1

Metal Cutting

1.1. Introduction

Metal cutting is a manufacturing process by which a workpiece is given :

(*i*) a desired shape

(*ii*) a desired size

(*iii*) a desired surface finish.

To achieve one or all of these, the excess (or undesired) material is removed (from the workpiece) in the form of chips with the help of some properly shaped and sized tools. The metal cutting processes are chip-forming processes. It comes under the broad heading of metal forming which embraces both chip-forming and chipless forming processes.

The chip forming (or metal cutting) processes are : Turning, Shaping, Planning, Boring, Drilling, Broaching, Milling, Honing, Grinding, etc.

The chipless forming processes are : Rolling, Spinning, Forging, Extrusion, etc.

1.2. Orthogonal and Oblique Cutting – (Refer Figs. 1.1 and 1.2)

In Fig. 1.1, the workpiece is held (holding devices is not shown in the Fig.) and rotated. The tool (*i.e.* the cutting device) is held by some

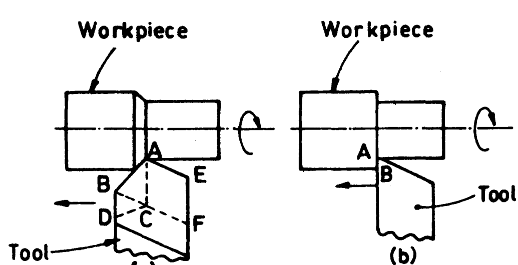


Fig. 1.1

other holding device. It is moved against the work in the direction of the arrow (*i.e.* towards left hand side in this case) and some material is removed from the work piece. The removed material is called chips. There is reduction in the diameter of the workpiece. On careful obser

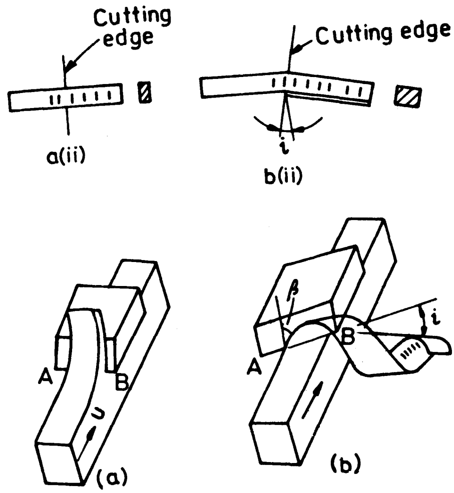


Fig. 1.2

vation we find that the edge *AB* (of the tool) only is in touch with the work. In fact only this edge takes part in cutting. It is called the cutting edge. The edge *AB* in Fig. 1.1(*b*) is at 90° to the direction of motion of the tool whereas the edge AB in Fig. 1.1 (*a*) is at some acute angle to the direction of motion of the tool.

In Fig. 1.2 (*a*) and (*b*), the workpieces do not rotate but move in a linear direction (in the direction of the arrow). The tool may be assumed to be stationary. The cutting edge *AB* in Fig. 1.2 (*a*) is at 90° to the direction of cutting and in Fig. 1.2 (*b*) it is at some acute angle *i*.

The cutting in Figs. 1.1 (*b*) and 1.2(*a*) is called two dimensional or orthogonal cutting. The cutting in Fig. 1.1 (*a*) and Fig. 1.2 (*b*) is called three dimensional or oblique cutting.

Basic Differences Between Orthogonal and Oblique Cutting

	Orthogonal cutting	*Oblique cutting*
1.	The cutting edge of the tool remains at 90° to the direction of feed (of the tool or the work).	The cutting edge of the tool remains inclined at an acute angle to the direction of feed (of the work or tool).

Contd.

2.	The chip flows in a direction normal to cutting edge of the tool.	The direction of the chip flow is not normal to the cutting edge. Rather it is at an angle β to the normal to the cutting edge.
3.	The cutting edge of the tool has zero inclination with the normal to the feed.	The cutting edge is inclined at the angle *i* to the normal to the feed. This angle is called inclination angle.
4.	The chip flows in the plane of the tool face. Therefore, it makes no angle with the normal (in the plane of the tool face) to the cutting edge.	The chip flows at an angle β to the normal to the cutting edge. This angle is called chip flow angle.
5.	The shear force (to be explained later) acts on a smaller area, so shear force per unit area is more.	The shear force acts on a larger area, hence the shear force per unit area is smaller.
6.	The tool life is smaller than that in oblique cutting (for same conditions of cutting).	The tool life is higher than obtained in orthogonal cutting.
7.	There are only two mutually perpendicular components of cutting forces on the tool.	There are three mutually perpendicular components of cutting force on the tool.
8.	The cutting edge is bigger than the width of cut.	The cutting edge is smaller than the width of cut.

1.3. Chip Formation

As the tool moves against the workpiece a layer of the metal is separated from the workpiece, which comes out sliding over the tool face in the form of chips. The separation of the material from the parent body (*i.e.* from the workpiece) could be due to

(*i*) Fracture process

(*ii*) Peeling process

(*iii*) Crack formation and propagation, ahead of the tool cutting edge.

(*iv*) Shearing and plastic flow process.

To ascertain the exact process involved in cutting many scientists allover world did much investigations. The process of cutting was *frozen* with the help of quick stop mechanism and the chips were subsequently examined with the microscope. Photographs of the side surface of the chip with a high movie camera fitted with microscope were also taken for examination by some investigators. Some examined the grid deformations on the side surface of the workpiece. These investigations revealed certain common facts which convinced the scientists that the process of metal removal was shear process. That is the chip formation is as a result of shearing of the metal by the tool. The shearing does not occur across a line (or plane) but there exists a zone (shown as *ABCD*

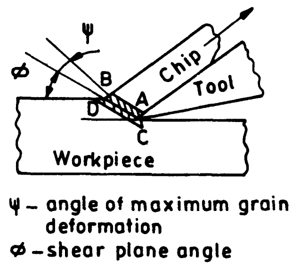
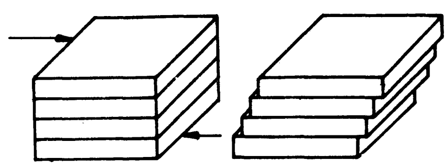


Fig. 1.3

in Fig. 1.3) known as shear zone or primary deformation zone. The width of the shear zone is very small and is only about 1 to 10 microns. Fig. 1.3 shows ϕ as angle of maximum deformation of the grains. The photo micrographs of the chip indicates that the grains deform along the anglec ψ But for all calculations and practical purposes only ϕ is considered. Even the shear zone idea is not utilised for calculation of forces or power in cutting but the shear plane only is considered for any such analysis.

Piispanen assumed the work material to consist of thin lamalle of metal. This he compared with a pack of cards (Fig. 1.4). Under a shear force the cards slide over each other as shown in Fig. 1.4(*b*). He



(*a*) (*b*)

Fig. 1.4

argued that the sections of layer of the metal being cut get successive slip and slide over the tool face in form of chips as shown in Fig. 1.5.

The mechanics of chip formation can be best understood by this. As the cutting tool or workpiece progresses, the metal immediately ahead of the tool is compressed resulting into deformation or elongation of the crystal structure. This elongation does not take place in the direction of shear. As the process continues, the material above the cutting edge is forced along the tool face (rake face).

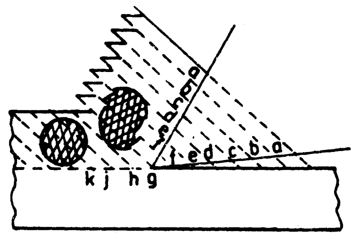


Fig. 1.5

1.3.1. Types of Chip

The chips are broadly divided into three categories :

(*i*) discontinuous

(*ii*) continuous

(*iii*) continuous with built up edge.

The type of chip produced in a particular operation depends on the following variables :

(*i*) Properties of the material cut, (*i.e.* ductile or brittle)

(*ii*) Cutting speed

(*iii*) Depth of cut

(*iv*) Feed rate

(*v*) Rake angle

(*vi*) Type and way of application of the cutting fluid

(*vii*) Surface roughness of the tool face

(*viii*) Coefficient of friction between the chip and tool interface,

(*ix*) Temperature of the chip on tool face.

(*x*) Nature of cutting *i.e.* continuous or intermittent.

1.3.2. Discontinuous Chip (Refer Fig. 1.6.)

The chips are produced as the tool advances in the direction of the feed due to plastic deformation of the material ahead of the tool nose and in the vicinity of the cutting edge But the chips are coming out as

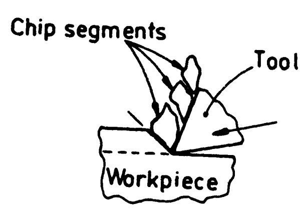


Fig. 1.6

small pieces or segments as compared to chips shown in Figs. 1.7 and 1.8. The reason for such type is that as the material gets sheared off due to advancement of the tool, it ruptures intermittently, and thus producing segmented or discontinuous chips. The conditions favourable for production of such chips are :

(*i*) Brittle work material

(*ii*) Small or negative rake angle

(*iii*) Large chip thickness, *i.e.* large depths of cut and high feed rate.

