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Technical terms, Overall understanding, Mathematics in instrumentation and control, science of automatic control, Process characteristics and Process dynamics.

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## TECHNICAL TERMS

The following are some of the technical terms that come quite often in instrumentation and control.

- |                              |                           |                             |
|------------------------------|---------------------------|-----------------------------|
| accuracy, 153                | frequency response, 110   | open loop, 53               |
| adaptive control, 75         | feedback, 66              | on-off control, 51          |
| bimetal thermometer, 168     | first order system, 5, 84 | optical pyrometer, 168      |
| bode diagrams/plots, 99, 110 | floating control, 52      | pressure gauge, 283         |
| capacitance, 17              | gain (A.R.), 125, 108     | phase margin, 101           |
| closed loop, 66              | hydraulic controller, 144 | pneumatic controller, 142   |
| controlled variable, 12      | impulse change, 6         | pneumatic, 144              |
| conductivity cell, 192       | integral control, 50      | polorograph, 202            |
| chromotograph, 197           | ionisation gauge, 283     | ramp, 156                   |
| control valve, 148, 306      | Laplace transform, 3      | root locus diagram, 117     |
| colorimetry, 216             | linear system, 30         | ruth-method, 96             |
| dynamics, 21, 25             | lag, dead time, 13        | resistance thermometer, 168 |
| delay, 13                    | load variable, 12         | radiation pyrometer, 168    |
| derivative control, 50       | Liapunov method, 260      | rotameter, 288              |
| degrees of freedom, 12       | liquid seal, 168, 287     | spectrometer, 210           |
| dead zone, 154               | lumped parameter, 130     | step change 4               |
| dead time, 13                | mode of control, 49       | sinusoidal change, 5        |
| distributed                  | manipulated variable, 12  | second order system, 85     |
| parameter, 131, 315          | manometer, 283            | stability, 96               |
| electronic controller, 165   | mcleoid gauge, 165        | state equation, 236         |
| error, 49                    | non-linear system, 30     | span, 153                   |
| forcing function, 5          | Nyquist diagram, 120      |                             |

## **Overall understanding of the subject : Instruments & Controllers are used for Safety, Operation and Maintenance of a Process and plant**

**Instrumentation section.** Deals with the principles of working, responses of temperature, pressure, composition and liquid level measuring instruments and their selection for various conditions of measurements. Instruments are selected based on the range of operation, time constants and responses. Pneumatic and electric transmission are used in transferring the measured variable to any point that is necessary. Instrumentation diagrams cover location of measuring instruments in a process or in entire plant, transmission lines and automatic controllers. Standard symbols are used in writing instrumentation diagrams, to represent or to show measuring instruments, controllers etc. Signalling, control centres, recording instruments and panel boards come under instrumentation section.

**Automatic control.** Automatic control is used to increase the quality and quantity. Automatic control is done with the information of process-dynamics, deviation that can be tolerated, various types of automatic controllers available, control valves, stability methods and block diagrams for analysis. Automatic control means, if the outlet variable (required quantity) is changing due to some disturbance in the process, then the controller reads it and changes the other variable (manipulated variable) to the required point. Once the equipment is installed, the technical person has to inspect only for their working or not. Computers are used to solve tedious equations or lengthy calculations like non-linear equations. Adaptive control, optimal control, stochastic control and methods of solutions for non-linear systems are the advanced sections in automatic control.

Optimal control aims for the best performance of the controlled system (better than the conventional feed back control). Adaptive controller adjusts the controller set up (like gain) by measuring the working conditions (operating). Stochastic means probabilistic type or uncertainty type. Nonlinear techniques are used when the system behaviour (equations) is non-linear type for control analysis. Modern control theory (state equations and state variables) is for multi-inputs-outputs, optimal control and for time domain analysis. Z-transform is used for sampled data (discrete data) systems for analysis. Recently computer control is used. Artificial intelligence, neural networks, fuzzy control etc. have come.

The following are some of the journals which are to be seen to be upto date in this subject.

(1) A.I.Ch.E. Journal (2) Canadian Journal of Chemical Engineering (3) I.E.C. Process design and development (4) International Journal of Chemical-Engineering (5) Chemical Engineering Science (6) International Journal of Control (7) British Journal of Chemical Engineering (8) Indian Chemical Engineer (9) I.E.E.E. Transactions on Automatic Control (10) Automatica.

**Table 1.1**  
**Laplace transforms of commonly used functions**

$f(t)$	$Lf(t) = f(s)$
1	$1/s ; s > 0$
$t$	$1/s^2 ; s > 0$
$t^n$	$n!/s^{n+1}$
$e^{at}$	$1/(s - a)$
$\sin at$	$a/(s^2 + a^2) ; s > 0$
$\cos at$	$s/(s^2 + a^2) ; s > 0$
$\sin h at$	$a/(s^2 - a^2) ; s >  a $
$\cos h at$	$s/(s^2 - a^2) ; s >  a $
$u(t - a)$	$e^{-as}/s$
unit impulse	1
$(t - a)$	$e^{-as} (\text{lag})$
$f^n(t)$	$s^n f(s) - s^{n-1} f(0) - s^{n-2} f'(0) \dots$ $\dots - f^{n-1}(0)$
$t^n f(t)$	$(-1)^n d^n f(s)/ds^n$
$t \sin at$	$2as/(s^2 + a^2)$
$t \cos at$	$(s^2 - a^2)/(s^2 + a^2)$
$e^{bt} \sin at$	$a/[(s - b)^2 + a^2]$
$e^{bt} \cos at$	$(s - b)/[(s - b)^2 + a^2]$
$e^{bt} t^n$	$n!/(s - b)^{n+1}$
$e^{-bt} t^n$	$n!/(s + b)^{n+1}$
$f^2(t)$	$s^2 f(s) - sf(0) - f'(0)$
$e^{-at} t^n$	$n!/(s + a)^{n+1}$
$e^{-at} \cos kt$	$s + a/[(s + a)^2 + k^2]$
$e^{-at} \sin kt$	$k/[(s + a)^2 + k^2]$
$f^1(t)$	$sf(s) - f(0)$

**1.1. Solving :**  $y^{111} - 3y^{11} + 3y^1 - y = t^2 e^t$  ;  $y(0) = 1$  ;  $y'(0) = 0$  ;  
 $y^{11}(0) = 2$

Taking laplace transforms on both sides

$$L(y^{111}) - 3L(y^{11}) + 3L(y^1) - L(y) = L(t^2 e^t)$$

$$s^3 y(s) - s^2 y(0) - sy'(0) - y^{11}(0) - 3[s^2 y(s) - sy(0) - y^1(0)]$$

$$+ 3[sy(s) - y(0)] - y(s) = 2/(s-1)^3$$

Substituting the values of  $y^1(0)$ ,  $y^{11}(0)$ ,  $y(0)$  and simplifying,

$$y(s) = \frac{s^2 - 3s + 5}{(s-1)^3} + \frac{2}{(s-1)^6}$$

$$= \frac{1}{(s-1)} - \frac{1}{(s-1)^2} + \frac{3}{(s-1)^3} + \frac{2}{(s-1)^6}$$

Taking inverse laplace transforms,

$$y(t) = e^t - t e^t + t^2 e^t / 2 + t^5 e^t / 60 \quad \text{Ans.}$$

**1.2. Solving :**  $dx/dt = 2x - 3y$  ;  $x(0) = 8$  ;  $y(0) = 3$ ,

$$dy/dt = y - 2x.$$

Taking laplace transforms on both sides,

$$sx(s) - 8 = 2x(s) - 3y(s),$$

$$sy(s) - 3 = y(s) - 2x(s)$$

Solving simultaneously, both equations

$$y(s) = 3s - 22/(s^2 - 3s - 4)$$

$$= \frac{5}{(s+1)} - \frac{2}{(s-4)}$$

$$y(t) = 5e^{-t} - 2e^{4t} \quad \text{Ans.}$$

$$x(s) = 8s - 17/(s^2 - 3s - 4)$$

$$= \frac{5}{(s+1)} + \frac{3}{(s-4)}$$

$$x(t) = 5e^{-t} + 3e^{4t} \quad \text{Ans.}$$

**1.3. For a first order instrument deriving the response equation if the step input is introduced :**

For a first order instrument the differential equation is

$$Td\theta/dt + \theta = \theta_f$$

where  $\theta$  = value indicated by instrument,

$\theta_f$  = final steady value ;

$t$  = time ;

$T$  = time constant ; For step change of  $\theta_f$  of magnitude  $A$

$$Td\theta/dt + \theta = A$$

Taking laplace transforms on both sides of the equation

$$Ts \theta(s) - \theta(0) + \theta(s) = A/s,$$

Assuming  $\theta(0) = 0,$

$$\theta(s) = \frac{A}{s(Ts + 1)}$$

Using partial fractions,

$$\theta(s) = A \left[ \frac{1}{s} - \frac{1}{(s + 1/T)} \right]$$

Taking inverse laplace transforms

$$\theta(t) = A (1 - e^{-t/T}). \quad \text{Ans.}$$

**1.4. Deriving the response equation for first order system or instrument with sinusoidal input as forcing functions.**

$$T \frac{d\theta}{dt} + \theta = \theta_f (A \sin \omega t)$$

where

$\theta$  = value indicated by instrument ;

$A$  = amplitude of sine wave ;

$\omega$  = frequency of sine wave ;

$T$  = time constant.

Taking Laplace transforms on both sides of the above equation and assuming  $\theta(0) = 0;$

$$\theta(s) = \frac{A\omega}{T(s + 1/T)(s^2 + \omega^2)}$$

By partial fractions  $\theta(s)$  can be written as

$$\begin{aligned} \theta(s) &= \frac{B_1}{(s + 1/T)} + \frac{B_2}{(s + i\omega)} + \frac{B_3}{(s - i\omega)} \\ B_1(s + i\omega)(s - i\omega) + B_2(s - i\omega)(s + 1/T) \\ &\quad + B_3(s + 1/T)(s + i\omega) = A\omega/T, \end{aligned}$$

Putting

$$s = iW$$

$$B_3 = \frac{A}{2i(i\omega T + 1)} \quad \dots (1)$$

Putting

$$s = -iW$$

$$B_2 = \frac{A}{2i(i\omega T - 1)} \quad \dots (2)$$

Let

$$s = -1/T, B_1 = A\omega T/(\omega^2 T^2 + 1)$$

$$\begin{aligned} \theta(s) &= \frac{A\omega T}{(s + 1/T)(\omega^2 T^2 + 1)} + \frac{A}{2i(i\omega T - 1)(s + i\omega)} \\ &\quad + \frac{A}{2i(i\omega T + 1)(s - i\omega)} \end{aligned}$$

$$\begin{aligned}
&= \frac{A\omega T}{(s + 1/T)(\omega^2 T^2 + 1)} \\
&\quad + \frac{A(i\omega T + 1)}{2i(i\omega T - 1)(i\omega T + 1)(s + i\omega)} \\
&\quad + \frac{A(i\omega T - 1)}{2i(i\omega T + 1)(i\omega T - 1)(s - i\omega)} \\
&= \frac{A\omega T}{(s + 1/T)(\omega^2 T^2 + 1)} + \frac{A}{(\omega^2 T^2 + 1)} \\
&\quad \left[ \frac{-i\omega T}{2i(s + i\omega)} - \frac{1}{2i(s + i\omega)} - \frac{i\omega T}{2i(s - i\omega)} + \frac{1}{2i(s - i\omega)} \right]
\end{aligned}$$

