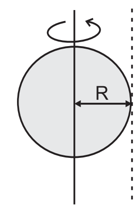
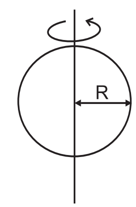
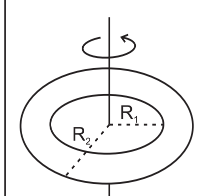
	Different Types of Systems	Results
(a)		(i) $a = \frac{m_2 g - \mu m_1 g}{(m_1 - m_2)}$ (ii) $T = \frac{m_1 m_2 g}{(m_1 + m_2)}(1 + \mu)$
(b)		(i) $a = \frac{\text{unbalanced force}}{\text{total mass}} = \frac{(m_2 g - m_1 g \sin\theta - \mu m_1 g \cos\theta)}{(m_1 + m_2)}$ (ii) $T = \frac{m_1 m_2 g}{m_1 + m_2}(1 + \sin\theta + \mu\cos\theta)$ 
(c)		(i) $a = \frac{m_2 g(\sin\theta_2 - \mu\cos\theta_2) - m_1 g(\sin\theta_1 + \mu\cos\theta_1)}{(m_1 + m_2)}$ (ii) Calculate tension using the following equations $T - m_1 g(\sin\theta_1 + \mu\cos\theta_1) = m_1 a$ $m_2 g(\sin\theta_2 - \mu\cos\theta_2) - T = m_2 a$

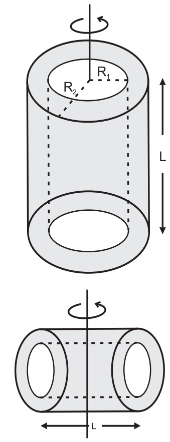
(d)		(i) $a = \dfrac{m_1 g \sin\theta + m_2 g \sin\theta - g\cos\theta(\mu_1 m_1 + \mu_2 m_2)}{m_1 + m_2}$ (ii) According to Newton's 2nd law, $m_1 g \sin\theta - \mu_1 m_1 g \cos\theta - T = m_1 a$ $m_2 g \sin\theta + T - \mu_2 m_2 g \cos\theta = m_2 a$

Moment of Inertia of Some Regular Bodies

S.N.	Bodies	Axis	Figure	Moment of Inertia
1.	Ring	(i) passing through the centre of perpendicular to the plane (ii) any diameter (iii) any tangent		(i) MR^2 (ii) $\frac{1}{2}MR^2$ (iii) $\frac{3}{2}MR^2$
2.	Disc	(i) passing through in centre and perpendicular to the plane (ii) any diameter		(i) $\frac{1}{2}MR^2$ (ii) $\frac{1}{4}MR^2$

3.	Uniform rod of length 'L'	(i) passing through the centre and perpendicular to the length (ii) perpendicular to the length and passing through one end		(i) $\frac{ML^2}{12}$ (ii) $\frac{ML^2}{3}$
4.	Rectangular sheet of length l and breadth b	perpendicular to the length and passing through the centre		(i) $\frac{M\ (l^2+b^2)}{12}$
5.	Solid cylinder of length L and radius R	(i) axis of the cylinder (ii) through the centre and perpendicular to the length		(i) $\frac{MR^2}{2}$ (ii) $M\left[\frac{L^2}{12}+\frac{R^2}{4}\right]$

6.	Solid sphere	(i) any diameter (ii) any tangent		(i) $\frac{2}{5}MR^2$ (ii) $\frac{7}{5}MR^2$
7.	Hollow sphere	(i) any diameter (ii) any tangent		(i) $\frac{2}{3}MR^2$ (ii) $\frac{5}{3}MR^2$
8.	Circular disc of inner radius R_1 and outer radius R_2	through centre and perpendicular to the plane		(i) $\frac{1}{2}M\ (R_1^2 + R_2^2)$

9.	Hollow cylinder of inner radius R_1 outer radius R_2 and length	(i) about axis of cylinder (ii) through midpoint and perpendicular to axis		(i) $\frac{1}{2}M\,(R_1^2 + R_2^2)$ (ii) $M\left(\frac{R_1^2 + R_2^2}{4} + \frac{L^2}{12}\right)$