(*iv*) Low cutting speed.

(*v*) Dry cutting *i.e.* cutting without the application of cutting fluid.

This type of chip is formed when cutting cast iron brass, bronze. etc.

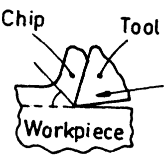


Fig. 1.7

1.3.3. Continuous Chips

The type of chip shown in Fig. 1.7, is called continuous chip. Discontinuous chips, as we discussed earlier are fragmented chips, whereas continuous chip does not break while passing on the tool face. So chip in big length are produced. Sometimes they get curled, sometimes not, depending upon the cutting conditions. In hot machining of alloy steel we can observe chip of few feet length before it breaks. Regarding the formation of such chips, one theory suggests that shear occurs ahead of the tool (as the tool advances) continuously without fracture. Conditions favourable for production of such chips are –

(*i*) Ductile material

(*ii*) Large rake angles

(*iii*) High cutting speed

(*iv*) Small depth of cut

(*v*) Small feed rate

(*vi*) Efficient way of application of cutting fluid to prevent built-up edge formation

(*vii*) Low coefficient of friction at chip-tool interface

(*viii*) Polished face of the cutting tool

(*ix*) Use of material having low coefficient of friction as cutting tool, such as cemented carbide.

Continuous chips pose difficulty while machining. One has to be very careful and quick in its disposal otherwise if it gets wrapped over the machined portion of the work, its surface would be spoiled. So during machining a device known as 'chip breaker' is attached over the tool post (near the tool nose) which breaks the chip into small fragments.

1.3.4. Continuous Chips with Built-up edge

This category of chip is shown in Fig. 1.8. The term built-up-edge and its formation we shall explain first. As the chip moves over the tool face due to high normal load on the tool face, high temperature, and high coefficient of friction between the chip and the tool interface a portion of chip get welded on the tool face forming the embryo of built-up edge. The strain hardened chip is so hard that now it becomes practical by the cutting edge and starts cutting the material. Since this built-up edge is irregular in shape, the surface produced becomes quite rough. As the machining continues more and more chip material gets welded on the embryo built-up edge, this increasing its size and ultimately, it becomes unstable and gets sheared off. This cycle is repeated.

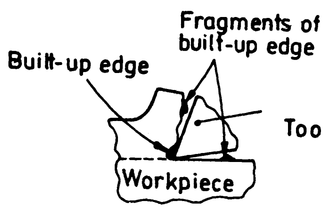


Fig. 1.8. Machining with a tool having built-up edge.

During the unstable stage, some fragments of the built-up edge are carried along the under surface of the chip while some escape along the flank thus worsening the surface finish of the machined surface.

Built-up Formation

Fig. 1.9 shows the formative cycle of built-up edge. After the embryo of built-up edge reaches the final stage, it is sheared off. Again

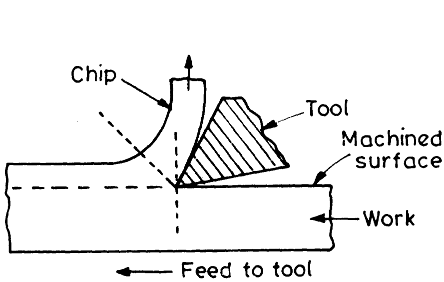


Fig. 1.9. (*a*) Machining with fresh ground tool
(no wear, no built-up edge)

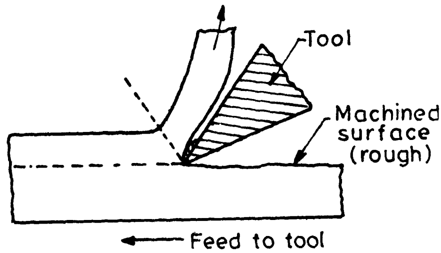


Fig. 1.9 (*b*) Embryo of built-up edge formed.
The finish of machined
surface deteriorates.

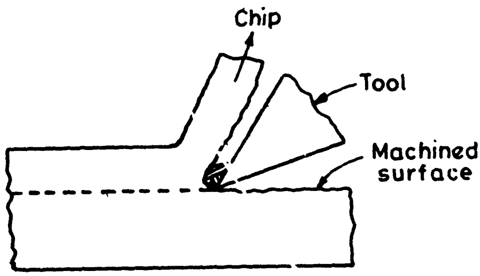


Fig. 1.9 (*c*) Built-up edge, increases in size.
The machined surface
further deteriorates.

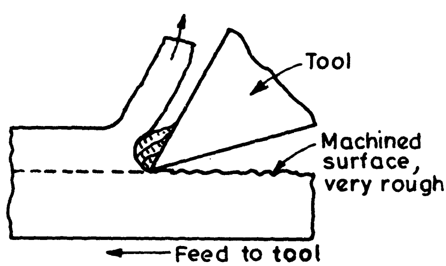


Fig. 1.9 (*d*) The built-up edge further grows in
size and becomes unstable
and gets sheared off.

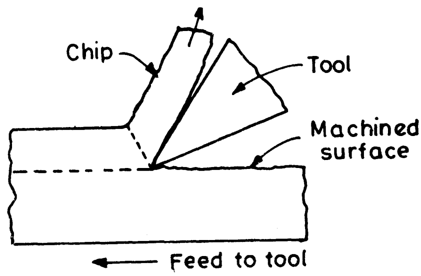


Fig. 1.9 (*e*) The machining continues. The built-up edge gets sheared off. Very small portion of it remains welded to the tool. The machined surface improves.

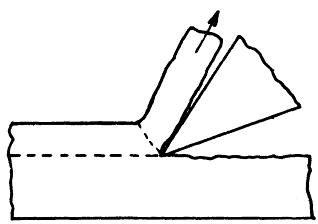


Fig. 1.9 (*f*) The built-up edge re-forms. The cycle continues.

embryo is formed and the whole cycle is repeated. Fig 1.10 shows the three forces acting on the built-up edge, where

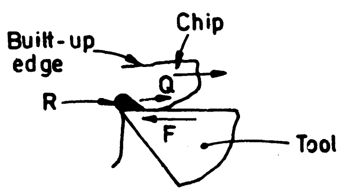


Fig. 1.10

F – frictional force between the built-up edge and the tool rake face (over which it slides)

Q – force between the built-up edge and the chip

R – Reaction on the built-up edge.

IF $F > (Q+R)$, the embryo forms and grows

when $F < (Q+R)$, the built-up edge is sheared off.

Thus the ratio of F and $(Q+R)$ changes periodically and the cycle of built-up edge formation repeats.

Under this category the chips come out without fracture as in the case of continuous chips but there is formation of built-up edge. The particles of built up edge, after breaking comes out with the chip as shown in Fig. 1.8. The conditions for formation of such chips are same as for the formation of continuous chips.

1.4. Tool Geometry (Tool-in-Hand Nomenclature)

Cutting tool is a device with which a material could be cut to the desired size, shape or finish. So a cutting tool must have atleast

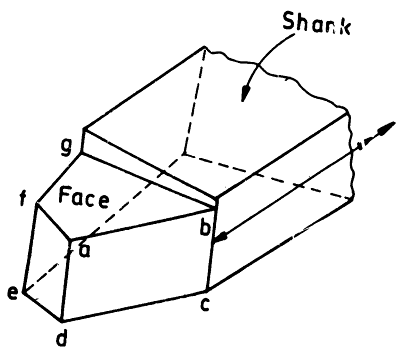


Fig. 1.11. Tool-in-Hand Nomenclature.

a sharp edge. We can take a blade or a knife, as simple examples of cutting tools having only one thin cutting edge. But in the industry where a workpiece say of steel is to be shaped on a lathe, (*i.e.* to be given a surface of revolution), a cutting tool of proper size, shape and length is essential. A cutting tool shown in Fig. 1.11 has an edge *ab*, which cuts the workpiece. This edge is called cutting edge.

We said earlier that a tool can have one or more than one cutting edges.

The tool having only one cutting edge is called single point tool. Example : Shaper tool, lathe tools, planer tool, etc.

The tool having more than one cutting edge is called multipoint cutting tools. For example drills, milling cutters, broaches, grinding wheel honing tool etc.

We shall restrict our discussion to single point cutting tools only.

A typical single point cutting tool is shown in the Fig. 1.11, where
ab–Cutting edge

af – Auxiliary cutting edge (or end cutting edge)

abcd – Principal flank surface

adef – Auxiliary flank surface

abgf – Face or Rake surface.

Cutting edge – This is the edge which cuts the metal. Fig. 1.11(*a*) shows the tool in cutting position.

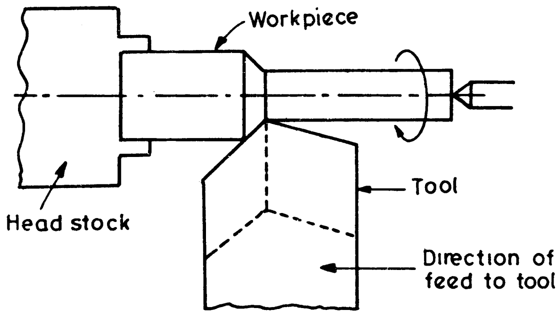


Fig. 1.11(*a*). Tool in Cutting Position.