Taking inverse Laplace transforms,

$$\begin{aligned}
\theta(t) &= \frac{A\omega T}{(\omega^2 T^2 + 1)} e^{-t/T} + \frac{A}{(\omega^2 T^2 + 1)} \\
&\quad \left[ \frac{1}{2i} (e^{i\omega t} - e^{-i\omega t}) - \frac{\omega T}{2} (e^{i\omega t} + e^{-i\omega t}) \right] \\
\theta(t) &= \frac{A\omega T e^{-t/T}}{(\omega^2 T^2 + 1)} + \frac{A}{(\omega^2 T^2 + 1)} (\sin \omega t - \omega T \cos \omega t)
\end{aligned}$$

Using trigonometric identity of

$$P \cos A + Q \sin A = r \sin(A + \phi)$$

where  $r = (P^2 + Q^2)^{1/2}$  and  $\phi = -\tan^{-1}(\omega T)$ ;  $Q/P = \omega T$

$$\theta(t) = \frac{A\omega T e^{-t/T}}{(\omega^2 T^2 + 1)} + \frac{A \sin(\omega t + \phi)}{(\omega^2 T^2 + 1)^{1/2}} \text{ as } t \rightarrow \infty$$

$$\theta(t) = \frac{A \sin(\omega t + \phi)}{(\omega^2 T^2 + 1)^{1/2}}; \phi = -\tan^{-1}(\omega T) \quad \text{Ans.}$$

**1.5. For a first order instrument, deriving the response equation for impulse input :**

**Hint :** put  $\theta_f(s) = 1$ ; **Ans.** for unit impulse

$$\theta(t) = \frac{1}{T} e^{-t/T}$$

**1.6. Given**  $x(s) = \frac{3s}{(s^2 + 1)(s^2 + 4)}$ , determining  $x(t)$  :

Using partial fractions,

$$x(s) = \frac{3s}{(s^2 + 1)(s^2 + 4)}, \text{ determining } x(t) :$$

Using partial fractions,

$$\begin{aligned}
x(s) &= \frac{3s}{(s + i)(s - i)(s + 2i)(s - 2i)} \\
&= \frac{A}{(s + i)} + \frac{B}{(s - i)} + \frac{C}{(s + 2i)} + \frac{D}{(s - 2i)}
\end{aligned}$$

equating the numerators,  $A = \frac{1}{2}$ ,  $B = \frac{1}{2}$ ,  $C = -\frac{1}{2}$ ,  $D = -\frac{1}{2}$ .

$$x(s) = \frac{1}{2(s+i)} + \frac{1}{2(s-i)} - \frac{1}{2(s+2i)} - \frac{1}{2(s-2i)}$$

Taking inverse Laplace transforms

$$x(t) = \frac{e^{-it}}{2} + \frac{e^{it}}{2} - \frac{e^{-2it}}{2} - \frac{e^{2it}}{2}$$

Using the relations,

$$e^{it} = \cos t + i \sin t,$$

$$e^{-it} = \cos t - i \sin t$$

$$x(t) = \cos t - \cos 2t \quad \text{Ans.}$$

**1.7. Solving using Laplace transforms the Bessel equation of type :**

$ty^{11} + y^1 + ty = 0$ , initial conditions are

$$y = 1; y^1 = 0, \text{ let } g(s) = Lf(t)$$

$$L(ty^{11}) = -d/ds(s^2 g(s) - s)$$

$$= -s^2 g^1(s) - 2sg(s) + 1$$

$$L(y^1) = sg(s) - 1; L(ty) = -d/ds g(s) = -g^1(s)$$

$$-s^2 g^1(s) - 2sg(s) + 1 + sg(s) - 1 - g^1(s) = 0$$

$$(s^2 + 1) \frac{dg}{ds} + sg = 0$$

integrating,  $\ln g + \frac{\ln(s^2 + 1)}{2} = \ln C_1$  where  $C_1$  is a constant ;

$$g(s) = \frac{C}{(s^2 + 1)^{1/2}} \text{ For } s > 1, \text{ by binomial series.}$$

$$g(s) = \frac{C}{s} \left( 1 + \frac{1}{s^2} \right)^{-1/2}$$

$$g(s) = \sum_{n=0}^{\infty} \frac{(-1)^n !^{2n}}{2^{2n} (n!)^2 s^{2n+1}}$$

$$y = f(t) = C \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n}}{2^{2n} (n!)^2}$$

$C = 1$  for the given initial conditions. This is the series for Bessel function.

$$LJ_0(t) = \frac{1}{(s^2 + 1)^{1/2}}, y(t) = J_0(t). \quad \text{Ans.}$$

**1.8. Problem.** Given  $x(s) = \frac{1}{s(s+1)^3}$ , determining  $x(t)$  :

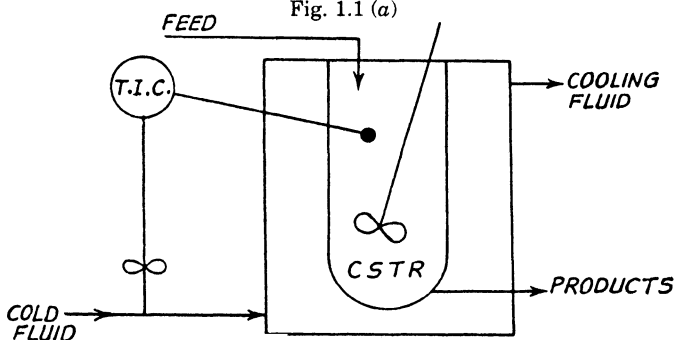
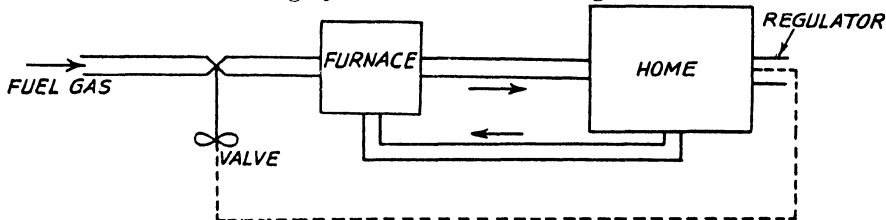
$$x(s) = \frac{A_1}{s} + \frac{A_2}{(s+1)} + \frac{A_3}{(s+1)^2} + \frac{A_4}{(s+1)^3}$$

$$\begin{aligned}
 A_1 &= sx(s) \Big|_{s=0} = \frac{1}{(s+1)^3} \Big|_{s=0} = 1 \\
 A_4 &= (s+1)^3 x(s) \Big|_{s=-1} = \frac{1}{s} \Big|_{s=-1} = -1 \\
 A_3 &= \frac{d}{ds} (s+1)^3 x(s) \Big|_{s=-1} = \frac{d}{ds} \left( \frac{1}{s} \right) = -\frac{1}{s^2} \Big|_{s=-1} = -1. \\
 A_2 &= \frac{1}{2!} \frac{d^2}{ds^2} (s+1)^3 x(s) \Big|_{s=-1} \\
 &= \frac{d}{2ds} \left( -\frac{1}{s^2} \right) \Big|_{s=-1} \\
 &= \frac{2}{2s^3} \Big|_{s=-1} = -1 \\
 x(s) &= \frac{1}{s} - \frac{1}{(s+1)} - \frac{1}{(s+1)^2} - \frac{1}{(s+1)^3} \\
 x(t) &= 1 - e^{-t} - \frac{te^{-t}}{2} - \frac{t^2 e^{-t}}{6} \quad \text{Ans.}
 \end{aligned}$$

### 1.9. Definition of automatic control and examples :

Automatic control is the maintenance of desired value by measuring the existing value, comparing with the desired value and using this difference to initiate action by a controller for reducing the difference. Examples are :

(a) Home heating system as shown in Fig. 1.1 (a).





Depending upon the temperature difference between the required value and measured value, a valve is operated to adjust the fuel gas supply to the furnace, to bring the home temperature to the wanted value.

(b) Temperature control in exothermic chemical reactor, by changing the flow rate of cooling fluid shown in figure 1.1 (b).

The temperature in the reactor is measured by TIC (temperature indicating controller) and changes the inlet cooling fluid flow rate, depending on the error (difference between measured value and desired value).

(c) Temperature control in endothermic chemical reactor by increasing or decreasing heat supply (electric or steam) shown in Fig. 1.6.

(d) Temperature control in water heater by changing heat flow shown in Fig. 1.4.

(e) Temperature control in heat exchangers (outgoing hot fluid temperature for example) by changing inlet cold fluid flow rate, as shown in Fig. 1.2.

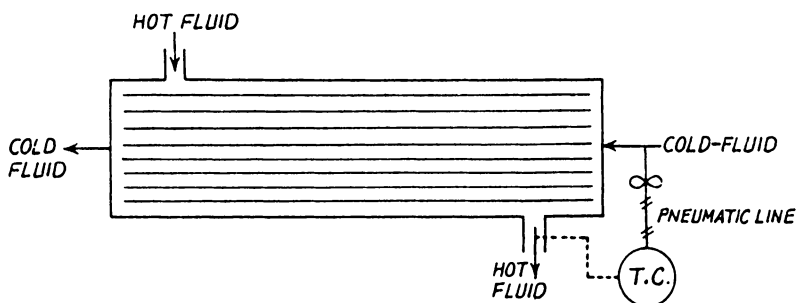


Fig. 1.2

In this example hot fluid temperature is measured and compared with desired value. Depending on the error, the inlet cold fluid amount is changed.

### 1.10. The uses of automatic control

- (a) increases in quantity or number of products
- (b) improves the quality of products
- (c) improves the uniformity of the products
- (d) savings in processing materials
- (e) savings in energy or Power requirements
- (f) savings in plant equipment
- (g) decreases human errors
- (h) economical in some processes.

### 1.11. Operational diagram and physical diagram with examples.

Operational diagram represents the various units present in the process or system during operation, including the automatic controller also. The physical diagram does not include controller. The operational diagram for water heater is shown in Fig. 1.3 and physical diagram in Fig. 1.4.

