1
Network Elements

1.1. Introduction

Various laws of nature are explained by theories, put forth from time to time by scientists. Such theories are based on observed facts and are able to explain most of the observations. Each theory is in fact nothing but imaginary picture or a "conceptual scheme". Any conceptual scheme which is in a position to explain most of the existing phenomena or observations, is considered true until superceded by a more powerful conceptual scheme. Thus the well known atomic theory is nothing but a well accepted conceptual scheme protraying an atom as consisting of a central nucleus with electrons moving in orbits around it. This conceptual scheme explains most of the observed phenomena and is, therefore, considered true. Similarly we have the conceptual schemes of conservation of energy and conservation of charge.

One of the most celebrated conceptual schemes is the Maxwell equations which explain all electric and magnetic phenomena in terms of fields resulting from charge and current. Results deducted from Maxwell's equations have agreed with the observations for over a century. It should then be expected that this conceptual scheme should be exclusively used for describing every electric and magnetic phenomenon. But we find that although Maxwell's equations are most fundamental in nature, and have stood the test of time, they fail to provide description and computation of various electric and magnetic phenomena in a simple and lucid manner. Accordingly we embark upon another conceptual scheme which permits simplicity of description and computation of various electric and magnetic phenomena, namely the concept of electric circuit. This concept of electric circuit has great practical utility. Thus in most of the cases, we are not so much interested in electric and magnetic fields in an electric device as in voltages and currents. This scheme of electric circuit permits analysis of an electric system in terms of voltages and currents and permits calculation of such quantities as charge, field, energy, power etc. Both the field concept and circuit concept have their respective utilities. But whereas the field concept is more fundamental, the circuit concept is more practical. Of course, the two concepts are intimately related. Thus the circuit concept arises from the same experimental facts as does concept described by Maxwell's equations. However, the field theory in more general and exact in nature, whereas the circuit concept involves several approximations. But inspite of this basic limitation, circuit theory retains its utility. It is significant, no doubt, that the nature of these approximations be clearly understood.

1.2. Network Elements

The basic electrical quantities in terms of which we may describe the function of an electrical circuit are : charge and energy. A physical circuit serves to transfer and transform energy. This energy transfer is achieved through charge transfer. Thus in any electrical network, there takes place energy transfer (through charge transfer) from the point of supply (*i.e.* from the energy source) to a point of transformation or conversion, referred to as the load or the sink. Of course, in this process, energy may be stored. Evidently then a physical circuit must provide means for such transfer and transformation. Accordingly a physical circuit is constituted by (*i*) sources of energy (*ii*) circuit elements like resistors, capacitors, inductors (including mutual inductance) and (*iii*) connecting wires etc.

Network elements may be (*i*) active or passive (*ii*) unilateral or bilateral (*iii*) linear or non-linear (*iv*) lumped or distributed. An electrical network is simply an interconnected aggregate of network elements of assorted nature. Network theory studies the behaviour of the entire network as a whole with respect to specified terminals and not the behaviour of the elements alone. The details of these elements are not of direct consequence. However, since behaviour of entire network depends directly on the behaviour of the elements, these network elements deserve careful consideration. Accordingly these network elements will be carefully classified and individually studies.

Active elements. An energy source, *i.e.* a voltage source or a current source, is said to be an active elements. Vacuum tubes and transistors are also active elements but they are not ideal energy sources.

Passive elements. An element, which is not an energy source, is a passive element. Thus a passive element serves to transform or store energy. Resistors, inductors and capacitors form typical passive elements.

Bilateral and unilateral elements. Bilateral elements are those elements which transmit equally well in either direction. Unilateral elements on the other hand, transmit widely unequally in the two directions. Elements made of high conductivity materials are in general, bilateral, while vacuum tubes, crystal rectifiers and metal rectifiers are unilateral elements.

Lumped and distributed elements. Physically separate elements such as resistors, capacitors and inductors are referred to as lumped elements. On the other hand, network elements which are inseparable for analytical purposes are called distributed elements. Thus a transmission line has distributed resistance, capacitance and inductance along its length.

Linear and non-linear elements. A linear element is one which is governed by a linear differential equation for all values of applied stimulus. Failing this, the element is said to be non-linear.

1.3. Scope of Network Analysis and Network Synthesis

Depending upon the character of the constituent elements, network systems may be classified into 3 basic categories namely, class A, class B and class C.

Class A network system is one in which all constituent elements are linear. An element (or a network system of elements) is said to be linear when its behaviour can be governed by linear differential equation. Thus for any given network system, let θ_i be the stimulus (or input) as a function of time and let θ_o be the corresponding response (or output) also expressed as a function of time. Now if the stimulus is increased to $k.\ \theta_i$, where k is a constant, the network system will be said to be linear if the response (or output) is $k.\ \theta_o$. Thus $k\theta_i$ and $k\theta_o$ form an admissible pair of stimulus and response functions for the linear system. Again if (θ_{i1}, θ_{o1}) and (θ_{i2}, θ_{o2}) denote any two admissible input-output pair functions for a network, then if the network is linear, then ($k_1\ \theta_{i1} + k_2\ \theta_{i2}$, $k_1\ \theta_{o1} + k_2\ \theta_{o2}$) will also form an admissible input-output pair function, where k_1 and k_2 are constants.

If one or more elements of the network are non-linear, the network degenerates to either class B or class C type. If the non-linear element/elements can be linearized atleast over a limited range of operation, the network system is said to be of class B type. On the other hand, if the non-linear elements cannot be linearized even over the limited range of operation, the network is said to be of class C type. The present work deals only with linear network system.

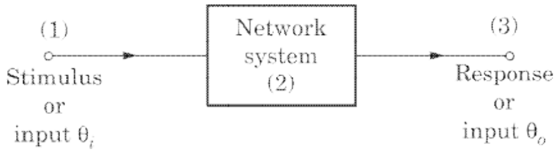


Fig. 1.1. The essential factors involved in network theory.

The study of linear network systems may be divided into two main heads : network analysis and network synthesis. Network analysis consists in finding the response (or output), when the stimulus (or input) and the network system components are given. On the other hand, network synthesis consists in finding the network system, which will give the prescribed output for a given stimulus (or input). Thus with reference to Fig. 1.1, network analysis and network synthesis are given by the following :

Network analysis : Given (1) and (2), to find (3).

Network synthesis : Given (1) and (3), to find (2).

This work deals with analysis of network composed of linear bilateral elements, mainly of the lumped type. However, some attention is afforded to analysis of network consisting of distributed linear bilateral elements as well. Finally synthesis of one port L-C networks is taken up.

1.4. Electric Charge and Electric Current

Electric charge and energy form the two most fundamental electrical quantities.

The basic unit of charge is the charge of an electron. But this charge is very small. Hence a larger unit of charge is used in practice. In M.K.S. system, coulomb is the unit of charge. An electron has a negative charge of 1.601×10^{-19} coulomb.

Charge of a particle or body may be positive or negative. Like charges repel each other while unlike charges attract each other. If there be two charges spaced a distance d apart, then the force of attraction or repulsion is inversely proportional to the square of the distance d between them.

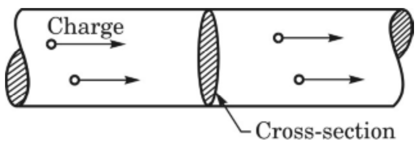


Fig. 1.2. Motion of charge in a conductor.

The charges may be at rest or may be in motion. A charge in motion constitutes an electric current. An electric current is defined as the "time rate of net motion of electric charge across a given cross sectional area". Thus con-sider the cross-section A of a conductor shown in Fig. 1.2. Electrons flow across this cross-section from say left to right. This constitutes an electrons current from left to right within the conductor. The conventional electric current, being the time rate of flow of positive charge will be considered to be flowing in the opposite direction, *i.e.* from right to left within the conductor.

Expressed mathematically, current i is given by

$$i = \frac{dq}{dt} \qquad ...(1.1)$$

where q is the charge in motion and t is the time.

If the charge q is in coulombs and the time i is in seconds, the current is in amperes.

The charge of an electron is 1.601×10^{-19} coulomb. Hence current of one ampere signifies motion of

$$\frac{1}{1.601 \times 10^{-19}} = 6.25 \times 10^{18}$$

electrons per second across the cross section of the conductor.

An electric current requires a net transfer of charge across a cross-section in one direction. Accordingly a random motion of electrons in a metal does not constitute a current.

1.5. Electric Energy : Electrical Potential

Energy forms another basic electrical quantity. A conceptual scheme commonly used is the "conservation of energy" which states that energy cannot either be created or destroyed, but can only be converted in form. Thus electrical energy can be obtained from various sources on transformation. Electrical energy may be obtained by :

(*i*) *Magnetic induction.* As in a rotating generator. Mechanical energy of rotation is converted into electric energy.

(*ii*) *Voltaic method.* As in electric batteries converting chemical energy into electric energy.

(*iii*) *Electrostatic method.* As in friction machines like Van de Graaf generator converting mechanical energy into electric energy.

(*iv*) *Thermo-electric method.* Producing electricity by heating the junction of dissimilar metals like bismuth and copper. Heat energy is converted into electric energy.

(*v*) *Photo-electric method.* Converting light energy into electric energy by letting light fall on photo-electric devices.

Whatever be the source of creation of electric energy, the source or generator has two terminals, referred to as positive and negative terminal in Fig. 1.3. If now the two terminals are connected by an external conductor, then electric current flows through this external conductor from the positive terminal to the negative terminal. Of course, work is done by the source of energy supply in transporting the electric charge round the external circuit. The "energy spent (or work done) per unit charge" in transporting it from one terminal to the other is called the "potential" (or voltage).

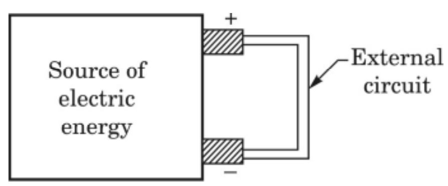


Fig. 1.3. Source of electric energy.

Thus mathematically, voltage v in volts is given by,

$$v = \frac{w}{q} \qquad ...(1.2)$$

where w is the work done (or energy spent) expressed in joules and q is the charge transported in coulombs.