Face – It is that portion of the tool, on which the cut metal, *i.e.* the chip flows.

Flank – The portion *abcd,* which faces the machining portion of workpiece [(Fig. 1.8(*a*)] and contains the cutting edge is principal flank.

The other flank *adef,* containing the auxiliary cutting edge is auxiliary flank surface. If faces the machined portion of the workpiece.

Shank– It is the main body of the tool which is held in the tool holding device for cutting purpose.

Base– It is the bearing or support of the shank. (That is the bottom most surface of the tool.)

The face and the flank are plain surfaces. The cutting edge can be assumed to be a line. These surfaces and the edges are inclined with respect to some reference plane or line. The inclination are called tool angles. These angles are defined by various names. Take the case of the face *abgf.* It is a plane surface no doubt, but can have inclinations. This surface may be parallel to the base or say to horizontal surface, or it can be inclined upward or downward with respect to the horizontal plane. Again it may have inclination sideward also. So in general the face can have two inclinations simultaneously, backward and sideward, for example.

Similarly the principal flank *abcd* or auxiliary flank *adef* can have two inclinations.

These angles have been defined in various ways by various systems. We shall see these one by one.

Fig. 1.11 does not depict any angles. It describes the nomenclature when held in hand or kept in the space. So it is called *tool in hand system or nomenclature.*

1.4.1. Orthogonal Rake System (ORS) or International Orthogonal

Fig. 1.12 shows the tool angles in ORS system. The tool is engaged with the workpiece. The plan view of the tool is shown in the figure. The angles are defined in the planes, $x - x, y - y, z - z$ and A-A. The plane $x_0 - x_0$ is perpendicular to the plan of principal cutting edge *ab*.

The plane $y_0 - y_0$ is along the plane of principal cutting edge *ab*.

The plane $z_0 - z_0$ is perpendicular to both these planes and is vertical.

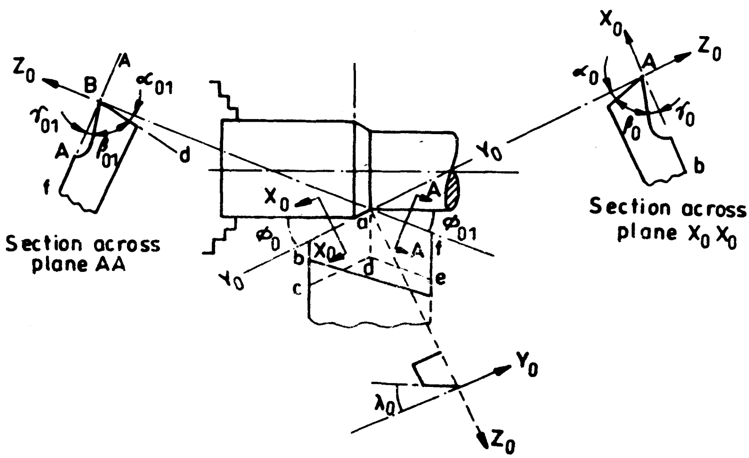
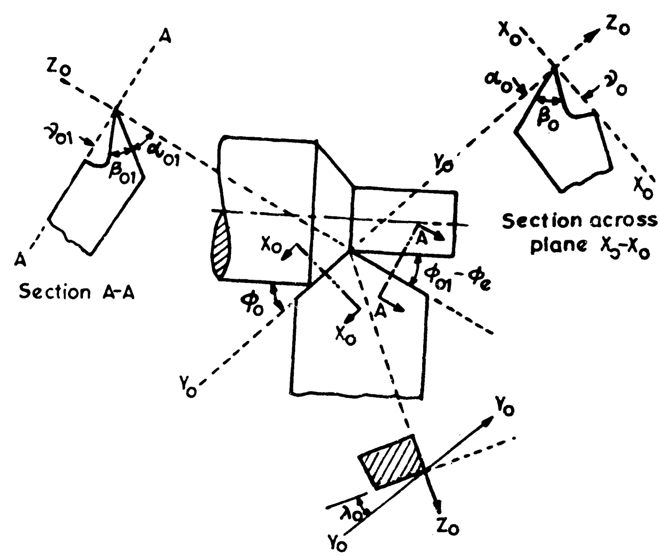


Fig. 1.12



Orthogonal rake system (ORS) or
International orthogonal system

Fig. 1.12(*a*)

1.4.2. Tool angles – ORS system

$\lambda_o \rightarrow$ Orthogonal rake
$\gamma_{o1} \rightarrow$ Side rake
$\beta_o \rightarrow$ wedge angle
$\beta_{o1} \rightarrow$ Side wedge angle
$\propto_0 \rightarrow$ Side relief angle
$\propto_{01} \rightarrow$ End relief angle
$\phi \rightarrow$ Principal cutting edge angle (or plan approach angle)
$\phi_{o1} \rightarrow$ Auxiliary cutting edge angle
$\lambda_o \rightarrow$ Inclination angle
$r \rightarrow$ Nose radial

However only following angles are mentioned in tool signature

$\lambda_0 \rightarrow$ Inclination angle (0 for orthogonal cutting)
$v_o \rightarrow$ Orthogonal rake
$\alpha_o \rightarrow$ Side relief angle
$\alpha_{01} \rightarrow$ End relief angle
$\phi_0 \rightarrow$ Principal cutting edge *ay*
$\phi_\varepsilon = \phi_{o1} \rightarrow$ Auxiliary cutting edge angle
$r \rightarrow$ nose radius

$\lambda_o - v_o - \alpha_o - \alpha_{o1} - \phi_{o1} - \phi_o - r$

$0 - 10 - 5 - 5 - 8 - 75 - 1$ mm (0RS)

Angles are in degrees

1.4.3. Tool Signature

$7 - 14 - 6 - 6 - 8 - 15 - \frac{1}{8}$ (ASA)

In ORS System

Plane $X_0 - X_0$ is perpendicular to the plan of principal cutting edge *ab*.

The $Y_o - Y_a$ is plane along the plan of the principal cutting edge *ab*

The plane $Z_o - Z_o$ is perpendicular to both these planes and is vertical. Plane A-A is perpendicular to the plan of auxiliary cutting edge *a f*.

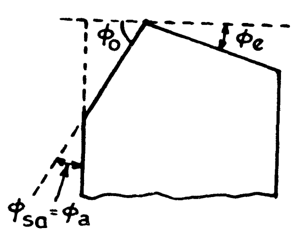


Fig. 1.13(*a*)

Note :

$\lambda_o - \nu_o - \alpha_o - \alpha_{o1} - \phi_{o1} - \phi_0 - r -$ (ORS)

$\nu_a - \nu_{a1} - \alpha_a - \alpha_{a_1} - \phi_{\alpha\varepsilon} - \phi_{sa} - r -$ (ASA)

$\phi_a = \phi_{sa} = 90 - \phi_o$ and $\phi_{ae} = \phi_o$; $\phi_{sa} = \phi_a$

1.4.4. Relationships

(1) To get inclination angle λ_0

$\tan \lambda_0 = \tan \nu_a \,.\, \cos \phi_{sa} - \tan \nu_{a1} \,.\, \sin \phi_{sa}$

(2) Orthogonal Rake. ν_0

$\tan \nu_0 = \tan \nu_a - \sin \phi_{sa} + \tan \nu_{a1} \,.\, \cos \phi_{sa}$

(3) Side Rake ν_{a1}

$\tan \nu_{a1} = \tan \nu_o \,.\, \cos \phi_{sa} - \tan \lambda_o \sin \phi_{sa}$

(4) Back rake. ν_a

$\tan \nu_a = \tan \lambda_0 - \cos \phi_{sa} + \tan \nu_o \sin \phi_{sa}$

Prob. 1.1. Calculate the Inclination angle and Orthogonal rake angle (ν_o)

Given : Side rake = 10° = (ν_{a1})

Back rake = 8° (ν_a)

Side cutting edge angle = 15° (ϕ_{sa})

(a) $\tan \lambda_0 = + \tan \nu_a \,.\, \cos \phi_{sa} - \tan \nu_{a1} \,.\, \sin \phi_{sa}$

$= \tan 8° - \cos 15° - \tan 10° \,.\, \sin 15°$

$= + (.1405)\,(.96593) - (.17633)\,(.25882)$

$= + .09001, \quad \lambda_0 = + 5°\, 9'$

(b) $\tan \nu_0 = \tan \nu_a \,.\, \sin \phi_{sa} + \tan \nu_{a1} \,.\, \cos \phi_{sa}$

$= \tan 8° \,.\, \sin 15° + \tan 10° \,.\, \cos 15°$

$= (.1405)\,(.25882) + (.17633)\,(.96593)$

$= .03637 + .17032$

$= .20669$

$\nu_0 = 11°\, 40'$

Prob 1.2. *Calculate Side rake angle and back rake angle* (ν_a). *Given*

Inclination angle $\lambda_0 = 5°\, 9'$

orthogonal rake $\nu_0 = 11^0\, 40'$

Side cutting edge angle = $15° = \phi_{sa}$

Sol. Side Rake angle ν_{a1}

(a) $\tan \nu_{a1} = \tan \nu_0 \,.\, \cos \phi_{sa} - \tan \lambda_0 \,.\, \sin \phi_{sa}$

$= \tan 5^0\, 9' \,.\, \cos 15° - \tan 11°\, 40' \,.\, \sin 15°$

$= (.20669)\,(.96593) - (+.09011)\,(.25882)$

$= .19945 - 0.02333 = .17612$

$\nu_{a1} = 10°$

(*b*) Back rake , ν_a

$\tan \nu_a = \tan \lambda_0 \,.\, \cos \phi_{sa} + \tan \nu_0 \,.\, \sin \phi_{sa}$

$=$

$= .14095$

$\nu_a = 8^\circ$

Prob. 1.3. *During machining of C-40 steel a double carbide cutting tool of 0 – 10 – 6 – 6 – 8 – 75 (ORS)*

shape has been used.