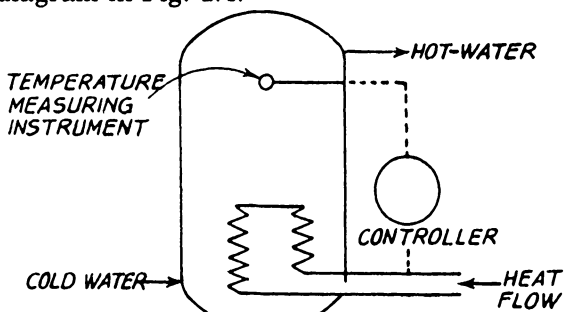


Fig. 1.3

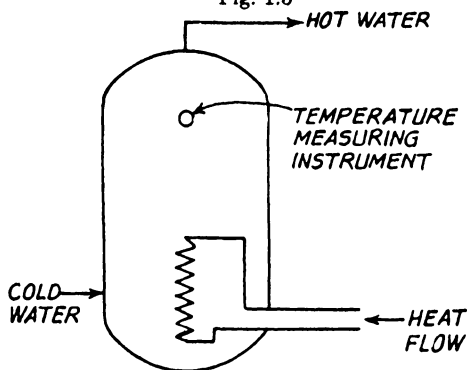


Fig. 1.4

### 1.12. The elements of block diagrams and drawing block diagram for one example :

Dynamic function :

$$x = (y, t) \text{ where } t \text{ is time}$$

Algebraic function :

$$x = z - y$$

The block diagram for water heater is shown in Fig. 1.5 (b).

$R$ —set temperature,  
 $m$ —manipulated variable,  
 $C$ —controlled variable,  
 $l$ —load variable,  $e$ —error.

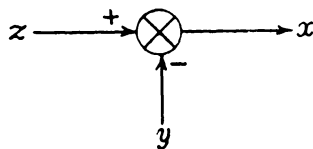
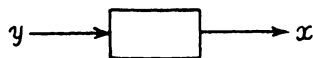


Fig. 1.5 (a)

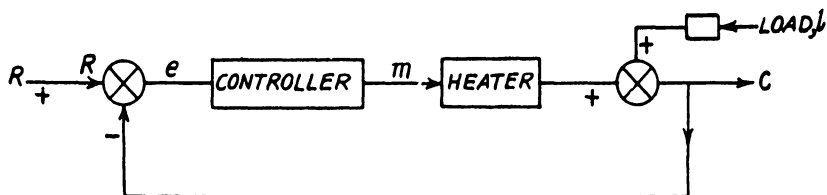


Fig. 1.5 (b)

**1.13. Drawing block diagram for control of temperature in Chemical reactor.**

The operational diagram is shown in Fig. 1.6 and the block diagram in Fig. 1.7.

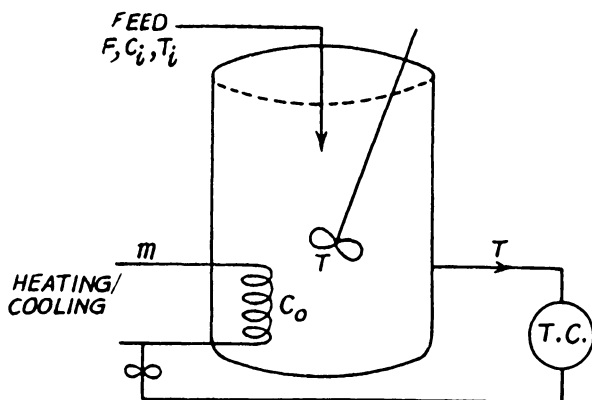


Fig. 1.6

T.C. — temperature controller,

F — Feed rate,

$C_i$  — inlet concentration,

$C_o$  — outlet concentration,

T — outlet temperature (temperature inside the reactor),

$T_i$  — inlet temperature.

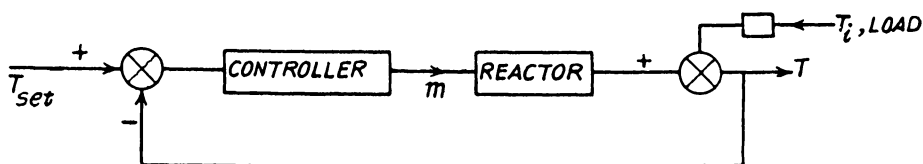


Fig. 1.7

#### 1.14. Short notes on block diagrams :

Block diagrams represent the various units present in a controlled Process. Analysis is done with the help of these block diagrams. Open loop transfer function and closed loop transfer

function are calculated from these block diagrams for control analysis. This is writing like flow diagrams in Process Control for calculations. Examples of writing block diagrams are given in 1.12 and 1.13.

**1.15. The process variables in process control and examples :**

(a) Controlled variable : is that variable which directly or indirectly indicates the form or state of the product. Examples are temperature in a chemical reactor, outlet temperature of water in water heater.

(b) manipulated variable : is that variable which is selected for adjustment by controller so as to maintain the controlled variable at the desired value. Examples are cooling by changing cold water flow rate or changing steam flow rate for heating in Chemical reactor and heat flow changing ( $m$ ) in water heater.

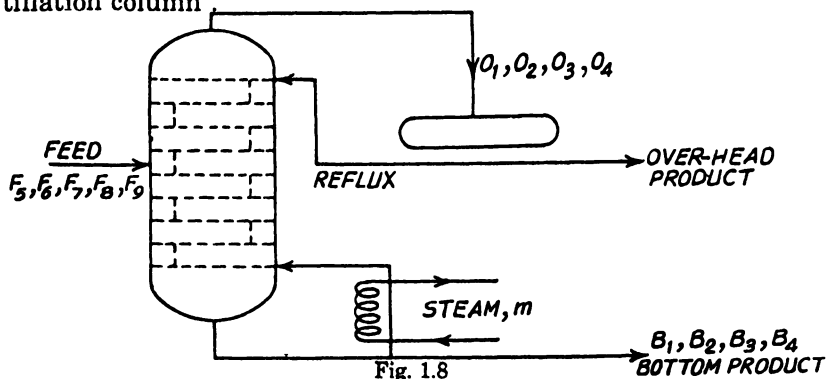
(c) Load variable : All other independent variables except controlled variable and manipulated variable. Examples are inlet concentration and temperature of reactants in a chemical reactor and inlet temperature of cold water in water heater.

**1.16. Process degrees of freedom and its uses :**

Process degrees of freedom means the number of independent variables in a process.

$n_p = n_v - n_e$  ;  $n_p$  = process degrees of freedom,  $n_v$  = number of variables in the process ;  $n_e$  = number of equations in the process. The number of controllers to be used in a process should not exceed the process degrees of freedom.

**1.17. The process degrees of freedom in a continuous distillation column**



Let  $O_1$  = overhead temperature,  $O_2$  = overhead pressure  
 $O_3$  = overhead composition;  $O_4$  = overhead flow rate

$B_1$  = bottom temperature ;  $B_2$  = bottom pressure

$B_3$  = bottom composition;  $B_4$  = bottom flow rate;

$F_5$  = feed temperature,  $F_6$  = feed pressure ;

$F_7$  = feed composition ;  $F_8$  = feed per cent vapour;

$F_9$  = feed flow rate ;  $m$  = steam flow rate.

The diagram is shown in Fig. 1.8.

Using Gibb's phase rule at the overhead, there are two components and two phases  $n = 2 - 2 + 2 = 2$ . So any two of three variables  $O_1, O_2, O_3$  are independent. The same is true at the bottom where any two of the three of the variables  $B_1, B_2, B_3$  are independent. For the feed also any two of  $F_5, F_6, F_7$  are independent. The number of variables are

Overhead—two out of  $O_1, O_2, O_3$  and  $O_4$  —3

Bottom — two out of  $B_1, B_2, B_3$  and  $B_4$  —3

Feed — any two of  $F_5, F_6, F_7$  and  $F_8, F_9$  —4

Heat input  $-m$ ..... —1

The number of equations are three which are material balance, energy balance and equilibrium relationship. Then  $n_p = 11 - 3 = 8$ .

Maximum number of controllers are — 8. **Ans.**

**1.18. Process control variables** in an exothermic chemical reactor. Controlled variable : temperature in the reactor ; manipulated variable : flow rate of cooling water ; load variable : inlet temperature of reactants, inlet concentrations or flow rate changes.

**1.19. Dead time, transportation lag** and examples :

Dead time is defined as any definite time delay between two related actions. Dead time is found in chemical reactions when a finite time should be waited before the reaction begins to take place as shown in Fig. 1.9. Sometimes transportation lag is also accounted as dead time. For example, if the temperature measuring instrument is placed at a distance, downstream of a heat exchanger, a time delay occurs before the fluid reaches from the

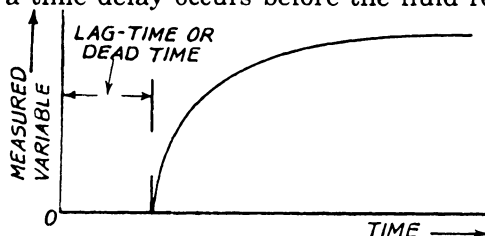


Fig. 1.9

outlet of the heat exchanger to the temperature measuring instrument. Any delay until a variable begins to change is accounted as dead time without being specific. Another example is conductivity meter for concentration measurement at down stream of dialyser. This is represented in Laplace form as  $e^{-Ls}$  where  $L$  is lag time.

**1.20. For a single tank** with resistance  $R$  and capacitance  $C$  deriving

$$q'_o(s)/q'_i(s) = \frac{1}{Ts + 1}, \text{ where } T = AR$$

Using unsteady mass balance equation :

Rate of input — Rate of output  $\pm$  change due to reaction = Rate of change of accumulation.

In this case chemical reaction term is zero. The single tank arrangement is shown in Fig. 1.10.

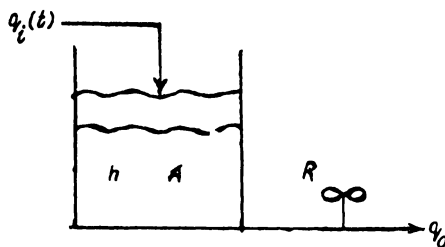


Fig. 1.10

$$q_i - q_o = \frac{d}{dt} (Ah), \text{ where } h \text{ is level in tank}$$

$A$  is area of cross section of tank and  $t$  is time.

Using  $h/q = R$  (linear relation)

$$q_i - q_o = AR \frac{dq_o}{dt} \quad \dots (1)$$

at steady state equation 1 can be written as

$$q_{is} - q_{os} = AR \frac{dq_{os}}{dt} \quad \dots (2)$$

Eq. (1)–Eq. (2) is

$$(q_i - q_{is}) - (q_o - q_{os}) = AR \frac{d}{dt} (q_o - q_{os}) \quad \dots (3)$$

Substituting  $q_i - q_{is} = q'_i$  and  $q_o - q_{os} = q'_o$ , equation 3 becomes

$$q'_i - q'_o = AR \frac{dq'_o}{dt} \quad \dots (4)$$

Taking laplace transforms on both sides of equation (4) gives

$$q'_i(s) - q'_o(s) = sAR q'_o(s)$$

collecting common terms gives

$$\frac{q_o'(s)}{q_i'(s)} = \frac{1}{(Ts + 1)} \quad \text{Ans.}$$

Similarly it can be shown, replacing  $q_o'$  by  $h'/R$

$$\frac{h'(s)}{q_i'(s)} = \frac{R}{(Ts + 1)} \quad \text{Ans.}$$

**1.21. For a stirred tank chemical reactor** shown in Fig. 1.11 deriving  $C_o'(s)/C_i'(s) = R/(Ts + 1)$ . Assume the reaction is of first order and isothermal.  $F$  = feed rate ;  $V$  = volume of reactor,  $C$  = concentration,  $t$  = time.

