1

Mechanics

1.1. FORCE

A force is the cause which changes or tends to change the state of rest or of uniform motion of a body. The force may be in the form of tension, pull, weight, gravity, friction, stress, pressure, action or reaction. A force always has a magnitude, direction, point of application and line of action.

Force is a vector quantity and hence can be represented by a straight line. The extremity of a line denotes the point of application of the force. The direction of the line denotes the direction of the force. The arrow head placed on the line indicates the sense of the force, and the length of the line denotes the magnitude of the force.

When two or more forces act upon a body, it may happen that these forces neutralise the effect of one another and the body continues in its state of rest or of uniform motion in a straight line. Such a system of forces is said to be in equilibrium.

Moment of a Force (Torque)

When a force is applied to a body, it has a tendency to turn the body about some point. The turning tendency of a force about a point is called the moment of the force about that point. It is measured by the product of the force and the perpendicular distance of its line of action from the point. The length of the perpendicular from the point on the line of action of the force is called the arm of the moment. The given point is called the "moment centre" or the "Fulcrum."

The moment of a force about a point = Force $\times$ Arm of the moment.

Couples

Two equal unlike parallel forces whose lines of action are not the same, are said to form a couple. The perpendicular distance between the lines of action of the forces of the couple is called the "arm of the couple". The product of either force of the couple with the arm of the couple is called the "moment of the couple".

Let each force of the couple be P and the perpendicular distance between their lines of action be p. Then, the moment of the couple = P_p. A couple is therefore denoted by P_p.

1.2. PRINCIPLES OF SIMPLE MACHINES

Mechanical Advantage. The ratio of load lifted to the effort applied is known as Mechanical Advantage (MA),

Thus, $$\text{MA} = \frac{\text{Load lifted by a machine (W)}}{\text{Effort applied (P)}}$$

Velocity Ratio. The ratio of the distance moved by the effort to the distance moved by the load is known as Velocity Ratio (V.R.),

Thus, $$\text{V.R.} = \frac{\text{Distance moved by the effort (D)}}{\text{Distance moved by the load (d)}}$$

Work Output. It is the work done by the machine on the load. Therefore, output of a machine = $W \times d$.

Work Input. It is the work done by the effort during the process. Therefore, input of the machine = $P \times D$.

Efficiency. It is the ratio of the output of a machine to its input is known as its efficiency.

Thus, efficiency $$(\eta) = \frac{W \times d}{P \times D}$$

$$= \frac{W/P}{D/d} = \frac{M.A.}{V.R.}$$

When the output and the input of a machine are equal, it is said to be the ideal (or perfect) machine. But in practice output is always less than the input because some of the energy is lost in overcoming friction.

S.I. Unit of Force. One newton, is that force, which, when applied to a body having a mass of one kilogram, gives it an acceleration of one metre per second squared—1 m/s^2. The unit of moment is Newton-metre.

Conventional Unit of Force. One kilogram force, is the force which, when applied to a body having a mass of 1 kg, gives it an acceleration of 9.80665 m/s^2 = standard acceleration of free fall. The unit of moment is kg-metre.

Weight. The weight of a body is that force which, when applied to the body, would give it an acceleration equal to the local acceleration of force fall. Unit of weight is kgf.

The quantity defined here is commonly called the local "gravitational" force on the body. It should be noted that the "weight" arises not only from the gravitational forces which exist at the place where the body is, but also from the centrifugal force.

The effect of atmospheric buoyancy being excluded, the weight defined is the weight in vacuum.

Mass. Unit of mass is *kilogram.* 1 kilogram is the mass of the platinum-iridium cylinder deposited at the International Bureau of Weights and Measures and declared international proto-type of the kilogram by the First General Conference of Weights and Measures.

Force = Mass × Acceleration

$$\boxed{F = m \times a}$$

m = mass

a = acceleration (*e.g.* in m/s^2)

F = force (*e.g.* in kgf),

Example 1. *Mass of a train is 15,000 kilograms. How much tractive force a locomotive should exert so as to accelerate the train starting from rest to reach a velocity of 60 km / h after 1 minute.*

Solution.

$$\text{Acceleration } a = \frac{V}{t} = \frac{60 \text{ km/h}}{1 \text{ min}}$$

$$= \frac{60000}{3600 \times 60} = \frac{5}{18} \text{ m/s}^2$$

Tractive force $F = m \times a$

$$= \frac{15000}{1} \times \frac{5}{18} = 4166.7 \text{ kgf.}$$

Graphical Representation of Forces. Effect of a force depends upon its magnitude, its direction and its point of application.

Forces are represented graphically by pointed arrow.

Point of application of force = Origin of arrow.

Magnitude of force = Length of arrow.

e.g. 1 cm = 10 kgf

5 cm = 50 kgf

Direction of force = Direction of arrow.

Law of Parallelogram of Forces. When two forces F_1 and F_2 act on a body at an angle, their resultant can be found out by representing the forces graphically and by completing the parallelogram. Length and direction of the diagonal between F_1 and F_2 will represent the magnitude and direction of the resultant.

Resolving a Force. By using the parallelogram, a force can be resolved into two (or more) components.

Example 1. If a rope R is to be replaced by two other ropes R_1 and R_2, the forces F_1 and F_2 acting on the individual ropes can be determined by representing graphically the force F in the rope R by the diagonal of the parallelogram and measuring the magnitudes F_1 and F_2 on the adjacent sides of the parallelogram.

Example 2. If a body weighing 150 kgf rests on an inclined plane having an inclination of 30°, the component of the weight acting normal to the inclined plane can be found by resolving the force G caused by the weight of 150 kgf into two components E along the plane and N normal to the plane by completing the parallelogram having its diagonal G.