Calculate (*i*) Back rake (*ii* Side rake (*iii*) **Front clearance.**

Hint. $\lambda_0 = 0$, $\nu_0 = 10^\circ$, $\phi_0 = 75^\circ$

Prob. 1.4. *In a turning operation the principal cutting edge angle is 60°, and back rake is 6°. Calculate the side rake so that the cutting can be considered as orthogonal.*

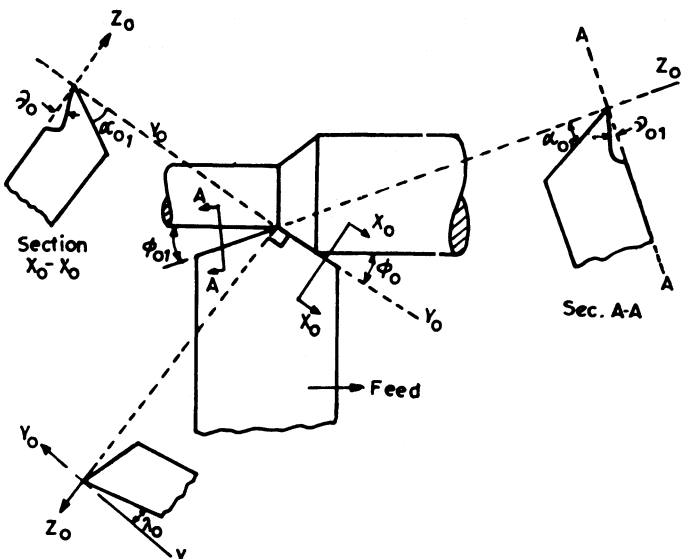
Hint. The problem emplies that $\nu_0 = 0$

Prob. 1.5. *The following tool geometry is used in turning operation 7 - 10 - 6 - 6 - 8 - 15° A.S.A Calculate* (*i*) *Orthogonal rake* (*ii*) *inclination angle.*

Prob. 1.6. *Calculate the inclination angle and orthogonal rake angle. Given :*

Side rake = – 5° , Back rake = – 7°,

Side cutting edge angle = 15°.



$\lambda_0 - \gamma_0 - \alpha_0 - \alpha_{01} - \phi_{01} - \phi_0 - \gamma$

Fig. 1.13 (*b*) Left Hand Tool (ORS)

Sol. (*a*) $\tan \lambda_0 = + \tan \nu_a \cdot \cos \phi_{sa} - \tan \nu_{a1} \cdot \sin \phi_{sa}$

$$= (-.12278)(.96593) - (-.08749)(.2588)$$
$$= -.09596, \quad \lambda_0 = 5^\circ 29'$$

(*b*) orthogonal rake,

$$\tan \nu_0 = \tan \nu_a \cdot Sin\ \phi_{sa} + \tan \nu_{a1} \cdot Cos\ \phi_{sa}$$
$$= (-.12278)(.25882) + (-.08749)(.965)$$
$$= -.03178 - .08451 = -.11629$$

Therefore orthogonal rake, $\nu_o = -6^\circ 38'$ **Ans.**

1.4.5. Left Hand Tool

The Fig. 1.13 (*b*) depicts left hand tool in ORS.

1.4.6. Relation for clearance angle

(1) Front clearance $\rightarrow \alpha_a$

$$\text{Cot} \propto_a = \text{Cot} \propto_0 - \sin \phi_{sa} + \tan \lambda_0 \cdot \text{Cos}\ \phi_{sa}$$

(2) Side clearance $\rightarrow \alpha_{a1}$

$$\text{Cot}(\propto_{a_1}) = \cot \alpha_0 \cdot \cos \phi_{sa} - \tan \lambda_0 \cdot \sin \phi_{sa}$$

The angles for V-thread in ORS are shown in figure 1.13 (*c*)

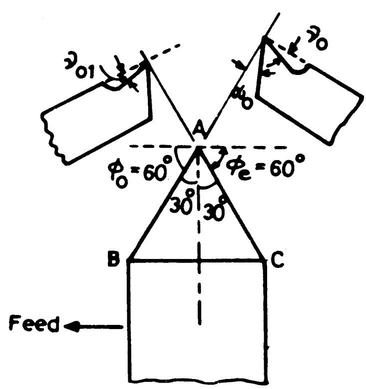


Fig. 1.13(*c*) V-Thread cutting Tool. (ORS system)

Example : Thread angle = $2\alpha = 60$

α = flank anlge = 30°

ϕ_p = Plan approach angle

ϕ_ε = End cutting edge angle = Plan trial angle

$\nu_0 \rightarrow$ Orthogonal rake

AB $\rightarrow$ Primary cutting edge of tool

AC $\rightarrow$ Secondary cutting edge.

1.4.3. In American Standards Association (ASA) System and tool angles are nomenclated as follows (or the tool signature in American System).

ν_a – Back rake (or top rake)

ν_{a1} –Side rake

α_a –End relief (or front clearance)

α_{a1} –Side relief (or side clearance)

ϕ_{ae} – End cutting edge angle

ϕ_{sa} – Side cutting edge angle

r – Nose radius.

It is also written as follows :

$$\nu_a - \nu_{a1} - \alpha_a - \alpha_{a1} - \left[\phi_{ae} - \phi_{sa}{}^{-r}\right]$$

For a particular turning tool, the values of the angles, are shown below

7–14–6–6–8–15–1/8" (ASA)

In orthogonal system (also called orthogonal rake system or ORS) the name of the angles are different. Their values are also different than those in ASA system. But a relationship could be established among the angles expressed in the two system so that the equivalent angles in any system could be obtained.

1.5. Force Relationship

Fig. 1.14 represents a turning operation. It is an oblique cutting since the principal cutting edge ab makes an angle with the direction of the feed. Since the metal is being cut, there must be a cutting force. And the cutting force is R. This cutting force can be **resolved** into any three mutually perpendicular directions. But they are invariably **resolved** in the following directions :

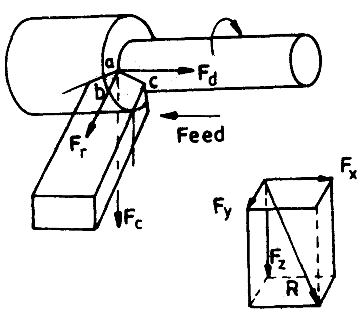


Fig. 1.14

(*i*) In the direction of feed of the tool. It is represented by F_d. It is the horizontal component of the cutting force. It is also called feed force. It remains tangent to the generated surface. It looks vertical in Fig. 1.15.

(*ii*) In the direction perpendicular to the feed direction. It is represented here by F_r. It is in the radial direction (*i.e.* in the direction perpendicular to the generated surface.) It may be considered due to the reaction between the tool and the workpiece. **It is called thrust force.**

(*iii*) In the verticular direction. It is represented here by F_c. It is the vertical component of the cutting force and in fact is the *main cutting* force. it is in the direction of movement of the tool in Fig. 1.15.

(Note that these forces in Fig. 1.14 have been shown in the reverse directions.)

Fig. 1.16 shows an orthogonal machining process. In this process the cutting force has two components only. One in the feed direction

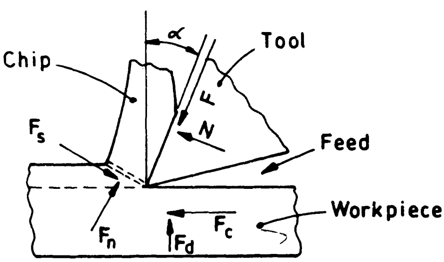


Fig. 1.15

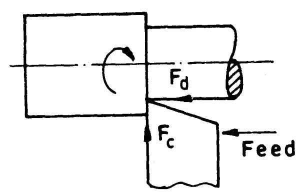


Fig. 1.16

F_d and other in vertical direction *i.e.* in the direction of cutting, F_c .

In all our discussion we shall consider only two components of forces.