Using unsteady mass balance equation for the CSTR shown in Fig. 1.11.

$$FC_i - FC_o - KC_oV = V \frac{dC_o}{dt} \quad \dots (5)$$

Where  $K$  is the first order rate constant. At steady state the equation is

$$FC_{is} - FC_{os} - KC_{os}V = V \frac{dC_{os}}{dt} \quad \dots (6)$$

Equation 5–Equation (6) is

$$F(C_i - C_{is}) - F(C_o - C_{os}) - K(C_o - C_{os})V = \frac{Vd}{dt}(C_o - C_{os}) \quad \dots (7)$$

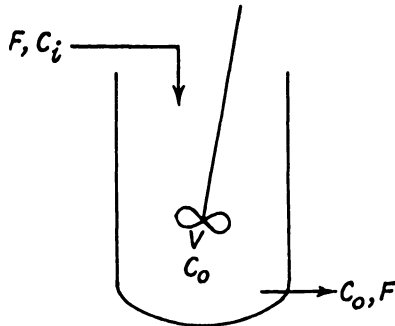


Fig. 1.11

Using deviation variables,

$$C_i - C_{is} = C_i', \quad C_o - C_{os} = C_o';$$

Equation 7 becomes

$$FC_i' - FC_o' - KVC_o' = V \frac{d}{dt} C_o' \quad \dots (8)$$

Taking laplace transforms on both sides of equation (8),

$$FC_i'(s) - FC_o'(s) - KVC_o'(s) = VsC_o'(s)$$

collecting common terms gives,

$$\frac{C_o'(s)}{C_i'(s)} = \frac{F/(F + KV)}{V/(F + KV)s + 1} = \frac{R}{Ts + 1} \quad \text{Ans.}$$

**1.22.** Developing the process dynamic equations for a continuous stirred tank chemical reactor with conditions of (a) first order type reaction (b) reaction is exothermic (c) rate constant varies with temperature.

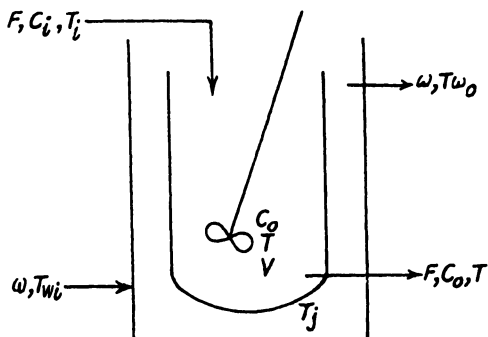


Fig. 1.12

Mass balance equation :

$$FC_i - FC_o - K_o VC_o e^{-dE/RT} = \frac{dC_o}{dt} V \quad \dots (9)$$

Heat balance equation :

$$F\rho(T_i - T)C_p + K_o VC_o e^{-dE/RT} dH - UA(T - T_j) = V\rho C_p dT/dt \quad \dots (10)$$

$F$  = flow rate ;

$T$  = temperature

$C$  = concentration ;

$K_o$  = constant for reaction rate

$V$  = volume of reactor ;

$t$  = time ;

$\rho$  = density ;

$C_p$  = specific heat

$dH$  = heat due to reaction;

$U$  = overall heat transfer coefficient ;

$A$  = area of cooling surface ;

$T_j$  = jacket temperature ;

$dE$  = energy term in reaction rate.

**1.23.** Number of degrees of freedom in CSTR having exothermic reaction :



The set up is shown in Fig. 1.12.

Inlet variables  $= T_i, C_i, F, TW_i, W = 5$

Outlet independent variables

$= T, C_o, TW_o, F, W - 2$  out of 5; equations

Mass balance, energy balance

$= 2$

Number of degrees of freedom

$= 7 - 2 = 5$ . **Ans.**

#### 1.24. Resistance and capacitance and an example for each.

Resistance  $(R) = \text{driving force/flow}$

Capacitance  $(C) = \text{storage/driving force}$ .

For liquid level system :

storage = volume of fluid,

flow = flow of liquid,

driving force = height of liquid ;

$R = dh/dq$  ;  $C = hA/h = A$

where  $A$  is the cross-sectional area of tank.

**1.25. Analogy** between thermal, electrical, gas and liquid systems. The analogous quantities of voltage, current and charge in electrical systems are marked with those in liquid, thermal and gas systems. From these, relations for resistance and capacitance are developed.

#### Electrical systems : Resistance

$(R) = de/di$

where  $e$  is the voltage and  $i$  is the current.

Capacitance  $(C) = dV/de$

where  $V$  is electrical charge and  $d$  is differential.

**Liquid systems.**  $h$  (height of liquid) is similar to  $e$ ,  $q$  (flow rate) is similar to  $i$ , volume of liquid is similar to charge ( $V$ ).

Liquid resistance  $(R) = dh/dq$ .

The equation for flow rate ( $q$ ) in turbulent flow is given by,

$$q = KA [2g (h_1 - h_2)]^{1/2}$$

where

$K$  = flow coefficient,

$A$  = area of restriction,

$g$  = acceleration due to gravity

Turbulent resistance  $(R)$  is  $= 2(h_1 - h_2)/q$

The laminar flow rate equation is

$$(h_1 - h_2) = 128 \mu l q (\pi \rho d^4)$$

where

$\mu$  = absolute viscosity,  
 $d$  = inside diameter of pipe,  
 $\rho$  = liquid density,  
 $l$  = length of pipe;

Laminar resistance ( $R$ )

$$= \frac{dh}{dq} = \frac{128 \mu l}{\pi \rho d^4}.$$

**Gas systems.** Pressure ( $P$ ) of gas is similar to  $e$ , flow rate of gas ( $w$ ) is similar to  $i$ , weight of the gas in vessel ( $W$ ) is similar to charge ( $V$ ). Turbulent gas flow rate equation,

$$w = KAY [2g(P_1 - P_2)]^{1/2}$$

where

$K$  = flow coefficient,  
 $A$  = area of restriction,  
 $Y$  = expansion factor,  
 $\rho$  = density of gas.

Turbulent gas resistance ( $R$ )  $= dP/dw = \frac{2(P_1 - P_2)}{w}$ , gas capacitance ( $C$ )  $= dW/dP = Vd\rho/dP$ , using ideal gas law  $Vd\rho/dP = V/(nRT)$

where

$V$  = volume of vessel,  
 $R$  = gas constant,  
 $T$  = absolute temperature of gas.

**Thermal systems.** Temperature ( $\theta$ ) is similar to  $e$ , heat flow rate ( $q$ ) is similar to  $i$ , heat content in the object ( $m C_p d\theta$ ) is similar to charge  $V$ .

Resistance ( $R$ )  $= d\theta/dq$

Conduction heat equation is,  $q = KA(\theta_1 - \theta_2)/dx$  where  $K$  = thermal conductivity,  $A$  = area normal to heat flow,  $dx$  = thickness of conductor.

Thermal resistance by conduction  $= R = dx/(KA)$ . In heat transfer by convection,  $q = hA d\theta$

where

$h$  = heat transfer coefficient,  
 $A$  = area of heat transfer

Thermal resistance by convection

$$= 1/(hA)$$

In radiation heat transfer,

$$q = KAE (\theta_1^4 - \theta_2^4)$$

where

$K = \text{constant,}$

$E = \text{emissivity, } A = \text{surface area,}$

Thermal resistance by radiation

$$(R) = d\theta/dq$$

$$\frac{d\theta}{dq} = \frac{1}{KAE (\theta_1^3 + \theta_2^3 + \theta_1^2 \theta_2 + \theta_2^2 \theta_1)}$$

taking

$$\theta_a = (\theta_1 + \theta_2)/2$$

$$\frac{d\theta}{dq} = \frac{1/KAE}{(2\theta_a)^3 - 2\theta_1\theta_2(\theta_1 + \theta_2)} = \frac{1}{4\theta_a^3 KAE}$$

if  $\theta_1 \equiv \theta_2$ , so  $d\theta/dq = 1/(4KAE\theta_a^3)$

Thermal capacitance

$$(C) = m C_P d\theta/d\theta = C_P m$$

where

$m = \text{weight of block,}$

$C_P = \text{Specific heat of block.}$

**1.26. Problem :** Determine the capacitance of the vessel of 5 sq. ft.

Cross-sectional area and 8 ft. high, storing carbon dioxide. The storage conditions are absolute pressure = 150 PSi, absolute temperature = 540 degrees R.

For theory refer 1.25 under gas systems.

Gas capacitance  $(C) = dW/dP = V/(nRT)$  for ideal gas.

$W = \text{weight of gas, } P = \text{pressure of gas.}$

Here  $n = 0.826$  using ideal gas behaviour

$R = 10.65 \text{ cu. ft. PSi/(lb. mole. or)}$

$$C = \frac{5 * 8}{10.65 * 0.826 * 540} = 8.420 * 10^{-3}.$$

**1.27. Time constant.** This comes for first order systems only. Time constant of any unit is the product of resistance and capacitance of it. The response of a set of units depends upon the time constants of the individual units and their interconnections, say Parallel or series. Equations are different for different connections.

For a first order system the response reaches 63.2% of final value for one time constant. For a mixer the time constant is = Volume of fluid in the mixer/Flow rate of fluid.

**1.28. Problem :** Steady heat flow occurs through a one inch thick wall of carbon plate with an area of 2.0 sq. ft. The temperature drop is from 330°F to 180°F. Calculate the thermal resistance ?

Thermal conductivity ( $K$ ) of carbon plate

$$= 29.0 \text{ BTU/(ft. deg. F. hr.)}$$

$$R = d\theta/dq = dx/(KA)$$

$$= \frac{1.0 \times 3600}{12 \times 29 \times 2} = 5.20 \text{ deg. Sec./BTU.}$$

### 1.29. Transfer function of a Process

The transfer function of a process represents the variation of one variable with any other variable in the Process in the Laplacian form. For example in a liquid tank, if  $q_i$  is the inlet flow rate,  $q_o$  is the outlet flow rate,  $T$  is time constant, then  $G(s) = q_o(s)/q_i(s)$ , where  $G(s)$  is the transfer function of  $q_o$  and  $q_i$ .

These transfer functions are used for analysis of linear systems.

**1.30. Problem :** For the heat transfer figure shown below, in Fig. 1.13,  $q$  (heat flow) = 100 cal. per minute, mass of water including water equivalent is 1000 gm.

Heat transfer

$$= hA dT = 10 (T - 70) \text{ cal./minute.}$$

- (a) Find  $T_s$  (steady state temperature),  
 (b) find time constant,  $\tau$  (c) find  $T'(s)/q'(s)$ , (d) find  $T$  at 2 minutes after  $q$  changes to 90.