Comparison of Rolling, Sliding and Falling Motions

S.N.	Physical Quantity	Rolling Motion ($\beta > 1$)	Sliding Motion ($\beta = 1$)	Falling Motion ($\beta = 1, \theta = 90°$)
1.	Velocity	$v_R = \sqrt{\frac{2gh}{\beta}} = \sqrt{\frac{2gs\sin\theta}{1 + \frac{k^2}{R^2}}}$	$v_S = \sqrt{2gh} = \sqrt{2gs\sin\theta}$	$v_F = \sqrt{2gh}$

2.	Acceleration	$a_R = (g \sin\theta / \beta) = \dfrac{g \sin\theta}{1 + \dfrac{k^2}{R^2}}$	$a_S = g \sin\theta$	$a_F = g$
3.	Time of descent	$t_R = \dfrac{1}{\sin\theta}\sqrt{\beta\left(\dfrac{2h}{g}\right)} = \dfrac{1}{\sin\theta}\sqrt{\dfrac{2h\left(1 + \dfrac{k^2}{R^2}\right)}{g}}$	$t_S = \dfrac{1}{\sin\theta}\sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2s}{g \sin\theta}}$	$t_F = \sqrt{\dfrac{2h}{g}}$

Acceleration, Velocity and Time of Descend for Different Bodies Rolling down an Inclined Plane

S.N.	Body	$a = \dfrac{g \sin\theta}{\left(1 + \dfrac{I}{Mr^2}\right)}$	$v = \sqrt{\dfrac{2gh}{\left(1 + \dfrac{I}{Mr^2}\right)}}$	$t = \dfrac{1}{\sin\theta}\sqrt{\dfrac{2h}{g}\left(1 + \dfrac{I}{Mr^2}\right)}$
1.	Solid sphere	$\dfrac{5}{7} g \sin\theta$	$\sqrt{\dfrac{10gh}{7}}$	$\dfrac{1}{\sin\theta}\sqrt{\dfrac{14h}{5g}}$
2.	Hollow sphere	$\dfrac{3}{5} g \sin\theta$	$\sqrt{\dfrac{6gh}{5}}$	$\dfrac{1}{\sin\theta}\sqrt{\dfrac{10h}{3g}}$
3.	Disc	$\dfrac{2}{3} g \sin\theta$	$\sqrt{\dfrac{4gh}{3}}$	$\dfrac{1}{\sin\theta}\sqrt{\dfrac{3h}{g}}$
4.	Cylinder	$\dfrac{2}{3} g \sin\theta$	$\sqrt{\dfrac{4gh}{3}}$	$\dfrac{1}{\sin\theta}\sqrt{\dfrac{3h}{g}}$

5.	Hollow cylinder	$\frac{1}{2}g \sin\theta$	$\sqrt{gh}$	$\frac{1}{\sin\theta}\sqrt{\frac{4h}{g}}$
6.	Ring	$\frac{1}{2}g \sin\theta$	$\sqrt{gh}$	$\frac{1}{\sin\theta}\sqrt{\frac{4h}{g}}$

Ratios of Rotational KE (K_R); Translational KE (K_r) and Total KE of Different Bodies

S.N.	Body	Value of $k^2 (Mk^2 = I)$	$\frac{K_R}{K_T} = \frac{\frac{1}{2}Mv^2 \cdot \frac{k^2}{r^2}}{\frac{1}{2}Mv^2} = \frac{k^2}{r^2}$	$\frac{K_T}{K} = \frac{\frac{1}{2}Mv^2}{\frac{1}{2}Mv^2\left(1+\frac{k^2}{r^2}\right)} = \frac{1}{1+\frac{k^2}{r^2}}$	$\frac{K_R}{K} = \frac{\frac{k^2}{r^2}}{1+\frac{k^2}{r^2}}$
1.	Ring and hollow cylinder	r^2	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{2}$
2.	Hollow sphere	$\frac{2}{3}r^2$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{2}{5}$
3.	Disc and solid cylinder	$\frac{1}{2}r^2$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{3}$
4.	Solid sphere	$\frac{2}{5}r^2$	$\frac{2}{5}$	$\frac{5}{7}$	$\frac{2}{7}$

Plane Motion

When a particle moves in *a* plane, it is said to have plane motion, otherwise its motion is in three dimensions.