In the above treatment, we have considered the potential of one terminal of a source with respect to the order. As a general case, we may consider any two points say P_1 and P_2. Then the potential of point P_2 with respect to that of point P_1 is the amount of work done in moving a unit positive charge from point P_1 to point P_2. We may as well define the potential of a point as the amount of work done in moving a unit positive charge from infinity to that point. If the unit charge is one coulomb, the work done is expressed in joules, then the potential is given is joules/coulomb, *i.e.,* in volts. In practice, however, the absolute potential of a point is of little significance. In most of the applications in Electrical Engineering, we are interested in the potential difference between two points.

Thus if w joules of work is done in moving q coulombs of positive charge from point P_1 to point P_2, the potential of P_2 with respect to that of P_1 in volts is given by Eq. (1.2).

Considering the same two points, let the charge transported be increased from q to $(q + dq)$. Then the work done in transportation also gets increased say from w to $(w + dw)$. Now since the potential difference has remainded unaltered,

$$v = \frac{w + dw}{q + dq} \qquad \text{...(1.3)}$$

From Eqs. (1.2) and (1.3),

$$v = \frac{dw}{dq} \qquad \text{...(1.4)}$$

The time rate of transfer of energy is called power. Thus power is given by,

$$p = \frac{dw}{dt} = \frac{dvq}{dt} \text{ from Eq. (1.2)}$$

$$= v.\ \frac{dq}{dt} \text{ if } v \text{ is constant}$$

$$= vi \qquad \text{...(1.5)}$$

The M.K.S. unit of power is watt. Energy is given by integral equations

$$w = \int pdt = \int vi\ .\ dt \qquad \text{...(1.6)}$$

1.6. R, L, C Parameters

Any network consists of one or more of the following passive elements : resistors, inductors and capacitors. Under ideal circumstances, resistors, inductors and capacitors are pure elements. Thus, an ideal resistor possesses only resistance and no inductance or capacitance. Similarly, in ideal cases, inductors and capacitors possess only inductance and capacitance respectively. In practice, ideal elements hardly ever exist. A physical resistor possesses not only resistance but also inductance and capacitance. Similarly a physical inductor possesses resistance and capacitance besides inductance and a physical condenser possesses resistance and inductance besides capacitance. A pure resistance obeys definite laws relating the voltage across it and the current through it. Similarly a pure inductance and pure capacitance follow different laws relating the associated voltage and current. These three elements *viz.* resistance, inductance and capacitance thus form the basic network elements or network parameters in terms of which any network may be expressed by replacing every physical passive network element by its equivalent network parameters. Then the response of the entire network may be calculated since we know :

(*i*) the response of each idealized network element and

(*ii*) the arrangement of these elements in the network.

Accordingly we proceed to study the current-voltage behaviour and other properties of these basic network parameters namely resistance, inductance and capacitance represented respectively by R, L and C.

1.7. The Resistance Parameter

Passage of electric current through a material results in collisions of the electrons with other atomic particles. These collisions may be elastic or inelastic. An inelastic collision results in change in the internal energy of the colliding particles and a corresponding loss in the total energy of the colliding particles. The loss in energy per unit charge manifests itself as potential drop across the material. This amount of energy lost depends upon the physical properties of a given substance.

For a given conductor, Ohm's law (Eq. 1.7) relates the current, voltage and resistance :

$$v = Ri \qquad \text{...(1.7)}$$

where R is the resistance of the conductor in ohms,
v is the voltage drop across the resistor,
and i is the current in amperes.

Eq. (1.7) may be alternatively be written as :

$$v = R \cdot \frac{dq}{dt} \qquad ...(1.8)$$

or

$$q = \frac{1}{R}\int_0^t v\, dt \qquad ...(1.9)$$

Eq. (1.7) is sometimes written as :

$$i = Gv \qquad ...(1.10)$$

where G is the conductance of the conductor and equals $\frac{1}{R}$.

In the M.K.S. system, the unit of resistance is the ohm and that of conductance is mho or Siemen.

The energy lost by the electrons in passage through a conductor dissipates as heat. Resistance thus forms an energy dissipative ele-ment. At a given temperature, resistance of a conductor remains con-stant. In network analysis, resistance is assumed to remain constant, *i.e.* the temperature is assumed to remain constant.

Power absorbed in a resistor is given by,

$$P = vi \qquad ...(1.11)$$

where v, i and P are expressed in volts, amperes and watts respectively.

Eq. (1.11) may alternatively be put as :

$$P = \frac{v^2}{R} = i^2 . R \qquad ...(1.12)$$

Power P indicates the time rate at which energy is absorbed in the system.

Hence energy lost in a resistance in time t is given by,

$$W = \int_0^t Pdt = Pt = i^2\, Rt = \frac{v^2}{R}t \qquad ...(1.13)$$

where v is in volts, R is in ohms, t is in seconds and energy W is in joules.

Example 1.1. *A solid copper sphere 8 cm in diameter is deprived of 10^{12} electrons. Calculate the charge of the sphere in coulomb and the sign of the charge.*

Solution. The charge removed

$= -\ 10^{12} \times q$ coulomb $= -\ 10^{12} \times 1.601 \times 10^{-19}$ coulomb

$= -\ 1.601 \times 10^{-7}$ coulomb.

Hence net charge on the sphere is positive and is equal to the charge removed in magnitude. Thus the charge on the sphere = + 1.601×10^{-7} coulomb.

Example 1.2. *A conductor carries a current of 1.2 amperes. Find the number of electrons that pass past its cross-section per second.*

Solution. Charge flow across the cross-section

= 1.2 coulomb/second.

Charge of an electron = 1.6×10^{-19} coulomb.

Hence the number of electrons that flow past the cross-section per second = $\frac{1.2}{1.6 \times 10^{-19}}$

= 7.5×10^{18}.

Example 1.3. *The current in a conductor varies according to the equation = $3e^{-t}$ amp. for t greater than zero and is zero for t less than zero. Find the total charge in coulomb that passes through the conductor.*

Solution. $\frac{dq}{dt} = i^3 e^{-t}$

Hence $q = \int_{t=0}^{t=\infty} i\,dt = \int_0^{\infty} 3e^{-t}\,dt = \left[-3e^{-t}\right]_0^{\infty} = [0 - 3(-1)] = 3$ coulombs

Example 1.4. *Current of three amperes flows through a resistor of 20 ohms. Find : (i) the power absorbed in the resistor ; (ii) energy dissipated in resistor per minute ; and (iii) charge flow through the resistor per minute.*

Solution. Power $P = i^2 \times R = (3)^2 \times 20 = 180$ watts

Energy dissipated per minute

$P \times t = 180 \times 60 = 10{,}800$ joules.

Charge flow per minute

$= q = i \times t = 3 \times 60 = 180$ coulombs.

1.8. The Inductance Parameter

In the year 1820, Oerstead observed that the needle of a mag-netic compass got deflected when current flowed in a neighbouring conductor. This showed that the effect was related to magnetism. He also discovered that the force was directed at right angles to the conductor for carrying the current. French physicist Andre Ampere, in the same year measured this force caused by the current in the conductor and derived a mathematical equation relating the force with the current, charge, distance etc.

With reference to Fig. 1.4, the magnetic field intensity, *i.e.,* the force per unit magnetic pole due to the current i in length dl of the conductor at a point P distant r from the conductor length dl is given by

$$dB = \frac{\mu i \cos \alpha \, dl}{4\pi r^2} \quad \text{...(1.14)}$$

where μ is the magnetic permeability of the medium in which the magnetic field exists.

$\propto$ is the angle that the line joining point with length dl makes with the normal to the conductor drawn form P,

or $$B = \frac{\mu i \cos \alpha}{4\pi r^2} dl \quad \text{...(1.14 } a\text{)}$$

From Eq. (1.14) it may be seen that the magnetic field density in constant at any fixed distance from the conductor. As a conceptual aid, solid lines with arrow-heads may be drawn to give the direction of B as Fig. 1.4 (b). These lines are referred to as the "magnetic field density lines" or the "lines of force".

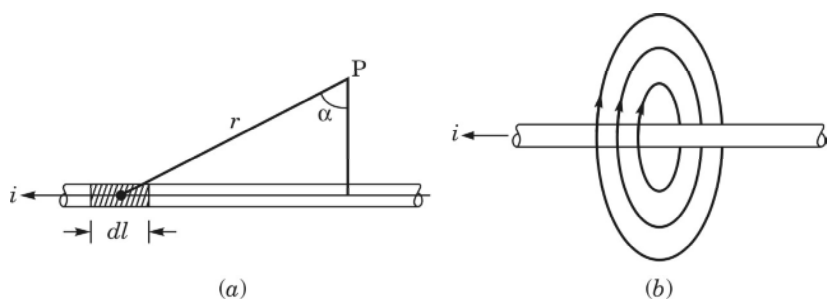


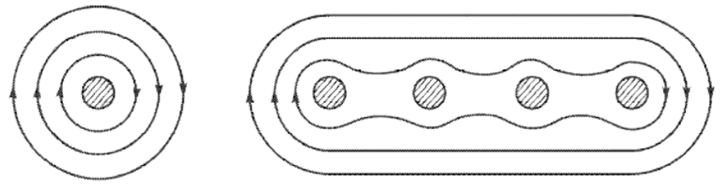
Fig. 1.4. The magnetic field of current carrying conductor.

Fig. 1.5 (*a*) shows cross section of a current carrying conductor along with the lines of force. Circular lines of force indicate that the magnetic field density is constant at constant distance from the conductor. The positions of the lines may be found experimentally by moving a point magnetic pole from place to place in space, while a magnetic compass will show the direction.

The magnetic flux is given by,

$$\phi = \int_S B \cos\theta \, ds \qquad \text{...(1.15)}$$

where θ is the angle between the surface of integration and the field density B.



(*a*) Magnetic field of a single conductor. (*b*) Magnetic field of N parallel conductors.

Fig. 1.5. Magnetic fields of a single conductor and N parallel conductors.

Let us next consider the case of N parallel conductor as shown in Fig. 1.5 (*b*) carrying currents in such directions that the magnetic fluxes add. The $N\,\phi$ flux linkages are said to exist.

The flux linkage is then given by,

$$\psi = N\phi = N \int_S B \cos\theta \, ds \qquad \text{...(1.16)}$$

In case, however, only ϕ_1 lines of flux link N_1 conductors, ϕ_2 lines of flux link N_2 conductors and so on, then the total flux linkage is the algebraic sum of these flux linkages and is, therefore, given by

$$\psi = \sum_{j=1}^{j=n} N_j \, \phi_j \qquad \text{...(1.17)}$$

If the magnetic field density B remains constant, then the total flux linkage ψ also remains constant. If, however, the magnetic field density at any point is varying with time, then a voltage is induced in a conductor placed at the point. This experimental discovery was made by Faraday. He placed conducting circuits in close proximity and found that when the magnetic field produced by one circuit is altered, voltage is induced in the second circuit. A change in the magnetic field may be produced in the following two ways : (*i*) By moving the conductor in space or (*ii*) by changing the current with time.