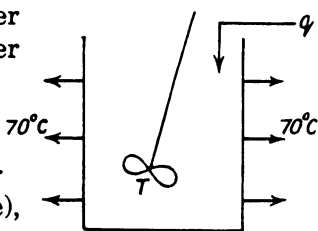


Fig. 1.13

(a) Using heat balance, at steady state,

Input heat rate = output heat rate

$$q = 10 (T_s - 70) = 100 ; T_s = 80^\circ\text{C. Ans.}$$

(b, c) Using unsteady heat balance,

$$q - 10 (T - 70) = 1000 C_p \frac{dT}{dt} \quad \dots (11)$$

where  $C_p$  = specific heat of water = 1.0,  $t$  = time. At steady state equation 11 becomes,

$$q_s - 10 (T_s - 70) = 0 \quad \dots (12)$$

Subtracting equation (12) from equation (11),

$$(q - q_s) - 10 (T - T_s) = 1000 \frac{dT}{dt} (T - T_s) \quad \dots (13)$$

Let  $q - q_s = q'$ ,  $T - T_s = T'$ ,

$$\text{Eq. 13 becomes, } q' - 10T' = 1000 dT'/dt \quad \dots (14)$$

taking Laplace transforms on both sides of equation 14,

$$q'(s) - 10T'(s) = 1000 sT'(s)$$

$$T'(s)/q'(s) = 0.1/(100s + 1) \text{ Ans.}$$

Time constant = 100 minutes, Resistance = 0.1

(d)  $q'(s) = -10/s$ ,  $T'(t) = -[1 - e^{-t/100}]$   
 at 2 minutes,  $T'(t) = -0.02$   
 $T - T_s = -0.02$ ;  $T_s = 80^\circ\text{C}$ ;  
 $T = 80 - 0.02 = 79.98^\circ\text{C}$ . **Ans.**

**1.31. Problem :** For the block diagram below, calculate the transfer functions of, (a)  $t_1$  to  $t_2$  (b)  $r$  to  $t_2$  as shown in Fig. 1.14.

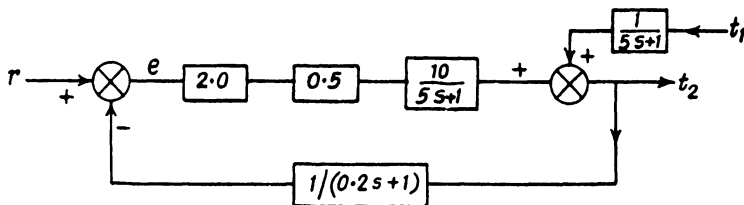


Fig. 1.14

$$(a) \left[ r - \frac{t_2}{(0.2s+1)} \right] 2 * 0.5 * \frac{10}{(5s+1)} + \frac{t_1}{(5s+1)} = t_2$$

taking

$$t_1 = 0$$

$$\frac{t_2}{r} = \frac{10/(5s+1)}{1 + \frac{10}{(5s+1)(0.2s+1)}}$$

simplifying,

$$t_2/r = \frac{s+5}{(0.5s^2 + 2.6s + 5.5)} \quad \text{Ans.}$$

$$(b) \text{ taking } r = 0, t_2/t_1 = \frac{1/(5s+1)}{1 + \frac{10}{(0.2s+1)(5s+1)}}$$

$$t_2/t_1 = \frac{0.2s+1}{(s^2 + 5.2s + 11)} \quad \text{Ans.}$$

**1.32. Process dynamic equations of a counter current dialyser, shown in Fig. 1.15.**

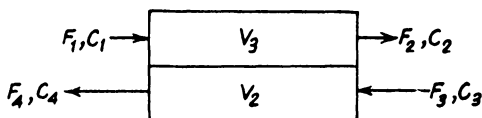


Fig. 1.15

$F$  = flow rate,  $C$  = concentration,

$K$  = permeation constant,  $A$  = permeation area

The mass balance equations are :

$$\left. \begin{aligned} F_3 C_3 - F_4 C_4 + KA (C_1 - C_4) &= V_2 dC_4/dt \\ F_1 C_1 - F_2 C_2 - KA (C_1 - C_4) &= V_3 dC_2/dt \end{aligned} \right\} \text{model — 1}$$

$$\left. \begin{aligned} F_3 C_3 - F_4 C_4 + KA (C_2 - C_3) &= V_2 dC_4/dt \\ F_1 C_1 - F_2 C_2 - KA (C_2 - C_3) &= V_3 dC_2/dt \end{aligned} \right] \text{model — 2}$$

Keeping flow rates as constants,

$$F_1 = F_2; F_3 = F_4$$

At steady state model 1 equations become

$$F_3 C_3 s - F_3 C_4 s + KA (C_1 s - C_4 s) = V_2 dC_4 s/dt$$

$$F_1 C_1 s - F_1 C_2 s - KA (C_1 s - C_4 s) = V_3 dC_2 s/dt$$

Subtracting steady state equations from unsteady state equations and using  $C_1 - C_1 s = C_1'$ ,  $C_2 - C_2 s = C_2'$ ;  $C_4 - C_4 s = C_4'$ , and  $C_3$  is constant.

$$F_1 C_1' - F_1 C_2' - KA (C_1' - C_4') = V_3 dC_2'/dt$$

$$F_3 C_3' - F_3 C_4' + KA (C_1' - C_4') = V_2 dC_4'/dt$$

Taking Laplace transforms on both sides of the equations and solving simultaneously,

$$\frac{C_4'(s)}{C_1'(s)} = \frac{KA/(F_3 + KA)}{V_2/(F_3 + KA) s + 1}$$

$$\frac{C_2'(s)}{C_1'(s)} = \frac{(F_1 - KA)(F_3 - KA) + K^2 A^2 + (F_1 - KA) s}{s^2 V_3 V_2 + s (F_1 V_2 + F_3 V_3 + K A V_3) + F_1 (F_3 + K A)}$$

Similarly using the same procedure, transfer functions are obtained using model — 2 equations.

### 1.33. Quiz questions

- (a) Laplace transform of frequency input of magnitude,  $A$  :

**Ans.**  $AW/(s^2 + W^2)$

- (b) Controlled variable in a heat exchanger.

**Ans.** Temperature of cooling or heating fluid (outlet temperature)

- (c) Manipulated variable in heat exchanger.

**Ans.** Flow rate of heating or cooling fluid.

- (d) Load variable in heat exchanger.

**Ans.** Temperature of entering hot or cold fluid.

- (e) Transfer function can be derived for highly non-linear systems or not ?

**Ans.** No.

- (f) Time constant of mixer.

**Ans.** Volume of mixer/flow rate of solution.

(g) What this symbol stands for in block diagrams.  $\begin{matrix} +x \\ \rightarrow \otimes \rightarrow z \\ +\uparrow y \end{matrix}$

**Ans.** Summing function.

(h) General transfer function of a first order system.

**Ans.**  $R/(Ts + 1)$ .

(i) General transfer function of a second order system.

**Ans.**  $\frac{R}{(T^2 s^2 + 2\xi Ts + 1)}$ .

### 1.34. Obtaining frequency response values in a simple way :

For first order system :

$$y(s)/x(s) = \text{output/input}$$

$$= R/(Ts + 1) = G(s), \text{ putting } s = iW; G(iW) = \frac{R}{(TiW + 1)}$$

Multiplying and dividing by  $(1 - TiW)$ ,

$$G(iW) = R(1 - iWT)/(1 + W^2T^2)$$

This  $G(iW)$  is a complex number. Magnitude of a complex number is :

$$\sqrt{(\text{Real Part})^2 + (\text{imaginary part})^2}$$

$$= R/(1 + W^2T^2)^{\frac{1}{2}}$$

$$(\text{Phase}) = \tan^{-1} (\text{imaginary part/Real part})$$

$$= \tan^{-1} (-WT) = -\tan^{-1} (WT)$$

$$y(t) = \frac{R}{(1 + W^2T^2)^{1/2}} \sin(Wt + \phi) \quad \text{Ans.}$$

Same procedure is applied for obtaining frequency response of other transfer functions shown in later chapters.

### 1.35. Developing dynamic equations (complicated example) :

Thermal dynamics of a distributed parameter non-adiabatic humidification Process : (Ref : R.R. Stewart and D.F. Bruley, Vol. 13, No. 4, A.I.Ch.E.J. 1967).

#### Assumptions :

1. Flat velocities and temperature profiles exist in both phases.
2. Gas phase mass velocity is constant.
3. Film theory applies.
4. Mass transfer sensible heat is negligible.
5. Heat loss from the liquid.

Phase to the surroundings is negligible.

$$\frac{\partial t_g}{\partial \theta} + V_g \frac{\partial t_g}{\partial z} = \frac{UP}{\rho_g CP_g A_g} (t_l - t_g) \quad \dots(15)$$

$$\frac{\partial t_l}{\partial \theta} - V_l \frac{\partial t_l}{\partial z} = \frac{UP}{\rho_l CP_l A_l} (t_g - t_l) - \frac{\lambda K_H P}{\rho_l CP_l A_l} (H_{gi}' - H_g') \quad \dots(16)$$

For simplification,  $H_g' = 0$ ,  $H_{gi}' = \alpha t_i$

Putting time constant ( $T$ ) and taking Laplace transforms,

$$\frac{d\bar{t}_g}{dz} + \frac{s(1/\tau_g)}{V_g} \bar{t}_g = -\frac{1}{V_g T_g} \bar{t}_l \quad \dots(17)$$

$$\frac{d\bar{t}_l}{dz} - \frac{s + (1/\tau_l) + (1/\tau_{mg})}{V_l} \bar{t}_l = \frac{(1/\tau_l) - (1/\tau_{ml})}{V_l} \bar{t}_g \quad \dots(18)$$

Model 1 is obtained by solving equations (17) and (18) with boundary conditions,  $\bar{t}_L = 0$  at  $z = L$ ,  $\bar{t}_g = \bar{t}_{gO}$  at  $z = 0$ .

$$\frac{\bar{t}_{gL}}{\bar{t}_{gO}}(s) = \frac{1}{B(s)} e^{-C(s)L} [B(s) \cos [B(s)L + \sin [B(s)L] \left[ \frac{D \sin B(s)L}{B(s) \cos B(s)L + E(s) \sin B(s)L} - E(s) \right] \dots \text{Model 1}$$

Model 2 neglects mass transfer, that is  $\alpha = 0$ .

$$\text{Model 1} \mid_{\tau_{mg} = \tau_{ml} = \infty} \dots \text{Model 2.}$$

Model 3 considers the effect of gas phase temperature changes on  $\tau_g$  through linearizing expression :

$$\frac{1}{\tau_g} = \frac{1}{T_{gss}} \left( 1 + \beta \frac{t_g}{T_{gssO}} \right)$$

$$\frac{\bar{t}_{gl}}{\bar{t}_{gO}}(s) = \exp \left\{ \left[ \frac{M}{K} (1 - \exp (KL)) - [F(s) + N(s)]L \right] \right\} \dots \text{Model 3.}$$

Model 4 assumes that mass transfer, heat effect and liquid temperature change  $t_l$  are negligible. These reduce equations (17) and 18 to

$$\frac{d\bar{t}_g}{dz} + \frac{s + (1/\tau_g)}{V_g} \bar{t}_g = 0$$

$$\frac{\bar{t}_{gL}}{\bar{t}_{gO}}(s) = \exp - \frac{s + (1/\tau_g)}{V_g} L \quad \dots \text{Model 4.}$$

Nomenclature :  $A$  = cross sectional area, sq. ft.

$$B = (D - E^2)^{\frac{1}{2}}$$

$C = (F - G)/2$ ;  $CP$  = heat capacity, BTU/(lb.°F)



$$D = 1/(\tau_g \tau_l V_g V_l) + \frac{1}{(\tau_g \tau_{ml} V_g V_l)}$$

$$E = (F + G)/2; F = (s + 1/\tau_g)/V_g$$

$$G = [(s + 1/\tau_l) + (1/\tau_{mg})]/V_l$$

$h$  = film heat transfer coefficient, BTU/(hr.sq.ft. °F)

$H'$  = dynamic humidity, lb. vapour/lb. gas

$K_H$  = gas phase mass transfer coefficient, lb/(hr. sq. ft.  $\Delta H$ )

$L$  = length of tower, ft.;  $M = \beta j / (T_{gSS} V_g T_{gSSO})$

$N = \beta I / T_{gSS} V_g T_{gSSO}$

$P$  = circumference of gas-liquid interface, ft.