The Basic Equations For velocity and acceleration are

1. $v = \frac{dx}{dt}$ [time rate of displacement]

2. $a = \frac{dv}{dt}$ [time rate of velocity] (or) $\frac{d^2x}{dt^2}$

3. $a\,dx = v\,dv$

Important Facts

- The motion of rigid body is said to be translation, if every line in the body remains parallel to its original position at all times.
- In translation motion, all the particles forming a rigid body move along parallel paths.
- If all particles forming a rigid body move along parallel straight line, it is known as rectilinear translation.
- If all particles forming a rigid body does not move along a parallel straight line but they move along a curve path, then it is known as curvilinear translation.

Straight Line Motion

It defines the three equations with the relationship between velocity, acceleration, time and distance traveled by the body. In straight line motion, acceleration is constant.

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

where, u = Initial velocity

v = Final velocity

a = Acceleration of body

t = Time

s = Distance traveled by body

Distance traveled in *n*th second

$$s_n = u + \frac{1}{2}a(2n - 1)$$

Important Facts For Solving Problem

- If a body starts from rest,its initial velocity is zero *i.e* u=0

- If a body comes to rest,its final velocity is zero *i.e* $v=0$
- If a body is projected vertically upwards, the final velocity at the highest point is zero *i.e* $v=0$.
- If a body starts moving vertically downwards, its initial velocity is zero *i.e* $u=0$.
- Equation of motion of body under uniform acceleration due to gravity can be expressed as

(a)For Downward motion:

$$a=+g$$
$$v=u+at$$
$$h=ut+\frac{1}{2}gt^2$$
$$v^2-u^2=2gh$$

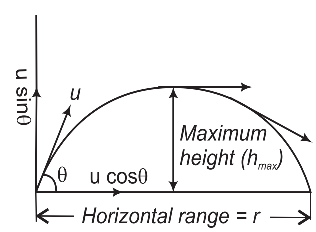
(b)For Upward motion(against gravity)

$$a=-g$$
$$v=u-gt$$
$$h=ut-\frac{1}{2}gt^2$$
$$v^2-u^2=-2gh$$

Projectile Motion

Projectile motion defines that motion in which velocity has two components, one in horizontal direction and other one in vertical direction. Horizontal component of velocity is constant during the flight of the body as no acceleration in horizontal direction.

Let the block of mass is projected at angle θ from horizontal direction



Projectile motion

$$\text{Maximum height } h_{\max}=\frac{u^2\sin^2\theta}{2g}$$

$$\text{Time of flight } T=\frac{2u\sin\theta}{g}$$

$$\text{Range } R = \frac{u^2 \sin 2\theta}{g}$$

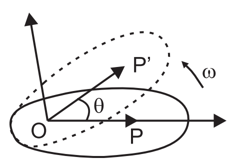
where, u = Initial velocity

Important Facts

- At maximum height vertical component of velocity becomes zero.
- When a rigid body move in circular paths centered on the same fixed axis, then the particle located on axis of rotation have zero velocity and zero acceleration.
- Projectile motion describe the motion of a body, when the air resistance is negligible.

Rotational Motion with Acceleration

The Fixed axis of rotation is defined as that motion of a rigid body in which particle moves in a circular path with their centres on a fixed straight line called axis of rotation.



Rigid body

θ – Angular displacement

Angular velocity (ω) $= \frac{d\theta}{dt}$ [Change in angular displacement per unit time]

Angular acceleration $\alpha = \frac{d\omega}{dt} \Rightarrow \alpha = \frac{d^2\theta}{dt^2}$

$$\omega = \frac{2\pi N}{60} \quad [N = \text{no of revolution per minute}]$$

In case of angular velocity, the equations with the relationships between velocity, displacement and acceleration are as follows.

$$\theta = \omega t$$
$$\alpha = 0$$
$$\omega = \omega_0 + \alpha t$$
$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$
$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

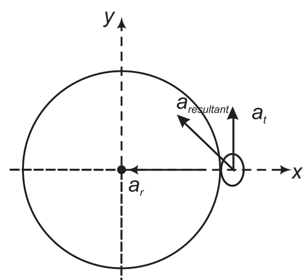
where ω_0 = Initial angular velocity
ω = Final angular velocity

α = Angular acceleration
θ = Angular displacement

Angular displacement in nth second $\theta_n = \omega_0 + \frac{1}{2}\alpha(2n - 1)$

Relation between Linear and Angular Quantities

There are following relations between linear and angular quantities in rotational motion.