This problem can also be solved by applying trigonometry.

1.3 DYNAMICS

We know that the 'statics' deals with bodies at rest while the dynamics deals with study by bodies in motion. In the study of dynamics, at times the body is treated as a particle and is defined as an object of small dimension in comparison with its range or flight track during motion. Dynamics is generally divided into two main branches as given under :

(*i*) **Kinematics** is the study of geometry of motion. It is used to relate displacement, velocity, acceleration and time, without reference to the forces causing the motion.

(*ii*) **Kinetics** is the study of the relation existing between the forces acting on a body, the mass of the body, and the motion of the body. It is used to predict the motion caused by given forces or to determine the forces required to produce a given motion.

Uniform Velocity

A body is said to have a uniform velocity if it moves equal distances (s) in equal interval of time. The velocity remains constant. Velocity (v) is the distance (s) travelled in unit time (t).

Linear and Curvilinear Motion

$$\text{Velocity} = \frac{\text{Distance covered}}{\text{Time}} \qquad \boxed{v = \frac{s}{t}}$$

$$\text{Distance} = \text{Velocity} \times \text{Time} \qquad \boxed{s = v \times t}$$

$$\text{Time} = \frac{\text{Distance covered}}{\text{Velocity}} \qquad \boxed{t = \frac{s}{v}}$$

Example 2. Planing operation

$$s = \text{Length of workpiece} = L = 1500 \text{ mm} = \frac{1500}{1000} \text{ m}$$

$$t = \text{Time required per stroke} = 5 \text{ seconds} = \frac{5}{60} \text{ min}$$

$$v = \frac{L}{t} = \frac{\frac{1500}{1000} \text{ m}}{\frac{5}{60} \text{ min}} = \frac{1500 \times 60}{1000 \times 5} = 18 \text{ m/min}$$

Rotary Motion

Peripheral speed = Circumference of workpiece multiplied by revolutions per unit time.

$$\boxed{v = \pi d \times n}$$

$$\text{Diameter} = \frac{\text{Peripheral speed}}{\pi \times \text{r.p.m.}}$$

$$\boxed{d = \frac{v}{\pi \times n}}$$

$$\text{r.p.m.} = \frac{\text{Peripheral speed}}{\text{Circumference of work}}$$

$$\boxed{n = \frac{v}{\pi \times d}}$$

Uniformly Accelerated Motion

The velocity (v) gradually increases per second at a constant rate a. The acceleration (or retardation) is the gradual rate of increase or decrease of the velocity per unit of time.

Let, a body moves from one point to another with a uniform acceleration, whereas,

u = initial velocity,

v = final velocity after time t,

s = distance travelled during time t,

f = acceleration or retardation (also denoted by a).

Then, velocity acquired in t units of time,

$$\boxed{v = u + ft} \qquad ...(i)$$

Distance travelled in t units of time

$$= \text{Average velocity} \times \text{Time}$$

$$S = \left(\frac{v+u}{2}\right) \times t$$

$$= \left(\frac{u+ft+u}{2}\right) \times t$$

$$\therefore \quad S = ut + \frac{1}{2} ft^2 \qquad \text{...(ii)}$$

Now, squaring equation (i), we get

$$v^2 = u^2 + 2u\,ft + f^2t^2$$

$$= u^2 + 2f\left(ut + \frac{1}{2} ft^2\right)$$

$$\therefore \quad v^2 = u^2 + 2fS \qquad \text{...(iii)}$$

Vertical Motion Under Gravity

Neglecting resistance of air, if a body dropped from a height will fall freely with a uniform acceleration g. Then, above equations will be as under :

$$v = u + gt \qquad \text{...(i)}$$

$$S = ut + \frac{1}{2} gt^2 \qquad \text{...(ii)}$$

$$v^2 = u^2 + 2gs \qquad \text{...(iii)}$$

When considering retardation *i.e.* throwing the body vertically upwards, the values of g will be negative.

Let a body is projected vertically upward with initial velocity u, and let h be the height upto which it rises, so that $v = 0$ when the distance travelled is h.

Substituting these values, in the formula, $v^2 + u^2 + 2fS$:

$$o = u^2 - 2gh$$

or $$h = \frac{u^2}{2g} \qquad \text{...(i)}$$

If the body take t second to reach the maximum height, substituting the values in the formula, $v = u + ft$.

$$o = u - gt$$

$$t = \frac{u}{g} \qquad \text{...(ii)}$$

Let a body is dropped with initial velocity zero and let v be its velocity after falling through distance h. Substituting these values in the formula $v^2 = u^2 + 2fS$.

$$v^2 = o + 2gh$$

or $$v = \sqrt{2gh} \qquad ...(iii)$$

Example 3. *If the velocity increases per second by 2 m/s, the acceleration* $a = \frac{2\ m/s}{s} = \frac{2m}{s^2} = 2\ ms^{-2}$

If initial velocity $u = 0$,

Velocity after t seconds $\boxed{v = at}$ m/s

Acceleration, $\boxed{a = \frac{v}{t}}$ m/s^2

Distance covered, $s = \frac{v}{2} \times t = \frac{a \times t}{2} \times t$

$$= \frac{a}{2} \times t^2 \text{ m}$$

Example 4. (*i*) *If $u = 0$, $a = 2\ cm/s^2$, $t = 75\ s$, $v = ?$*

$$r = a \times t = 2 \text{ cm/s}^2 \times 75\ s$$

$$= \frac{2 \text{ cm} \times 75\ s}{s^2} = 150 \text{ cm/s}$$

(*ii*) *If $u = 0$, $v = 8\ m/s$, $t = 4\ s$, $a = ?$, $s = ?$*

$$a = \frac{v}{t} = \frac{8 \text{ m/s}}{4\ s} = 2 \text{ m/s}^2 \text{ or } a = 2 \text{ ms}^{-2}$$

$$s = \frac{v}{2} \times t = \frac{4 \text{ m}}{s} \times 4\ s = 16 \text{ m.}$$

Angular Velocity

Angular velocity of a rotating body is the distance covered per second by a point lying at a distance of 1 metre from the axis of rotation, along the periphery of the circle of 1 metre radius and having its centre on the axis of rotation.