Faraday found that the voltage induced is proportional to the time rate of change of flux linkage. Thus Faraday's law states that,

$$v = k \cdot \frac{d\psi}{dt} \qquad \text{...(1.18)}$$

where k is the constant of proportionality.

In M.K.S. system, ψ is in weber-turns, t is in seconds, v is in volts and $k = 1$.

Hence the M.K.S. system, Eq. (1.18) giving the Faraday's law reduces to,

$$\psi = \frac{d\psi}{dt} \qquad \text{...(1.19)}$$

Eq. (1.19) may be put in the following integral form :

$$\psi = \int v dt \qquad \text{...(1.20)}$$

Thus flux linkage ψ is time integral of voltage v.

But from Eq. (1.1)

$$\theta = \int i \, dt \qquad \text{...(1.21)}$$

i.e. the charge q is the time integral of current i.

By comparing Eqs. (1.20) and (1.21) we see that what flux linkage ψ is to voltage v, charge q is to current i.

Substituting the value of B from Eq. [1.14 (*a*)] into Eq. (1.16), we get

$$\psi = N\int \left(\int \frac{\mu \, i \cos \alpha \, dl}{4\pi r^2} \right) ds \qquad \text{...(1.22)}$$

Making the assumption that i is constant throughout the length of the conductor, it may be removed from the integral. Hence, we get

$$\psi = \left[N\int \left(\int \frac{\mu \cos \alpha \, dl}{4\pi r^2} \right) ds \right] i \qquad \text{...(1.23)}$$

The integral $N\int \left(\int \frac{\mu \cos \alpha \, dl}{4\pi r^2} \right) ds$ is defined as the "inductance parameter" or the coefficient of inductance and may be computed mathematically for simple systems of current carrying conductors or may be obtained experimentally by measuring ψ and i.

If the flux linkage ψ and the current i refer to the same physical system, the parameter is called the self-inductance and is symbolised by letter L. Then from Eq. (1.23),

$$\psi = Li \qquad \text{...(1.24)}$$

where

$$L = N\int \left(\int \frac{\mu \cos \alpha \, dl}{4\pi r^2} \right) ds \qquad \text{...(1.25)}$$

On the other hand, if a current i_1 in one circuit produces flux linkage ψ_2 in another circuit, the parameter is called the "mutual inductance", symbolized by letter M. We then get

$$\psi_2 = M_{21} \cdot i_1 \qquad \text{...(1.26)}$$

Substituting Eq. (1.24) into Eq. (1.19), we get

$$v = \frac{d}{dt}(Li) \qquad \text{...(1.27)}$$

If the inductance L does not vary with time, Eq. (1.27) reduces to,

$$v = L\frac{d}{dt} \qquad \text{...(1.28)}$$

Eq. (1.28) on integration yields,

$$i = 1/L \int v dt \qquad \text{...(1.29)}$$

For mutual inductance,

$$v = \frac{d}{dt}(Mi) \qquad \text{...(1.30)}$$

If the mutual inductance M does not vary with time,

$$v = M\frac{di}{dt} \qquad \text{...(1.31)}$$

Eq. (1.31) may be put in the following integral form :

$$i = \frac{1}{M}\int v dt \qquad ...(1.32)$$

The M.K.S. unit of inductance is henry.

Eq. (1.28) may also be written as

$$v = L\frac{d}{dt}\left(\frac{dq}{dt}\right)$$

$$= L \cdot \frac{d^2q}{dt^2} \qquad ...(1.33)$$

From the relation $\psi = Li$, we see that the product Li cannot change instantaneously, since an instantaneous change in ψ would mean an infinitely large voltage, which is an impossibility in any physical circuit. Let the product Li or ψ change by finite amount in interval of time $\Delta t = t_2 - t_1$ as shown in Fig. 1.6. Then time interval Δt cannot be zero. Eq. (1.20) may be put in the definite integral form :

$$\psi = \psi_0 + \int_0^t v dt \qquad ...(1.34)$$

The integral portion of this equation can not have a finite value in zero time with finite v, *i.e.*

$$\lim_{t \to 0}\int_0^t v dt = 0 \quad v \neq \infty \qquad ...(1.35)$$

This integration process is shown in Fig. 1.7 as summation of infinitesimal areas, v in height and dt in width. The interval from $t = 0$ to $t = t_1$ must be greater than zero for any area to be summed.

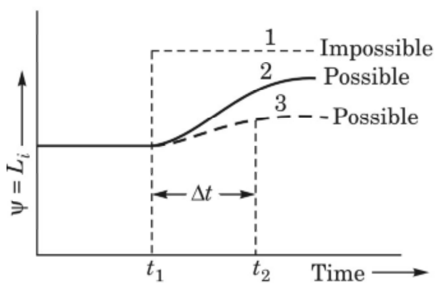


Fig. 1.6 Change of Li with time.

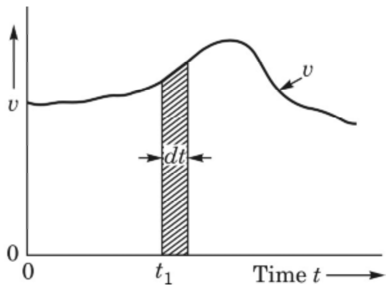


Fig. 1.7. Integration of voltage.

Eq. (1.35) aids in visualizing the requirement that the flux linkage ψ in an inductive system cannot change in zero time. Of course, either inductance L or current i may change instantaneously so as to maintain the product of L and i constant,

i.e. provided $\quad L_1 i_1 = L_2 . i_2 \qquad ...(1.36)$

where subscripts 1 and 2 refer to conditions existing at time t_1 and t_2 extremely close to each other. This is called the "principle of constant flux linkage" and is similar to the "principle of conservation of momentum" in mechanics.

Thus in mechanics, Newton's law of force gives :

Force $\quad F = \frac{d}{dt}Mv \qquad ...(1.37)$

where M is the mass, v is the velocity and Mv is the momentum.

Momentum Mv of a system cannot change instantaneously.

From Eq. (1.36) we conclude the "if inductance remains constant, current cannot change instantaneously".

Power in an inductive element is given by,

$$P = vi = Li\frac{di}{dt} \qquad ...(1.38)$$

If L is in henry, i is in amperes and t in seconds, then power P is in watts.

Hence energy in an inductance is given by,

$$W = \int P dt = \int Li\frac{di}{dt}dt$$

$$= \int Li di = \frac{1}{2}Li^2 \qquad ...(1.39)$$

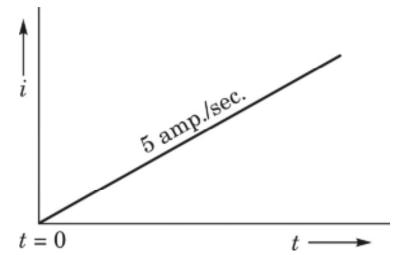
If L and i are respectively in henrys and amperes, then energy W is in joules.

Eq. (1.39) may be put as :

$$W = \frac{1}{2}\frac{\psi^2}{L} \text{ joules} \qquad ...(1.40)$$

This energy W is said to be "stored in the magnetic field" set up by the current through the inductor element. In this regard, inductance is analogous to mass or inertia in a mechanical system.

Example 1.5. *The current in a 3 henry inductor varies as shown in following figure. Find the following quantities after the current has flown for two seconds : (i) flux linkage in the system (ii) the time rate of change of flux linkages in the system and (iii) the quantity of charge having passed through the inductor.*



Solution. (*i*) After 2 seconds, current i = 1 amp.

Hence flux linkages $\psi = Li$

= 3 henry × 1 amp. = 3 henry-amp. = 3 weber-turn.

(*ii*) After 2 seconds, di/dt still remains constant at the value 0.5 amp/second.

Hence $$\frac{d\psi}{dt} = L\frac{di}{dt}$$

= 3 henry × 0.5 amp/sec. = 1.5 weber-turns/sec.

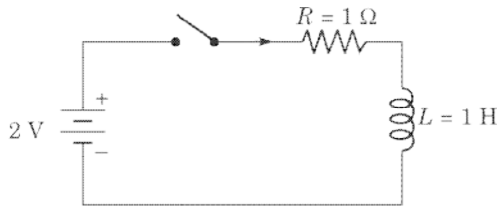
(*iii*) During 2 seconds, the charge that has passed through the inductor is given by,

$$q = \int_0^2 i di = \int_0^2 kt\, dt = \left[\frac{1}{2}kt^2\right]_0^2$$

where $k = 0.5$ amp/sec.

Hence $\quad q = \dfrac{1}{2} \times 0.5$ amp/sec $\times$ (2 sec)^2 = 1 amp/sec = 1 coulomb.

Example 1.6. *In the given circuit, switch K is closed at time t = 0. The current in the circuit is given by the equation : $i = 2\ (1 - e^{-t})$ amp, $t > 0$. At time t = 1 second, find (i) value of current (ii) rate of change of current (iii) total flux linkage (iv) rate of change of flux linkages (v) voltage across the inductor (vi) voltage across the resistor (vii) energy stored in the magnetic field of the inductor (viii) rate at which energy is being stored in the magnetic field of the inductor (ix) rate at which energy is being dissipated as heat and (x) rate at which the battery is supplying the energy.*



Solution.

At time $\quad t = 1$ second,

(*i*) $\quad i = 2\ (1 - e^{-t})$ amp. $= 2\left(1 - \dfrac{1}{e}\right) = 1.264$ amp.

(*ii*) $\quad \dfrac{di}{dt} = 2(-e^{-t})\ (-1) = 2e^{-t} = 2e^{-1} = 0.7357$ amp/sec.

(*iii*) Total flux linkages

$= Li = 1$ henry $\times$ 1.264 amp. = 1.264 weber turns.

(*iv*) Rate of change of flux linkage

$= L\ \dfrac{di}{dt} = 1$ henry $\times$ 1.7357 amp/sec. = 0.7357 weber-turn/sec.

(*v*) Voltage across the inductor

$= \dfrac{d}{dt}\ (Li) = 0.7357$ volt.