Position of radial and tangential vectors

$$|a_r| = |a_t| = 1$$

e_r and e_t are radial and tangential unit vector.

Linear velocity $v = r\omega e_t$

Linear acceleration (Net)

$$a = -\omega^2 r\, e_r + \frac{dv}{dt} e_t$$

Tangential acceleration $a_t = \frac{dv}{dt}$ (rate of change of speed)= $r \times \alpha$

Centripetal acceleration $a_r = \omega^2 r = \frac{v^2}{r}$ ($\because v = r\,\omega$)

Net acceleration $a = \sqrt{a_r^2 + a_t^2}$

$$= \sqrt{\left(\frac{v^2}{r}\right)^2 + (r\alpha)^2}$$

where, a_r = Centripetal acceleration
a_t = Tangential acceleration

Important Facts (Translation of Rigid Body)

When a rigid body is in translation the motion of a single point completely specifies the motion of the whole body.

Important Facts (Rotation about a Fixed Axis)

The angular acceleration α and angular velocity ω are valid for any line perpendicular to the axis of rotation of the rigid body at a given instant.

Centre of Mass of Continuous Body

➠ Centre of mass of continuous body can be defined as

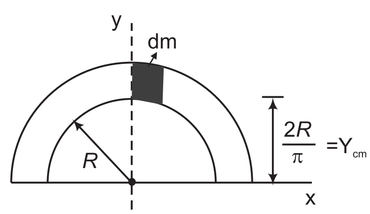
➠ Centre of mass about x, $x_{CM} = \dfrac{\int x\,dm}{\int dm} = \dfrac{\int x\,dm}{M}$

➠ Centre of mass about y, $y_{CM} = \dfrac{\int y\,dm}{\int dm} = \dfrac{\int y\,dm}{M}$

➠ Centre of mass about z, $z_{CM} = \dfrac{\int z\,dm}{\int dm} = \dfrac{\int z\,dm}{M}$

➠ CM of uniform rectangular, square or circular plate lies at its centre.

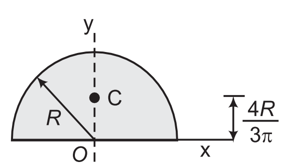
➠ CM of semicircular thin wire or radius R and mass m.



Note: For an uniform circular wire (whit its centre coinciding with the origin) then you would get

$$X_{cm} = 0$$
$$Y_{cm} = 0$$

➠ CM of semicircular disc



Note:

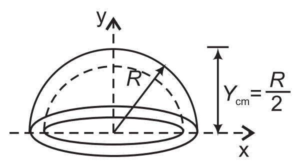
➠ Centre of mass of complete circular disc (uniform) lies at the centre of disc.

➠ While choosing elements, you must take care of the following:

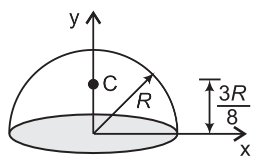
(a) The centre of mass of the element itself must be known.

(b) By using simple integration your element must be able to cover entire body.

➠ CM of hemispherical shell



➠ CM of solid hemisphere



Volume = $\frac{2}{3}\pi r^3$

Important Facts

➠ The first moment of area about the centroidal axis is zero.

➠ Any axis of symmetry is automatically the centroidal axis.

➤ **Radius of Gyration**

1. In mass moment of Inertia $k = \sqrt{\frac{I_{\text{mass (or) min}}}{m}}$ $m \rightarrow$ mass

2. In Area moment of Inertia $I = K^2 A$ (or) $K = \sqrt{\frac{I}{A}}$

➠ The radius of gyration, is thus a measure of spread of area about an axis.