Angular velocity ω (rad/s) $= \frac{3.14 \text{ r.p.m.}}{30}$

$$\boxed{\omega = \frac{\pi \times n}{30} = 0.105 \times n}$$

n = r.p.m., ω = rad/s

ω is approximately $= \frac{1}{10} \times$ r.p.m.

Angular velocity ω (rad/s)

$$= \frac{\text{Peripheral speed } v \text{ (m/s)}}{\text{Radius } r \text{ (m)}}$$

$$\boxed{\omega = \frac{v}{r}}$$

v = m/s

r = m

ω = rad/s

Example 5. *n = 750 r.p.m., d = 300 mm = 0.3 m, ω = ?, v = ?*

Solution.

$$\omega = \frac{\pi \times n}{30} = \frac{3.142 \times 750}{30} = 78.5 \text{ rad/s},$$

$$v = \omega \times r = 78.5 \times 0.15 = 11.78 \text{ m/s}.$$

1.4 WORK

When a body is displaced due to the action of the force, then force is said to do the work. If the force remains constant in magnitude and direction, the work done by it is measured by the product of the force and the displacement in the direction of the force.

In mechanics, work is defined as a product of a force acting on a body and the distance through which the force moves the body along its direction.

Work = Force × Distance

Work A = Force F × Distance covered s (s is measured in the direction of force)

$\boxed{A = F \times s}$ A, the work done is expressed in kgfm.

Example 6. *When a body weighing 20 kgf is lifted to a height s = 5 metres or alternatively a force of 20 kgf applied uniformly to a body displaces it by 5 metres, the work done.*

$$A = F \times s = 20 \textit{ kgf} \times 5 \textit{ m} = 100 \text{ kgf-m}.$$

Solution.

The same amount of work is done while lifting a load along an inclined plane, as would have been done in lifting the same load vertically upwards.

Example 7. *How much work is done in moving a vehicle weighing 4000 kgf, through a distance of 0.8 km, along an inclined plane having a gradient 1 : 100 ?*

Solution.

$$G = 4000 \text{ kgf}, \ h = \frac{0.8 \times 1000}{100} = 8 \text{ m}$$

Work done $A = G \times h = 4000 \text{ kgf} \times 8 \text{ m} = 32{,}000$ kgf-m.

Force F in the upward direction of vehicle along the inclined plane can be found out by using the parallelogram of forces.

G can be resolved into two components G_1 and G_2. $F = G_2$, but acts in opposite direction.

$$\therefore \quad F = G_2 = G \times \sin \alpha = 4000 \times \frac{1}{100} = 40 \text{ kgf},$$

or

$$A = F \times s = 40 \text{ kgf} \times 800 \text{ m} = 32{,}000 \text{ kgf-m}.$$

Power

Power is the rate of doing the work. An agent which performs 550 ft-lb per second is said to be working at the rate of 1 horse power (written as H.P., and termed as British Horse Power).

Power P = Work A per unit of time.

If A is in kgf-m, t in sec, then power is expressed in kgf-m/s,

thus $$\boxed{P = \frac{A}{t}}$$

If F is in kgf-m, s in m, t in sec, then P is expressed in kgf-m/s,

thus $$\boxed{P = \frac{F \times s}{t}}$$

If F is in kgf, v in m/s, P is expressed in kgfm/s,

thus $$\boxed{P = F \times v}$$

Power P = Force F × Velocity v.

Example 8. *A load of 20 kgf lifted vertically upwards to a height $h = 5$ m in 4 seconds corresponds to a power $P = 25$ kgfm / s.*

Solution.

$$P = \frac{A}{t} = \frac{F \times s}{t} = F \times v$$

$$= \frac{20 \text{ kgf} \times 5}{4} = 25 \text{ kgf m/s}$$

75 kgf m/s = 1 metric horse power = 0.736 kW

$$\boxed{\text{HP} = \frac{F \times v}{75}}$$

102 kgf m/s = 1 kW = 1.36 metric H.P.

$$\text{kW} = \frac{F \times v}{102}$$

Example 9. *What should be the rating of a motor for lifting a load of 1200 kgf to a height of 10 metres in 50 seconds ?*

Solution. $A = F \times s = 1200 \text{ kgf} \times 10 \text{ m}$

$= 12{,}000$ kgf-m

$$P = \frac{A}{t} \; \frac{12{,}000 \text{ kgfm}}{50\ s} = 240 \text{ kgfm/s}$$

$$= \frac{240}{102}\ kW = 2.35 \text{ kW approx.}$$

Potential and Kinetic Energy

Energy of a body is the capacity for doing work.

(*i*) Potential energy E_p = Work, a body can do as a result of its position.

(*ii*) Kinetic energy E_k = Work, a body can do as a result of its velocity.

Potential energy $\boxed{E_p = G \times h} = m \times g \times h$

where m = Mass.

g = Local acceleration of free fall.

h = Height in m,

Then E_p is expressed in kgfm.