(*vi*) Energy stored in the magnetic field

$= \dfrac{1}{2}\ Li^2 = \dfrac{1}{2} \times 1 \times (1.264)^2 = 0.7987$ joule.

(*vii*) Voltage across the resistor

$= i \times R = 1.264$ amp $\times$ 1 ohm = 1.264 volts.

(*viii*) Rate of storage of energy in the magnetic field

= Voltage across the inductor × Current = 0.7357 × 1.264 watt

= 0.93 watt.

(*ix*) Rate of energy dissipation in resistor

= Power in resistor = Voltage across resistor × Current

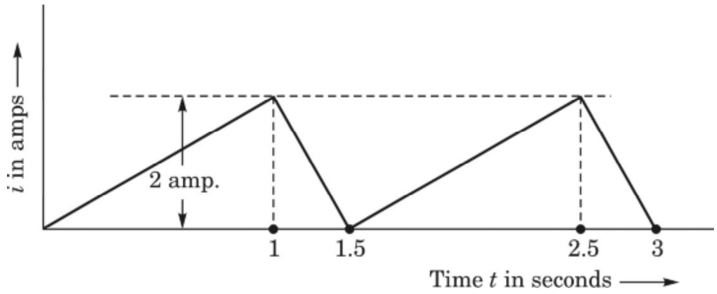
= 1.264 volts × 1.264 amp. = 1.597 watts.

(*x*) Rate of supply of energy by battery

$= \dfrac{d}{dt}$ (energy) = Voltage × Current

= 2 × 1.264 watts = 2.528 watts.

Example 1.7. *The following figure gives the variation of current i in an inductor of 2 henry. Calculate the voltage across the inductor and the charge in inductor at time t = 1 and 1.5 seconds.*



Solution. During time $t = 0$ to $t = 1$, current $i = 2t$ amps and $\frac{di}{dt} = 2$ amp/sec.

Voltage across the inductor

$$= L\frac{di}{dt} = 2 \times 2 = 4 \text{ volts}$$

Charge $\quad q = \int idt = \int 2\,t\,dt = t^2$

At, $\quad t = 1,\ q = (1)^2 = 1$ coulomb.

During $\quad t = 1$ to $t = 1.5$ seconds,

$$\frac{di}{dt} = \frac{-2 \text{ amps.}}{0.5 \text{ sec.}} = -4 \text{ amps./sec.}$$

Voltage across the inductor

$$= L\frac{di}{dt} = 2(-4) = -8 \text{ volts.}$$

Current $\quad i = 2 - 4\,(t - 1) = (6 - 4t)$

At time $t = 1.5$ seconds, charge q is given by

$$q = 1 + \int_{1.0}^{1.5} idt = 1 + \int_{1.0}^{1.5} (6 - 4t)\,dt = 1 + \left[\frac{(6-4t)^2}{2}\left(-\frac{1}{4}\right)\right]_{1.0}^{1.5} = 1.5 \text{ coulombs.}$$

1.9. The Capacitance Parameter

If there be charges q_1 and q_2 on two objects, placed say a distance r apart, as shown in Fig. 1.8, there results a force between the two objects. This forms a fundamental property of nature. Further it is found that the like charges repel while unlike charges attract each other. The force of attraction F is given by the relation,

$$F = \frac{-q_1 q_2}{4\pi\varepsilon r^2} \qquad ...(1.41)$$

where F is the force in newtons,

r is the distance in metres between the two point charges q_1 and q_2,

q_1 and q_2 are charges expressed in coulombs, and ε is the permittivity (For free space, $\varepsilon = 8.854 \times 10^{-12}$ Farad per metre in M.K.S. units).

Eq. (1.41) is true for point charges only. The same may be applied to find the force between two objects and geometry or known charge distribution by vectorially adding all the forces.

Consider a unit positive charge placed in the space between these two objects carrying charges q_1 and q_2 of opposite polarities.

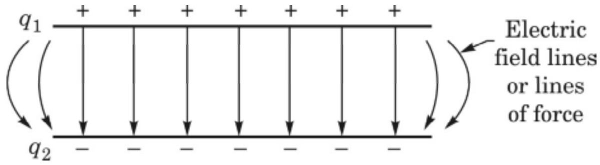


Fig. 1.8. Electric field between two charged objects.

The force exerted on this unit positive charge is vector quantity and is called the "electric field". If $q_1 = q_2 = q$ say, then the electric field is given by

$$E = \frac{F}{q} \qquad \text{...(1.42)}$$

As a conceptual aid, often electric field is represented by line drawn in the direction of the field, *i.e.,* in the direction in which force will be exerted on a unit positive charge. Fig. 1.8 gives such electric field line or lines of force for parallel plane plates carrying charges q_1 and q_2. It may, of course, be borne in mind that these lines of force are simply conceptual aids and are not actually present.

The voltage v between the plates, *i.e.* the work done in moving a unit charge from one plate to the other is given by,

$$v = \int E \cos\theta \, dt \qquad \text{...(1.43)}$$

where dr is the increment of distance between the plates.

and θ is the angle between the force and the direction of movement of r.

Substituting the value of force F from Eq. (1.41) into Eq. (1.43) and putting $q_1 = q_2 = q$, we get

$$v = \int \frac{q \cos\theta}{4\pi\varepsilon r^2} dr \qquad \text{...(1.44)}$$

or

$$v = \left(-\int \frac{\cos\theta}{4\pi\varepsilon r^2} dr\right) q \qquad \text{...(1.45)}$$

The quantity within the brackets may be evaluated for simple geometry of the plates or bodies carrying charges and may be obtained experimentally by measuring q and $\boldsymbol{v}$. For a given geometry, the integral is a constant and is called "elastance", denoted by the letter S. Hence from Eq. (1.45)

$$v = Sq \qquad \text{...(1.46)}$$

where

$$S = -\int \frac{\cos\theta}{4\pi\varepsilon r^2} dr \qquad \text{...(1.47)}$$

The reciprocal of elastance is the capacitance denoted by letter C.

Thus

$$C = \frac{1}{S} \qquad \text{...(1.48)}$$

Hence from Eqs. (1.46) and (1.48), we get

$$q = Cv \qquad ...(1.49)$$

If q is in coulombs and v is the volts, capacitance C is given in "farads". The unit of elastance is daraf (farad spelled backwards). The quantity C, *i.e.* the capacitance forms a circuit parameter of the system.

Let there be an initial charge q_0 on the system, shown in Fig. 1.8 and let the charge increase linearly with time at the rate of k coulombs/second. Then charge on the system at any time t is given by,

$$q = q_0 + kt \qquad ...(1.50)$$

The current is then given by,

$$i = \frac{dq}{dt} = \frac{d}{dt}(q_0 + kt)$$

or

$$i = k \qquad ...(1.51)$$

We thus conclude that the current in a capacitive system is independent of the initial charge on the system but depends entirely on the rate of accumulation of charge.

Starting once again from the equation

$$i = \frac{dq}{dt}, \text{ we get}$$

$$dq = i\ dt \qquad ...(1.52)$$

On integration $\displaystyle\int_{q_0}^{q} dq = \int_0^t i\,dt$

$$d = q_0 + \int_0^t i\,dt \qquad ...(1.53)$$

From Eq. (1.52) we see that the total charge q on a capacitive system is equal to the initial charge q_0 plus the additional charge

$\displaystyle\int_0^t i\,dt$ deposited by the flow of current i for time t.

In order to find a relationship between current and voltage in capacitive system, we revert to the equation $q = Cv$. Current i is given by,

$$i = \frac{dq}{dt} = \frac{d}{dt}(Cv) \qquad ...(1.54)$$

If the capacitance C does not vary with time, then

$$i = C\frac{dv}{dt} \qquad ...(1.55)$$

On the other hand, if C is a time varying quantity, then we get

$$i = \frac{d}{dt}(Cv) = C\frac{dv}{dt} + v\frac{dC}{dt} \qquad ...(1.56)$$

Equations (1.55) and (1.56) give current i in terms of v or its derivative.

Similary from Eq. (1.48), we get

$$v = \frac{q}{C} = \frac{1}{C}\int i\,dt = S\int i\,dt \qquad ...(1.57)$$

Equation (1.57) gives voltage v across the capacitive system in terms of (*i*) integral of current i and (*ii*) circuit capacitance C or elastance S.

Voltage of a capacitive system cannot change instantaneously. Let us assume that the product Cv of a capacitive system has changed instantaneously.

Then

$$\frac{d}{dt}(Cv) = \infty \quad \text{or} \quad \frac{dq}{dt} = \infty \quad \text{or} \quad i = \infty.$$

We thus see that an instantaneous change in product Cv implies an infinite current, a physical impossibility. Thus with reference to Fig. 1.9, if the q or product Cv changes by a finite amount in time Δt, then this Δt cannot be made zero. Thus curve 1 is inadmissible while curves 2 and 3 are admissible.

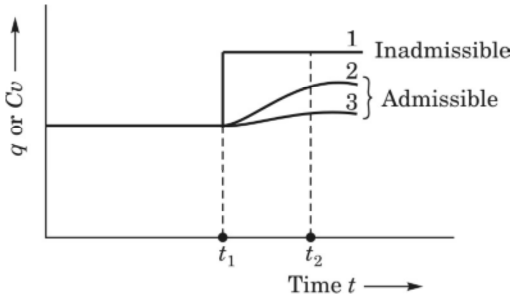


Fig. 1.9. Change of q or Cv with time.

We have already seen that,

$$q = q_0 + \int_0^t i\,dt \qquad \text{...(1.53)}$$

With reference to Eq. (1.53), the above statement implies that for any finite value of current i, *i.e.* for $i = \infty$.

$$\lim_{t \to 0} \int_0^t i\,dt = 0 \qquad \text{...(1.58)}$$

Fig. 1.10 shows a plot of current i as a function of time t for an arbitrary capacitive system. The area under the curve gives the charge added to the system, *i.e.* the quantity $\int_0^t i\,dt$. The integration process then amounts to summation of infinitesimal areas i in height and dt in width. Eq. (1.58), then implies that the interval from $t = 0$ to $t = t_1$ must be greater than zero for any area to be summed.

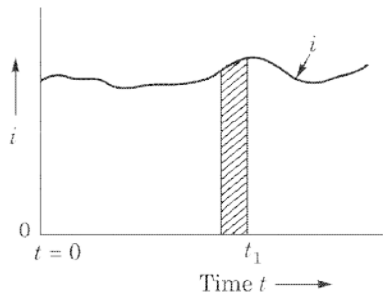


Fig. 1.10. Summation of current i to give addition of charge.