➤ **Law of Conservation of Linear Momentum**

The product of mass and velocity of a particle is defined as its linear momentum (p)

$$p = mv$$

Linear impulse (I) = $\int F\,dt$

➠ The magnitude of impulse (I) can be represented by the shaded area under the curve of force - time.

$$F = \frac{dp}{dt} = \frac{d}{dt}(m\vec{v}) = m; \quad \frac{d\vec{v}}{dt} = m\vec{a}$$

where, K = Kinetic energy of the particle
F = Net external force applied to body
P = Momentum

Principle of linear impulse and momentum.

$$\int_0^t (\Sigma Fx)\, dt = m(V_{fx} - V_{ix})$$

$$\int_0^t (\Sigma Fy)\, dt = m(V_{fy} - V_{iy})$$

$$\int_0^t (\Sigma Fz)\, dt = m(V_{fz} - V_{iz})$$

Important Facts

The relation between impulse and linear momentum can be under stood by the following equation

$$\int_0^t Fdt = m(V_f - V_i)$$

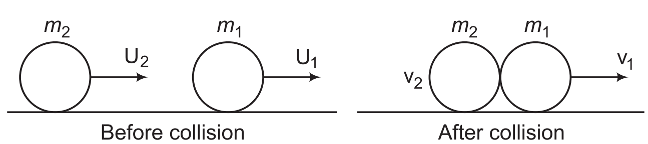
Collision

A Collision is an isolated event in which two or more moving bodies exert forces on each other for a relatively short time.

Collision between two bodies may be classified in two ways:

Head-on Collision

Let the two balls of masses m_1 and m_2 collide directly with each other with velocities U_1 and U_2 in direction as shown in figure. After collision the velocity become v_1 and v_2 along the same line.



$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)U_1 + \left(\frac{m_2 + em_2}{m_1 + m_2}\right)U_2$$

$$v_2 = \left(\frac{m_2 - em_2}{m_1 + m_2}\right)U_2 + \left(\frac{m_1 + em_1}{m_1 + m_2}\right)U_1$$

where, m_1 = Mass of body 1

m_2 = Mass of body 2

U_1 = Velocity of body 1

U_2 = Velocity of body 2

v_1 = Velocity of body 1 after collision

v_2 = Velocity of body 2 after collision

where, e = Co-efficient restitution

$$e = \frac{\text{Separation speed}}{\text{Approach speed}}$$

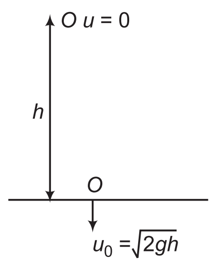
$$e = \frac{v_1 - v_2}{U_2 - U_1}$$

➠ In case of head-on elastic collision $e = 1$.

➠ In case of head-on inelastic collision $0 < e < 1$.

➠ In case of head-on perfectly inelastic collision $e = 0$.

If e is coefficient of restitution between ball and ground, then after nth collision with the floor, the speed of ball will remain $e^n v_0$ and it will go upto a height $e^{2n} h$.



Collision of a ball with floor

$$v_n = e^n v_0 = e^n \sqrt{2gh}$$

$$h_0 = e^{2n} h$$

Important Facts

1. Inelastic collision : $\vec{P}$ conserved, but not K.E.

Example: Number ball on a hard surface (Ball deforms $\rightarrow$ internal elastic P.E.)

2. Perfectly inelastic collision : Two objects stick together $V_1 = V_2 = V_2$ $\vec{P}$ conserved but not K.E. conservation of $\vec{P}$ gives

$$m_2 U_2 + m_1 U_1 = (m_1 + m_2)\, v$$

Example : Two lumps of clay.

3. Elastic collision : $\vec{P}$ and K.E. are conserved.

Example : Two billiard balls (no deformation).

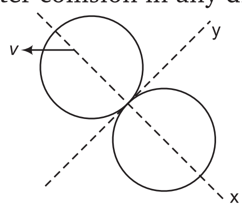
We have $m_1U_1 + m_2U_2 = m_1v_1 + m_2v_2$

$$\frac{1}{2}m_1U_1^2 + \frac{1}{2}m_2U_2^1 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_1v_2^2$$

By combining these two equation, we obtain a third (dependent) equation that tells us that the relative velocity before collision is the negative of the relative velocity after collision.