$s$  = Laplace transform;

$t$  = temperature change from steady state;

$\bar{t}$  = Laplace transformed temperature change;

$T$  = temp. °F;  $U$  = overall heat transfer

Coefficient =  $h_g h_l / (h_g + h_l)$

$V$  = velocity, ft/hr;  $z$  = axial position, ft;

$I = (T_{ISSO} - T_{iSSL} - T_{gSSO} + T_{gSSL}) / [1 - \exp. (KL)]$

$$j = \frac{(T_{iSSL} - T_{gSSL}) + (T_{gSSO} - T_{iSSO}) \exp (KL)}{1 - \exp (KL)}$$

$\alpha$  = Constant relating change of interface liquid temperature to change in interface humidity, (lb. vapour/lb. dry air)/ °F

$\beta$  = dimensionless constant relating change in gas phase time constant to change in gas phase temperature.

$\theta$  = time, hr. ;  $\rho$  = density, lb<sub>m</sub>/ft<sup>3</sup>;  $\lambda$  = latent heat;

$\tau_g = \rho_g CP_g A_g / (UP)$ , hr;  $\tau_l = \rho_l CP_l A_l / UP$ , hr ;

$\tau_{mg} = h_g \tau_g / (\lambda K_H^2)$ , hr. ;  $\tau_{ml} = h_l \tau_l / (\lambda K_H \alpha)$ , hr;

$g$  = gas Phase;  $l$  = liquid phase ;  $i$  = interface;

$ss$  = steady state;  $m$  = mass transfer ;

$o$  = air inlet,  $L$  = air outlet ;

**1.36. Dynamics of a packed liquid extraction column,** shown in Fig. 1.16.

**Ref :** J.E. Doninger, W.F. Stevans, A.I. Ch. E.J., Vol. 14, No. 4, 1965. Mixing cell model is used.

The dynamic equations are

$$x_{n-1} L - x_n L - Q_a H = H h_L \frac{dx_n}{dt}$$

$$y'_{n+1} G - y'_n G + Q_a H = H h_G \frac{dy'_n}{dt}$$

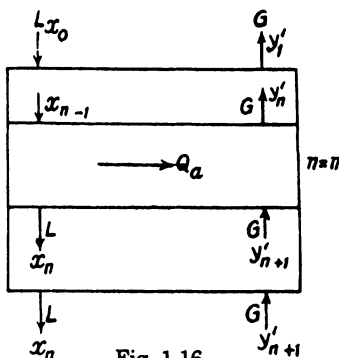


Fig. 1.16

where  $Q_a = K_L a(x_n - x_n^*)$ .

$$y' = mx^* - b = 0.768 x^* - 0.0056$$

(HAC-water—organic Phase)

$L$  = water phase flow rate, gm/(min. sq. cm.)

$G$  = organic phase, gm/(min. sq. cm.)

$x$  = mass fraction of acetic acid in water phase,  
gm. acid/gm. solution

$y$  = mass fraction of acetic acid in organic phase, gm  
HAC/gm solution

$a$  = mass transfer area, sq. cm/c.c. packing

$Q$  = mass transfer rate, gm/(sq. cm. min. transfer area)

$x^*$  = equilibrium value of  $x$

$K_{La}$  = mass transfer coefficient, water phase, gm/(min. c.c.)

$$y_n = y'_n + b$$

$h_L$  = hold up of raffinate ;  $h_G$  = hold up of extract

By taking Laplace transformation of the first four equations, the following equations are obtained.

$$T_L \bar{x}_{n+2} - (T_L T_G - fg + 1) \bar{x}_{n+1} + T_G \bar{x}_n = 0$$

$$T_L = \frac{H h_{LS}}{L} + \frac{H K_{La}}{L} + 1 ; f = \frac{H K_{La}}{mL}$$

$$T_G = \frac{H h_{GS}}{G} + \frac{H K_{La}}{mG} + 1 ; g = \frac{H K_{La}}{G}$$

$H$  = packing height, cm. (mixing cell)

$\bar{x}, \bar{y}$  = Laplace transform of concentration variable.

$y'$  = mass fraction of acetic acid in organic phase.  
gm acid/gm. solution

$m$  = distribution coefficient

$\bar{x}_n = \bar{x}_0$  at  $n = 0$  ;  $\bar{y}_{n+1} = 0$  at  $n = N$  (boundary values)

$$G(s) = \frac{\bar{x}_N}{\bar{x}_0} = \frac{T^{GN} \sqrt{(T_L T_G - fg + 1)^2 - 4 T_L T_G}}{T_L T_G (D_2^N - D_1^N - D_2^{N-1} + D_1^{N-1}) - fg (D_2^N - D_1^N)}$$

where  $D_{1,2} = \frac{T_L T_G - fg + 1 \pm \sqrt{(T_L T_G - fg + 1)^2 - 4T_L T_g}}{2}$

### 1.37. Well mixed isothermal crystallizer :

Ref : M.B. Sherwin, R. Sinnar and S. Katz, A.I. Ch. E.J., Vol. 13, No. 6, 1967, shown in Fig. 1.17.

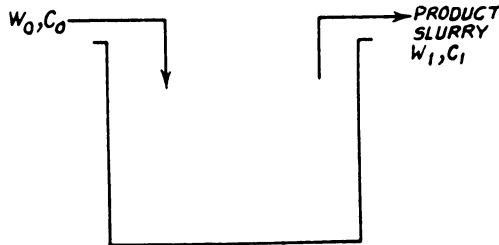


Fig. 1.17

Conservation equation :

$$\begin{aligned} \text{Particle balance : } & \frac{\partial}{\partial t} (V^1 f) + \frac{\partial}{\partial r} (G^2 \phi V) \\ & = \Sigma V B \delta(r^3 - r_0) - W_1^4 f + W_O^5 \psi \end{aligned}$$

Solute and crystal balance :

$$\begin{aligned} & \frac{d}{dt} [V (\epsilon C^6 + (1 + \epsilon) - (1 - \epsilon) \rho)] \\ & = W_O [\epsilon_O C_O^7 + (1 - \epsilon_O) \rho] - W_1 [\epsilon C^8 + (1 - \epsilon) \rho] \end{aligned}$$

Term 1 = accumulation of crystals at size  $r$  ; term 2 = net flux of crystals away from size  $r$  due to growth, term 3 = input of particles at size  $r_0$  due to nucleation ; term 4 = withdrawal of particles of size  $r$  due to product removal ; term 5 = input of particles of size  $r$  due to solids in feed ; term 6 = accumulation of solute and crystals, term 7 = input of solute and crystal by feed stream ; term 8 = removal of solute and crystal due to product withdrawal.

The above equations were simplified and solved with some assumptions.

### 1.38. Matrices and determinants.

$$\text{Matrix } A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{bmatrix}$$

Horizontal—row, vertical—column

$$\text{Vector} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \text{ Matrix } A = [A] \\ \text{Determinant } A = |A|$$

Square matrix is the one where the number of rows is equal to the number of columns.

$$\text{Diagonal matrix } A = \begin{bmatrix} a_{11} & 0 & 0 & \dots & 0 \\ 0 & a_{12} & 0 & \dots & 0 \\ 0 & 0 & a_{13} & \dots & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \dots & a_{nn} \end{bmatrix}$$

$$[A] + [B] = [(a_{ij} + b_{ij})]; [A] - [B] = [a_{ij} - b_{ij}]$$

$$K[A] = \begin{bmatrix} Ka_{11} & Ka_{12} & \dots & Ka_{1m} \\ Ka_{21} & Ka_{22} & \dots & Ka_{2m} \\ \vdots & & & \\ Ka_{n1} & Ka_{n2} & \dots & Ka_{nm} \end{bmatrix} \quad \text{where } K \text{ is a scalar quantity}$$

$$[A] [B] = [C] = C_{ij} = \sum_{K=1}^n a_{iK} b_{Kj}$$

$$[AB] [C] = [A] [BC]$$

$$[A + B] [C] = [AC] + [BC]$$

$$[C] [A + B] = [CA] + [CB]; [AB] \neq [BA]$$

$$\text{Inverse of a matrix : if } A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$A^{-1} = \frac{1}{(ad - bc)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$\text{Adjoint matrix : } adj A = \begin{bmatrix} A_{11} & A_{21} & \dots & A_{n1} \\ A_{12} & A_{22} & \dots & A_{n2} \\ \vdots & & & \\ A_{1n} & A_{2n} & \dots & A_{nn} \end{bmatrix}$$

$$A^{-1} = \frac{\text{adjoint } A}{|A|}$$

where  $|A|$  is determinant of  $A$ .

where  $A_{11}, A_{21}, \dots, A_{nn}$  are cofactors.

$$\text{Transpose of } A = \begin{bmatrix} a_{11} & a_{21} & \dots & a_{n1} \\ a_{12} & a_{22} & \dots & a_{n2} \\ \vdots & & & \\ a_{1m} & a_{2m} & \dots & a_{nm} \end{bmatrix}$$

$$\frac{d}{dt} A(t) = \begin{bmatrix} \frac{d}{dt} a_{11}(t) & \frac{d}{dt} a_{12}(t) & \dots & \frac{d}{dt} a_{1n}(t) \\ \frac{d}{dt} a_{21}(t) & \frac{d}{dt} a_{22}(t) & \dots & \frac{d}{dt} a_{2n}(t) \\ \vdots & & & \\ \frac{d}{dt} a_{n1}(t) & \frac{d}{dt} a_{n2}(t) & \dots & \frac{d}{dt} a_{nm}(t) \end{bmatrix}$$

$$\int A(t) dt = \left[ \int a_{ij}(t) dt \right]$$

$$= \begin{bmatrix} \int a_{11}(t) dt & \int a_{12}(t) dt & \dots & \int a_{1n}(t) dt \\ \vdots & & & \\ \int a_{1m}(t) dt & \int a_{2m}(t) dt & \dots & \int a_{nm}(t) dt \end{bmatrix}$$