Thus we clearly establish that the product Cv or charge q in a capacitive system cannot either increase or decrease in zero time. But in most of the systems, capacitance does not change with time. Hence the above discussion establishes that "the voltage of a capacitive system cannot change instantaneously."

Assuming that capacitance C of a capacitor is time invariant, power in the capacitive element is given by the relation,

$$P = vi = vC\frac{dv}{dt} \text{ watts} \qquad \text{...(1.59)}$$

and the energy E is given by,

$$E = \int Pdt = \int vCdv$$

$$= \frac{1}{2} Cv^2 \text{ joules} \quad \text{...(1.60)}$$

This energy E is stored in the electric field set up by the charges on the capacitive elements. If, however, voltage v is constant and capacitance C is time variant, then power in the capacitive element is given by,

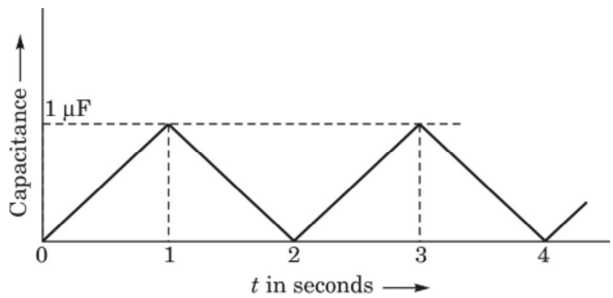
$$P = vi = V \frac{dq}{dt}$$

$$= V \cdot V \frac{dc}{dt} \text{ watts} \quad \text{...(1.61)}$$

and energy E is given by,

$$B = \int Pdt = \int V^2 dc = V^2 c \quad \text{...(1.62)}$$

Example 1.8. A *condenser has one plate so rotated that the capacitance varies as shown in the following figure. A battery of constant voltage 10 volts is connected across the condenser. Find (i) current during time t = 0 to t = 1 second, (ii) Charge accumulated across the condenser at t = 1 second (iii) power in the condenser at t = 1 second and (iv) energy stored in the condenser at t = 1 second.*



Solution.

(*i*) Current $i = \frac{dq}{dt} = \frac{d}{dt}(cV) = V\frac{dc}{dt}$

During time $t = 0$ to $t = 1$,

$$\frac{dc}{dt} = 1 \ \mu \text{ F/sec}$$

Hence $i = V \times 1 \ \mu\text{F/sec}$

$= 10 \text{ volts} \times 10^{-6} \text{ Farad/sec} = 10^{-5} \text{ amp.}$

(*ii*) $q = cv$

At $t = 1$ second

$c = 1 \ \mu\text{F}.$

Hence $q = 10^{-6}$ Farad $\times$ 10 volts $= 10^{-5}$ coulombs.

(*iii*) Power $P = Vi = V \cdot \frac{dq}{dt} = V \cdot \frac{d}{dt}(cV) = V^2 \cdot \frac{dc}{dt}$

$$\frac{dc}{dt} = 1 \ \mu\text{F/sec.}$$

Hence during time $t = 0$ to $t = 1$ second.

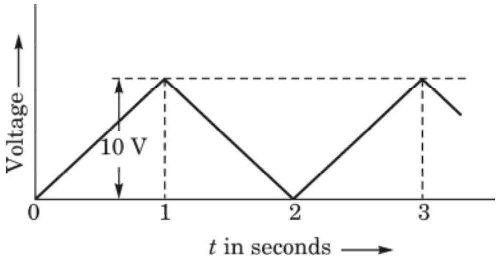
Power $= V^2 \times 1 \ \mu\text{F/sec} = (10^2) \times 10^{-6}$ watt $= 10^{-4}$ watt.

(*iv*) Energy stored $= \int Pdt = \int V^2 \, dc = V^2 c$

At $t = 1$ sec, $c = 1 \ \mu$F.

Hence energy $= (10)^2 \times 10^{-6} = 10^{-4}$ joule.

Example 1.9. *To a 2μF condenser is applied a voltage of the form shown in the following figure. Find (i) the current during time t = 0 to t = 1 second, (ii) charge accumulated across the condenser at t = 1 second, (iii) power in the condenser at t = 1 second and (iv) energy stored in the condenser at t = 1 second.*



Solution.

(*i*) Current $i = \frac{dq}{dt} = \frac{d}{dt}(Cv) = V\frac{dv}{dt}$

During time $t = 0$ to $t = 1$ second

$$\frac{dv}{dt} = \frac{10 \text{ volts}}{1 \text{ sec}} = 10 \text{ volts/sec.}$$

Hence $i = C\frac{dv}{dt} = 2 \times 10^{-6} \times 10$ amp. $= 20$ μA.

(*ii*) At $t = 1$ second, $v = 10$ volts.

Hence $q = Cv = 2 \times 10^{-6} \times 10 = 2 \times 10^{-5}$ coulomb

(*iii*) Power $= v \times i = v\frac{dq}{dt} = v\frac{d}{dt}(Cv) = vC\frac{dv}{dt}$

At $t = 1$,

$$\frac{dv}{dt} = 10 \text{ volts}$$

and $v = 10$ volts/sec.

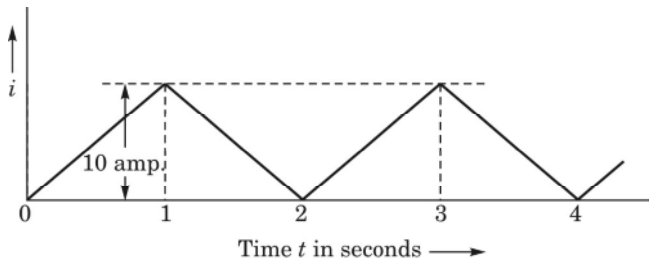
Hence power $= 10 \times 2 \times 10^{-6} \times 10 = 2 \times 10^{-4}$ watts.

(*iv*) Energy stored $= \int P dt = \int vC\frac{dv}{dt} . dt = \int vC\, dv = C\int v\, dv = \frac{1}{2}Cv^2$

At $t = 1$ second,

$$\text{energy} = \frac{1}{2} \times 2 \times 10^{-6} \times (10)^2 = 10^{-4} \text{ joule.}$$

Example 1.10. *A current shown in the following figure flows through a 10 μF condenser. Calculate voltage, charge, power and energy stored at time t = 1 and 2 seconds.*



Solution. $q = Cv$

Hence $v = \frac{1}{C}q = \frac{1}{C}\int i\, dt$

During time t = *0 to t = 1 second,*

$i = 10\,t$ amps, where t is in seconds.

Hence $v = \frac{1}{C}\int 10\,t\, dt = \frac{10}{C} . \frac{t^2}{2} = \frac{5t^2}{C}$

At $t = 1$ second,

$$v = \frac{5}{10^{-5}} = 5 \times 10^5 \text{ volts.}$$

Charge $q = \int i\, dt = \int 10\, t = 10\, \frac{t^2}{2} = 5 \times 1 = 5$ coulombs.

Power $= vi = \frac{5t^2}{C} \times 10\, t = \frac{50t^3}{C}$

At $t = 1$ second,

$$\text{Power} = \frac{50}{10^{-5}} = 5 \times 10^6 \text{ watts}$$

$$\text{Energy} = \int P\, dt = \int \frac{50t^3}{C}\, dt = \frac{50}{C} \cdot \frac{t^4}{4} = \frac{25t^4}{2C}$$

At $t = 1$ second,

$$\text{Energy} = \frac{25}{2 \times 10^{-5}} = 12.5 \times 10^5 \text{ joules.}$$

During time $t = 1$ to $t = 2$ seconds :

$$i = 10 - 10\,(t - 1) = 20 - 10t$$

Charge $q = q_{t-1} + \int i\, dt = 5 + \int (20 - 10t)\, dt = 5 + \left[20t = 5t^2\right]_{t=1}^{t=t}$

At $t = 2$, $q = 5 + [(40 - 20) - (20 - 5)] = 10$ coulombs.

Hence at $t = 2$ seconds,

$$v = \frac{q}{C} = \frac{10}{10^{-5}} = 10^6 \text{ volts.}$$

$$\text{Power} = vi = 10^{-6} \times 0 = 0$$

$$\text{Energy} = \int P\, dt = \int vi\, dt$$

$$= E_{t=1} + \int_{t=1}^{t=2} (20 - 10t)\, \frac{1}{C}\, (5 + 20t - 5t^2)\, dt$$

$$= 12.5 \times 10^5 + \frac{50}{C} \int_{t=1}^{t=2} (2 - t)\,(1 + 4t - t^2)\, dt$$

$$= 12.5 \times 10^5 + \frac{50}{C} \int (2 + 7t - 6t^2 + t^3)\, dt$$

$$= 12.5 \times 10^5 \times \frac{50}{C} \left[2t + \frac{7t^2}{2} - 2t^3 + \frac{t^4}{4}\right]_{t=1}^{t=2}$$

$$= 12.5 \times 10^5 + 50 \times 10^5\, [2.25] = 10^5\, (12.5 + 112.5) = 125 \times 10^5 \text{ joules.}$$

1.10. Energy Sources

By energy source, used in this context, is meant a device for generating electrical energy. Of course such generation of electrical energy is done by transformation from some other type of energy. Energy sources form active network elements.

Energy sources are classified in accordance with their current-voltage characteristics. Thus we have two types of energy sources. One category of energy sources approximate "ideal voltage sources" while the other category of sources approximate "ideal current sources".

Ideal voltage source. An ideal voltage source generates volt-age of a given time variation but neither the magnitude nor the time variation of the generated voltage changes with the magnitude of the current drawn from it. Thus the terminal voltage of this source remains constant for all values of output current from zero current condition to blunt short circuit. Evidently such a performance can be achieved by an energy source only when it has zero internal resistance, inductance and capacitance. The symbol for ideal voltage surce is generally a circle with polarity marks + and – denoting the positive and negative terminals of the source as shown in Fig. 1.11(*a*) and (*b*). The lower case letter *v* indicates a time varying source while the upper case letter *V* indicates a time invariant source. An approximate time variation of voltage is sometimes sketched within the circle. Thus the symbol having ~ placed within the circle indicates a source of sinusoidal voltage. However a battery is generally symbolized as in Fig. 1.11 (*c*).