Consider Fig. 1.15. In this figure two components of cutting force have been shown and the forces acting on the chip etc. have **been shown. We shall discuss it now. (It is also a case of orthogonal cutting).**

The cutting tool moves along the feed direction. The metal gets plastically deformed along the shear plane (as explained earlier). The chips (deformed metal) move along the rake surface of the tool. The chip being rough gets resistance in movement and hence a frictional force F.

So F is the frictional resistance of the **chip** acting on the tool.

Force N is the reaction provided by the tool, acting in a direction normal to the rake face of the tool.

Force F_s. is the shear force of the metal. It can also be called the resitance of the metal to shear in forming the chip.

Force F_a. is normal to the shear plane. It is a backing up force provided by the workpiece on the chip. It causes compressive stress on the shear plane.

Fig. 1.17 is a free body diagram of the chip with the forces acting on it.

R – is the resultant of forces F_s and F_a.

R' – is the resultant of forces F and N.

Since the chip is in equilibrium, the resultant force R and R' are equal in the magnitude, opposite in direction and collinear.

For a fixed geometry of the cutting tool there exists a definite relationship among these forces. The component of the cutting forces could be measured by a dynamometer and all other forces could be calculated.

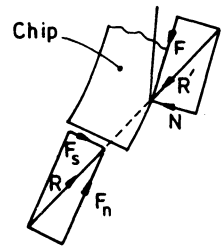


Fig. 1.17

Merchant represented these forces in a circle, known as *Merchant's circle diagram*. One such diagram has been shown in the Fig. 1.18.

where α = back rake angle

ϕ = shear angle

η = angle of friction.

ϕ, the shear angle can be measured by two methods.

(*i*) By taking a photomicrograph of the cutting process.

(*ii*) By measuring the thickness of the chip and the depth of cut. (This method will be discussed in the next article.)

α– the back rake of the tool can be measured and the forces F_d and F_c, could be measured with dynamometer.

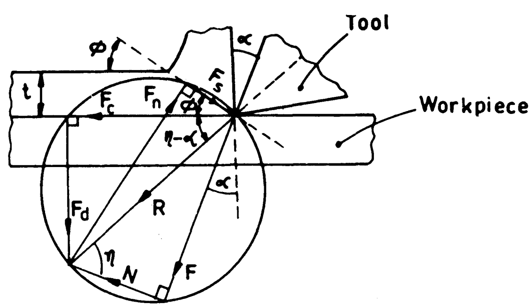
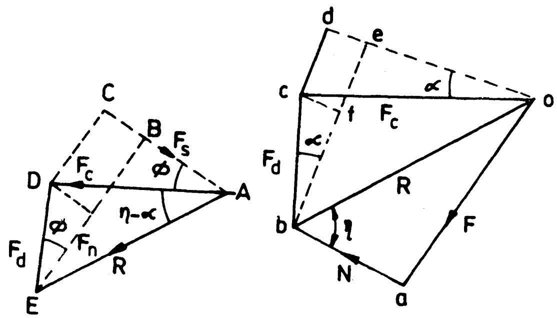


Fig. 1.18

Therefore, once F_d, F_c, α and ϕ are known, all other components of forces acting on the chip can be determined with the geometry shown in Fig. 1.18.

To get the relationships, we draw Fig. 1.19 and relate the unknown forces with F_d and F_c only. From the Fig. 1.19 (*a*)



(*a*) (*b*) $F_s = AB = AC - BC$

$F_c = AD; F_n = BE$

Fig. 1.19

$$F_s = F_c \,.\, \cos\phi - F_d \,.\, \sin\phi \qquad ...(1.5)$$

$$F_n = F_c \,.\, \sin\phi + F_d \,.\, \cos\phi \qquad ...(1.6)$$

$$= F_s \,.\, \tan(\phi + \eta - \alpha) \qquad ...(1.7)$$

Again,

$$F_c = R \,.\, \cos(\eta - \alpha)$$

$$F_a = R \,.\, \cos(\phi + \eta - \alpha)$$

or $$R = F_s/\cos(\phi + \eta - \alpha)$$

$\therefore$ $$F_c = \frac{F_s}{\cos(\phi + \eta - \alpha)} \,.\, \cos(\eta - \alpha) \qquad ...(1.8)$$

or $$F_s = f_c \,.\, \frac{cso\ (\phi + \eta - \alpha)}{\cos(\eta - \alpha)}$$

From Fig. 1.19 (*b*)

Since, $N = ab = oe = od - de$

$N = F_c \cos\alpha - F_d \,.\, \sin\,.\,\alpha$

Since $F = ao = be$

$= ef + fb = cd + fb.$

So $F = F_c \,.\, \sin\alpha + F_d \,.\, \cos\alpha$

Let μ = coefficient of friction

$F = \mu N.$

$\therefore$ $$\mu = \frac{F}{N} = \tan\eta$$

$$= \frac{F_c \,.\, \sin\alpha + F_d \,.\, \cos\alpha}{F_c \,.\, \cos\alpha - F_d\ \sin\alpha}$$

Dividing the numerator and denominator by cos α, we get

$$\mu = \frac{(F_c \,.\, \tan \alpha + F_d)}{F_c - F_d \tan \alpha} \qquad ...(1.9)$$

Conditions for maximum cutting force (F_c) from the equation 1.8.

$$F_c = F_s \,.\, \frac{\cos (\eta - \alpha)}{\cos (\phi + \eta - \alpha)}$$

where, F_s = shear force

= Shear stress × Area of shear plane

$= S_s. \dfrac{b \,.\, t}{\sin \phi}$

So $$F_c = S_s \,.\, \frac{b \,.\, t}{\sin \phi} \,.\, \frac{\cos (\eta - \alpha)}{\cos (\phi + \eta - \alpha)} \qquad ... (1.9a)$$

For F_c to maximum, $\dfrac{d F_c}{d \phi} = 0$

or $$\frac{dF_c}{d \phi} = \frac{d}{d \phi}\left[\frac{S_c \,.\, b.t}{\sin \phi} \,.\, \frac{\cos (\eta - \alpha)}{\cos (\phi + \eta - \alpha)}\right] = 0$$

$$= S_s \,.\, b.t \cos (\eta - \alpha) \,.\, \frac{d}{d\phi}\left[\frac{1}{\sin \phi \,.\, \cos (\phi + \eta - \alpha)}\right] = 0$$

or $$= S_s \,.\, b.t. \cos (\eta - \alpha)$$

$$\frac{[-\cos \phi \,.\, \cos (\phi + \eta - \alpha) + \sin \phi \,.\, \sin (\phi + \eta - \alpha)]}{[\sin \phi \,.\, \cos (\phi + \eta - \alpha)]^2} = 0$$

or $-\cos \phi \,.\, \cos (\phi + \eta - a) + \sin \phi \,.\, \sin (\phi + \eta - \alpha) = 0$

or $\cos \{\phi + (\phi + \eta - \alpha)\} = 0$

or $\cos (2\phi + \eta - \alpha) = 0 = \cos \pi/2$

or $2\phi + \eta - \alpha = \pi/2$

or $$\phi = \frac{\pi}{4} - \frac{\eta}{2} + \frac{\alpha}{2} \qquad ...(1.10)$$

The above relationship is based on Earnst Merchant theory, which makes the following assumptions :

(*i*) The stress is maximum at the shear plane and it remains constant.

(*ii*) The shear takes place in a direction in which the energy required for shearing is minimum.

1.6. Merchant Theory

Merchant modified the relationship derived by Earnst-Merchant, by assuming that the shear stress along the shear plane varies linearly with normal stress. it is given as, (from Fig. 1.20)

$$S_s = S_o + K \,.\, S_n$$

where, S_o – Static stress

S_n – Normal stress

S_s – Shear stress

K – Constant.

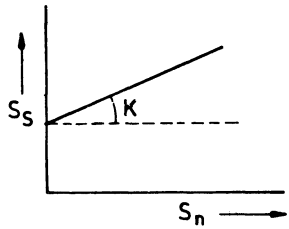


Fig. 1.20

$$F_c = F_s . \frac{\cos(\eta - \alpha)}{\cos(\phi + \eta - \alpha)}$$

$$= (S_o + K . S_n) . \frac{b.t}{\sin \phi} . \frac{\cos(\eta - \alpha)}{\cos(\phi + \eta - \alpha)}$$

From Eqn. (1.9*a*)

Putting, $\frac{dF_c}{d\phi} = 0$, we get

$$\cos(2\phi + \eta - \alpha) = K$$

or $\quad 2\phi + \eta - \alpha = \cot^{-1} K$

or $\quad \phi = \frac{\cot^{-1} K}{2} - \frac{\eta}{2} + \frac{\alpha}{2}$ ...(1.11)

1.6.1. Lee and Shaffer Theory. According to this theory the shear occurs on a single plane. So for a cutting process according to this theory, the following are supposed to hold good :

(*i*) The material ahead of the cutting tool, behave as ideal plastic material.

(*ii*) The chip does not get hardened.

(*iii*) The chip and parent work-material are separated by a shear plane.

Lee and Shaffer derived the following relationship as :

$$\eta + \phi - \alpha = \frac{\pi}{4}$$

This relationship was further modified as,

$$\eta + \phi - \alpha - \theta = \frac{\pi}{4}$$

where θ is a factor to take into account the changes due to built-up edge formation.

