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Particle Technology

1.1. Particle Size and Shape

There is a reputed saying that a liquid is a hen-pecked husband whereas a solid particle is a male chauvinist pig. The hint behind this statement is obvious. A liquid does not have any definite shape, it takes the shape of its container; whereas every solid particle has a specific shape of its own. Thus, whenever we handle any system involving solid particles, which we encounter quite often in chemical process industries, for defining the system, first we have to specify the size and shape of the particles involved. The task however, is not an easy one.

If the particle conforms itself to any of the standard configurations we are familiar with such as spherical, cubical, cylindrical etc., then its size can be easily specified. For example, the size of a spherical particle is nothing but its diameter and that of a cubical particle the length of its side. However, many of the particles commonly encountered in industrial practices do not conform to any of these standard configurations. We can call them, in general, irregular particles. How to define the size of such an irregular particle is now the question before us.

During the early stages of development of particle technology, a number of authors proposed empirical definitions to particle size. For example Martin (1931) defined the size of an irregular particle as the length of the line bisecting the maximum cross-sectional area of the particle. Similarly, Feret (1929) defined particle size as the distance between the two most extreme points on the particle surface. The limitations of such definitions are obvious. For instance, if the distance between the farthest edges on the particle surface remains the same but the rest of its configuration changes, its Feret's diameter shall

remain unaltered. Obviously, such a definition cannot describe the actual size or shape of an irregular particle.

It was in the later years, the latest system of defining particle size by comparison with a standard configuration (normally by comparing with a spherical particle) got developed. Thus, we utilise what is called the equivalent size or equivalent diameter of an irregular particle which can be broadly defined as *the size of a spherical particle having the same controlling characteristic as the particle under consideration.* Naturally, to apply this definition, we must first specify what this "controlling characteristic" is. This depends on the system and the process in which the particle is involved. For example, for catalyst particles, the surface area is the most controlling parameter. Therefore, for defining the size of a catalyst particle we can use the surface diameter (d_s) which will thus be defined as the diameter of a spherical particle having the same surface area as the particle. If S_p is the surface area of the particle, then

$$S_p = \pi d_s^2$$

or,

$$d_s = \sqrt{S_p/\pi} \qquad ...(1.1.1)$$

The gravitational free settling velocity of a particle in a liquid is very much controlled by the mass of the particle (or, for a given density, its volume). We can therefore define the particle size for such a case by the volumetric diameter (d_v) which is once again defined as the diameter of a spherical particle having the same volume as the particle under consideration. Thus, if V_p is the volume of the particle,

$$V_p = \pi d_v^3/6$$

or,

$$d_v = (6V_p/\pi)^{\frac{1}{3}} \qquad ...(1.1.2)$$

The dynamics of gas bubbles in a liquid or that of liquid drops in a liquid or gas depend not only on the bubble or drop volume but also on the interfacial tension at the gas-liquid or liquid-liquid interface. Thus, both the volume as well as the surface area of the bubble or drop are controlling parameters here. In such cases, the bubble size or drop size is defined using the volume-surface diameter or more commonly called the Sauter diameter (d_{vs}). This is accordingly defined as the diameter of a spherical particle having the same specific surface (surface area per unit volume) as the particle (bubble or drop) under consideration. Thus,

$$s_p = \frac{(\pi\, d_{vs}^2)}{(\pi\, d_{vs}^3/6)} = \left(\frac{6}{d_{vs}}\right)$$

or, $$d_{vs} = (6/s_p) \quad ...(1.1.3)$$

where s_p is the specific surface (surface area per unit volume) of the particle (bubble or drop).

Thus, once the controlling characteristic is specified, we can define the size of any irregular particle using the above methodology. Another popularly employed definition of particle size is the screen size or the screen average size, d_{avg}. This in fact is the aperture size of a standard screen through which the particle just passes or more correctly, the arithmetic average of the aperture sizes of two successive standard screens, one of which lets the particle pass through whereas the other retains it. We shall be discussing about screening and particle size analysis using screening in more detail in the subsequent sections.

Once we have defined the particle size, let us now discuss how the particle shape can be defined. Again, the exact shape of an irregular particle is difficult to specify. One of the methods of defining particle shape is by using the term *sphericity* (ψ_s). It is defined as the ratio of the surface area of a spherical particle having the same volume as the particle to the surface area of the particle. Since the diameter of a spherical particle having the same volume as the particle is d_v,

$$\psi_s = \frac{\pi d_v^2}{S_p} \quad ...(1.1.4)$$

It must be noted that since sphericity compares the surface area of the particle to that of the equivalent spherical particle, it defines only the particle shape and is independent of particle size. Sphericity of a spherical particle is obviously 1.0. The sphericity of a cubical particle of side a can be determine as follows :

From equation (1.1.2), the volumetric diameter of the cubical particle,

$$d_v = \left[\frac{6V_p}{\pi}\right]^{1/3} = \left(\frac{6a^3}{\pi}\right)^{1/3}$$

$$= (1.24)a \quad ...(1.1.5)$$

Now, from equation (1.1.4),

$$\psi_s = [\pi (1.24\, a)^2/6a^2] = 0.806 \quad ...(1.1.6)$$

Similarly, the value of sphericity for a cylindrical particle of length equal to its diameter will be 0.874. Typical values of sphericities for some common materials are given in Table (1.1). Every mineral has a given crystallographic structure, thus a given shape or configuration and therefore a specific sphericity.

The reciprocal of sphericity is commonly called the shape factor or more precisely, the *surface shape factor* (λ_s). Thus,

$$\lambda_s = (1/\psi_s) \qquad ...(1.1.7)$$

Table 1.1

Material	*Sphericity,* (ψ_s)
Sand (rounded)	0.83
Fused flue dust	0.89
Fused flue dust (aggregates)	0.55
Tungsten powder	0.89
Sand (angular)	0.73
Pulverised coal	0.73
Coal dust (upto 10 mm)	0.65
Flint sand (jagged flakes)	0.43
Mica flakes	0.28
Berl saddles	0.3 (average)
Raschig rings	0.3 (average)

Another popularly used parameter in this connection is the *specific surface ratio* (n). It is defined as the ratio of the specific surface (surface area per unit mass) of the particle to the specific surface of a spherical particle of the same diameter. Let the average size of the particle be d_{avg}. Then,

$$n = s_p/(6/\rho_s\, d_{avg}) \qquad ...(1.1.8)$$

where ρ_s is the density of the particle.

or

$$s_p = \frac{6n}{\rho_s\, d_{avg}} \qquad ...(1.1.9)$$

It will be interesting to find out how the specific surface ratio (n) and the sphericity (ψ_s) are inter-related. From equation (1.1.4),

Surface area of the particle,

$$S_p = \pi d_v^2/\psi_s \qquad ...(1.1.10)$$

Therefore, the specific surface (s_p) of the particle

$$= \frac{S_p}{(\pi d_v^3/6)\,\rho_s}$$

$$= (6\pi d_v^2)/\psi_s\,(\pi d_v^3)\,\rho_s$$

$$= \frac{6}{\rho_s \, d_v \, \psi_s} \qquad ...(1.1.11)$$

Comparing equations (1.1.9) and (1.1.11), we get

$$\frac{(n)}{d_{avg}} = \frac{1}{\psi_s \, d_v}$$

or

$$n = \left(\frac{d_{avg}}{d_v}\right)\frac{1}{\psi_s} = \left(\frac{d_{avg}}{d_v}\right)\lambda_s \qquad ...(1.1.12)$$

If the screen diameter of the particle is very nearly equal to its volumetric diameter, then,

$$(d_{avg}/d_v) \approx 1$$

and

$$n = \lambda_s \qquad ...(1.1.13)$$

In the above case, n can be taken as a constant for a given material since it is now equal to its shape factor. However, this is true only for the specific case when $d_{avg} \approx d_v$. Otherwise, it must be noted that unlike sphericity, the specific surface ratio n is a function of the particle size (d_{avg}). Standard log-log plots of n versus d_{avg} for different materials are available in the literature[1,2]. One such set of plots is reproduced in Fig. (1.1).

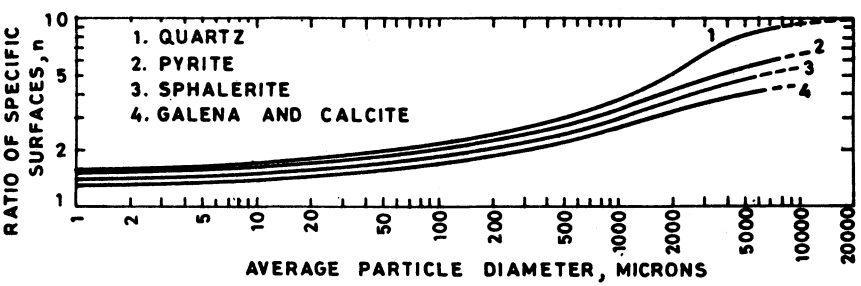

Fig. 1.1. Plots of Specific Surface Ratio (n) Versus Average Particle Size for Different Materials (From G.G. Brown and Associates, Unit Operations, Wiley, New York, 1950, by permission).

Another shape factor, designated as the *volume shape factor* (λ_v), is sometimes used for calculating the volume of an irregular particle. We know that the volume of a spherical particle is proportional to the cube of its diameter. If we assume the same is true for an irregular particle as well, then

$$V_p \, \alpha \, d_{avg}^3$$

or, $$V_p = \lambda_v \, d_{avg}^3 \qquad ...(1.1.14)$$

where λ_v is the constant of proportionality and is called the volume shape factor. Its value is ($\pi/6$) for spherical particles. In the strict sense, λ_v is not a constant (though designated as shape factor), but is a function of the particle size. However, for a number of cases, its variation with particle size is not very large and therefore an average value of λ_v could be used for a number of process calculations.