Kinetic energy $\boxed{E_k = \dfrac{m \times v^2}{2}}$

where $m = \dfrac{G \text{ (kgf)}}{g \text{ (m/s}^2)}$

$v^2 = 2 \times g \times h$

Then E_k is expressed in kgfm.

Example 10. *Weight G of drop forge hammer = 200 kgf,*

Height h of fall = 2.5 m,

then potential energy $E_p = G \times h = 200 \text{ kgf} \times 2.5 \text{ m}$

$= 500$ kgfm.

Kinetic energy attained by the hammer after falling down through the distance of 2.5 m (*i.e.* at the time of striking the piece to be forged) must also be the same as its original potential energy.

Example 11. $E_k = \frac{mv^2}{2}$

$$= \frac{200 \text{ kgf} \times 2.5\ m \times g \times 81 \text{ m/s}^2 \times 2}{g \times 81\ m/s^2 \times 2} = 500 \text{ kgf-m}$$

g = Local acceleration of free fall, taken as 9.81 m/s^2.

Efficiency

While transmitting power or changing energy from one form to another, part of the power or energy is lost due to friction, radiation, etc. Hence the output is always less than input.

$$\text{Efficiency } \eta = \frac{\text{Output work}}{\text{Input work}} = \frac{\text{Output power}}{\text{Input power}}$$

$$\text{Percentage } \eta = \frac{\text{Output}}{\text{Input}} \times 100$$

The more η approaches unity better the machine.

Example 12. *200 revolutions of the winch handle raise a load of 500 kgf upto a height of 20 m. Force applied on winch handle is 25 kgf and radial length of handle is 0.4 m. Find the efficiency.*

Solution. Output work = $G \times h$ = 500 kgf × 20 m

= 10,000 kgfm

Input work $F \times s = F \times \pi \times d \times 200$

= 25 kgf × π × 0.8 × 200

= 12,560 kgfm

$$\eta = \frac{\text{Output work}}{\text{Input work}} = \frac{10{,}000 \text{ kgf}}{12{,}560 \text{ kgf}} = 0.7961$$

= 79.61%.

Example 13. *Calculate the cutting force required while turning a work piece of 250 mm dia. at 25 r.p.m. using 5 kW drive motor and assuming efficiency of drive as 75%.*

Solution. $\eta = \frac{\text{Output power}}{\text{Input power}}$

Output = η × Input = 0.75 × 5 kW = 3.75 kW

Output = $F \times v$,

$$\therefore \quad F = \frac{\text{Output}}{v} = \frac{3.75 \text{ kW}}{\pi d n} \times s$$

$$\therefore \qquad F = \frac{3.75 \times 102 \text{ kgf} \times \text{m/s}}{\pi \times 0.25 \text{ m} \times \frac{25}{60}} = 1168.36 \text{ kgf}$$

Table 1.1
Efficiency of Various Machines

S. No.	*Name of the Machine*	*Efficiency*
1.	Electric motors	0.8 to 0.9
2.	I.C. engines	0.25 to 0.30
3.	Steam engines	0.15 to 0.2
4.	Steam turbines	0.18 to 0.22
5.	Water turbines	0.85 to 0.9

1.5 FRICTION

The surfaces which appear perfectly smooth to the naked eye, on examination under a microscope are found to consist of raised portion and the depressions. If an attempt is made to move one body over another, a certain resistance to motion is experienced ; this resistance which is due to the roughness of the surface is called as “friction” and the force exerted is called the “force of friction”.

If a little lubricant is added to the surface, this fills up the microscopic depressions in the apparently smooth surface and thus makes the surface more smooth and reduces the friction between the surfaces in contact.

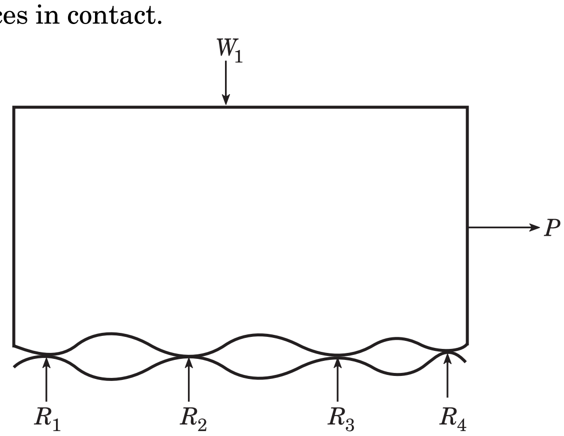


Fig. 1.1. Magnified view of contact surfaces.

Laws of Dry Friction

Dry friction is the force of friction between unlubricated surfaces. It is governed by the following laws :

1. The direction of the force of friction is opposite to that in which it is moving or has the tendency to move relative to the other.

2. So long as there is no relative motion between the bodies, the force of friction between them is a self-adjusting force, only that much comes into play as is just sufficient to prevent motion.
3. There is a limit to the magnitude of static friction. The maximum value F_s is called limiting static friction.
4. When maximum friction is acting, the angle between the resultant reaction and the normal to the surface is called the angle of friction.
5. The coefficient of friction depends on nature of surfaces and does not depend on area of contact.

Sliding Friction. The frictional resistance F_f depends upon material of the body and sliding surface, its surface finish and the weight of the sliding body. For a body sliding uniformly on a horizontal plane, the driving force F equals the frictional resistance F_f. The ratio of the frictional force F_f to the normal reaction R_N (*i.e.,* the reaction caused due to the weight of the body at right angles to the supporting plane XX). This ratio is known as "Coefficient of friction μ".

$$\mu = \frac{F_f}{R_N}$$

where F_f = Force of friction kgf = F

R_N = Normal reaction, acting at right angles to supporting plane in kgf.