Multiplication of matrices :

$$[A] [B] = [C] = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}$$

$$C_{11} = a_{11} b_{11} + a_{12} b_{21} ; C_{12} = a_{11} b_{12} + a_{12} b_{22}$$

$$C_{21} = a_{21} b_{11} + a_{22} b_{21} ; C_{22} = a_{21} b_{12} + a_{22} b_{22}$$

$$C_{jk} = \sum_{i=1}^2 a_{ji} b_{ik} = a_{j1} b_{1k} + a_{j2} b_{2k}$$

**Determinants :**

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = (a_{11} a_{22} - a_{21} a_{12})$$

$$|A| |B| = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix} \begin{vmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \vdots & & & \\ b_{n1} & b_{n2} & \dots & b_{nn} \end{vmatrix}$$

$$= \begin{vmatrix} C_{11} & C_{12} & \dots & C_{1n} \\ C_{21} & C_{22} & \dots & C_{2n} \\ \vdots & & & \\ C_{n1} & C_{n2} & \dots & C_{nn} \end{vmatrix}$$

$$C_{ik} = a_{i1} b_{1k} + a_{i2} b_{2k} + \dots + a_{in} b_{nk}.$$

**1.39. Determine the transfer function  $H(s)/Q(s)$  for the liquid level system shown in Fig. 1.18. Resistances are linear ( $R_1$  and  $R_2$ ). The flow rate from tank 3 is maintained constant at  $b$  by means of a pump i.e. the flow rate from tank 3 is independent of head  $h$ . The tanks are non-interacting.**

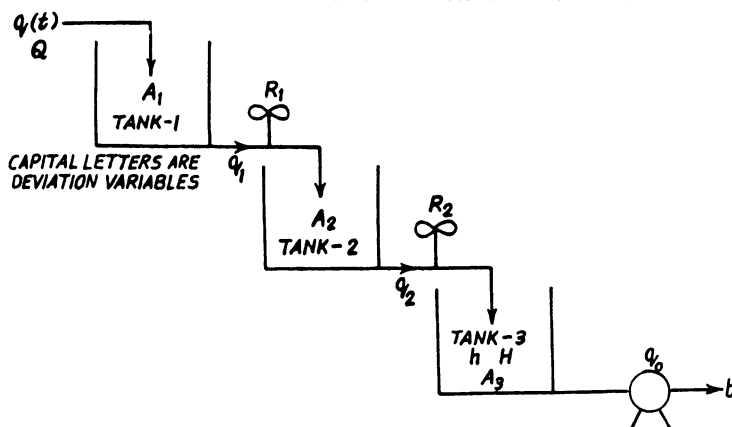


Fig. 1.18

$$\frac{H(s)}{Q(s)} = \frac{H(s)}{Q_2(s)} \cdot \frac{Q_2(s)}{Q_1(s)} \cdot \frac{Q_1(s)}{Q(s)}$$

To obtain  $H(s)/Q_2(s)$ : Applying mass balance for tank—3

$$q_2 - q_o = A_3 \frac{dh}{dt}$$

at steady state,

$$q_2s - q_0s = A_3 \frac{dhs}{dt}$$

in deviation form

$$(q_2 - q_2s) - (q_o - q_0s) = A_3 \frac{d}{dt} (h - h_s)$$

$$Q_2 = A_3 dH/dt$$

taking Laplace transforms on both sides

$$Q_2(s) = sA_3 H(s), \quad Q_2(s)/Q_1(s) = \frac{1}{(T_2s + 1)}, \quad \frac{H(s)}{Q_2(s)} = \frac{1}{A_3s}$$

$$\frac{Q_1(s)}{Q(s)} = \frac{1}{(T_1s + 1)}$$

$$\frac{H(s)}{Q(s)} = \frac{1}{A_3s} \cdot \frac{1}{(T_1s + 1)} \cdot \frac{1}{(T_2s + 1)}$$

where  $T_1 = A_1R_1$ ,  $T_2 = A_2R_2$  **Ans.**

#### 1.40. Linearizing non-linear behaviour.

Non-linear terms are linearised by using Taylor's series expansion.

(1) Linearization of one variable :

Neglecting higher order terms (derivatives)

$$Z(x) = Z(x_s) + \left. \frac{\partial Z}{\partial x} \right|_{x_s} (x - x_s) \quad \dots(19)$$

where  $x_s$  is steady state value.

**For Example :**  $q = C\sqrt{h}$  in liquid — level system (flow rate and level relation)

where  $q$  = flow rate,  $C$  = constant and  $h$  = liquid level in tank.

For single tank shown in Fig. 1.19,

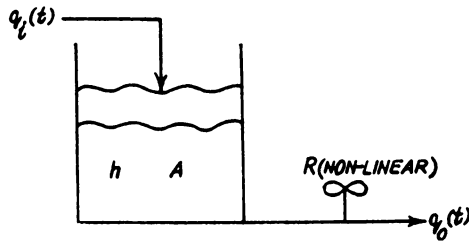


Fig. 1.19

The unsteady-state material balance equation is (assuming constant area of cross-section,  $A$  and density of fluid in the tank)

$$q_i - q_o = A \, dh/dt \quad \dots(20)$$

$$q_o = C \sqrt{h} \quad (\text{non-linear relation}) \quad \dots(21)$$

$$q_i - C\sqrt{h} = A \, dh/dt$$

We can not take Laplace transforms on both sides for the above equation and obtain transfer function since  $\sqrt{h}$  is present. Since  $q_o$  is a function of  $h$ , using Taylor's series expansion,

$$q_o = q_{os} + \left. \frac{\partial q_o}{\partial h} \right|_{h_s} (h - h_s)$$

$$\frac{\partial q_o}{\partial h} = \frac{\partial}{\partial h} (C\sqrt{h}) = \frac{C}{2\sqrt{h}}$$

$$\left. \frac{\partial q_o}{\partial h} \right|_{h_s} = \frac{C}{2\sqrt{h_s}}; q_{os} = C\sqrt{h_s}$$

$$\text{So} \quad q_o = C\sqrt{h_s} + (h - h_s) \frac{C}{2\sqrt{h_s}}$$

Eq. 20 becomes,

$$q_i - C\sqrt{h_s} - (h - h_s) \frac{C}{2\sqrt{h_s}} = A \frac{dh}{dt} \quad \dots(22)$$

In deviation form,

$$(q_i - q_{is}) - \frac{C}{2\sqrt{h_s}} (h - h_s) = A \frac{d}{dt} (h - h_s) \quad \dots(23)$$

In deviation form,

$$Q_i - \frac{C}{2\sqrt{h_s}} H = \frac{AdH}{dt} \quad \dots(24)$$

Now, taking Laplace transforms on both sides of equation 24,

$$Q_i(s) - \frac{C}{2\sqrt{h_s}} H(s) = sA H(s) \quad \dots(25)$$

$$Q_i(s) = H(s) \left[ As + \frac{C}{2\sqrt{h_s}} \right]; \frac{H(s)}{Q_i(s)} = \frac{2\sqrt{h_s}}{(As2\sqrt{h_s} + C)}$$

If we compare with the standard form of first order transfer function,

$$\frac{H(s)}{Q_i(s)} = \frac{R}{Ts + 1}; R = 2h_s^{1/2}/C; T = RA$$

(2) Linearization of a term having two variables :

Taylor's series expansion is

$$\begin{aligned} Z(x, y) = Z(x_s, y_s) &+ \left. \frac{\partial Z}{\partial x} \right|_{x_s, y_s} (x - x_s) \\ &+ \left. \frac{\partial Z}{\partial y} \right|_{y_s, x_s} (y - y_s) + \text{higher order terms in} \quad \dots(26) \\ &\quad (x - x_s) \text{ and } (y - y_s) \end{aligned}$$

Neglecting higher order terms,

$$\begin{aligned} Z(x, y) \approx Z(x_s, y_s) &+ \left. \frac{\partial Z}{\partial x} \right|_{x_s, y_s} (x - x_s) \\ &+ \left. \frac{\partial Z}{\partial y} \right|_{x_s, y_s} (y - y_s) \quad \dots(27) \end{aligned}$$

If  $Z$  is a function of 3 or more variables, the linearized form is same as that of equation 27 with additional terms for each variable.

For example : In a dialyser using model 1 equations as shown in item 1.32.

$$F_3 C_3 - F_4 C_4 + KA(C_1 - C_4) = V_2 dC_4/dt$$

$$F_1 C_1 - F_2 C_2 - KA(C_1 - C_4) = V_3 dC_2/dt$$

We have seen, the equations are linear if flow rates are constants.

Suppose if flow rates ( $F$ 's) are also changing, then the first two terms in the equations become non-linear. Then using Taylor's series expansion,

$$F_3 C_3 = F_{3s} C_{3s} + (F_3 - F_{3s}) C_{3s} + (C_3 - C_{3s}) F_{3s}$$

$$F_4 C_4 = F_{4s} C_{4s} + (F_4 - F_{4s}) C_{4s} + (C_4 - C_{4s}) F_{4s}$$



$$F_1 C_1 = F_{1s} C_{1s} + (F_1 - F_{1s}) C_{1s} + (C_1 - C_{1s}) F_{1s}$$

$$F_2 C_2 = F_{2s} C_{2s} + (F_2 - F_{2s}) C_{2s} + (C_2 - C_{2s}) F_{2s}$$

using the above linearized relations, transfer functions can be determined.

#### 1.41. Process dynamics of plate absorbers

The treatment of absorbers and distillation columns is same. The arrangement of a  $n$  stage absorber is shown in Figure 1.20.