1.2. Mixture of Particles

In the earlier section, we have considered the case of a single particle only. However, in actual industrial practices, we normally come across mixtures of particles of different sizes. In such a case, the mixture can be separated into a number of fractions each fraction consisting of particles of a given size d_{avg_i}. Let m_i designate the total mass of the *i-th* fraction. Since each fraction contains particles of the same size, the specific surface of *i-th* fraction can be computed using equation (1.1.9). Thus,

$$s_i = \frac{6n_i}{\rho_s \, d_{avg_i}} \qquad ...(1.2.1)$$

and, surface area of *i*th fraction,

$$S_i = m_i \, s_i = (6 \, n_i \, m_i / \rho_s \, d_{avg_i}) \qquad ...(1.2.2)$$

Therefore, the specific surface of the mixture of particles will be,

$$s_m = \frac{\text{Total surface area of all fractions}}{\text{Total mass of mixture}}$$

$$= \frac{(6n_1 \, m_1/\rho_s \, d_{avg1}) + (6n_2 \, m_2/\rho_s \, d_{avg2}) + \ldots.}{m_1 + m_2 + \ldots..} \qquad ...(1.2.3)$$

Let, $m_1 + m_2 + \ldots. = M$

Then, $$s_m = \frac{6n_1}{\rho_s \, d_{avg1}}\left(\frac{m_1}{M}\right) + \frac{6n_2}{\rho_s \, d_{avg2}}\left(\frac{m_2}{M}\right) + \ldots...(1.2.4)$$

We know, $(m_i/M) = x_i$ = mass fraction of *i*th fraction in the mixture.

Thus, $(m_1/M) = x_1$, $(m_2/M) = x_2$ and so on. Equation (1.1.18) therefore becomes,

$$s_m = \frac{6n_1 \, x_1}{\rho_s \, d_{avg1}} + \frac{6n_2 \, x_2}{\rho_s \, d_{avg2}} + \ldots.$$

$$= \frac{6}{\rho_s}\left[\frac{n_1 x_1}{d_{avg1}} + \frac{n_2 x_2}{d_{avg2}} + \ldots\right]$$

$$= (6/\rho_s) \sum_i \frac{n_i x_i}{d_{avg_i}} \qquad \ldots(1.2.5)$$

The total number of particles in the mixture can also be computed in a similar way. Thus, the number of particles in the ith fraction will be,

$$N_i = \frac{\text{Total mass of } i\text{th fraction}}{\text{Mass of each particle in the fraction}}$$

$$= \frac{m_i}{\rho_s \text{ (volume of each particle in the fraction)}}$$

Volume of each particle in the fraction can be computed using equation (1.1.14). Thus,

$$N_i = \frac{m_i}{\rho_s\,[\lambda_{vi}\, d^3_{avg_i}]} \qquad \ldots(1.2.6)$$

The total number of particles per unit mass of the mixture will be therefore,

$$\left(\frac{N}{M}\right) = \overline{N} = \frac{\dfrac{m_1}{\rho_s\,\lambda_{v1}\, d^3_{avg1}} + \dfrac{m_2}{\rho_s\,\lambda_v 2\, d^3_{avg2}} + \ldots.}{M}$$

$$= \frac{x_1}{\rho_s\,\lambda_{v1}\, d^3_{avg1}} + \frac{x_2}{\rho_s\,\lambda_{v2}\, d^3_{avg2}} + \ldots.$$

$$= \frac{1}{\rho_s} \sum_i \frac{x_i}{\lambda_{vi}\, d^3_{avg_i}} \qquad \ldots(1.2.7)$$

If we assume that the variation of λ_v within the size range under consideration is small and a constant average value of λ_v could be therefore assumed, then

$$\left(\frac{N}{M}\right) = \overline{N} = \frac{1}{\lambda_s\,\rho_s} \sum_i \frac{x_i}{d^3_{avg_i}} \qquad \ldots(1.2.8)$$

The specific surface and the number of particles per unit mass of a mixture of particles can be thus computed, once the size distribution is known.

1.3. Statistical Mean Diameters

While handling mixtures of particles, if it is desired to express the average size of the entire mixture, then we can do it using any of the statistical averages. However, if particles with a wide range of sizes are present in the mixture, a statistical mean of all the sizes may not give any realistic picture. Also, the statistical mean can be defined in a very large number of ways such as the length mean, surface mean, volume mean, square root mean, cube root mean, harmonic mean etc. It is therefore practically impossible to propose in advance which type of mean shall be best suitable for a given industrial application. The matter can be decided only from experience. Also, it is seen on many occasions that if the average size of a mixture is computed using all the above definitions, each yields an entirely different value of average size, clearly indicating that their applicabilities cannot be mutually compared with each other. Some of the popular methods of defining statistical average are given below.

(*a*) Arithmetic or number mean, D_A or $D_N = \dfrac{\Sigma N_i d_i}{\Sigma N_i}$

where N_i = number of particles of size d_i. If d_i is defined by the screen size d_{avg_i}, then N_i in the above equation can be re-expressed using equation (1.2.6). Thus,

$$D_A \text{ or } D_N = \frac{\dfrac{M}{\rho_s} \sum_i \left(\dfrac{x_i}{\lambda_{vi}\, d^3_{avg_i}} \right) d_{avg_i}}{\dfrac{M}{\rho_s} \sum_i \dfrac{x_i}{\lambda_{vi}\, d^3_{avg_i}}}$$

$$= \frac{\sum_i [x_i/\lambda_{vi}\, d^2_{avg_i}]}{\sum_i [x_i/\lambda_{vi}\, d^3_{avg_i}]} \qquad \text{...(1.3.1)}$$

(*b*) Length mean, $D_L = \dfrac{\Sigma N_i d_i^2}{\Sigma N_i d_i}$ ...(1.3.2)

(*c*) Surface mean, $D_s = \dfrac{\Sigma N_i d_i^3}{\Sigma N_i d_i^2}$ ...(1.3.3)

(*d*) Volume mean, $D_v = \dfrac{\Sigma N_i d_i^4}{\Sigma N_i d_i^3}$ ...(1.3.4)

(*e*) Square root mean, $D_{SR} = \left[\frac{\Sigma N_i d_i^2}{\Sigma N_i} \right]^{1/2}$ ...(1.3.5)

This is designated as surface mean by Brown.[4]

(*f*) Cube root mean, $D_{CR} = \left[\frac{\Sigma N_i d_i^3}{\Sigma N_i} \right]^{1/3}$ ...(1.3.6)

This is designated as volume mean by Brown.[4]

(*g*) Harmonic mean, $D_H = \frac{\Sigma N_i}{\Sigma (N_i/d_i)}$ ...(1.3.7)

Let us now consider an illustrative example in which we shall compute the statistical average diameter of a mixture of particles using all of the above methods. Consider the following size distribution :

Number of particles, N_i	*Particle size, d_i (microns)*
155,000	2.0
25,600	5.0
6,200	10.0
1,750	20.0
660	30.0
156	40.0
100	50.0
87	60.0

The computed values of statistical average diameters are given below :

D_A or D_N = 3.0 microns (1 micron = 10^{-3} mm)

D_L = 7.0 microns

D_S = 21.0 microns

D_V = 36.4 microns

D_{SR} = 4.6 microns

D_{CR} = 7.6 microns

D_H = 2.3 microns

It can be easily noticed that the value of statistical average diameter computed using each method is significantly different from that computed by other methods.

1.4. Determination of Particle Size

The size of a particle or the size distribution of a mixture of particles can be determined using a number of methods such as screening (best applicable for particles of size above 40 microns), gravity sedimentation (for size 1—100 microns), centrifugal sedimentation (for sizes 0.005 to 3 microns) and elutriation under gravity (for sizes 5—100 microns) and under centrifugal field (for sizes 1—60 microns). More sophisticated techniques such as ultra microscopy (for sizes 0.005 to 0.2 microns), electron microscopy (for sizes 0.0005 to 5.0 microns), light scattering (for sizes 0.1 to 10 microns) and X-ray scattering (for sizes 0.005 to 0.05 microns) are used when high precision is desired in the particle size determination. In this section, we shall confine our discussion to screening which is one of the cheapest and easiest methods of particle size analysis. Sedimentation and Elutriation have been discussed in subsequent chapters.

The importance of particle size analysis in industrial practices need not have to be over-emphasized. Particles must be separated according to their size before using them for any industrial operation since the process efficiency very much depends on the particle size employed. As for example, the effectiveness of catalyst particles is dependent on their surface area which in turn depends on their size. The hiding power of a paint pigment is controlled by the projected area of the pigment particles which once again is a function of their size. The setting rate of Portland cement and the final strength of concrete depend substantially on the uniform size of cement dust and the size of sand and gravel used to prepare the concrete mix. In the combustion of solid fuels like coal, uniformity of the fuel bed is a very crucial parameter. Even in practices like carbonisation and gasification, it is absolutely necessary that the coal bed is composed of particles of uniform size to ascertain satisfactory yields and quality of products. As stated earlier, screening offers to be one of the easiest and most rapid method of particle size analysis and separation.

1.4.1. Screening

What is a screen ?