μ = Coefficient of friction.

$$F = F_f = \mu \times R_N$$

$$\mu = \tan \alpha = \frac{h}{l}$$

where α = Angle of repose

= Angle of inclination of the inclined plane when the body just starts sliding down.

Example 14. *Find the driving force F, which will just make the body weighing 50 kgf to slide on a horizontal plane, assuming* $\mu = 0.1$.

Solution. $F = F_f = \mu \times R_N = 0.1 \times 50$ kgf

= 55 kgf.

Rolling Friction. In rolling, the contact of roller with the surface on which the roller rolls is theoretically a point or line contact. However in practice, due to deformation of roller or surface or both, the contact is not a line or point contact.

The weight G will act through the centre of the roller at right angles to contact surface, and will have a lever arm = f, where f is the coefficient of rolling friction (For steel wheel upon steel rail, $f = 5$ mm).

By Law of Moments,

$$\boxed{F_f = F = \frac{f}{r} \times R_N = \frac{f}{r} \times G}$$

where r = Radius of roller in cm,

F_f = Frictional resistance

= Driving force, F

G = Weight

= Normal reaction, R_N.

Pulley

By using a machine, we do not save work. We can only lift or move a bigger load by applying a smaller force. But if we save in force, this force has to be applied for a larger distance. Thus, neglecting the losses, the input work done by force = output work done by load.

In the figures given below :

F = Force and Q = Load.

Fixed pulley

$$F = Q$$

Movable pulley (Movable block)

$$F = \frac{1}{2} Q$$

Set of pulleys

$$F = \frac{Q}{n}$$

Differential pulley block

$$F = Q\left(\frac{R—r}{2R}\right)$$

where n = number of blocks

Belt drives through pulley have been dealt in a separate chapter on "Transmission of Power from Prime Mover".

Lever

It is the simplest form of a machine, and consists of a straight rigid bar or a rod of any strong material. It is made to turn about a fixed point and is used to raise loads. The point where it is turned is called the "fulcrum".

In lever systems, Mechanical Advantage = Leverage = $\frac{W}{P}$

where, W is the load lifted by applying the effort P.

The distance of the effort and of the load from the fulcrum are D (or a) and d (or b), which are called effort arm (or lever arm) and load arm respectively.

First Class Lever — Second Class Lever

Bell Crank Lever — Wheel and Axle, Winch

According to Law of Moments,

$$\boxed{F \times a = Q \times b}$$

Force $F \times$ lever arm a = Load $Q \times$ Load arm b

Moment of force = Moment of load.

Sum of clockwise moments = Sum of anticlockwise moments.

Lever arm is the perpendicular distance of the fulcrum from the line of action of the force acting on the lever.

Inclined Plane and Wedge

$$F \times l = Q \times h \qquad F_H \times l = Q \times h \qquad F \times l = Q \times h$$

$$\therefore \boxed{F = \frac{Q \times h}{l}} \qquad \therefore \boxed{F_H = \frac{Q \times h}{l}} \qquad \therefore \boxed{F = \frac{Q \times h}{l}}$$

Equilibrium Situation

Forces acting on a body at rest may be in equilibrium or may give a resultant force or moment. For determining which event obtains the forces indicated on the free body diagram, are resolved into components in three mutually perpendicular directions x, y and z. Components are summed giving ΣF_x, ΣF_y and ΣF_z. And the moments of the components about some convenient point are also summed giving ΣM_x, ΣM_y and ΣM_z.

The body is in equilibrium.

$$\left.\begin{array}{ll} \Sigma F_x = 0 & \Sigma M_x = 0 \\ \Sigma F_y = 0 & \Sigma M_y = 0 \\ \Sigma F_z = 0 & \Sigma M_z = 0 \end{array}\right\} \quad ...(1)$$

If Eqn. (1) does not satisfy, then the magnitude of the resultant force

$$R = \left[(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2\right]^{1/2}$$

and the resultant moment

$$M = \left[(\Sigma M_x)^2 + (\Sigma M_y)^2 + (\Sigma M_z)^2\right]^{1/2}$$

When only coplanar forces are involved, the problem is reduced to one of two dimensions and we may use an alternative graphical solution by using vector addition of forces and moments.

When concurrent forces are involved, the sum of moments is always zero and only the forces require to be investigated.

Above methods give solution to situations which are statically determinate, *i.e.* situations having minimum number of constraints for supporting an equilibrium position.

For statically indeterminate situations, other texts may be consulted.

1.6. NEWTON'S LAWS OF MOTION

1. A particle remains at rest or continues to move in a straight line with uniform velocity until acted upon by an external force.
2. Sum of components of external forces ; which act on a particle in a given direction, is equal to the rate of change of momentum in that direction.
3. When particles interact, the forces of action and reaction are equal in magnitude and line of action but opposite in direction.

Centre of gravity (G)

It is the point at which the distributed weight of a system acts as a single force. Position of centre of gravity is found by analysing or experiment. In the former case, moments are taken about three mutually perpendicular axes giving coordinates of G,

$$\bar{x} = \frac{\Sigma\,\delta_{mg}\,.\,x}{\Sigma\,\delta_{mg}},$$

$$\bar{y} = \frac{\Sigma_{mg}\,.\,y}{\Sigma\,\delta_{mg}}, \text{ and}$$

$$\bar{z} = \frac{\Sigma\,\delta_{mg}\,.\,z}{\Sigma\,\delta_{mg}}$$

where δ_m = an elementary mass at distance x, y or z from the respective axes. Table 1.2 gives some centre of gravity data.