Oblique Collision

In case of oblique collision linear momentum of individual particle do change along the common normal direction. No component of impulse act along common tangent direction. So, linear momentum or linear velocity remains unchanged along tangential direction. Net momentum of both the particle remain conserved before and after collision in any direction.



Oblique collision

Important Facts

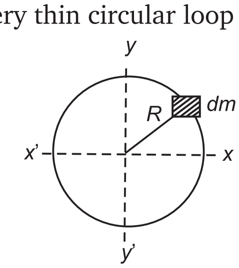
➠ The components of velocity, normal to line of impact remain unchanged before and after impact.

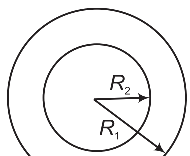
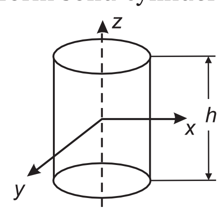
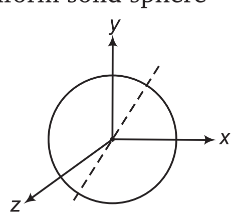
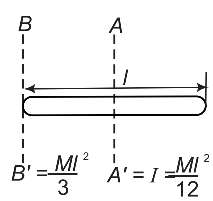
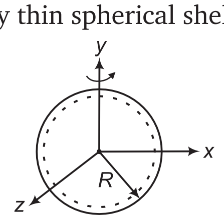
Moment of Inertia

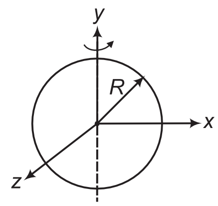
Momentum of inertia can be defined as

r = Distance of body of mass, m from centre of axis.

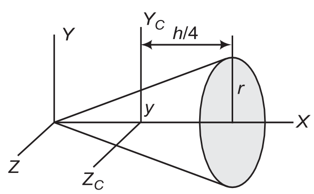
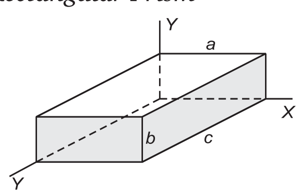
$$I = \sum_i m_i r_i^2 = \int r^2\, dm$$

Name of Geometrical Body	Formula of Moment of Inertia
1. Very thin circular loop (ring) 	Using $\perp^r$ axis theorem $I_{xx} + I_{yy} = I_{zz}$ $\frac{MR^2}{2} + \frac{MR^2}{2} = MR^2$ $I = MR^2$

<table>
<tr><td>2. Uniform circular loop
</td><td>$I = \frac{MR^2}{2}$
$I = M\left(\frac{R_1^2 + R_2^2}{2}\right)$</td></tr>
<tr><td>3. Uniform solid cylinder
</td><td>$I_x = I_y = \frac{1}{12}M(3R^2 + h^2)$</td></tr>
<tr><td>4. Uniform solid sphere
</td><td>$I = \frac{2}{5}MR^2 = I_x = I_y = I_z$</td></tr>
<tr><td>5. Uniform thin rod
</td><td>(AA') moment of inertia about the centre and perpendicular axis to the rod moment of inertia about the one corner point and perpendicular (BB') axis to the rod.
Thin rod about axis through one end $\perp^r$ to length.
$BB' = I = \frac{1}{3}Ml^2; AA' = I = \frac{Ml^2}{12}$</td></tr>
<tr><td>6. Very thin spherical shell
</td><td>$I = \frac{2}{3}MR^2$</td></tr>
</table>

7. Thin circular sheet 	$I = \frac{MR^2}{4}$

where, M = Mass of body,
R = Radius of the ring
I = Moment of inertia

Geometrical Body	Axis	Mass Moment of Inertia
Right Circular Cone Radius at base r and altitude h 	(i) About axis of revolution (x-axis) (ii) About centroidal y and z axis. (iii) About axis x perpendicular to axis of revolution and through vertex	$I_x = \frac{3}{10} mr^2$ $I_{yc} = I_{zc}$ $= \frac{3}{80} m(4r^2 + h^2)$ $I_y = I_z$ $= \frac{3}{5} m\left(\frac{r^2}{4} + h^2\right)$ $= \frac{3}{20} m(r^2 + 4h^2)$
Rectangular Prism 	(i) About centroidal axis	$I_x = \frac{1}{12} m(b^2 + c^2)$ $I_y = \frac{1}{12} m(c^2 + a^2)$ $I_z = \frac{1}{12} m(a^2 + b^2)$