1.7. Chip Thickness Ratio

Fig. 1.21 shows a cutting process

where, t = depth of cut

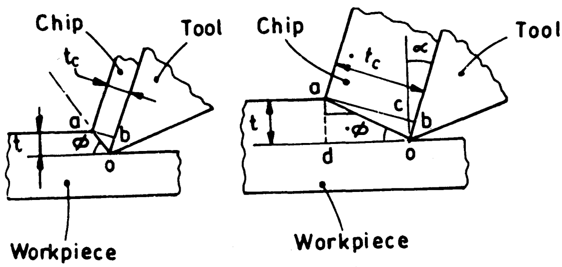


Fig. 1.21

t_c = chip thickness (*i.e.* thickness of the cut layer after deformation)

ϕ = shear angle

In Fig. 1.21 (*a*), $t = t_c$

In Fig. 1.21 (*b*), $t < t_c$

It is to say that the thickness of the chip varies with the shear angle. The smaller the shear angle the thicker is the chip and vice versa for same depth of cut.

The chip thickness ratio, r

$$= \frac{\text{Depth of cut}}{\text{Chip Thickness}} = \frac{t}{t_c}$$

where , t= feed (s) in Turning = depth of cut for shaping and planing.

$$= \frac{\text{Uncut chip thickness}}{\text{Chip thickness}} \text{ ... for Shaping and Planning operations.}$$

$$= \frac{S \times \phi_p}{\text{Chip thickness}} \quad ...$$

where ϕ_p = Plan approach angle or Principal cutting edge angle (Fig. 1.21 (*a*))

For ORTHOGONAL cutting

$$\phi_p = 90°$$

$$\sin \phi_p = 1$$

Therefore, $r = \frac{s}{t} = \frac{\text{feed}}{\text{chip thickness}}$ .... for Turning operation

(ORTHOGONAL Turning)

Uncut Chip thickness (for Turning operation)

$= S \times Sin\ \phi_p$

Note : The following relationships –

Uncut chip thickness = $S \times Sin\ \phi_p$ – For Turning operation

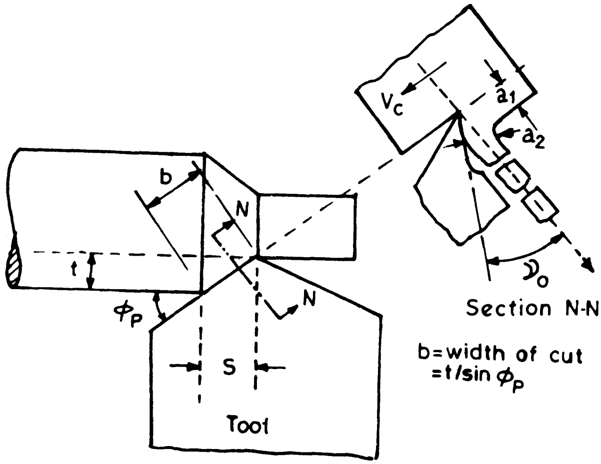


Fig. 1.21 (*a*)

Shear plane Area

$$= \frac{\text{Uncut chip cross–section}}{\text{Sin } \phi}$$

$$= \frac{t \times s}{Sin\ \phi} \quad (\phi = \text{shear angle})$$

$$= \frac{\text{depth of cut} \times \text{feed}}{\sin \phi}$$

Uncut chip cross-section

$= t \times s$

$=$ depth of cut $\times$ feed

Consider the triangles, Δoab, Δoac, Δobc.

ab is perpendicular on ob (*i.e.* rake force of the tool)

oc is a vertical line

$$\angle boc = \alpha$$

$$\angle aod = \phi$$

$$\angle oba = 90^\circ$$

$$\angle cod = 90^\circ$$

$$\angle ocb = 90 - \alpha$$

$\therefore$ $\angle oca = 180 - (90 - \alpha) = 90 + \alpha$

$$\angle aoc = 90^\circ - \phi$$

$\therefore$ $\angle oac = 180 - (90 + \alpha + 90 - \phi) = \phi - \alpha$

$$\therefore \qquad t = ad = oa \,.\, \sin \phi$$

$$t_s = ab = oa \,.\, \cos (\phi - \alpha)$$

Therefore, the chip thickness ratio.

$$r = \frac{oa \,.\, \sin \phi}{oa \cos (\phi - \alpha)} = \frac{\sin \phi}{\cos (\phi - \alpha)} \qquad \text{...(1.12)}$$

$$r = \frac{\sin \phi}{\cos \phi \,.\, \cos \alpha + \sin \phi \,.\, \sin \alpha}$$

$$= \frac{\sin \phi / \cos \phi}{\dfrac{\cos \phi \,.\, \cos \alpha}{\cos \phi} + \dfrac{\sin \phi \,.\, \sin \alpha}{\cos \phi}}$$

$$= \frac{\tan \phi}{\cos \alpha + \tan \phi \,.\, \sin \alpha}$$

or $$\tan \phi = \frac{r \,.\, \cos \alpha}{1 - r \,.\, \sin \alpha} \qquad \text{...(1.13)}$$

1. *Determination of r*

The depth of cut t is known.

The chip thickness t_c can be measured with a micrometer and hence r could be calculated. Since the chip is very rough, the measurement of t_c becomes approximate. Therefore, this method is not used.

2. *The volume of the metal removed*

= Volume of the chip

or $t.\, b.\, l.\, p = t_c.\, b_c.\, l_c.\, \rho_c.$

where, t – depth of the cut

t_c – thickness of chip

b – width of the cut

b_c – width of chip

l – length of the metal cut

l_c – length of the chip produced

ρ – density of the metal cut

ρ_c – density of the chip.

Since $$b = b_c$$

$$\rho = \rho_c$$

$$\therefore \qquad tl = t_c\, l_c$$

or $$\frac{t}{t_c} = \frac{l_c}{l}$$

l – the length of metal cut

$= \pi \,.\, D$ in one revolution

where D – outside diameter of the workpiece

l_c– the chip length is measured after annealing and straightening the chip.

The ratio of l_c to l gives r.

1.8. Velocity Ratio

Fig. 1.22 shows the velocity relation in metal cutting. As the tool advances, the metal gets cut and chip is formed. The chip glides over the rake surface of tool. With the advancement of the tool, the shear plane also moves.

Let

V_c – Velocity of the tool relative to the workpiece. It is called cutting velocity.

V_t – Velocity of the chip (over the tool face) relative to the tool. It is called chip flow velocity.

V_s – Velocity of displacement or formation of the newly cut chip elements, relative to the workpiece along the shear plane. It is called velocity of shear.

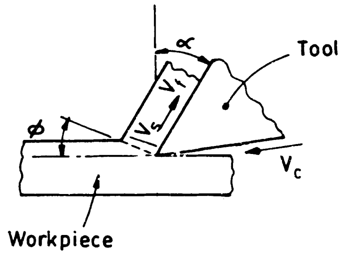


Fig. 1.22

With the help of velocity polygon (Fig. 1.23) we get, the velocity relationship.

In the triangle ABC

$$\angle ABC = 90°$$

$$\angle BAC = \alpha - \phi$$

$$\angle ACB = 90 - (\alpha - \phi)$$

$$\angle ACD = 180 - [90 -`(\alpha - \phi)]$$

$$= 90 + (x - \phi)$$

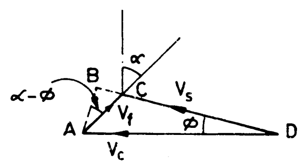


Fig. 1.23

$$\frac{V_f}{\sin\phi} = \frac{V_c}{\sin[90 + (\alpha - \phi)]}$$

or $$V_f = \frac{V_c \,.\, \sin\phi}{\cos(\phi - \alpha)}$$

or $$V_f = r \,.\, V_c \qquad \text{...(1.14)}$$

Since $r = \dfrac{\sin\phi}{si\,(\phi - \alpha)}$ [from Eqn. (1.12)]

Again,

$$\frac{V_s}{\sin(90 - \alpha)} = \frac{V_c}{\sin[90 + (\alpha - \phi)]}$$

or $$V_s = \frac{V_c \,.\, \cos\alpha}{\cos(\phi - \alpha)} \qquad \text{...(1.15)}$$

1.9. Energy of Cutting Process

Let P_c – Horse power (H.P.) in K.W. required for cutting.