Table 1.2. Centres of Gravity and Moments of Inertia

Figure	*G*	*I*
Plane circular sector	$\bar{x} = \frac{2r \sin\theta}{3\theta}$ (Centroid)	$I_{xx} = \frac{r^4}{4}\left(\theta - \frac{\sin 2\theta}{2}\right)$ $I_{yy} = \frac{r^4}{4}\left(\theta - \frac{\sin 2\theta}{2}\right)$ (Second moment of area)
Cylinder, radius *r*	—	$I_{xx} = \frac{mr^2}{4} + \frac{ml^2}{12}$ $I_{zz} = \frac{mr^2}{2}$
Half cylinder, radius *r*	$\bar{x} = \frac{4r}{3\pi}$	$I_{xx} = I_{yy}$ $= \frac{mnr^2}{4} + \frac{ml^2}{12}$ $I_{zz} = \frac{mr^2}{2}$
Rectangular parallelopiped	—	$I_{xx} = \frac{m}{12}(a^2 + l^2)$
Sphere	—	$I = \frac{2mr^2}{5}$
Hemisphere, radius *r*	$\bar{x} = \frac{3r}{8}$	$I_{xx} = \frac{2mr^2}{5}$
Circular cone, basic radius *r*	$\bar{x} = \frac{1}{4}$	$I_{xx} = \frac{3mr^2}{10}$ $I_{yy} = \frac{3mr^2}{20} + \frac{ml^2}{10}$
Slender rod	—	$I_{yy} = \frac{ml^2}{12}$

Moment of Inertia (I) about an axis *xx*, *i.e.,*

$$I_{xx} = \Sigma\, \delta_{mx}^2 = mk_{xx}^2,$$

where x = perpendicular distance of the elementary mass m from axis xx and k_{xx} = radius of gyration about axis xx.

Angular velocity (α)**.** It is the rate of change of angular distance (θ) with time, $\omega = \frac{d\theta}{dt} = \dot{\theta}$.

$$\text{Angular velocity} = \frac{\text{Linear velocity}}{\text{Radius of rotation}} = \frac{v}{x} = \omega$$

Angular acceleration (ω)**.** It is the rate of change of angular velocity with time,

$$\alpha = \frac{d\omega}{dt} = \frac{\theta d^2}{dt} = \ddot{\theta}.$$

$$\text{Angular acceleration} = \frac{\text{Linear acceleration}}{\text{Radius of rotation}}$$

i.e. $\alpha = \frac{a}{x}.$

Torque (T). It is the moment of force about the axis of rotation. In Fig. 1.2, the force required to accelerate $\delta_m = \alpha_x \delta_m$. Thus, torque to accelerate the body, $T = \Sigma_x (\alpha \cdot x \delta_m) = x\Sigma (\delta_m x^2)$ or $T = I_0 \alpha$.

A torque may also be due to couple, *i.e.* two forces equal in magnitude but opposite in direction having separate points of application.

Moment of momentum or angular momentum of a body. It is the moment of linear momentum of body about axis of rotation, 0.

In Fig. 1.2. Angular momentum $= \Sigma\delta_m \omega_x \cdot x = \Sigma\delta \omega x^2 = \omega I_0$. And by parallel axes theorem,

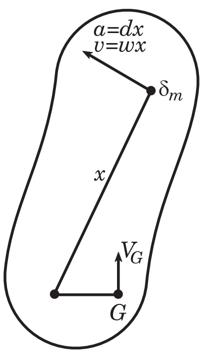


Fig. 1.2

$$I_0 = I_G + m \times OG^2$$

so that angular momentum $= \omega (I_G + M \times OG^2)$

Angular momentum of a system remains constant unless the system is acted on by an external force.

Angular impulse. It is the product of torque by time, *i.e.*

Angular impulse $= Tt = I\alpha.t = I(\omega_2 - \omega_1)$.

Angular kinetic energy about an axis

$$= \Sigma \frac{1}{2} \delta m (\omega x^2) = \frac{1}{2} I_0 \omega^2$$

But, $I_0 = I_G + m \times OG^2$

so that angular kinetic energy

$$= \frac{1}{2} \omega^2 (I_G + m \times OG^2) = \frac{1}{2} I_G \omega^2 + \frac{1}{2} mvG^2$$

Work due to torque. It is the product of torque by angular distance.

Power due to torque. It is the rate of work ;

$$P = \frac{T\theta}{t} = T\omega$$

Friction. Motion of one body relative to another body with which it is in contact is accompanied by a friction force. This force acts tangentially to the surfaces in contact in a direction opposing motion.

Value of force, $F = \mu R$,

where R = normal reaction between the surfaces and μ = coefficient of friction.

Value of μ depends on the nature of surfaces which are in contact.

1.7. MOTION IN TWO DIMENSIONS

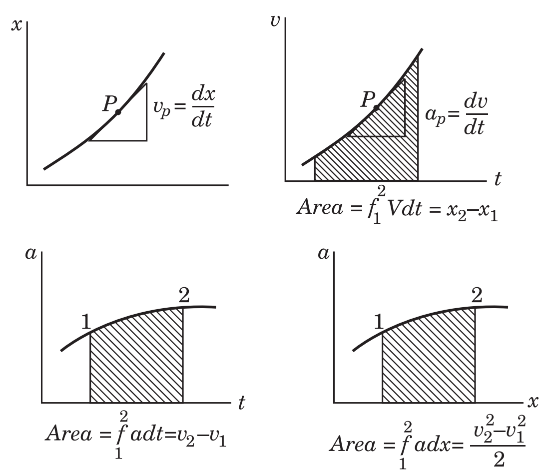
Basic equations of motion contained in above concepts where $v = \dot{x}$, $a = \ddot{x} = \dot{v}$, $\omega = \dot{\theta}$, $\alpha = \ddot{\theta} = \dot{\omega}$.