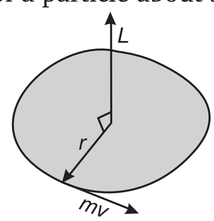
Torque and Angular Acceleration of a Rigid Body

For a rigid body, net torque acting $T = I\alpha$
where, α = Angular acceleration of rigid body
I = Moment of inertia about axis of rotation

➠ Kinetic energy of a rigid body rotating about fixed axis

$$KE = \frac{1}{2} I\omega^2 \quad (\omega = \text{angular velocity})$$

➠ Angular moment of a particle about same point



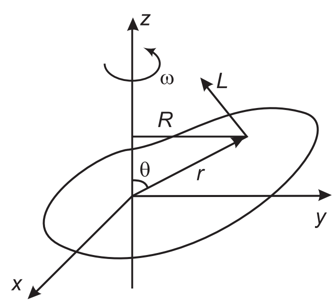
Angular moment

$$L = r \times p$$
$$L = m(r \times v)$$

where, L = angular displacement

➠ Angular moment of a rigid body rotating about a fixed axis.

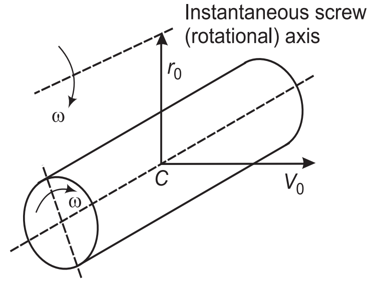
$$L = I\omega$$



Angular moment of a rigid body

➠ Angular moment of a rigid body in combined rotation and translation

$$L = L_{CM} + M(r_0 \times v_0)$$



Combined rotation and translation in a rigid body

➠ Conservation of angular momentum

$$T = \frac{dL}{dt}$$

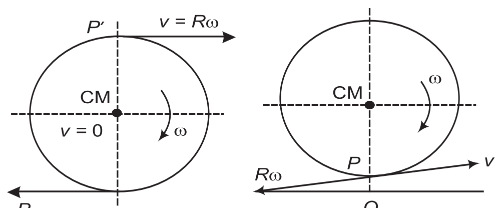
$$\frac{dL}{dt} = r \times F + v \times p$$

➠ Kinetic energy of rigid body in combined translational and rotational motion

$$K = \frac{1}{2} m v_{CM}^2 + \frac{1}{2} I_{CM} \omega^2$$

Uniform Pure Rolling

Pure rolling means no relative motion or no slipping at point contact between two bodies.



Uniform Pure Rolling

If $v_P = v_Q \Rightarrow$ No slipping

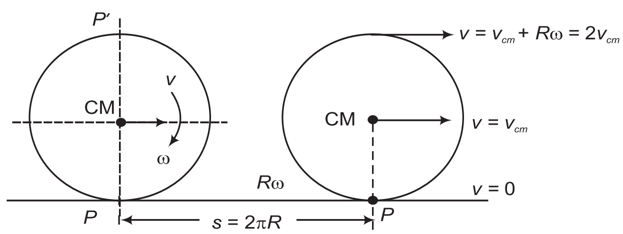
$$v = R\omega$$

If $v_P > v_Q \Rightarrow$ Forward slipping

$$v > R\omega$$

If $v_P < v_Q \Rightarrow$ Backward slipping

$$v < R\omega$$



Pure Rolling and translation

No slipping $s = 2\pi R$

Forward slipping $s > 2\pi R$

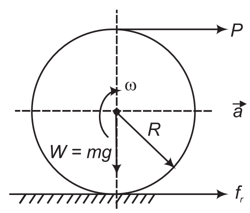
Backward slipping- $s < 2\pi R$

Accelerated Pure Rolling

A pure rolling is equivalent to pure translation and pure rotation. It follows a uniform rolling and accelerated pure rolling can be defined as

$$P + f_r = Ma$$

$$(P - f_r) \cdot R = I \times \alpha = \frac{MR^2}{2}\alpha$$



Accelerated pure rolling

Let force F be applied at the highest point in the horizontal direction. So that the sphere does not slip on the surface. Since the sphere is rolling the frictional force is in the same direction as the force applied at its highest point.