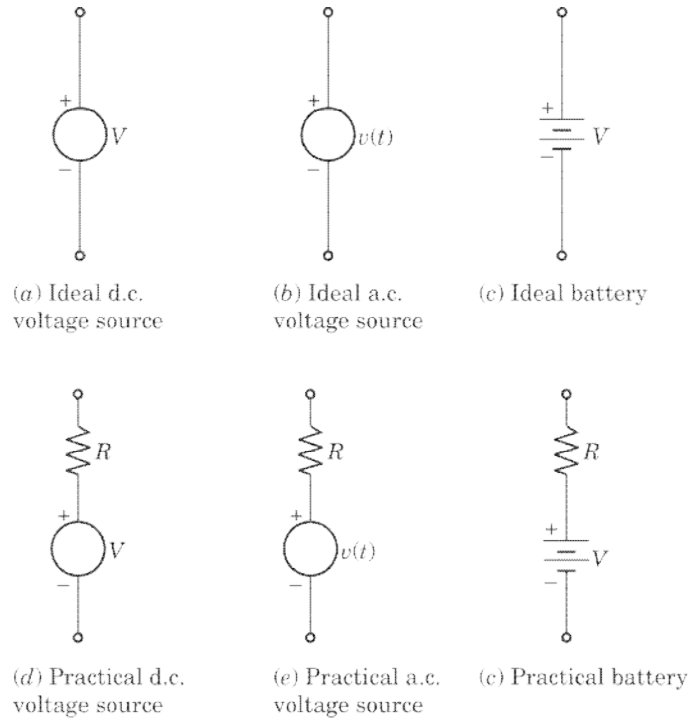


Fig. 1.11. Symbols for ideal and practical voltage source.

Almost all practical voltage sources fall short of the ideal and their terminal voltage falls with increase in the output current. However in most of the cases, a practical voltage source may be approximated as an ideal voltage source with a series resistance in case of d.c. sources or with a series impedance (usually inductor) in the case of a.c. voltage source. This series resistor then accounts for the fall in terminal voltage with increase of output current. Symbol for a practical voltage source is accordingly the same as for an ideal voltage source with a series resistor *R* (or impedance *Z*) as shown in Fig. 1.11(*d*), (*e*) and (*f*).

Ideal current source. An ideal current source generates current of a given time variation but neither the magnitude nor the time variation of the generated current changes with the load. Thus the output current of this source remains constant for all values of load ranging from zero resistance to infinite resistance. If this ideal current source gives zero output current, the source reduces to just an open circuit. The requirement of constant current in an ideal current source can be satisfied only provided it has zero internal resistance, inductance and capacitance. The symbol for ideal current source is generally a circle with an associated arrow indicating the positive direction of current flow as shown in Fig. 1.12 (*a*) and (*b*). In this case also, the lower case letter *i* or *i* (*t*) indicates the time varying source while the upper case letter *I* indicates the time invariant source. Again approximate time variation of current may be sketched within the circle. Sometimes a rectangle instead of a circle is used to symbolize current source, in order to clearly differentiate it from voltage source.

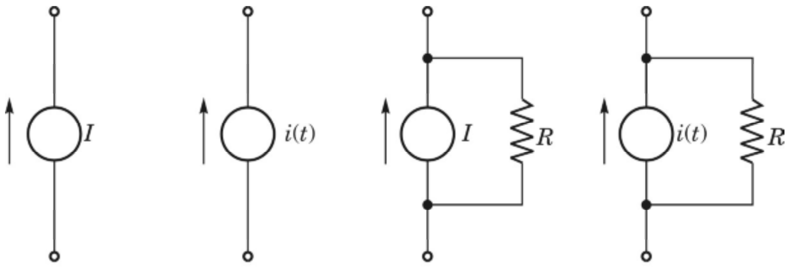


Fig. 1.12. Symbols for ideal and practical current sources.

Again all practical current sources fall short of the ideal and their output current falls with the increase of load resistance (or impedance). Accordingly a practical current generator may be approximated as an ideal current generator with a shunt resistor in case of d.c. current source or a shunt impedance in case of a.c. current source. This shunt resistor then accounts for the fall of output load current with increase of load resistance. Symbol for a practical current source is the same as that for the corresponding ideal current source with a shunt resistance (impedance) as shown in Fig. 1.12 (*c*) and (*d*).

Photo-electric cell and pentode vacuum tube amplifiers form practical current generators of common occurrence in electronic circuits.

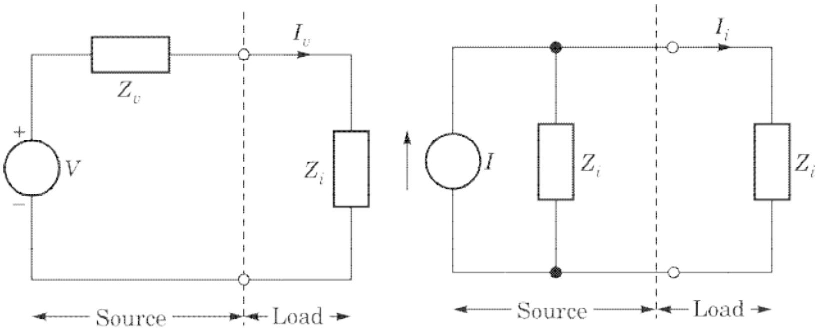
Although ideal current and voltage sources form two altogether different types of sources, practical sources cannot be so distinctly classified. In fact any source with suitable associated regulator circuit may be made to behave as constant voltage generator or constant current generator, closely approaching the ideal condition in either of the cases. The distinction made here is primarily for facility in network analysis and any particular energy source is treated either as a voltage source or a current source depending upon which type facilitates analysis of the network under consideration. Thus a practical voltage source may be transformed into its equivalent practical current source or *vice versa*.

1.11. Transformation of Energy Sources

Any practical energy source may be represented by either constant voltage source or constant current source. It is possible, therefore, to transform a practical voltage source into a current source and *vice versa*. Thus in Fig. 1.13 parts (*a*) and (*b*) show respectively the constant voltage source and constant current source with associated internal impedances Z_v and Z_i respectively. Each source drives a current through the load impedance Z_l.

Then with the voltage source connected to the load impedance Z_l, load current I_v through Z_l is given by,

$$I_v = \frac{V}{Z_v + Z_l} \qquad \text{...(1.63)}$$



(a) Voltage source (b) Current source

Fig. 1.13. Transformation of energy sources.

With current source connected to Z_l, the load current I_i through Z_l is given by,

$$I_i = \frac{Z_i}{Z_i + Z_l} . I \quad ...(1.64)$$

If the two energy sources are equivalent, then the load current I_v and I_i must be equal. Equating I_v to I_i, from Eqs. (1.63) and (1.64), we get

$$\frac{V}{Z_v + Z_l} = \frac{Z_i T}{Z_i + Z_l} \quad ...(1.65)$$

Also $\quad V = Z_i .I \quad ...(1.66)$

From Eqs. (1.65) and (1.66)

$$Z_v + Z_l = Z_i + Z_l$$

or $\quad Z_v = Z_i.$

Thus a constant voltage source of voltage V and series impedance Z may be equated to a constant current source of current I in parallel with an impedance Z, where $I = V/Z$.

1.12. Approximations in Circuit Representation

We have so far discussed the response of pure circuit elements namely resistance, inductance and capacitance. In analysing electrical systems we utilize the circuit concept and represent a physical system in terms of network parameters. The physical system involved may be an antenna, a transmission line or an electric motor. For the purpose of analysis, we study the properties of the physical system and replace it by equivalent circuit parameters. In so doing we put an inductor in the equivalent network if we observe the system to possess inductive effect. Similarly we may put in the equivalent network, a resistance and capacitance to represent resistive and capacitive effect respectively. These various elements then correspond to the various effects observed. Their magnitudes and placement in the equivalent network is also carefully decided.

Thus in an equivalent network, we account for all the large or prominent effects observed in the system. But there always remain a few smaller or secondary effects which often remain unrepresented in the equivalent electrical network. Thus an approximation has been made in representing the physical system. Obviously this lowers the accuracy of the results obtained after making calculations based on these approximate equivalent circuit.

To what extent should approximations be made ? Putting it otherwise, to what limit should the secondary effects be included in the equivalent network ? This decision entails engineering judgements and should be made considering the values of these secondary effects relative to the principal effects and the order of accuracy desired.

Again in the case of commercial components, such as resistors, inductors and capacitors, approximations are often involved in their representation in the equivalent circuit. Thus an inductor never behaves as a pure inductance. In most circumstances, it possesses unwanted resistive as well as capacitive effects. In several cases, these unwanted parasitic effects may be neglected in the equivalent network. Whether such an approximation should be made or not again forms an engineering decision.

In deriving expressions for *R, L* and *C* parameters, done in the earlier articles, we have assumed that (*i*) the charge does not vary with the dimensions of the conductor and (*ii*) the current does not vary with the length or the cross-sectional area of the conductor. These assumptions are not true for periodic currents flowing for brief intervals of time. Parameters calculated for these charged conditions will differ from the general parameter calculated under assumption of uniform current and charge. For accurate results, these parameters should be calculated for each type of current waveform. However, in most of the cases, an approximation is made that parameter values are the same as for non-varying or static conditions.

Further it is assumed that the system is linear. This forms a good approximation for most of the elements operating within their nominal operating range. Even vacuum tube, though basically non-linear, are considered linear for certain analysis if operated over a restricted range.

Again we assume the elements to be bilateral and lumped. Also as an approximation we assume that there is no interaction between electric and magnetic field, *i.e.* there is no radiation of energy.

1.13. Mutual Inductance

When two or more inductors are placed close together, there may result a mutual interaction of the magnetic fields to these inductors. This results in the mutual inductance parameters. We will first examine the case of two inductors mutually coupled and then subsequently examine the general case of n (n is an integer > 1) inductors mutually coupled.

1.13.1. Two Mutually Coupled Inductors and Dot Convention

Fig. 1.14 (*a*) shows two inductors wound on a common core. The magnetic field produced by current i_1 flowing in one coil, say coil 1 induces a voltage in the other coil. The coils are then said to be coupled and the windings constitute a transformer. The diagram also shows the detailed distribution of the windings on the core. Fig. 1.14 (*b*) gives the corresponding circuit diagram. Knowing the details of transformer construction for a given current i_1 in one coil, it is possible to compute magnitude and direction of the voltage induced in the other winding. The need for giving detailed construction as on a blue print is obviated by using the characterizing factor namely the coefficient of mutual inductance *M*. Manufacturers usually mark one end of each winding with a dot or some other symbol. These dots are equivalent to details of construction as far as the direction of the magnetic fluxes caused by currents entering the coils at the dotted ends are concerned, *i.e.* as far as the voltage directions are concerned. We will now discuss the meaning of these dot markings.