If formal equations to displacement, velocity or acceleration are available, the above relations may be, integrated or differentiated or graphically if experimental data evaluated. Examination of several cases :

(1) ***Constant Acceleration.***

Linear Motion	Angular Motion
Eqns. (2) below are obtained.	$\theta = \omega_1 t + \frac{1}{2} \alpha t^2$
Suffix 1 refers to values when $t = 0$.	$\omega_2 = \omega_1 + \alpha t$
Suffix 2 refers to values when $t = t$	$\omega_2^2 = \omega_1^2 + 2\alpha\theta$

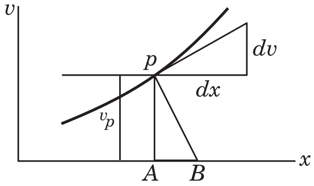
...(3)



(*b*)

Fig. 1.2 (*a*)

$$\left\{\begin{array}{l} x = v_1\, t + \frac{1}{2}\, at^2 \\ v_2 = v_1 + at \\ v_2^2 = v_1^2 + 2ax \end{array}\right\} \quad ...(2)$$



$AB = v_p \dfrac{dv}{dx} = a_p$

Fig. 1.3

2. ***Linear or Angular Motion with Variable Acceleration.*** Graphs of above figure indicate relations which apply to linear case.

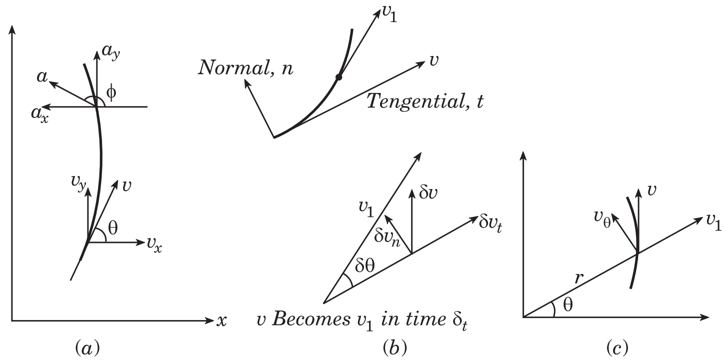
3. ***Curvilinear Motion.*** In which there are linear as well as angular components, may be described by different methods :

(*i*) Method *I* describes velocity and acceleration of a point by *x* and *y* components along two axes at right angles Fig. 1.4 (*a*) ;

$$v_x = v \cos\theta,\ v_y = v \sin\theta,$$

$$a_x = a \cos\phi,\ a_y = a \sin\phi,$$

(*ii*) Method II describes velocity and acceleration of a point by normal and tangential components at the point Fig. 1.4 (*b*) ;



(*a*) (*b*) (*c*)

Fig. 1.4

$$\left.\begin{array}{r} v_t = v = r\dot{\theta} = r\omega \\ a_t = r\ddot{\theta} + \dot{r}\dot{\theta} = r\alpha + \dot{r}\,\omega \\ v_n = 0,\ a_n = v\dot{\theta} = r\dot{\theta}^2 = r\omega^2 \end{array}\right\} \quad ...(4)$$

(*iii*) Method III describes velocity and acceleration of a point in polar coordinates *r* and θ, Fig. 1.4 (*c*) ;

$$\left.\begin{aligned} v_r &= \dot{r},\ a_r = \ddot{r} - r\,\dot{\theta}^2 \\ v_\theta &= r\dot{\theta},\ a_\theta = r\ddot{\theta} + 2\dot{r}\,\dot{\theta} \end{aligned}\right\} \quad ...(5)$$

4. ***Motion in a circle.*** It is a special case of curvilinear motion in which r is constant and above methods may be applied. If we consider acceleration $\omega^2 r$ towards the centre, there should be a force in this direction to maintain the motion. This is centripetal force. Outward centrifugal reaction to this force is important in stress analysis of rotating bodies.

5. ***Velocity and Acceleration in Mechanisms.*** For determining the motion of points in mechanisms, it is usually convenient to draw velocity and acceleration vector diagrams using the principles indicated above.

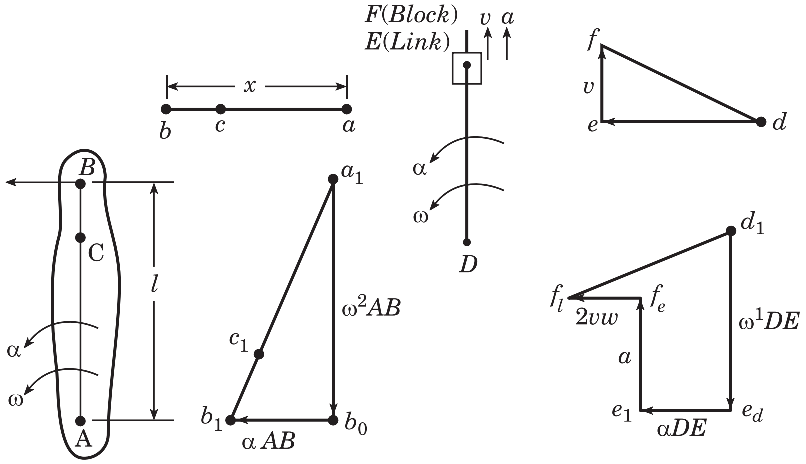


Fig. 1.5.