A screen can be called an open container usually cylindrical with uniformly spaced openings at the base. It is normally made of wire mesh cloth, the wire diameter and the interspacing between wires being accurately specified. The openings are commonly square though rectangular openings are not unusual. The size of the square opening (the length of clear space between individual wires) is called the *aper-*

Table 1.2. Indian Standard Test Sieves

Sieve Designation Number	*Sieve Opening or Width of Aperture*		*Permissible Variation in Width of Aperture*				*Wire Diameter*			*Equivalent Mesh Number of Other Standard Screens*		
			Average opening		*Maximum opening*							
	mm	*inch*	*Microns*	*Percent*	*Microns*	*Percent*	*mm*	*inch*	*SWG*	*ASTM*	*BSS*	*Tyler*
(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*	*(10)*	*(11)*	*(12)*	*(13)*
570	5.660	0.2230	± 170	± 3.0	+ 566	+ 10	2.184	0.086	13.5	3.5	—	3.5
480	4.760	0.1870	± 143	± 3.0	+ 476	+ 10	2.032	0.080	14	4	3/16"	4
400	4.000	0.1570	± 120	± 3.0	+ 400	+ 10	1.829	0.072	15	5	—	5
340	3.353	0.1320	± 108	± 3.2	+ 370	+ 11	1.727	0.068	15.5	6	5	6
320	3.180	0.1252	± 102	± 3.2	+ 350	+ 11	1.829	0.072	15	—	1/8"	—
280	2.818	0.1109	± 97	± 3.4	+ 309	+ 11	1.422	0.056	17	7	6	7
240	2.399	0.0945	± 91	± 3.8	+ 263	+ 11	1.219	0.048	18	8	7	8
200	2.032	0.0800	± 93	± 4.6	+ 244	+ 12	1.118	0.044	18.5	10	8	9
170	1.676	0.0659	± 55	± 3.3	+ 201	+ 12	0.864	0.034	20.5	12	10	—
160	1.600	0.0630	± 54	± 3.4	+ 192	+ 12	0.965	0.038	19.5	—	1/16"	10
140	1.405	0.0553	± 48	± 3.4	+ 168	+ 12	0.711	0.028	22	14	12	12
120	1.201	0.0473	± 46	± 3.8	+ 156	+ 13	0.610	0.024	23	16	14	14
100	1.000	0.0394	± 50	± 5.0	+ 150	+ 15	0.584	0.023	23.5	18	16	16
85	0.842	0.0332	± 44	± 5.2	+ 126	+ 15	0.559	0.022	24	20	18	20
80	0.790	0.0311	± 38	± 4.8	+ 111	+ 14	0.533	0.021	24.5	—	1/32"	—
70	0.708	0.0279	± 38	± 5.4	+ 106	+ 15	0.457	0.018	26	25	22	24

Sieve Designation Number	Sieve Opening or Width of Aperture		Permissible Variation in Width of Aperture				Wire Diameter			Equivalent Mesh Number of Other Standard Screens		
			Average opening		Maximum opening							
	mm	inch	Microns	Percent	Microns	Percent	mm	inch	SWG	ASTM	BSS	Tyler
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
60	0.592	0.0233	± 33	± 5.6	+ 95	+ 16	0.417	0.0164	27	30	25	28
50	0.500	0.0197	± 25	± 5.0	+ 85	+ 17	0.345	0.0136	29	35	30	32
40	0.420	0.0165	± 21	± 5.0	+ 106	+ 25	0.284	0.0112	31.5	40	36	35
35	0.351	0.0138	± 19	± 5.4	+ 88	+ 25	0.224	0.0088	34.5	45	44	42
30	0.296	0.0117	± 16	± 5.4	+ 74	+ 25	0.193	0.0076	36	50	52	48
25	0.251	0.0099	± 13	± 5.2	+ 63	+ 25	0.173	0.0068	37	60	60	60
20	0.211	0.0083	± 12	± 5.5	+ 53	+ 25	0.142	0.0056	38.5	70	72	65
18	0.177	0.0070	± 11	± 6.2	+ 71	+ 40	0.122	0.0048	40	80	85	80
15	0.151	0.0060	± 11	± 7.3	+ 61	+ 40	0.102	0.0040	42	100	100	100
12	0.124	0.0049	± 8	± 6.5	+ 50	+ 40	0.086	0.0034	43.5	120	120	115
10	0.104	0.0041	± 7	± 7.1	+ 43	+ 41	0.066	0.0026	45.5	140	150	150
9	0.089	0.0035	± 7	± 7.4	+ 44	+ 49	0.061	0.0024	46	170	170	170
8	0.075	0.0030	± 7	± 9.3	+ 46	+ 60	0.051	0.0020	47	200	200	200
6	0.064	0.0025	± 8	± 12.0	+ 58	+ 90	0.041	0.0016	48	230	240	230
5	0.053	0.0021	± 5	± 9.0	+ 48	+ 90	0.030	0.0012	49	270	300	270
4	0.044	0.0017	± 3	± 7.0	+ 40	+ 90	0.030	0.0012	49	325	—	325

ture size of the screen. Screens are usually designated by their mesh number. The mesh number indicates the number of apertures per linear length. For example, a screen having 10 square openings per cm may be called a 10 mesh screen and in that case, the aperture size of the screen will be 0.1 cm minus the wire diameter. Clearly, higher the mesh number, smaller will be the aperture size of the screen. For instance, a 200 mesh screen will have a very small aperture width, whereas a 20 mesh screen will have a large aperture size. This is the practice followed in British standard screens (BSS), American standard screens (ASTM and Tyler standard screens), German standard screens (DIN 1171) etc.

The Indian standard screens (ISS) however follow a different type of designation. For an IS screen, the mesh number is equal to its aperture size expressed to the nearest deca-micron (0.01 mm). Thus, an IS screen of mesh number 50 will have an aperture width of approximately 500 microns. Such a method of designation has the simplicity that the aperture width is readily indicated from the mesh number. The specification of Indian standard test screens and the equivalent specifications of other standard screens (such as ASTM, BS and Tyler) are given in Table (1.2).

Standard test screens are usually made of phosphor bronze wires. Brass or mild steel wires are also sometimes used. It is always preferable to maintain a *standard screen interval* between successive test screens. The screen interval is the factor by which the aperture size of a test screen is to be divided to get the aperture size of the next successive test screen. An internationally accepted standard screen interval is $(2)^{1/4}$, that is 1.189.

In a standard sieve shaker, test screens are stacked one above the other in the ascending order of their aperture size. That is, the top-most screen will have the largest aperture size and the bottom-most screen the smallest. A weighed amount of the feed material is fed to the top-most screen and the whole assembly is shaken continuously either manually or more preferably mechanically. A bottom pan is kept below the bottom-most screen to collect the fines and the top-most screen is provided with a cover to prevent loss of fines. After a period of time when no more fines are being collected in the bottom pan, the vibration is stopped and the screens are desembled. The material retained on each screen (including the material in the bottom pan) is weighed. The data collected is as given below :

Material retained on	*Mass (gm)*
ISS No. 480	0.0
400	m_1
340	m_2
320	m_3
280	m_4
240	m_5
200	m_6
—	—
—	—
—	—
—	—
8	m_{27}
Bottom pan	m_b

The material that passes through the screen is called the minus (–) material or the undersize and the material that is retained on the screen is called the plus (+) material or the oversize. It can be seen that m_1 is the mass of the material that has passed through the 480 mesh screen but retained by the 400 mesh screen. It is therefore designated as the (– 480 + 400) fraction. The average size of the particles in this fraction will be therefore the arithmetic average of the aperture sizes of these two screens. Thus, from table (1.2), the d_{avg} of this fraction will be $\frac{4.76 + 4.00}{2} = 4.38$ mm.

If M is the total mass of the feed material, then the mass fraction of material having average size 4.38 mm will be (m_1/M), say x_1. Similarly, m_2 is the mass of (– 400 + 340) fraction and its average size (d_{avg2}) will be 3.6765 mm and its mass fraction in the mixture is $x_2 = (m_2/M)$.

Thus, the above data can be reported in a better way as follows :

Table 1.3

ISS mesh	*Average size, mm*	*Mass fraction*
+ 480	> 4.760	0.0
– 480 + 400	d_{avg1} (= 4.380)	x_1
– 400 + 340	d_{avg2} (= 3.6765)	x_2

ISS mesh	*Average size, mm*	*Mass fraction*
– 340 + 320	d_{avg3}	x_3
– 320 + 280	d_{avg4}	x_4
– 280 + 240	d_{avg5}	x_5
– 240 + 200	d_{avg6}	x_6
—	—	—
—	—	—
—	—	—
– 9 + 8	d_{avg27}	x_{27}
– 8	< 0.075	x_b

The above method of representing size distribution of a mixture is called the differential size analysis. The data may also be represented graphically by plotting x versus d_{avg} on rectangular co-ordinates or on a semi-log graph paper with d_{avg} on the log scale. Representation on log-log graph paper is most recommended since this avoids crowding of points particularly in the small size range.

It may be noted that the size distribution given above is in fact incomplete since the size distribution of the material collected in the bottom pan is not known. All that we know is that all particles in this fraction will have a size less than the aperture size of the 8 mesh screen, that is, less than 0.075 mm. For estimating the size distribution of a fraction that contains such fine particles only, we can make use of the Gaudin-Schumann size distribution law. This distribution is precisely applicable in the case of well-ground products from a fine grinding unit. According to this law, for particles of very fine sizes, a log-log plot of mass fraction x versus average size d_{avg} shall give a straight line. Mathematically,

$$ln\ (x) = m\ ln\ (d_{avg}) + ln\ B \qquad ...(1.4.1)$$

where m and B are constants.

Thus, once the mass fraction x has been plotted against d_{avg} on a log-log graph paper, the graph can be extrapolated back to the y-axis and the slope of the extrapolated portion will be equal to m and its y-intercept will give the value of B. Once the constants m and B are evaluated, the size distribution of -8 fraction can be determined using equation (1.4.1).

It must be noted that in equation (1.4.1), d_{avg} is the screen size of the particles. It is therefore essential that while applying the above equation, the values of d_{avg} must conform to the standard screen interval employed for the test screens. Also, Gaudin-Schumann distribu-

tion is valid for products from a fine grinder, but it's applicability for a classified product cannot be guaranteed. We shall now illustrate a numerical example.

Example 1.1. *Estimate the specific surface and sauter diameter of a sample of galena (specific gravity = 7.43) having the screen analysis below :*

Mesh	*Mass fraction*
- 570 + 480	*0.01*
- 480 + 340	*0.04*
- 340 + 240	*0.081*
- 240 + 160	*0.115*
- 160 + 120	*0.160*
- 120 + 85	*0.148*
- 85 + 60	*0.132*
- 60 + 40	*0.081*
- 40 + 30	*0.062*
- 30 + 20	*0.041*
- 20 + 15	*0.036*
- 15 + 10	*0.022*
- 10 + 8	*0.019*
- 8	*0.053*

Assume Gaudin-Schumann distribution is valid for sizes below 8 mesh.