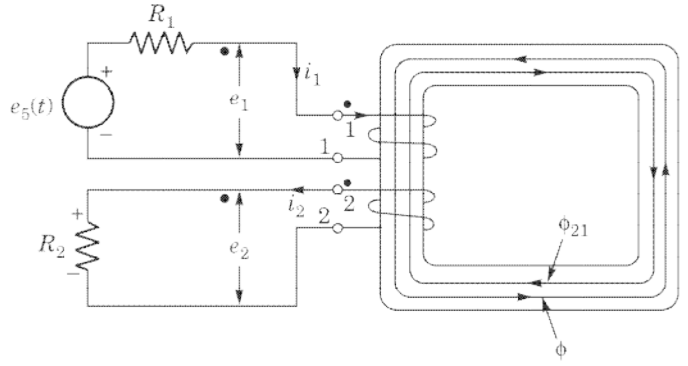
In Fig. 1.14 (*a*), a time varying voltage source of voltage $e_s(t)$ is connected to the winding 1-1 (now called the primary winding). At given instant of time, the voltage e_s has the polarity shown in Fig. 1.14 (*a*). The resulting current i_1 in the primary flows in the direction shown and enters the primary winding at the end marked with a dot.

The resulting phenomena are outlined below in steps :

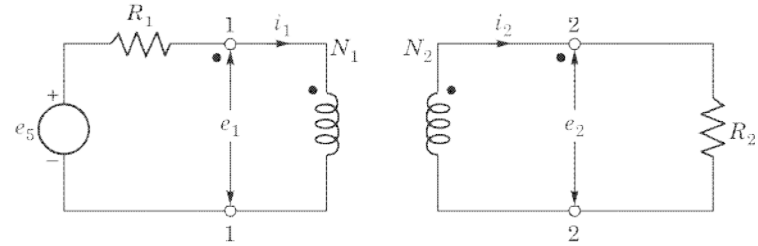
(*i*) *Field.* The current i_1 in primary winding 1-1 causes a magnetic field which is chiefly concentrated along the axis of the coil.

This field is given by,

$$dB = \frac{\mu i_1 dl \cos \alpha}{4\pi r^2} \qquad ...(1.67)$$



(*a*) Two winding magnetic circuit.



(*b*) Circuit diagram.

Fig. 1.14. Two winding magnetic circuit and its equivalent circuit diagram.

(*ii*) *Flux.* The magnetic flux associated with the magnetic field is given by,

$$\phi = \int_A B \cos \theta \, d\theta \qquad ...(1.68)$$

Direction of this flux is determined by the right handed screw rule, *i.e.* if the current is along the longitudinal motion of the screw, the flux is in the direction given by the circular motion of the screw. Of course almost the entire flux is confined to the magnetic core. Applying the above rule, the flux in the core is in direction shown by the arrow giving ϕ_{21}.

(*iii*) *Flux linkage.* The flux produced in the core due to current in the winding 1-1 links with the winding 2-2. Let the flux which so links with winding 2-2 be denoted by ϕ_{21}, where the subscripts 2 and 1 correspond to the order "effect-cause".

In this case, the same flux ϕ_{21} links with both the windings 1-1 and 2-2.

The magnitude of flux linkage with winding 1-1 is :

$$\psi_1 = N_1 . \phi_{21} \qquad ...(1.69)$$

where N_1 is the number of turns on the primary winding 1-1.

By Faraday's law, flux linkage ψ_1 is related with the voltage e_1 at terminals 1-1 by the relation :

$$\phi_1 = \int e_1 \, . \, dt \qquad ...(1.70)$$

From Eqs. (1.69) and (1.70), we get

$$\phi_{21} = \frac{\psi_1}{N_1} = \frac{1}{N_1} \int e_1 \, dt \qquad ...(1.71)$$

(*iv*) *Voltage induced in the secondary.* Since I_1 is a time varying quantity, ϕ_{21} is also time varying. Hence a voltage is induced in winding 2-2.

Flux linkage in winding 2-2 = $\psi_2 = N_2 \, \phi_{21}$ ...(1.72)

Hence according to Faraday's law, voltage e_2 induced in the winding 2-2 is given by,

$$e_2 = \frac{d\psi_2}{dt} \qquad ...(1.73)$$

The coefficient of mutual inductance relates flux linkage ψ_2 in the secondary winding 2-2 with the current i_1 in the primary winding 1-1.

Thus $\quad \psi_2 = M_{21} \, . \, i_1$ ...(1.74)

Hence $\quad e_2 = \dfrac{d\psi_2}{dt} = M_{21} \, . \, \dfrac{di_1}{dt}$ ...(1.75)

provided M_{21} is time invariant.

(*v*) *Direction of induced voltage.* The law regarding the direction of the induced voltage was established by the German physicist Lenz in 1834. Lenz's law estates that "the voltage induced in a coil by a time variant flux establishes a current in the coil in such a direction as to oppose the change in the initial flux which induced this voltage". The flux ϕ_{21} caused by current i_1 is directed clockwise in the core of the coils and this flux is increasing at the instant under consideration. The flux ϕ_{12} produced by current i_2 in coil 2-2 due to induced voltage v_2 in coil 2-2 must oppose the flux ϕ_{21}. Hence by right hand screw rule the current i_2 must flow in the direction shown by arrow head in Fig. 1.14. The reason why the induced flux ϕ_{12} must necessarily oppose the incident flux ϕ_{21} is quite apparent. If ϕ_{12} aids the flux ϕ_{21}, another increasing current would be induced in the coil 1-1 and so on forming a vicious, cycle to produce infinite current. This is against the principle of conservation of energy.

(*iv*) *Dot convention.* The induced current i_2 flows out of the top end of the winding. Hence top end of winding 2-2 becomes positive and is marked with a dot. Thus with a time varying voltage applied at terminals 1-1, when the dotted end of winding 1-1 is positive, the dotted end of winding 2-2 is also positive. Thus the dotted ends of coils become positive (or negative) concurrently. If the positions of source and load are interchanged, *i.e.,* the source is connected to winding 2-2 and the load is connected to winding 1-1, a step-by-step analysis will show that an increasing current flowing in the dotted terminal of coil 2-2 makes the upper end of coil 1-1 positive and hence the dotted end. Thus the dotted ends of coils become concurrently positive irrespective of the positions of the driving source and the load.

Again let us suppose that the source voltage has at any instant of time polarity reverse to that shown in Fig. 1.14 so that a current flows out of the dot. Then by inspection or by step-by-step analysis we may establish that the dotted terminal of winding 2-2 becomes negative.

We thus conclude that :

Current flowing into the dot on one winding includes a voltage in the other winding making the dotted end positive.

Or

Current flowing out of the dot on one winding induces a voltage in the other winding making the dotted end negative.

Or

Current simultaneously flowing into (or simultaneously flowing out of) dots on winding induce flux in the core which are additive.

1.13.2. Three Mutually Coupled Inductors

We have already studies two mutually coupled inductors. We now proceed to consider the case of several inductors magnetically coupled.

Fig. 1.15 (*a*) shows three mutually coupled inductors L_1 L_2 and L_3. The currents entering these coils are i_1, i_2 and i_3 respectively.

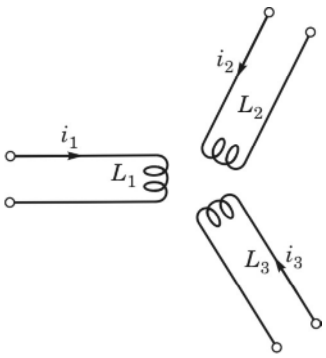


Fig. 1.15. Three magnetically coupled coils.

To facilitate understanding, we assume i_2 and i_3 to be zero and consider the effect of i_1 alone on the system. This current produces magnetic field B and a magnetic flux ϕ_1 whole of which links with the coil L_1 itself. The flux linkage ψ_1 produced with coil L_1 is related with the current i_2 in accordance with the relation :

$$.\psi_1 = L_{11} \cdot i_1 \qquad ...(1.76)$$

where L_{11} is the self-inductance of coil L_1.

Out of the total magnetic flux ϕ_1, a part ϕ_{21} links with N_2 turns of coil L_2 producing the flux linkage ϕ_{21}, so that

$$\psi_{21} = N_2 \cdot \phi_{21} \qquad ...(1.77)$$

where ϕ_{21} depends upon the mutual inductance M_{21}.

Eq. (1.77) may be put as : $\psi_{21} = M_{21} \cdot i_1$ ...(1.78)

where M_{21} is the mutual inductance between coils L_1 and L_2. The subscripts 2, 1 are placed in the order "effect, cause". Thus M_{21} gives the mutual inductance relating the cause *i.e.* the current i_1 in coil L_1 to effect *i.e.* flux linkage ψ_{21} in coil L_2.

Similarly flux linkage ψ_{31} with coil L_3, due to current i_1 in coil L_1 is given by,

$$\psi_{31} = M_{31} \cdot i_1 \qquad ...(1.79)$$

In general there will be either sources or loads connected to each of the coils so that current flows in each of the coils. We assume that the current directions and winding senses of the three coils are such that the flux linkages of any coil caused by all the coils are additive.