Velocities. In a link of fixed length, r is constant and velocity of one point relative to another on the link will be perpendicular to the line joining the points. Refer to Fig. 1.5 (a),

$$\omega = \frac{v_{ba}}{AB}$$

$$= \frac{ab}{AB} = \frac{x}{l}.$$

Also $\quad \dfrac{AC}{AB} = \dfrac{ac}{ab}.$

When a block slides on a rotating link, the velocity of the block v relative to the link is along the link. Velocity of the block relative to the fixed point D is df (Fig. 1.5).

Accelerations. In a link of fixed length Fig. 1.5, B will have a centripetal acceleration $\omega^2 AB = a_1 b_a$ towards A and a tangential acceleration $\alpha. AB$. Acceleration of B relative to A is $a_1 b_1$.

When a block slides along a rotating link, it may have a linear acceleration a along the link relative to the link. Whether a is + ve, – ve or zero, there will also be a tangential acceleration $2\, v\omega$. This is called "Coriolis acceleration" and its direction is determined by rotating the sliding velocity vector through 90° in the direction of the link angular velocity ω. Fig. 1.5 (b) indicates four acceleration components of sliding block F.

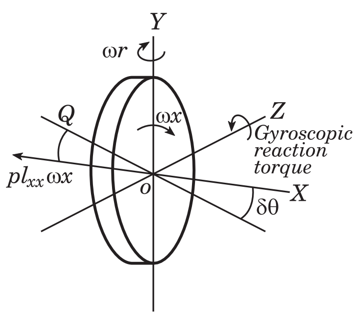


Fig. 1.6. A rotor with moment of inertia I_{xx} about axis OX.

6. ***Gyroscopic Effects.*** Fig. 1.6 indicates a rotor with moment of inertia I_{xx} about axis OX. Rotor has angular velocity ω_x about OX which itself rotates about axis OY (at right angles to OX with angular velocity ω_y.

Due to change (in direction) of angular momentum vector OP to OQ in time δt, there should be a torque which may be represented in direction by the vector PQ. Magnitude of the torque is PQ divided by δ_0. Gyroscopic reaction torque T is opposite in sign to this torque and is supported by the bearings of the rotor shaft,

$$T = I_{xz}\ \omega_x \frac{\delta\theta}{\delta_\theta}$$

$$= I_{xx}\ \omega_x w_y \qquad \text{...(6)}$$

Gyroscopic reaction torque occurs in all situations where a rotor is precessed *viz.,* automobile wheels on corners, aircraft turbine rotors in turns or loops, ship turbine rotors in pitch or turns.

Where precession is not *convenient* in OZX plane, angular momentum vectors are resolved. Gyroscopic effects are useful in design of missile and spacecrafts and in navigation.

Balancing

Rotating and reciprocating parts of machines at high rotational speeds must be balanced so as to avoid excessive stresses and vibration.

Rotating Masses

Forces required to be balanced are the centrifugal reactions to the circular motion. If all the rotating masses are in same plane (Fig.

1.6), balancing is achieved by placing a single extra mass in the plane. If this is inconvenient, more than one mass is used. These masses are placed at the greatest possible radius so as to reduce their volumes.

Conditions for balancing such two dimensional problems are :

$$\Sigma F_x = \Sigma mw^2 r \sin \theta = 0,$$

$$\Sigma F_y = \Sigma mw^2 r \cos \theta = 0,$$

where, w = angular velocity of the shaft,

r = radius of the mass m,

θ = its angular position relative to the axis y.

Axes x and y are perpendicular in the plane in question. Eqs. may be solved graphically with a single force polygon.

If all the masses are not in the same plane, the centrifugal reactions will have moments about any plane which should also be balanced. Two masses in different planes are to achieve complete balancing. Convenient planes are chosen for these masses and moments taken to give conditions for balancing ;

$$F_x = \Sigma mw^2 r \sin \theta = 0$$

$$F_y = \Sigma mw^2 r \cos \theta = 0$$

$$M_x = \Sigma mw^2 r \sin \theta \,.\, a = 0$$

$$M_y = \Sigma mw^2 r \cos \theta \,.\, a = 0$$

where a = distance of mass m from the chosen plane.

These Eqns. can be solved graphically by a moment or couple polygon and a force polygon.

Questions

1. Define Force. What is moment of a force ?
2. Under what conditions forces of different magnitudes produce equal moment with respect to one and the same centre.
3. Define Torque and Couples.
4. Distinguish between mass and weight.
5. What is graphical representation of forces.
6. Define dynamics. Explain briefly its main branches.
7. Distinguish between linear velocity and angular velocity.
8. Define work and friction.
9. Distinguish between sliding friction and rolling friction.

10. Explain vector and scalar quantities and give two examples of each.
11. (*a*) What are the laws of friction ?
    (*b*) Explain briefly coefficient of friction, angle of friction and angle of repose.
12. Distinguish between traction and friction.
13. What is a friction ? How it effects the performance of a machine ?
14. (*a*) Lever is the simplest machine. Justify.
    (*b*) What are different kinds of levers.
15. What do you understand by machine, velocity ratio, mechanical advantage and efficiency of machine ? What is the relationship between them ?
16. Define the following.
    Linear velocity and linear acceleration. Angular velocity and angular acceleration.
17. Explain the following.
    (*i*) Friction, (*ii*) Centrifugal force, and (*iii*) Coefficient of friction.
18. Define the following.
    Speed, velocity, uniform velocity, acceleration and retardation.
19. Distinguish between angular and linear acceleration.
20. Define Work, Power, and energy.
21. Explain friction, laws of dry friction.
22. Write a short note on equilibrium situation.
23. What are the Newton's laws of motion.
24. Write short notes on centre of gravity, moment of Inertia.