Solution. The Specific surface of the sample can be computed from equation (1.2.5)

$$s_m = \frac{6}{\rho_s} \sum_i \frac{n_i\, x_i}{d_{avg_i}} \text{ where } \rho_s = 7.43 \text{ gm/cc.}$$

If D_{vs} is the volume surface diameter (sauter diameter) of the sample, then by definition,

$$s_m = 6/(\rho\, D_{vs})$$

or

$$D_{vs} = 1/\sum_i \frac{n_i\, x_i}{d_{avg_i}}$$

The values of specific surface ratio n can be obtained from the standard plot given in Fig. (1.1). The average size d_{avg} of each fraction is computed from the aperture sizes of screens given in Table (1.2). The results are given below :

Mesh	*Average size, d_{avg}* (cm)	*Specific surface ratio, n*	*Mass fraction, x*
– 570 + 480	$\frac{0.566 + 0.476}{2} = 0.521$	4.0	0.01
- 480 + 340	0.4056	3.8	0.04
– 340 + 240	0.2876	3.55	0.081
– 240 + 160	0.19995	3.2	0.115
– 160 + 120	0.14005	2.8	0.160

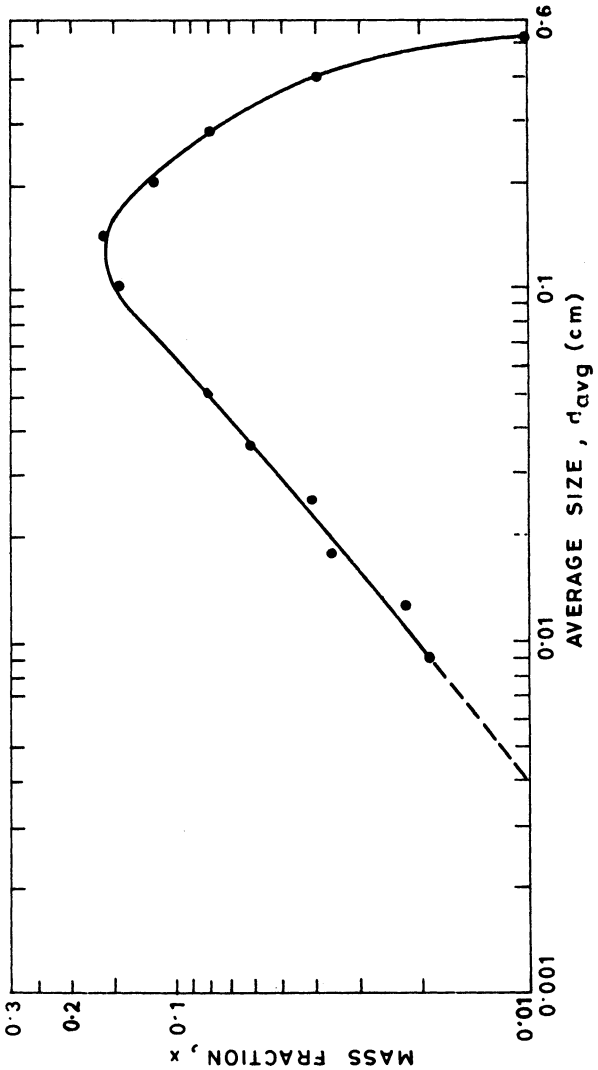

Fig. 1.1.1. Log-Log Plot of Mass Fraction Versus Average Particle Size.

Mesh	*Average size, d_{avg} (cm)*	*Specific surface ratio, n*	*Mass fraction, x*
– 120 + 85	0.10215	2.7	0.148
– 85 + 60	0.0717	2.5	0.132
– 60 + 40	0.0506	2.3	0.081
– 40 + 30	0.0358	2.1	0.062
– 30 + 20	0.02535	2.0	0.041
– 20 + 15	0.0181	1.85	0.036
– 15 + 10	0.01275	1.75	0.022
– 10 + 8	0.00895	1.70	0.019

Since it is given in the problem that for sizes below 8 mesh, Gaudin-Schumann size distribution law is applicable, we can plot x versus d_{avg} on log-log co-ordinates and then extrapolate back the lower portion of the graph (corresponding to low size range) since the plot within this portion must be linear. The plot is given in Fig. (1.1.1). From the figure, the average slope of the extrapolated portion is,

$$m = 0.7777$$

We can therefore write equation (1.4.1) as,

$$\log x = 0.7777 \log d_{avg} + \log B$$

or

$$x = B\,(d_{avg})^{0.7777}$$

It can be seen that the screen data reported above follows a screen interval of $(2)^{1/2}$ and not $(2).^{1/4}$ Therefore, in applying above relation, we must use d_{avg} in the interval of $(2).^{1/2}$ Thus, at $d_{avg} = \dfrac{0.0895}{1.41} = 0.063475$ mm, the mass fraction (x) from the graph is 0.0142. Using these two values, we can compute the value of constant B as,

$$0.0142 = B\,(0.063475)^{0.7777}$$

or

$$B = 0.21$$

The relation between x and d_{avg} for sizes below 8 mesh thus becomes,

$$x = 0.121\,(10\,d_{avg})^{0.7777}$$

where d_{avg} is in cm.

To compute the size distribution below 8 mesh, we proceed to calculate x for different values of d_{avg} (without forgetting to maintain the screen interval of 1.41) from the above correlation until all the computed values of x add closely to 0.053 (that is, total mass fraction of – 8 material). The results are given below :

d_{avg}, cm	x	*Specific Surface ratio (n)*
0.0063475	0.0142	1.65
0.0045	0.0108	1.60
0.00319275	0.0083	1.5
0.002264	0.00636	1.5
0.001606	0.00486	1.45
0.0011389	0.003725	1.40
0.000810	0.002852	1.35
0.000573	0.002183	1.30
	Total = 0.05328	

Having thus made the size distribution complete, let us now compute (mx/d_{avg}) for all the fractions :

Mass Fraction (x)	(nx/d_{avg})
0.01	0.07677
0.04	0.37475
0.081	0.9998
0.115	1.84046
0.160	3.1988
0.148	3.9120
0.132	4.6025
0.081	3.6818
0.062	3.6368
0.041	3.2347
0.036	3.6795
0.022	3.0196
0.019	3.6089
0.0142	3.6912
0.0108	3.84
0.0083	3.8995
0.00636	4.2138
0.00486	4.3879
0.003725	4.579
0.002852	4.75^
0.002183	4.9527
	$\Sigma \frac{n_i x_i}{d_{avg_i}} = 70.18378 \text{ cm}^{-1}$

Specific surface of sample,

$$s_m = \frac{6}{7.43}\,[70.18378] = 56.676 \text{ cm}^2/\text{gm} = 5.667 \text{ m}^2/\text{kg}$$

Sauter diameter,

$$D_{vs} = 1/\Sigma \frac{n_i\, x_i}{d_{avg_i}} = 1/[70.18378] = 0.14248 \text{ mm.}$$

Example 1.2. *In the above example, estimate the total number of particles per kg of the sample, if the volume shape factor (λ_v) of the material is 2.0 and may be assumed to be essentially constant within the size range under consideration.*

Solution. The number of particles per unit mass of the sample can be computed from equation (1.2.8) :

$$(N/M) = \frac{1}{\lambda_v\, \rho_s} \sum_i \frac{x_i}{d^3_{avg_i}}$$

where $\lambda_v = 2.0$ and $\rho_s = 7.43$ gm/cc. Let us therefore compute the value of (x/d^3_{avg}) for all the fractions.

x	d_{avg}, cm	$[x/d^3_{avg}]$, cm^{-3}
0.01	0.521	0.0707
0.04	0.4056	0.5995
0.081	0.2876	3.405
0.115	0.19995	14.386
0.16	0.14005	58.247
0.148	0.10215	138.850
0.132	0.0717	358.110
0.081	0.0506	625.221
0.062	0.0358	1351.27
0.041	0.02535	2516.807
0.036	0.0181	6071.09
0.022	0.01275	10,614.318
0.019	0.00895	26,502.352
0.0142	0.0063475	55,523.928
0.0108	0.0045	118,518.50
0.0083	0.00319275	255,037.29
0.00636	0.002264	548,059.50

x	d_{avg} cm	$[x/d_{avg}^3]$, cm^{-3}
0.00486	0.001606	1,173,274.50
0.003725	0.0011389	2,521,560.20
0.002852	0.00081	5,366,540.30
0.002183	0.000573	11,603,524.0
		$\Sigma \frac{x_i}{d_{avg_i}^3} = 21{,}690{,}292.0$

Therefore,

$$(N/M) = \frac{1}{(2.0)\,(7.43)}\,[21{,}690{,}292.0] = 14{,}59{,}643 \text{ per gm.}$$

Number of particles per kg = **1459.643 × 10⁶**

Another popular method of reporting screen data is on the cumulative basis. In *cumulative analysis*, our interest will be to estimate the total mass of that fraction in which all particles have sizes below (or sometimes above) a particular value.

For example, cumulative mass fraction X_i corresponds to mass fraction of the material having size less than d_i, or all particles in that fraction pass through a screen of aperture d_i mm. Consider the differential analysis reported in Table (1.3).

Since there is no + 480 material, it is evident that all material fed have passed through the 480 mesh screen, or all material in the feed have sizes below the aperture size of 480 mesh screen. In other words, the cumulative mass fraction for 480 mesh is 1.0. The mass fraction of material retained by 400 mesh screen is x_1 and therefore cumulative mass fraction for 400 mesh will be $(1 - x_1)$. Similarly we can compute cumulative mass fraction corresponding to each screen and the data can be tabulated as shown in Table (1.4).

Let $X_1 = 0.8$, $X_2 = 0.6$ and $X_3 = 0.4$. This means 80% of particles in the mixture have size less than 4 mm, 60% have size less than 3.353 mm and 40% less than 3.18 mm. The data of cumulative analysis can also be represented graphically by plotting X versus the aperture size (d_p) either on rectangular co-ordinates or on a semi-log graph paper (with aperture size on the log scale). One of the applications of cumulative size analysis we shall be discussing in the next section. Incidentally, if the cumulative analysis is available, then equation (1.2.8) that determines the number of particles in unit mass of a mixture can be rewritten as,

Table 1.4

Screen Mesh	*Aperture Size, mm*	*Cumulative Mass Fraction, X*
480	4.760	1.00
400	4.000	$(1-x_1)=X_1$
340	3.353	$(1-x_1-x_2)=X_2$
320	3.180	$(1-x_1-x_2-x_3)=X_3$
280	2.818	$(1-x_1-x_2-x_3-x_4)=X_4$
240	2.399	$(1-x_1-x_2-x_3-x_4-x_5)=X_5$
200	2.032	$(1-x_1-x_2-x_3-x_4-x_5-x_6)=X_6$
—	—	—
—	—	—
—	—	—
8	0.075	$x_b=X_b$

$$(N/M)=\frac{1}{\lambda_v\,\rho_s}\int_0^1\frac{dX}{d_p^3} \qquad ...(1.4.2)$$

where d_p is the aperture size. The integral on the right hand side of the above equation can be evaluated either numerically or graphically.