Then the total flux linkage ψ_1 in say coil L_1 is the sum of the flux linkages ψ_{11}, ψ_{12} and ψ_{13} where ψ_{11} ($= L_{11}\, i_1$) is the flux linkage produced by the current i_1 in the coil L_1 itself, and ψ_{12} ($= M_{12} \cdot i_2$) and ψ_{13} ($= M_{13}\, i_3$) are the flux linkages produced by the current i_2 and i_3 flowing in the coils L_2 and L_3 respectively. Thus,

$$\psi_1 = L_{11} \cdot i_1 + M_{12} \cdot i_2 + M_{13} \cdot i_3 \qquad ...(1.80)$$

Similarly for coil L_2, flux linkages ψ_2 is given by,

$$\psi_2 = M_{21} \cdot i_1 + L_{22}\, i_2 + M_{23}\, i_3 \qquad ...(1.81)$$

and for coil L_3, flux linkages ψ_3 is given by,

$$\psi_3 = M_{31} \cdot i_1 + M_{32}\, i_2 + L_{33}\, i_3 \qquad ...(1.82)$$

Assuming inductance parameters (both self and mutual) to be time invariant, by Faraday's law, the voltage induced in each, being the time rate of change of flux linkage, is given by,

$$e_1 = L_{11} \cdot \frac{di_1}{dt} + M_{12} \cdot \frac{di_2}{dt} + M_{13} \cdot \frac{di_3}{dt} \qquad ...(1.83)$$

$$e_2 = M_{21} \cdot \frac{di_1}{dt} + L_{22} \cdot \frac{di_2}{dt} + M_{23} \cdot \frac{di_3}{dt} \qquad ...(1.84)$$

and $$e_3 = M_{31} \cdot \frac{di_1}{dt} + M_{32} \cdot \frac{di_2}{dt} + L_{33} \cdot \frac{di_3}{dt} \quad ...(1.85)$$

When more than two coils are magnetically coupled, the magnetic flux induced by any one coil is related with that of every other by the usual dot convention. Of course dots of different shapes such as ● ■ ▮ ▲ etc. are used for each pair of coils. This scheme, however, has the drawback of confusion resulting from the use of a large number of similar dots. As an alternative to the use of dots for giving the information regarding the direction of resulting magnetic flux, or induced voltage, the same information may be contained in the sign of the coefficient of mutual inductance. This system is preferable to the dot system when the number of magnetically coupled coils is large. Of course, both systems have their advantages peculiar to certain problems and hence both systems are extensively used.

EXERCISE

1.1. 5×10^{11} electrons are removed from a solid copper sphere of diameter 6 cm. Find the charge of the sphere in coulombs and its sign. [**Ans.** $+ 8 \times 10^{-20}$ coulomb]

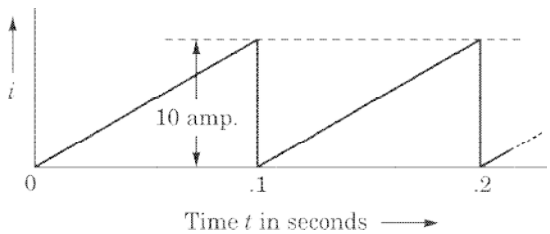
1.2. In a copper conductor, 3×10^{-18} electrons/second move past its cross-sectional area. Find the current in amperes. [**Ans.** 0.48 ampere]

1.3. A copper conductor carries a current of 400 mA. Find the net flow of electrons past any cross-section of the conductor per second. [**Ans.** 2.5×10^{18}]

1.4. In a copper conductor, there are 5×10^{22} free electrons in 1 cubic centimetre and the current density is 240 amp/cm^2. Find the mean electron drift in cm/second. [**Ans.** 0.03 cm/sec]

1.5. The current in a circuit varies according to the equation $i = 5 \sin \omega t$ amp., where $\omega = 2\pi \times 10^3$ radians/second. Find the charge in coulombs that passes through the circuit during the interval current i is positive. [**Ans.** 1.59×10^{-3} coulomb]

1.6. The current in a circuit varies as shown in the diagram. Calculate the charge in coulombs that passes through the circuit in one second. [**Ans.** 5 coulombs]



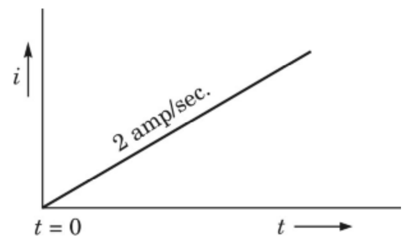
1.7. An energy source is connected to a circuit resulting in a current i of 5 amperes. The voltage difference across the output terminals of the source is 50 volts. Calculate (*i*) the power drawn from the source (*ii*) the energy supplied by the source in one minute (*iii*) charge transported in a minute.
[**Ans.** (*i*) 250 watts (*ii*) 15,000 joules (*iii*) 300 coulombs]

1.8. A current of 2 amperes flows through a resistor of 10 ohms. Find (*i*) the power absorbed in the resistor (*ii*) energy dissipated in the resistor per minute and (*iii*) charge flow through the resistor per minute.
[**Ans.** (*i*) 40 watts (*ii*) 2400 joules (*iii*) 120 coulombs]

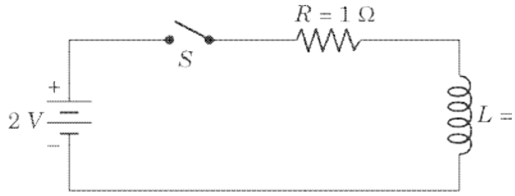
1.9. Find the maximum current that may be allowed through a 40 ohm, 10 watt resistor. With this maximum current, find the energy lost in the resistor in 5 minutes. [**Ans.** 0.5 amp ; 3000 joules]

1.10. The current in a 2-henry inductor varies as shown in the following figure. Find (*i*) the flux linkage in the system after one second (*ii*) the time rate of change of flux linkages in the system after 10 seconds and (*iii*) the quantity of charge having passed through the inductor after 4 seconds.

[**Ans.** (*i*) 4 weber-turns (*ii*) 4 weber turns/sec (*iii*) 16 coulombs]

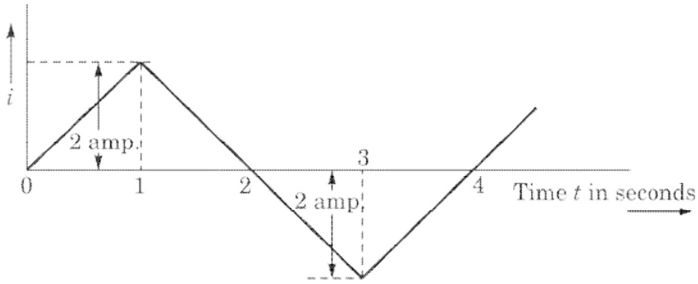


1.11. In the given circuit, the switch S is closed at time $t = 0$. The current in the circuit is given by the equation : $t = 2\,(1 + e^{-t/2})$ amp. $t > 0$. At time $t = 2$ seconds, find : (*i*) value of current (*ii*) rate of change of current (*iii*) total flux linkages (*iv*) rate of change of flux linkages (*v*) voltage across the inductor (*vi*) energy stored in the magnetic field (*vii*) voltage across the resistor (*viii*) rate at which energy is being stored in the magnetic field of the inductor (*ix*) rate of dissipation of energy as heat (*x*) rate of supply of energy by the battery.

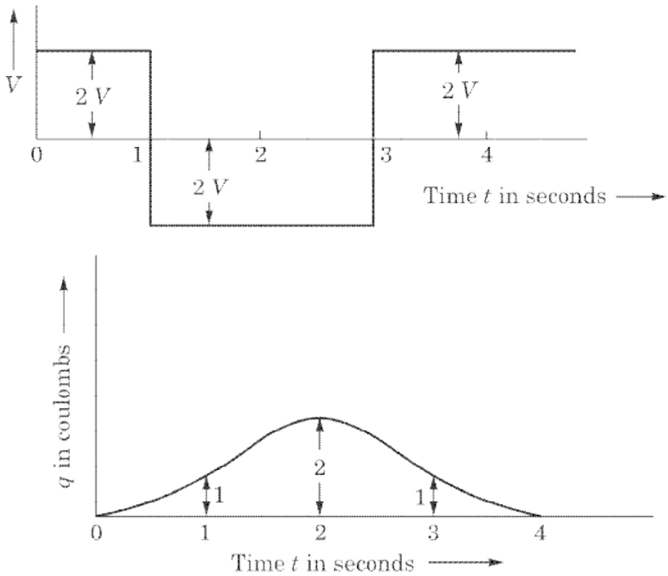


[**Ans.** (*i*) 1.264 amps (*ii*) 0.368 amp/sec (*iii*) 2.258 weber-turns (*iv*) 0.736 weber-turn/sec (*v*) 0.736 volt (*vi*) 1.6 joules (*vii*) 1.264 volts (*viii*) 0.93 watts (*ix*) 1.6 watts (*x*) 2.528 watts]

1.12. The following figure gives the variation of current i in an inductor of 1 henry. Plot the waveforms of charge stored in the magnetic field and the voltage across the inductor. Calculate voltage across the inductor and charge stored at time $t = 1, 2, 3$ and 4 seconds.



[**Ans.** (*i*) 2 volts ; change from + 2 volts to – 2 volts ; – 2 volts ; change from – 2 volts to + 8 volts (*ii*) 1, 2, 1, 0 coulomb]



1.13. A condenser consists of two plates, one of which rotates so that the capacitance between the plates is given by the equation :

$$C = C_0 (1 - \cos \omega t)$$

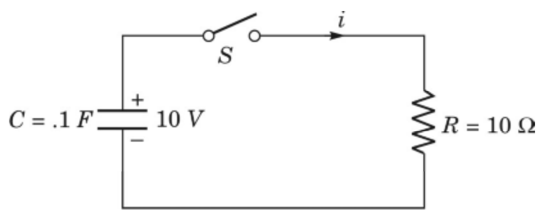
A battery of constant voltage V volts is connected across the condenser. Find the expression for the current through the condenser.

[**Ans.** $i = VC_0\, \omega \sin \omega t$]

1.14. A voltage $v = 10 \sin 2\pi \times 10^3 t$ is applied across a condenser C of 1 μF. Calculate (*i*) the amplitude of current through it (*ii*) maximum charge across the condenser (*iii*) maximum energy stored in the capacitor (*iv*) maximum power in the capacitor.

[**Ans.** (*i*) 62.8 mA (*ii*) 10^{-5} coulomb (*iii*) 5×10^{-5} joule (*iv*) 7.962×10^{-9} watt]

1.15. In the given circuit, the capacitor has been charged to a voltage of 10 volts and at time $t = 0$, switch S is closed. The resulting current in the circuit is known to be of the form $i = e^{-t}$ amp for $t > 0$. At time $t = 1$ sec, calculate : (*i*) rate of change of voltage v across capacitor C (*ii*) charge on the capacitor (*iii*) time rate of change of product Cv (*iv*) voltage across the condenser (*v*) energy stored in the electric field of the capacitor (*vi*) voltage across the resistor (*vii*) rate at which energy is being taken from the electric field of the capacitor (*viii*) rate at which energy is being dissipated as heat in the resistor.



[**Ans.** (*i*) 3.68 volts/sec (*ii*) 0.632 coulomb (*iii*) 0.368 amp (*iv*) 0.32 volts (*v*) 1.997 joule (*vi*) 3.68 volts (*vii*) 1.37 watts (*viii*) 1.37 watts]