P_o – Gross horse power in K.W. of the motor.
P_t – Tare horse power (or horse power consumed while running idle)

$$P_c = P_o - P_t$$

Again, $$P_c = \frac{F_c \times V_c}{60 \times 75 \times 1.36} \text{ K.W.}$$

where, V_c = cutting velocity

or $$F_c = \frac{P_c \times 6120}{V_c} \text{ kg.}$$

(Note here $F_c \,.\, V_c$ = work done kg. m/min)

where, F_c – kg.
V_c – metre/min.
p_c – KW.
$\frac{p_c}{P_o}$ = over all efficiency of machine tool
$= \eta_o$ (let)

1.10. Stress and Strain in the Chip

Let σ_{ga} – average shear stress on the shear plane
A_s – area of the shear plane
ω – width of the chip
t – thickness of the chip.

$$S_s = \frac{F_s}{A_s}, \qquad \text{where, } F_s \text{ – shear force}$$

Since, $$A_s = \frac{\omega \,.\, t}{\sin \phi}$$

Therefore, $$S_s = \frac{F_\phi \,.\, \sin \phi}{\omega \,.\, t}$$

Now $$F_s = F_c \,.\, \cos \phi - F_d \,.\, \sin \phi \text{ [from Eqn. (1.5)]}$$

Therefore $$S_a = \frac{(F_o \,.\, \cos \phi - F_4 \,.\, \sin \psi) \,.\, \sin \phi}{\omega t} \qquad \text{...(1.16)}$$

Also, $$F_s = F_c \,.\, \frac{\cos (\phi \,.\, \eta - \alpha)}{\cos (\eta \; \alpha)}$$

So, $$S_a = F_c \times \frac{\cos (\phi + \eta - \alpha)}{\cos (\eta - \alpha)}, \frac{\sin \phi}{\omega, t}$$

Similarly, average normal stress, σn_a,

$$\sigma n_a = \frac{F_n}{A_s} = \frac{F_c \,.\, \sin \phi + F_d \,.\, \cos \phi}{\omega . t} \qquad \text{...(1.17)}$$

1.10.1. Determination of Strain (Refer Fig. 1.24)

Consider the chip to consist of a large number of elements.

Let thickness of each element be
Δx –
Δs –

displacement suffered by each element after passing through the shear plane.

ε – strain

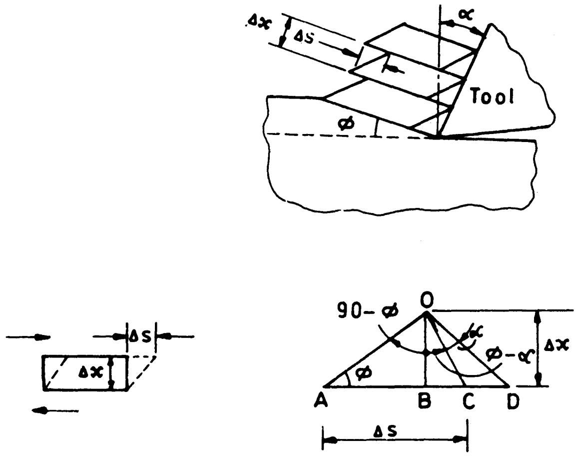


Fig. 1.24

Now,
$$\varepsilon = \frac{\Delta s}{\Delta x}$$
$$= \frac{Ac}{\Delta x}$$
$$= \frac{AB}{\Delta x} + \frac{BC}{\Delta x}$$
$$= \frac{\Delta x \,.\, \tan(\phi - \alpha)}{\Delta x} + \frac{\Delta x \,.\, \tan(90 - \phi)}{\Delta x}$$
$$= \tan(\phi - \alpha) + \cot \phi \qquad \text{...(1.18)}$$
$$= \frac{\sin(\phi - \alpha)}{\cos(\phi - \alpha)} + \frac{\cos \phi}{\sin \phi}$$
$$= \frac{\sin(\phi - \alpha) \,.\, \sin \phi + \cos(\phi - \alpha) \,.\, \cos \phi}{\sin \phi \,.\, \cos(\phi - \alpha)}$$
$$= \frac{\sin^2\phi \,.\, \cos \alpha - \cos \alpha - \cos \phi \,.\, \sin \alpha \sin \phi + \cos^2 \phi \,.\, \cos \alpha + \sin \pi \sin \alpha \cos \pi}{\sin \phi \,.\, \cos(\phi - \alpha)}$$
$$= \frac{\cos \alpha(\sin^2 \phi + \cos^2 \phi)}{\sin \phi \,.\, \cos(\phi - \alpha)}$$
$$= \frac{\cos \alpha}{\sin \phi \,.\, \cos(\phi - \alpha)} \qquad \text{...(1.19)}$$

Since,
$$V_s = V_c \,.\, \frac{\cos \alpha}{\cos(\phi - \alpha)} \qquad \text{from eq. (1.15)}$$

or $$\frac{V_s}{V_c} = \frac{\cos \alpha}{\cos (\phi - \alpha 0}$$

Substituting in the eqn. (1.19). we get

$$e = \frac{V_s}{V_c \,.\, \sin \phi}$$

or $$V_c = \varepsilon \,.\, V_c \,.\, \sin \phi \qquad \text{...(1.20)}$$

The total energy consumed per unit time, in cutting,

$$E = F_c \,.\, V_c$$

The total energy consumed per unit volume of metal removed

$$E_1 = \frac{E}{V_c \,.\, \omega \,.\, t} = \frac{F_c \,.\, V_c.}{V_c \,.\, \omega \,.\, t}$$

$$= \frac{F_c}{\omega \,.\, t} \,.\, \text{kg/mm}^2$$

The total energy is consumed in the following way :

a – Shear energy per unit volume (E_s)

b – Specific friction energy (E_f)

c – Surface energy per unit volume (E_a)

d – Momentum energy per unit volume (E_m).

The surface energy and momentum energy per unit volume are negligible relative to other components and so can be discarded in calculation.

The shear energy per unit volume

$$E_d = \frac{E_s \,.\, V_s}{A_o \,.\, V_c}$$

$$= \frac{V_s}{V_c \,.\, \sin \phi}$$

$$= \frac{S_x \,.\, \cos \alpha}{\sin \phi \cos (\eta - \alpha)} = S_s \,.\, \varepsilon \text{ kg/mm}^2 \qquad \text{...(1.21)}$$

The specific friction energy

$$E_f = \frac{F \,.\, V_f}{A_o \,.\, V_c} \text{ kg/mm}^2 \qquad \text{..(1.22)}$$

Practically all the energy required in metal cutting is consumed in the plastic deformation on the shear plane and the friction between the chip and the tool.

Example 1.6. *The dynamometer recorded the following : feed force 200 kg., cutting force, 300 kg. The rake angle of the tool used was 10°. The chip thickness ratio 0.35, Find.*

(*i*) Shear angle

(*ii*) Shear force

(*iii*) Co-efficient of friction at the chip tool interface and the friction angle

(*iv*) Compressive force at the shear plane.

Solution. Given :

$$F_d = 200 \text{ kg.}$$
$$F_c = 300 \text{ kg,}$$
$$\alpha = 10^\circ$$
$$r = 0.35$$

(*i*)
$$\tan \phi = \frac{r \cos \alpha}{1 - r \cdot \cos \alpha}$$
$$= \frac{0.35 \times \cos 10}{1 - 0.35 \times \cos 10}$$
$$= \frac{0.35 \times 0.985}{1 - 0.35 \times 0.985} = 0.367$$

$\therefore$ $\phi = 20.15^\circ$ **Ans.**

(*ii*) The shear force :

$$F_s = F_c \cdot \cos \phi - F_d \cdot \sin \phi$$

or
$$= 200 \cdot \cos (20.15) - 300. \sin (20.15)$$
$$= 200 \times 0.93 - 300 \times 0.34$$
$$= 84 \text{ kg.}$$ **Ans.**

(*iii*) The normal force :

$$F_n = F_c \cdot \sin \phi + F_d \cos\phi$$

or
$$= 300 \cdot \sin (20.15) + 200 \cdot \cos (20.15)$$
$$= 288 \text{ kg.}$$ **Ans.**

(*iv*) Coefficient of friction μ

$$= \frac{F_c \cdot \tan \alpha + F_d}{F_c - F_d \cdot \tan \alpha}$$
$$= \frac{300 \cdot \tan 10 + 200}{300 - 200 \cdot \tan 10}$$
$$= 0.95$$
$$= 0.95$$
$$= \tan \eta \qquad \text{where } \eta = \text{friction angle}$$

$\therefore$
$$\eta = \tan^{-1} 0.952$$
$$= 43^\circ\ 33'$$ **Ans.**

Example 1.7. *A seamless tube 3 cm outside diameter is reduced in length on a lathe with the help of a single point cutting tool. The cutting speed is 40 m per min. the depth of cut is 0.125 mm. The length of continuous chip, for one revolution of the tube, on measurement comes to be 17.77 cm. The cutting force is 200 kg. and the feed force is 75 kg. The rake angle of the tool is 35 degree.*

Calculate

(i) Coefficient of friction

(ii) Chip thickness ratio

(iii) Shear plane angle

(iv) Velocity of the chip along the tool face

(v) Velocity of shear along the shear plane.

Solution. Since a tube is being reduced in length, therefore, it is a case of orthogonal machining.