1.4.2. Screening errors and Screen Effectiveness

Screening is undoubtedly one of the easiest and most rapid methods of particle size analysis and separation. However, it cannot be called a very accurate method. It must be remembered that the probability of a particle passing through a screen depends very much also on the direction or configuration in which the particle approaches the screen. This is because for an irregular particle, its surface area exposed to the screen opening is different in different directions. As a result, it is very possible that when fed in one particular direction or configuration, the particle may pass through the screen but when fed in another direction the same particle may be retained by the screen. This therefore induces uncertainties in size analysis, since in screening the particle size is determined based on whether the particle passes through the screen or is retained by it. It is therefore absolutely necessary that while performing standard screen test on a mixture of particles, the test is repeated three to four times until consistent results are obtained. Also, the screens must be subjected to a type of vibratory, oscillatory, gyratory or rotary motion so that each particle gets chances of approaching the screen in almost all directions or configurations possible.

The presence of the so called "near mesh particles" always causes hindrances in screening operation. Near mesh particles to a screen are those particles having size very close to the aperture size of the screen. As a result, it is very possible that in one particular configuration, one of such particles passes only partly through the screen. It thus sits on the screen thereby causing clogging or blinding of the screen. Other disturbing factors are cohesion of particles to each other due to electrostatic forces or if the particles are moist, they tend to stick to the surface of the screen and also to each other causing difficulties in screening. If the particles are dry (very low moisture content) or if they are fed in a stream of water (wet screening), then the screening process will be faster and more efficient.

The effectiveness of a classifying screen is a parameter that has been defined in different ways by different authors[2,3,4]. In general, the efficiency of an industrial screen depends on two aspects : firstly, it must separate almost all particles of the desired size from the feed (recovery of desired material) and secondly, the classified product must contain very little number of particles having sizes other than the desired size (rejection of undesired material). Therefore, one of the most general methods of defining classifying screen effectiveness (E_c) is,

$$E_c = \text{(Recovery) (Rejection)} \qquad ...(1.4.3)$$

Let y_F be the mass fraction of the desired material (particles of desired size) in the feed, y_P that in the product and y_R that in the reject stream. It must be kept in mind that either the undersize (material that has passed through the screen) or the oversize (material that has been retained by the screen) can be our final product. If excess fines are not permissible in the product, then the oversize is collected as the product and if it is desired that the product must contain particles of below a particular size only, then the undersize stream shall constitute the final product. Now we can define rejection and recovery as,

$$\text{Recovery} = \frac{\text{Desired material in the product}}{\text{desired material in the feed.}}$$

$$= \frac{Py_P}{Fy_F} \qquad ...(1.4.4)$$

where F is the feed rate (kg/s) and P is the product rate.

$$\text{Recovery} = \frac{\text{Undesired material in the reject}}{\text{Undesired material in the feed}}$$

$$= \frac{R\,(1-y_R)}{F\,(1-y_F)} = \frac{F\,(1-y_F) - P\,(1-y_P)}{F\,(1-y_F)}$$

$$= 1 - \frac{P\,(1 - y_P)}{F\,(1 - y_F)} \qquad ...(1.4.5)$$

Thus,
$$E_c = \frac{Py_P}{Fy_F}\left[1 - \left(\frac{P}{F}\right)\frac{(1 - y_P)}{(1 - y_F)}\right] \qquad ...(1.4.6)$$

The (P/F) ratio in the above equation can be expressed in terms of mass fractions by making a material balance around the screen :

$$F = P + R \qquad ...(1.4.7)$$

Also, desired material in the feed

= (desired material in the product) + (desired material in reject)

That is,
$$Fy_F = Py_P + Ry_R \qquad ...(1.4.8)$$

Combining above two equations, we get

$$Fy_F = Py_P + (F - P)\,y_R$$

or,
$$F\,(y_F - y_R) = P\,(y_P - y_R)$$

$$(P/F) = (y_F - y_R)/(y_P - y_R) \qquad ...(1.4.9)$$

Substituting (1.4.9) in equation (1.4.6), we get

$$E_c = \frac{(y_F - y_R)\,y_P}{(y_P - y_R)\,y_F}\left[1 - \frac{(y_F - y_R)\,(1 - y_p)}{(y_p - y_R)\,(1 - y_F)}\right] \qquad ...(1.4.10)$$

Let us consider a few numerical examples :

Example 1.3. *Anthracite coal from a pulverization unit has been found to contain an excess of fine material (75% by weight). In order to remove these fines, it is screened using a 1.5 mm screen. Estimate the effectiveness of the screen from the following data :*

	Mass fraction	
Particle size, mm	*Oversize from Screen*	*Undersize from Screen*
– 3.33 + 2.36	*0.143*	*0.00*
– 2.36 + 1.65	*0.211*	*0.098*
– 1.65 + 1.17	*0.230*	*0.234*
– 1.17 + 0.83	*0.186*	*0.277*
– 0.83 + 0.59	*0.196*	*0.149*
– 0.59 + 0.42	*0.034*	*0.101*
– 0.42 + 0.29	*0.00*	*0.141*

Solution. Since the objective of screening is to remove the fine material, undersize from screen which will contain most of the fine material is the reject and the oversize material from the screen is the

product. The aperture size of classifying screen is 1.5 mm. It is obviously not a standard screen, but is an industrial screen constructed specifically for the purpose. Our desired material is therefore the + 1.5 material. Let X_P' and X_R' be cumulative mass fraction corresponding to the particle size of 1.5 mm in case of product (oversize from screen) and reject (undersize from screen) respectively. Therefore, by definition, X_P' is the total mass fraction of material having size less than 1.5 mm in the product. The mass fraction of + 1.5 material (the desired material) in the product will be then $(1 - X_P')$. Thus, $y_P = [1 - X_P']$.

Similarly, $y_R = [1 - X_R']$

Since the feed contains 75% fine material,

$$y_F = 0.25$$

To find the values X_P' and X_R', let us construct a cumulative mass fraction table from the data given :

Particle size, mm	X_P	X_R
3.33	1.0	1.0
2.36	0.857	1.0
1.65	0.646	0.902
1.17	0.416	0.668
0.83	0.230	0.391
0.59	0.034	0.242
0.42	0.0	0.141

We now plot both X_P and X_R against particle size on a semi-log graph paper. The plot is given in Fig. (1.3.1). From the Fig., the cumulative mass fractions corresponding to a particle size of 1.5 mm are obtained as,

$$X'_P = 0.57$$

$$X'_R = 0.85$$

Therefore, $y_P = (1 - X_P') = 0.43$

$$y_R = (1 - X_R') = 0.15$$

Substituting the values of y_P, y_F, and y_R in equation (1.4.10), we can estimate the screen effectiveness :

$$E_c = \frac{(0.43)\,(0.1)}{(0.25)\,(0.28)}\left[1 - \frac{(0.1)\,(0.57)}{(0.28)\,(0.75)}\right](100)$$

$$= 44.755\%$$

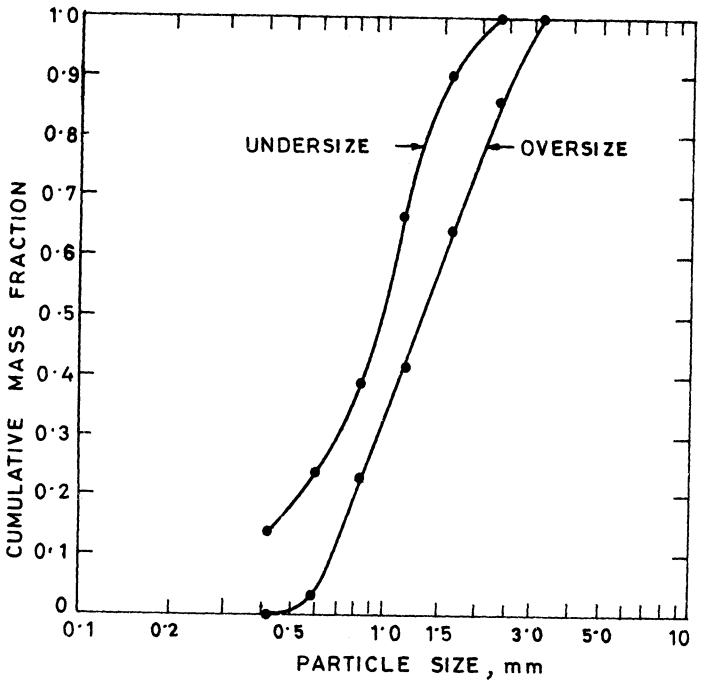

Fig. 1.3.1. Plots of Cumulative Mass Fraction Versus Particle Size on Semi-log Coordinates.

Example 1.4. *Table salt is being fed to a vibrating screen at the rate of 150 kg/hr. The desired product is – 30 + 20 mesh fraction. A 30 mesh and a 20 mesh screen are therefore used (double deck), the feed being introduced on the 30 mesh screen. During the operation, it was observed that the average proportion of oversize (from 30 mesh screen) : oversize (from 20 mesh screen) : undersize (from 20 mesh screen) is 2 : 1.5 : 1. Calculate the effectiveness of the screener from the following data :*

Mesh	*Mass fraction*			
	Feed	*Oversize from 30 Mesh Screen*	*Oversize from 20 Mesh Screen*	*Undersize from 20 Mesh Screen*
– 85 + 60	*0.097*	*0.197*	*0.026*	*0.0005*
– 60 + 40	*0.186*	*0.389*	*0.039*	*0.0009*
– 40 + 30	*0.258*	*0.337*	*0.322*	*0.0036*
– 30 + 20	*0.281*	*0.066*	*0.526*	*0.3490*
– 20 + 15	*0.091*	*0.005*	*0.061*	*0.2990*
– 15 + 10	*0.087*	*0.006*	*0.026*	*0.3470*

Solution. It is given that the desired product is the – 30 + 20 fraction. Since the feed is introduced on the 30 mesh screen, only the – 30 material will fall on the 20 mesh screen. Thus the oversize from the 20 mesh screen will be the – 30 + 20 fraction (the product). However, since both screens are not 100% efficient, other sizes than desired are also present in the product and reject streams.