$P =$ Force acting on a body,

$f_r =$ Friction on that body

Angular impulse : Angular impulse represents the effect of a moment (Force acting at a distance from the TBCM) on a system. It is defined as the moment of force acting over a specified period of time.

Angular impulse = $\Sigma M \Delta t$

Angular Momentum

Angular momentum describes the quantity of angular motion. It is defined as the moment of linear momentum.

$$H = I_{CM} \times \omega$$

where, H = Angular Momentum

I_{CM} = Moment of Inertia about the centre of mass

ω = Angular velocity

Important Facts

- TBCM (Total Body Center of Mass).
- Angular impulse equal to area under moment - time curve.
- Angular impulse are two types positive angular impulse and negative angular impulse.
- A net positive angular impulse indicates that the system will rotate in a counter clockwise direction.
- Angular momentum is directly proportional to angular velocity.

WORK AND POTENTIAL ENERGY

Work is scalar quantity. It is the product of force and the corresponding displacement. Potential energy is the capacity of system to do work on another system. These concepts are advantageous in the analysis of equilibrium of complex systems, in dynamics and in mechanics of materials.

Work of Force

The work U of a constant force F is

$$U = FS$$

Where, S = Displacement of body in the direction of the vector F.

For a displacement along an arbitary path from point 1 to 2, with dr tangent to the path,

$$U = \int_1^2 F.\, dr = \int_1^2 \left(F_x dx + F_y dy + F_z dz\right)$$

There is no work when :

1. A force is acting on a fixed, rigid body ($dr = 0$, $du = 0$)
2. A force acts perpendicular to the displacement ($F.dr=0$)

Work of a Couple

A couple of magnitude M does work.

$$U = M\theta$$

Where, θ = Angular displacement (radians) in the same plane in which the couple is acting in a rotation from angular position α to β.

$$U = \int_\alpha^\beta \text{M}.\, d\theta$$

$$= \int_\alpha^\beta (M_x d\theta_x + M_x d\theta_y + M_z d\theta_z)$$

Important Facts

Solving general plane motion problem, including those of interconnected rigid bodies

- The angular velocity of a rigid body in plane motion, independent of the reference point.
- The common point of two or more pin-jointed members must have the same absolute velocity, even though the individual members may have different angular velocities.
- The points of contact in members that are in temporary contact may or may not have the same absolute velocity. If there is sliding between members, the joints in contact have different absolute velocities. The absolute velocities of the contacting particles are always the same if there is no sliding.
- If the angular velocity of a member is not known, but some points of the member move along defined paths. (i.e. the end points of piston rod), these paths define the direction of the velocity vectors and are in useful in the solution.

Important Facts

(For solving acceleration in general plane motion).

- The common points of pin-jointed members must have the same absolute acceleration even though the individual members may have different angular velocities and angular acceleration.
- The points of contact in members that are in temporary contact may or may not have the same absolute acceleration. Even when there is no sliding between the members, only the tangential acceleration of the points in contact are the same, while the normal acceleration are frequently different in magnitude and direction.
- The Instantaneous center of zero velocity in general has an acceleration and should not be used as a reference point for acceleration unless its acceleration is known and included in the analysis.
- The geometric center of wheel rolling on a flat surface moves in rectilinear motion. If there is no slipping at the point of contact, the linear acceleration of the center point is parallel to the flat surface and equal to $r\alpha$ for a wheel of radius r and angular acceleration α.

Important Facts (For No Work)

- Forces that act at Fixed points on the body do not do work. For example the reaction at a fixed frictionless pin does no work on the body that rotates about that pin.
- A Force which is always perpendicular to the direction of the motion does no work.
- The weighty of a body does no work, when the body's center of gravity moves in horizontal plane.
- The friction force at a point of contact on a body that rolls without slipping does no work. This is because the point of contact is the instantaneous center of zero velocity.