(*i*) Coefficient of friction,

$$\mu = \frac{F_c \,.\, \tan \alpha + F_d}{F_c - F_d \,.\, \tan \alpha}$$

$$= \frac{200 \,.\, \tan 35 + 75}{200 - 75 \,.\, \tan 35} = \frac{200 \times 0.75 + 75}{200 - 75 \times 0.75}$$

$$= 1.45 \textbf{ Ans.}$$

(*ii*) Chip thickness ratio, r

$$= \frac{\text{length of chip for one revolution}}{\text{Length of uncut chip for one revolution}}$$

$$= \frac{17.77}{\pi \times \text{Diameter}}$$

$$= \frac{17.77}{\pi \times 3} = 0.53 \textbf{ Ans.}$$

(*iii*) Shear angle ϕ

$$\tan \phi = \frac{r \,.\, \cos \alpha}{1 - r \,.\, \sin \alpha} = \frac{0.53 \times 0.819}{(1 - 0.53 \times 0.574)}$$

$$= 0.624$$

$$\therefore \qquad \phi = 32^\circ \textbf{ Ans.}$$

(*iv*) Velocity of chip along the shear plane

$$V_f = V_c \,.\, \frac{\sin \pi}{\cos (\phi - \propto)} = 40 \times \frac{0.53}{0.999}$$

$$= 21.2 \textbf{ Ans.}$$

(*v*) Velocity of shear along shear plane

$$V_s = V_c \,.\, \frac{\cos \alpha}{\cos (\phi - \alpha)} = 40 \times \frac{0.819}{0.999}$$

$$= 0.328. \textbf{ Ans.}$$

Example 1.8. *During the machining of AISI-1025 steel, with 0—10—6—6—8—90—1 mm ORS shaped tool the following observations were taken :*

Feed : 0.5 mm. / rev.

Depth of cut 2 mm

Cutting speed = 40 m. / min

The shear angle = 20°

The power consumed while machining = 3 kW.

The power consumed while running idle = 0.5 k W

Calculate :

(i) The cutting force

(ii) Chip thickness ratio

(iii) Normal pressure on the chip

(iv) Chip thickness.

Solution. From the tool geometry, we get

Back rake or orthogonal rack (note that in the force calculations we use orthogonal rake only)

$= 10^\circ$ (it is α)

Since the principal cutting edge angle is 90° so the machining is orthogonal.

The net cutting power

= Gross power – Power (consumed) while running idle

$= 3.0 - 0.5 = 2.5$ **kW.**

$$\text{Power} = \frac{F_c \cdot V}{60 \times 75 \times 1.36} \text{ kW.}$$

$$= F_c \cdot V$$

or $$= F_c \frac{2.5 \times 6120}{40}$$

$= 382.5$ kg. **Ans.**

(ii) $$\tan \phi \frac{r. \cos \alpha}{1 - r . \cos \alpha}$$

or $$\tan 20 = \frac{r . \cos 10}{1 - r . \sin 10}$$

or $$0.365 = \frac{r \times 0.985}{1 - r \times 0.174}$$

or $r = 0.34$. **Ans.**

(iii) Normal pressure on the chip

$$= \frac{F_c}{\text{Chip area}} = \frac{F_c}{\omega \times t} = \frac{382.5}{2 \times 0.5}$$

$= 382.5$ kg./mm^2 **Ans.**

(iv) Chip thickness

$$= \frac{\text{Feed}}{\text{Chip thickness ratio}} = \frac{0.5}{0.34}$$

$= 1.4$ mm. **Ans.**

Example 1.9. *A seamless tubing 35 mm outside dia is turned orthogonally on a lathe. The following data are available – Rake angle = 35°, cutting speed 15 m/min, feed = 0.1 mm/ rev. length of continuous chip in one revolution = 50 mm. Cutting force = 200 kg, feed force = 80 kg. Calculate (i). Co-efficient of friction, (ii) Shear Plane angle (iii) velocity of chip along tool face and (iv) chip thickness.*

Sol. $= \text{Co-eff of friction, } \mu = \dfrac{Ft + F_c \tan \propto}{F_c - F_t \tan \propto}$

$$= \frac{80 + 200 \times 0.7}{200 - 80 \times .7} = \frac{210}{144} = 1.525$$

(*ii*) Shear plane angle ϕ-

$$\xi = r = \text{Chip thickness ration} = \frac{t_1}{t_2} = \frac{l_2}{l_1} = \frac{50}{\pi \times D} = \frac{50}{\pi \times 35}$$

$$= 0.525$$

$$\tan \phi = \frac{0.525 \times \cos \alpha}{1 - 525 \times Sin\ \alpha} = \frac{0.525 \times .82}{1 - 0.525 \times .575} = 2.62$$

$$\phi = 32^\circ$$

(*iii*) Chip velocity (V_f)

$$= V_c \times r = 15 \times 0.525 = 7.87 \text{ m/min}$$

(*iv*) Chip - thickness, t_2

$$t_2 = \frac{t_1}{r} = \frac{0.1}{.525} \doteq 0.191 \text{ mm.}$$

Example 1.10 : *In an orthogonal Cutting test on m-s tube of size 150 mm dia and depth of cut 2.1 mm conducted at 90 metres / min and 0.21 mm / rev. feed, the following data were recorded. Cutting force = 125 kg, Feed force = 30 kg.*

Chip thickness = 0.3 mm. Contact length = 0.75 mm. Net horse power = 2 kw., Back - rake = –10°

Compute – Shear strain, Strain energy / vol.

Sol. r = Chip thickness ratio = 0.21 / 0.3 = 0.70

$$\tan \phi = \frac{0.73 \times \cos(-10^\circ)}{1 - 0.70 \sin(-10^\circ)} = \frac{0.70 \times .984}{1 + .7 + 1.73} = \frac{.616}{1.21}$$

$$= .615$$

$$\phi = 3.15^\circ \left[\begin{array}{l} \text{for orthogonal cutting } \phi_p = 90^\circ, \text{ Sin } \phi_p = 1 \\ \because r = \dfrac{s \cdot \sin \phi_p}{t_c} \dfrac{s}{t_c} \end{array} \right]$$

(*ii*) Shear Strain = tan ($\phi - \alpha$) + cot ϕ

= tan (31.5° + 10) + (cot 31.5′ = .884 + 1.631

= 2.515

(*iii*) Strain Energy per unit volume

= Shear stress × Shear strain

$$\text{Shear stress} = \frac{fs}{A_s} = \frac{F_c \cdot \cos \phi - F_t \cdot \sin \phi}{A_s}$$

$$= \frac{125 \times .852 - 30 \times .522}{.21 \times 2.1}$$

feed = 0.21

delth of cut = 2.1

The thickness of tube = 2.1 mm

= depth of cut since nothing is given

$$= \frac{106.5 - 15.66}{.441} \times .0522 = 107.2\text{kg/mm}^2$$

Shear energy = $107.2 \times 2.515 = 270$ kg/mm^2

Example 1.11. *While machining SAE (1040 Steel with a double carbide tool of shape 0-10-8-8--75-1 mm (ORS), the following observations were made - Spindle Speed = 400 rpm.*

Tool feed rate = 80 mm / min, Depth of cut = 2. mm. Work dia = 60mm. Cut-chip thickness = 0.4 mm. Determine (i) Chip thickness ratio (ii) Shear plane anlge (iii) Dynamic Shear Strain (iv) Theortical continuous chip length per min. (v) Chip cross section.

Sol. Tool feed per revolution $= \frac{80}{400} = 0.2$ mm/rev.
$= S$

Depth of cut = 2.5 mm = t

Plan approach angle $\phi_p = 75°$

Rake angle $(\alpha_0) = 10°$

Inclination angle $(\lambda) = 0°$ (orthogonal cutting)

Uncut Chip Thickness = $S \sin \phi_p = 0.2 \times \sin 75$

$$t_1 = 0.9 \times 0.9659 = 0.19318 \text{ mm}$$
$$= 0.193\text{mm}$$

Cut-chip thickness = $t_2 = 0.4$

Chip-thickness ratio $\xi = \frac{t_1}{t_2} = \frac{0.193}{0.4} = 0.482$

$$\text{Tan } \phi = \frac{\xi . \cos \alpha}{1 - \xi \sin \alpha} = \frac{0.482 \times \cos 10}{1 - 0.482 . \sin 10} = 0.5183$$

ϕ = Shear plane angle

$= \tan^{-1} (0.5183) = 27.4°$

Shearing Strain = ϵ

$$= \frac{\xi^2 - 2\xi \sin \propto + 1}{\xi . \cos \alpha}$$

$$= \frac{1.0645}{0.4748} = 2.242$$

or

$$t = \tan(\phi - \propto) + \cot \phi = \tan (17.4) + \cot 27.4°$$
$$= 0.3134 + 1.929 = 2.242$$

Uncut Chip Cross-section $= t \times s$ (depth of cut $\times$ feed)

$= 2.5 \times 0.2 = 0.5$ mm^2

Cut Chip Cross Section $= \dfrac{t \times s}{\xi} = \dfrac{0.5}{0.462} = 1.041 \text{ mm}^2$

Uncut chip length (continuous) per min

$$= \frac{\pi \,.\, D.\, N}{1000} = \frac{3.14 \times 60 \times 400}{1000} = 75.36 \text{ m/min}$$

Cut-Chip length (continuous) per min

$$= 75.36 \times \xi = 75.36 \times 0.482$$

$$= 36.32 \text{ m/min.}$$