From the given table, mass fraction of the desired material (– 30 + 20 material) in the feed is 0.281. Thus,

$$y_F = 0.281$$

Similarly, y_P = 0.526 (since oversize from 20 mesh screen is the product stream).

A material balance around the double-deck screener gives,

Feed (F) = oversize from 30 mesh (R_1) + oversize from 20 mesh (P) + undersize from 20 mesh (R_2)

Since it is given that $R_1 : P : R_2 = 2 : 1.5 : 1$,

$$\left(\frac{P}{F}\right) = \frac{P}{P + R_1 + R_2} = \frac{1.5}{4.5} = \frac{1}{3}$$

Now substituting the values of y_F, y_P, and (P/F) in equation (1.4.6), we can compute the effectiveness of the screener :

$$E_c = \frac{(0.526)}{3\,(0.281)}\left[1 - \frac{(1 - 0.526)}{(1 - 0.281)\,(3.0)}\right]$$

$$= 0.486 = 48.6\%.$$

1.4.3. Industrial Screening Equipment

Grizzlies are the most rugged types of industrial screens. They are used for screening large sizes of rocks of 25 mm and above, as for example, for sizing the feed to a primary crushing unit. They consist of a set of parallel bars usually made of manganese steel and of trapezoidal in cross-section. Stationary grizzlies are the simplest, requiring no external power supply and needs little maintenance. They are used to handle dry materials of sizes 50 mm and above, but are not suitable for moist or sticky materials. Vibrating grizzlies use a cam arrangement or an eccentric to impart lengthwise reciprocal movement to alternate bars which permit easy flow of material and prevent clogging. Flat grizzlies are used above feed bins of coal and ores or below the unloading trestles to retain huge lumps. Inclined grizzlies maintain a slope of $20^0 - 50^0$ with the horizontal. Chain grizzlies in which the bars are replaced by endless chains passing over sheaves are used for handling sticky or clay-like material.

Vibrating screens are one of the most popular ones used in chemical industry. They can handle large tonnages of material, possess high efficiency, provide good accuracy of sizing, require less maintenance per ton of material handled and also provide a saving in weight and installation space. They can be mechanically vibrated using an eccentric or unbalanced flywheel which will constitute a circular motion in the vertical plane, or electrically vibrated using an electromagnet. The frequency or speed of vibration varies from 1500 to 7200 per minute. They can handle a wide variety of materials starting from 480 mesh to 4 mesh.

Oscillating screens are characterised by relatively low speed oscillations (300 to 400 per minute) in a plane parallel to the screen cloth. They are one of the cheapest types of industrial screens and are widely used for batch screening of coarse material of 5 to 15 mm and fine, light, free-flowing materials of 4 to 8 mesh.

Reciprocating screens are popularly used in the chemical industry for handling materials down upto 4 mesh. They are driven by an eccentric under the screen at the feed end. The motion varies from gyratory at the feed end to reciprocating motion at the discharge end. The speed varies from 500 to 600 per minute. They are usually kept inclined at around 5° with the horizontal. They are widely used for handling dry chemicals, light metal powders, powdered food and granular materials, but are not suitable for handling heavy tonnages of rock or gravel.

Gyratory screens are box-like machines either square or round with series of screen cloths nested one upon the other. Eccentrics impart oscillations in circular or near-circular orbits. Examples are the vibro-energy separators and the gyratory riddles (widely used in batch screening).

Trommels are revolving screens usually cylindrical or conical in shape, open at both ends and are normally inclined at 5 – 10° with the horizontal. They are rotated about their axis at around 15 – 20 rpm. Trommels have relatively low capacity and low efficiency. They are however quite efficient for coarse sizes. Conical trommels which have the shape of a truncated cone are normally mounted horizontal. Compound trommels can be constructed using a number of screens of gradually increasing aperture size along the length of the cylinder, the feed being introduced at the finest screen. Thus, a number of products of different sizes can be collected from a single trommel. However, it will be more efficient to use a number of simple trommels in series, with the undersize from one trommel passing to the next. In concentric trommels, a number of trommels are mounted one inside the other on

a common shaft. The innermost screen will be the coarsest and the outermost the finest, the feed being introduced into the innermost screen. Provisions are also made for separate removal of oversize from each screening surface. The capacity of a trommel increases with increase in speed of rotation upto a stage when blinding occurs due to overcrowding of particles at the screen surface. If the speed is further increased to the so-called "critical speed," then the material will no longer cascade over the screen surface but will be carried around by centrifugal force. Thus, there will be no size separation and the trommel becomes inoperative. Since the critical speed of rotation of a trommel is thus the speed at which the force of gravity on the particle just becomes equal to the centrifugal force acting on it, we can write,

$$mg = m\,\omega_c^2\,r \qquad \text{...(1.4.11)}$$

where m = mass of particle

g = acceleration due to gravity

ω_c = critical angular velocity of particle (equal to that of trommel)

$$= \frac{2\,\pi\,N_c}{60}$$

N_c = critical speed of rotation of trommel, rpm

r = radius of rotation of particle.

Since the maximum radius of rotation of particle (at critical speed) is the radius of the trommel, we can put $r = D/2$, where D is the diameter of the trommel. Thus, equation (1.4.11) becomes,

$$\left[\frac{4\pi^2 N_c^2}{3600}\right]\left(\frac{D}{2}\right) = g$$

or,

$$N_c = \frac{60}{2\pi}\sqrt{2g/D} \qquad \text{...(1.4.12)}$$

Thus, the critical speed is governed by the diameter of the trommel (for concentric trommels, diameter of the outermost screen). Obviously, the operating speed of trommel must be below its critical speed. Usually, 33 to 45% of N_c is preferred.

1.5. Computer-aided Analysis

Today's era is commonly called the "computer era." The computations discussed above involving specific surface of particle mixtures, screen effectiveness etc. can be made faster and more accurate if the help of high speed computers is sought.

The primary step in computer-aided analysis is the preparation of the computation flow graph. It is nothing but a flow sheet illustrating all the steps of the computation process and the detailed procedure followed in each step. The initial specifications of the problem (that are available at the outset) are kept stored in one of the memory files of the computer, commonly called the "father file." On-the-spot specifications to be supplied at different stages of the computation are supplied from the *"son files"*. Data bases are used for storing standard tables, charts or graphs from which the value of any particular parameter can be retrieved whenever necessary. These are particularly useful when the value of a parameter cannot be computed using any specific correlation, but is available from standard tables, charts or graphs.

We shall present here the computation flow graph corresponding to examples (1.1) and (1.2) discussed in the earlier sections. The flow sheet is given in Fig. (1.2).

In the flow sheet, M_j ($j = 1, 2, \ldots, 14$) are the mesh numbers of the screens. Thus, in example (1.1), $M_1 = 570$, $M_2 = 480$ and so on. Finally, $M_{14} = 8$. Data base – 1 stores the specifications of Indian standard screens (given in table 1.2). Thus, once the mesh number M_j is specified, its aperture size a_j can be retrieved from this data base. S_{int} is the screen interval followed and in the present problem, S_{int} = 1.41. The total mass fraction of – 8 material is denoted as x_{fine}. Thus, in the present case, $x_{fine} = 0.053$. Data base – 2 stores the standard plot of specific surface ratio (n) versus average size (d_{avg}), given in figure (1.1.).

Once the computation flow graph has been prepared, it can be easily translated into any of the popular computer languages such as FORTRAN or PASCAL and then executed on any of the available high speed computers. The computer program for fitting straight line using the method of least squares (used in the above computation flow graph under 'EXTRAPOLATION') is given in the appendix. Under 'NUMPART' the alternate method of computing number of particles per unit mass of the sample ($\overline{N}$) based on cumulative analysis, that is, using equation (1.4.2), is given.

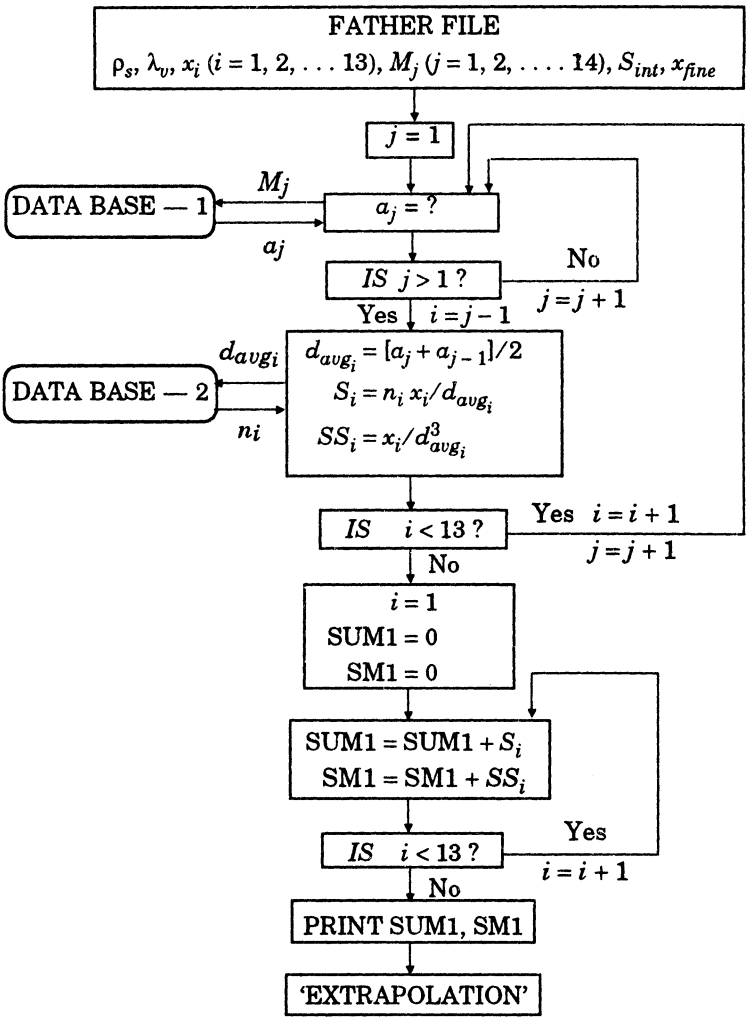
FATHER FILE
ρ_s, λ_v, x_i (i = 1, 2, . . . 13), M_j (j = 1, 2, ... 14), S_{int}, x_{fine}
$j = 1$
DATA BASE — 1
M_j
a_j
$a_j = ?$
IS $j > 1$?
No
$j = j + 1$
Yes
$i = j - 1$
DATA BASE — 2
d_{avg_i}
n_i
$d_{avg_i} = [a_j + a_{j-1}]/2$
$S_i = n_i\, x_i / d_{avg_i}$
$SS_i = x_i / d^3_{avg_i}$
IS $i < 13$?
Yes $i = i + 1$
$j = j + 1$
No
$i = 1$
SUM1 = 0
SM1 = 0
SUM1 = SUM1 + S_i
SM1 = SM1 + SS_i
IS $i < 13$?
Yes
$i = i + 1$
No
PRINT SUM1, SM1
'EXTRAPOLATION'

Fig. 1.2(*a*)

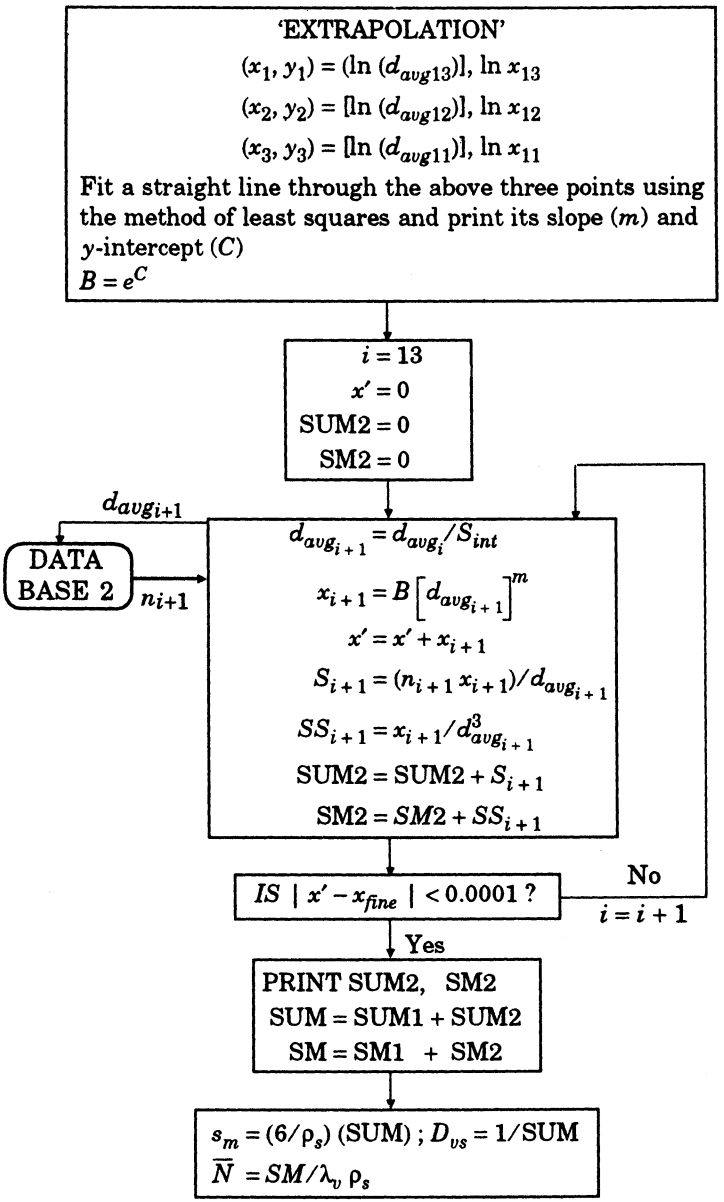
'EXTRAPOLATION'
$(x_1, y_1) = (\ln (d_{avg13})], \ln x_{13}$
$(x_2, y_2) = [\ln (d_{avg12})], \ln x_{12}$
$(x_3, y_3) = [\ln (d_{avg11})], \ln x_{11}$
Fit a straight line through the above three points using the method of least squares and print its slope (m) and y-intercept (C)
$B = e^C$
$i = 13$
$x' = 0$
SUM2 = 0
SM2 = 0
$d_{avg_{i+1}}$
DATA BASE 2
n_{i+1}
$d_{avg_{i+1}} = d_{avg_i}/S_{int}$
$x_{i+1} = B\left[d_{avg_{i+1}}\right]^m$
$x' = x' + x_{i+1}$
$S_{i+1} = (n_{i+1}\, x_{i+1})/d_{avg_{i+1}}$
$SS_{i+1} = x_{i+1}/d^3_{avg_{i+1}}$
SUM2 = SUM2 + S_{i+1}
SM2 = $SM2 + SS_{i+1}$
$IS \mid x' - x_{fine} \mid < 0.0001$?
No
$i = i + 1$
Yes
PRINT SUM2, SM2
SUM = SUM1 + SUM2
SM = SM1 + SM2
$s_m = (6/\rho_s)$ (SUM) ; $D_{vs} = 1/\text{SUM}$
$\overline{N} = SM/\lambda_v\, \rho_s$

Fig. 1.2(*b*)

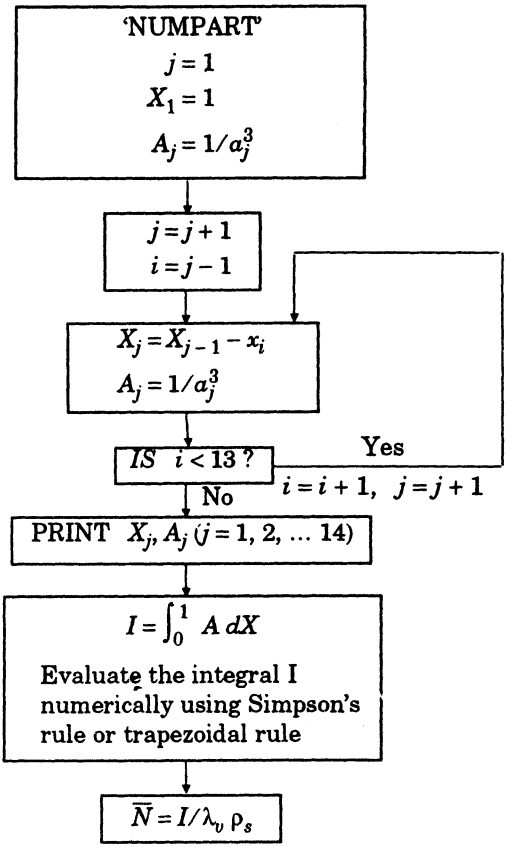

Fig. 1.2 (*c*)
Fig. 1.2 (*a*), 1.2 (*b*) and (1.2 (*c*) Computer Aided Analysis of a Screening Problem

For the interest of the readers, we shall present a few sample computer programs here. Let us first translate Figures 1.2(*a*) and 1.2(*b*) into FORTRAN. The resulting computer program is given below (Program 1.1). In the program, MESH(I) is used to indicate mesh number instead of M_j. Also, the values of specific surface ratio (n) at different values of average particle size (d_{avg}) are assumed available in the form of an elaborate table. For intermediate values of d_{avg}, the values of n are retrieved from this table by interpolation, as shown in the Program. It may also be noted that NC(I) and DAVC(I) are the calculated values of specific surface ratio and average particle size respectively.

PROGRAM 1.1

```
C**     COMPUTATION OF SPECIFIC SURFACE AND SAUTER
C**     DIAMETER OF A MIXTURE OF PARTICLES AND
C**     ALSO THE NUMBER OF PARTICLES PER UNIT
C**     MASS OF THE MIXTURE.
        REAL N(50), NC(50), LMDV, MM, NUMR, NUMP
        DIMENSION MESH(14), AP(14), X(50), DAV (50)
        DIMENSION DAVC(50)
        READ (*, *) ROWS, LMDV, SINT, XFINE
        NP = 14
        READ (*, *) [X(I), I =1, NP - 1]
        READ (*, *) [MESH(I), AP(I), I = 1, NP]
C**     THE VALUES OF SPECIFIC SURFACE RATIO AT DIFFERENT
C**     VALUES
C**     OF AVERAGE PARTICLE SIZE (DAV) ARE ASSUMED
C**     AVAILABLE.
        READ (*, *) [N(K), DAV(K), K = 1,50]
        SUM1 = 0.0
        SM1 = 0.0
        DO 10 J = 1, NP - 1
        DAVC(I) = [AP(I) + AP (I + 1)]/2.0
        DO 15 K = 1,50
        IF [DAV(K). GE. DAVC(I)] GO TO 20
 15     CONTINUE
 20     NC(I) = N (K - 1) + [N(K) - N (K - 1)] * [DAVC(I)
        - DAV (K - 1)]/[DAV(K) - DAV (K - 1)]
        SUM1 = SUM1 + NC(I) * X(I)/DAVC(I)
        SM1 = SM1 + X(I)/DAVC(I) * * 3
 10     CONTINUE
C**     GAUDIN-SCHUMANN DISTRIBUTION LAW IS ASSU-
C**     MED APPLICABLE FOR FINE SIZES BELOW DAVC (NP-4)
C**     THE CONSTANTS B AND MM OF G-S DISTRIBUTION
C**     LAW ARE EVALUATED BY FITTING A STRAIGHT
C**     LINE THROUGH THE POINTS {ALOG [DAVC(I)],
C**     ALOG[X(I)]}, I = NP-3, NP-2, NP-1 USING
C**     THE METHOD OF LEAST SQUARES.
```

```
      SUMX = 0.0
      SUMY = 0.0
      SUMXY = 0.0
      SUMXX = 0.0
      DO 25 I = NP-3, NP-1
      SUMX = SUMX + ALOG [DAVC(I)]
      SUMY = SUMY + ALOG[X(I)]
      SUMXY = SUMXY + ALOG [DAVC(I)] * ALOG[X(I)]
      SUMXX = SUMXX + ALOG [DAVC(I)] * * 2
      AN = 3.0
      NUMR = AN * SUMXY - SUMX * SUMY
      DENR = AN * SUMXX - SUMX * * 2
      MM = NUMR/DENR
      C = (SUMY - MM * SUMX)/AN
      B = EXP(C)
      I = NP - 1
      SX = 0.0
      SUM2 = 0.0
      SM2 = 0.0
30    DAVC (I + 1) = DAVC(I)/SINT
      X (I + 1) = B * DAVC (I + 1) * * MM
      DO 35 K = 1,50
      IF [DAV(K). GE. DAVC(I + 1)] GO TO 40
35    CONTINUE
40    NC (I + 1) = N (K - 1) + [N(K) - N(K - 1)] * [DAVC
      (I + 1) - DAV (K - 1)]/[DAV(K) - DAV (K - 1)]
      SUM2 = SUM2 + NC (I + 1) * X(I + 1)/DAVC (I + 1)
      SM2 = SM2 + X (I + 1)/DAVC (I + 1) * * 3
      SX = SX + X (I + 1)
      DIF = ABS (SX - XFINE)
      IF (DIF. LT. 0.0001) GO TO 45
      I = I + 1
      GO TO 30
45    SUM = SUM1 + SUM2
      SM = SM1 + SM2
      SS = (6.0/ROWS) * SUM
```

```
      DVS = 1.0/SUM
      NUMP = SM/(LMDV * ROWS)
      WRITE (*, *) SS, DVS, NUMP
      STOP
      END
```

Let us now consider computation of the number of particles in a mixture using cumulative size analysis (as illustrated in Figure 1.2*c*). The computer program is given below (Program 1.2). The size distribution table is first made complete by extrapolation and making use of Gaudin-Schumann size distribution law. The values of cumulative mass fraction, represented as CX(I) in the program, are now computed and the integral of equation (1.4.2) evaluated using trapezoidal rule (Simpson's rule cannot be used here since CX(I) values are not at equal intervals). The rule is given below :

$$\int_{x_0}^{x_n} f(x)\,dx = (1/2)\sum_{i=1}^{n} h_i\,[f(x_{i-1}) + f(x_i)] \qquad ...(1.5.1)$$

where

$$h_i = [x_i - x_{i-1}] \qquad ...(1.5.2)$$

PROGRAM 1.2

```
C**   COMPUTATION OF NUMBER OF PARTICLES IN A
C**   MIXTURE USING CUMULATIVE SIZE ANALYSIS
C**   (USING EQUATION 1.4.2)
      REAL LMDV, MM, NUMR, NUMP
      DIMENSION MESH(14), AP(50), X(50), DAV(3)
      READ (*, *) ROWS, LMDV, SINT, XFINE
      NP = 14
      READ (*, *) [X(I), I = 1, NP - 1]
      READ (*, *) [MESH(I), AP(I), I = 1, NP]
C**   DETERMINE SIZE DISTRIBUTION BELOW AP(NP)
C**   BY EXTRAPOLATION.
C**   GAUDIN-SCHUMANN DISTRIBUTION LAW IS
C**   ASSUMED VALID FOR FINE SIZES BELOW AP (NP - 3).
      DAV (NP - 3) = [AP (NP - 4) + AP (NP - 3)]/2.0
      DAV (NP - 2) = [AP (NP - 3 ) + AP (NP - 2)]/2.0
      DAV (NP - 1) = [AP (NP - 2) + AP (NP - 1)]/2.0
C**   FIND THE CONSTANTS B AND MM OF G-S
C**   DISTRIBUTION LAW BY FITTING A STRAIGHT
```

```
C**     LINE THROUGH THE ABOVE THREE VALUES
C**     OF DAV AND CORRESPONDING VALUES OF X
C**     ON LOG-LOG COORDINATES.
        SUMX = 0.0
        SUMY = 0.0
        SUMXX = 0.0
        SUMXY = 0.0
        DO 10 I = NP - 3, NP - 1
        SUMX = SUMX + ALOG [DAV(I)]
        SUMY = SUMY + ALOG [X(I)]
        SUMXY = SUMXY + ALOG [DAV(I)] * ALOG [X(I)]
 10     SUMXX = SUMXX + ALOG [DAV(I)] * * 2
        AN = 3.0
        NUMR = AN * SUMXY - SUMX * SUMY
        DENR = AN * SUMXX - SUMX * * 2
        MM = NUMR/DENR
        C = (SUMY - MM * SUMX)/AN
        B = EXP (C)
        I = NP
        SX = 0.0
 15     AP (I + 1) = AP (I) /SINT
        DAV(I) = [AP(I) + AP (I + 1)]/2.0
        X(I) = B * DAV(I) * * MM
        SX = SX + X(I)
        DIF = ABS (SX - XFINE)
        IF (DIF. LT. 0.0001) GO TO 20
        I = I + 1
        GO TO 15
 20     MP = I
C**     COMPUTE THE VALUES OF CUMULATIVE MASS
C**     FRACTIONS, CX(I).
        CX(1) = 1.0
        DO 25 I = 1, MP - 2
        CX (I + 1) = CX(I) - X(I)
 25     CONTINUE
        CX (MP) = X (MP)
```

```
      CX (MP + 1) = 0.0
C**   EVALUATE INTEGRAL OF EQUATION (1.4.2)
C**   USING TRAPEZOIDAL RULE.
      SUM = 0.0
      F(1) = 1.0/AP(1) * * 3
      DO 30 I = 1, MP
      H(I) = CX (I) - CX (I + 1)
      F (I + 1) = 1.0/AP (I + 1) * * 3
 30   SUM = SUM + [H(I)/2.0] * [F(I) + F (I + 1)]
      NUMP = SUM/(LMDV * ROWS)
      WRITE (*, *) NUMP
      STOP
      END
```

NOMENCLATURE

a_j aperture size of j-th screen, corresponding to mesh number M_j (used in CAD flowsheet), m

d_{avg} average screen size of a particle, m.

d_{avg_i} average screen size of the ith fraction of a mixture of particles, m.

d_p particle size/aperture size, m.

d_s surface diameter of an irregular (non-spherical) particle, m.

d_v volumetric diameter of an irregular particle, m.

d_{vs} volume-surface diameter (sauter diameter) of an irregular particle, m.

D diameter of trommel, m.

D_A (or D_N) arithmetic mean (or, number mean) size of a mixture of particles, m.

D_{CR} cube-root mean size of a mixture of particles, m.

D_H harmonic mean size of a mixture of particles, m.

D_L length mean size of a mixture of particles, m.

D_S surface mean size of a mixture of particles, m.

D_{SR} square-root mean size of a mixture of particles, m.

D_v volume mean size of a mixture of particles, m.

D_{vs} volume-surface diameter (sauter diameter) of a mixture of particles, m.

E_c screen effectiveness, dimensionless

F feed rate, kg/s.

g gravitational acceleration, m/s^2

m, B constants in Gaudin-Schumann size distribution law (see eq. 1.4.1).

m_i mass of the ith fraction of a mixture of particles, kg.

M total mass of a particle mixture, kg.

M_j mesh number of j-th test screen (see the CAD flowsheet) corresponding to aperture size a_j.

n specific surface ratio, dimensionless.

n_i specific surface ratio of ith fraction of a mixture of particles, dimensionless

$N, \overline{N}$ total number of particles and number of particles per unit mass respectively, in a mixture of particles.

N_i number of particles of size d_i (or d_{avg_i})

N_c critical speed of rotation of trommel, rpm.

P product discharge rate, kg/s.

R reject (or recycle) rate, kg/s.

s_i specific surface of ith fraction, m^2/kg.

s_p specific surface of particle, m^2/m^3 or m^2/kg.

s_m specific surface of a mixture of particles, m^2/kg.

S_i surface area of ith fraction, m^2.

S_{int} standard screen interval (see the CAD flowsheet), dimensionless.

S_p surface area of a particle, m^2

V_p volume of a particle, m^3

x_i mass fraction of particles of size d_{avg_i} in a mixture.

X cumulative mass fraction

y_F mass fraction of desired material (material of desired size) in the feed to the screen.

y_P mass fraction of desired material in the product from the screen.

y_R mass fraction of desired material in the reject stream from the screen.

λ_s shape factor (or, surface shape factor), dimensionless

λ_v volume shape factor, dimensionless

ψ_s sphericity, dimensionless

ρ_s solid density, kg/m^3

Computer Notations

AP(I) aperture size of i-th standard screen (a_i)

B constant in Gaudin-Schumann law (eq. 1.4.1)

CX(I) cumulative mass fraction corresponding to aperture size a_i

DAV(I) average screen size of i-th fraction (d_{avg_i})

DAVC(I) calculated value of DAV(I)

DVS volume-surface diameter of a mixture of particles (D_{vs})

LMDV volume shape factor (λ_v)

MM constant in Gaudin-Schumann size distribution law (m)

MESH(I) mesh number of i-th standard screen (M_i)

N(I) specific surface ratio (n_i) of particles of size d_{avg_i}

NC(I) calculated value of N(I)

NUMP number of particles per unit mass ($\overline{N}$)

ROWS solid density (ρ_s)

SINT screen interval (S_{int})

SS specific surface of a mixture of particles (s_m)

X(I) mass fraction of particles of size d_{avg_i} (x_i)

XFINE total mass fraction of fine material whose size distribution is not known (x_{fine}).

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