$V$  – vapour flow rate

$L$  – liquid flow rate

$x$  – liquid composition

$y$  – vapour composition

$b$  – constant

$H$  – liquid hold up on plate

$h$  – vapour hold up on plate

$T$  – time constant

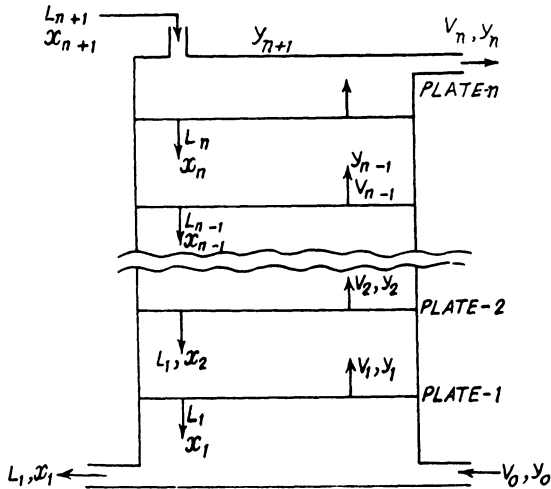


Fig. 1.20

The assumptions are (a) Perfect mixing of vapour and liquid phases (b) equilibrium relation of the form

$$y_n = mx_n + b \text{ holds. (c) no chemical reaction.}$$

Unsteady state mass balance of a component on plate  $n$  is :  
Using input-output = Rate of accumulation,

$$\frac{d}{dt} (H_n x_n + h_n y_n) = L_{n+1} x_{n+1} + V_{n-1} y_{n-1} - L_n x_n - V_n y_n$$

assuming

$$H_1 = H_2 = \dots = H = H_n$$

$$h_1 = h_2 = \dots = h_n = h$$

$$\frac{d}{dt} (Hx_n + mhx_n) = L_{n+1} x_{n+1} + V_{n-1} (mx_{n-1} + b) - L_n x_n - V_n (mx_n + b)$$

It is reported by experiments that the overall mass balance equations of form,

$$T \frac{dL_1}{dt} = L_2 - L_1, T \frac{dL_2}{dt} = L_3 - L_2, \dots, T \frac{dL_n}{dt} = L_{n+1} - L_n.$$

So for a  $n$ -plate column, there will be  $n$  equations of component balance and  $n$  equations of overall balance. If temperature also changes on each plate of the absorber, there will be  $n$  equations representing temperature variation. These equations are solved simultaneously to obtain the solutions with the help of boundary conditions.

*For I-plate absorber :* The equations become

$$\frac{d}{dt} (Hx_1 + mhx_1) = L_2 x_2 + V_0 (x_0 m + b) - L_1 x_1 + V_1 (m_1 x_1 + b) \quad \dots(1)$$

$$T \frac{dL_1}{dt} = L_2 - L_1 \quad \dots(2)$$

in deviation form, equations 1 and 2 can not be written because of the presence of product of two variables ( $L$  and  $x$ ). Either they have to be linearized by using Taylor's expansion or we have to assume  $L_1 = L_2 = L$ .

Assuming  $L_1 = L_2 = L = \text{constant}$  and  $V_0 = V_1 = V = \text{constant}$  equation 1 becomes in deviation form as,

$$\frac{d}{dt} (Hx_1' + mhx_1') = Lx_2' + V_0 x_0' m - L_1 x_1' - V_1 m x_1'$$

taking Laplace transforms on both sides gives,

$$sHx_1'(s) + shm x_1'(s) = Lx_2'(s) + V_0 x_0'(s) m - L_1 x_1'(s) - V_1 m x_1'(s)$$

collecting common terms, we get

$$x_1'(s) [sH + shm + V_1 m + L_1] = Lx_2'(s) + V_0 m x_0'(s)$$

$$\text{if } x_0 \text{ is constant } (y_0) \quad \frac{x_1'(s)}{x_2'(s)} = \frac{L}{s(H + mh) + (L_1 + V_1 m)}$$

$$\text{if } x_2 = \text{constant}, \quad \frac{x_1'(s)}{x_0'(s)} = \frac{mV_0}{s(H + mh) + (L_1 + V_1 m)}$$

So first order transfer functions are obtained for one stage. For two stage, second order transfer function is obtained and for  $n$  stage column,  $n$ th order transfer function will be obtained.

### 1.42. Heat exchangers (Fig. 1.21)

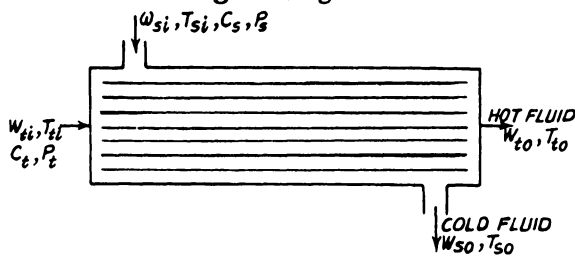


Fig. 1.21

$W_t$  = flow rate of tube side fluid,  $W_s$  = flow rate of shell side fluid,  $T_s$  = temperature of shell side fluid,  $T_t$  = temperature of tube side fluid,  $i$  = inlet,  $O$  = outlet,  $V_t$  = hold up volume on tube side,  $V_s$  = hold up volume on shell side,  $t$  = tube side,  $s$  = shell side.

Assumptions : (1) temperature variation along the axes is neglected (Lumped parameter is assumed) (2) flow rates are equal, that is  $W_{si} = W_s = W_{so}$  and  $W_t = W_{ti} = W_{to}$ . (3) Physical properties do not vary much.

The unsteady heat balance equation on tube side is :

$$\begin{aligned} W_t C_t \rho_t T_{ti} - C_t W_t \rho_t T_{to} - UA (T_{to} - T_{so}) \\ = V_t \rho_t C_t \frac{dT_{to}}{dt} \end{aligned} \quad \dots(1)$$

Where  $\rho$  = density,  $C$  = heat capacity,  $U$  = overall heat transfer coefficient,  $A$  = heat transfer area.

The heat balance equation on shell side is :

$$\begin{aligned} W_s C_s \rho_s T_{si} - C_s W_s \rho_s T_{so} + UA (T_{to} - T_{so}) \\ = V_s \rho_s C_s \frac{dT_{so}}{dt} \end{aligned} \quad \dots(2)$$

The equations (1) and (2) are the process dynamic equations (simplest type) of heat exchanger, obtained in simple form, with some unrealistic assumptions. In practice, in heat exchangers, temperature varies considerably along the length.

Equation 1 and 2 are of linear type, if one of the two variables (flow rate, temperature) is kept constant. So transfer functions can be obtained. For rigorous equations, distributed parameter model is to be used.

Equation (1) in deviation form is : (keeping  $W_s$ ,  $W_t$ ,  $T_{ti}$  as constant)

$$\begin{aligned} W_t C_t \rho_t T_{ti}' - W_t C_t \rho_t T_{to}' - UA (T_{to}' - T_{so}') \\ = V_t C_t \rho_t \frac{dT_{to}'}{dt} \end{aligned} \quad \dots(3)$$

Eq. 2 in deviation form is :

$$W_s C_s \rho_s T_{si}' - W_s C_s \rho_s T_{s0}' + UA (T_{t0}' - T_{s0}') = V_s \rho_s C_s dT_{s0}'/dt \quad \dots(4)$$

in Laplacian form, equations 3 and 4 are

$$- W_t C_t \rho_t T_{t0}'(s) - UA[T_{t0}'(s) - T_{s0}'(s)] = V_t \rho_t C_t s T_{t0}'(s) \quad \dots(5)$$

$$W_s C_s \rho_s T_{si}'(s) - W_s C_s \rho_s T_{s0}'(s) + UA[T_{t0}'(s) - T_{s0}'(s)] = V_s \rho_s C_s s T_{s0}'(s) \quad \dots(6)$$

Eliminating  $T_{t0}'(s)$  between equations (5) and (6), and using  $V_t \rho_t C_t s + UA + W_t \rho_t C_t = A_1 s + B$  gives,

$$\frac{T_{s0}'(s)}{T_{si}'(s)} = \frac{W_s C_s \rho_s (A_1 s + B)}{(s V_s \rho_s C_s + W_s C_s \rho_s) (A_1 s + B) - U^2 A^2 + UA(A_1 s + B)}$$

Similarly, transfer functions for other outlet/inlet variables can be obtained.

#### 1.43. Inputs :

(a) Step function :  $x(t)$

The step function is shown in Fig. 1.22

$$x = 0 ; t < 0$$

$$x = A ; t > 0$$

$$x(s) = A/s$$

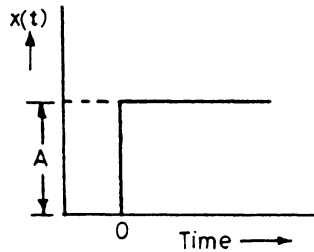


Fig. 1.22

(b) Impulse function : shown in

Fig. 1.23

at  $x = 0 ; t < 0$

$$x = A/b ; 0 < t < b$$

$$x = 0, t > b$$

$$L[A\delta(t)] = L(\text{Impulse}) = A$$

(c) Sinusoidal input :

Shown in Fig. 1.24

$$x = 0 ; t < 0$$

$$x = A \sin \omega t, t \geq 0$$

$$x(s) = A\omega/(s^2 + \omega^2)$$

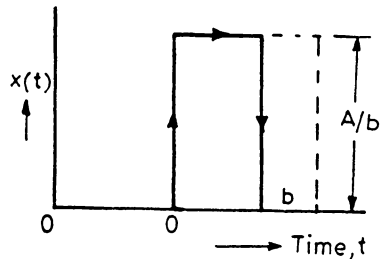


Fig. 1.23

where  $A$  is amplitude of sine wave

**1.44. Single Tank block diagram :** Shown in Figs. 1.25 and 1.26

Inflow rate ( $q_i$ ) is manipulated variable ( $m$ ). The vessel head ' $c$ ' is the controlled variable and ' $u$ ' is downstream head as load variable.

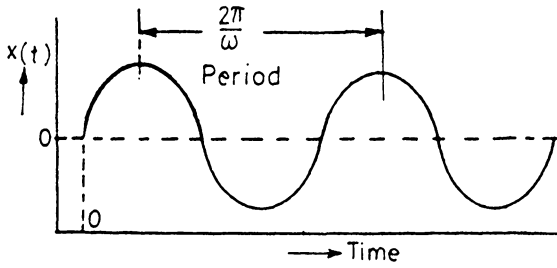


Fig. 1.24

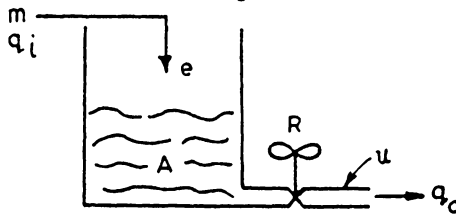


Fig. 1.25

By mass balance to the tank,

$$m - q_o = A \, dc/dt \quad \dots(1)$$

$$q_o = \frac{1}{R} (c - u) \quad \dots(2)$$

where  $R$  is valve resistance,  $A$  is area of cross-section of tank and  $q_o$  is outlet flow rate.

Replacing  $q_o$  in Eqn. (1) by Eqn. (2) gives,

$$A \dot{c} = m - \frac{1}{R} (c - u)$$

$$T \dot{c} + c = R_m + u$$

$$\text{where } T = RA$$

Taking Laplace transforms on both sides of the equation, and taking initial values as zeroes gives,

$$sTc(s) + c(s) = Rm(s) + u(s)$$

$$c(Ts + 1) = Rm(s) + u(s)$$

$$\frac{c(s)}{m(s)} = \frac{R}{Ts + 1} \quad [\text{taking } u(s) = 0] ; \quad \frac{c(s)}{u(s)} = \frac{1}{Ts + 1}$$

taking

$$m(s) = 0$$

The block diagram for the above single tank is, Fig. 1.26

$G_c$  = controller transfer function,

$G_m$  = Measuring instrument transfer function.

**1.45. Non-interacting systems :** An example is arranging tanks one below the other as shown in Fig. 1.27.

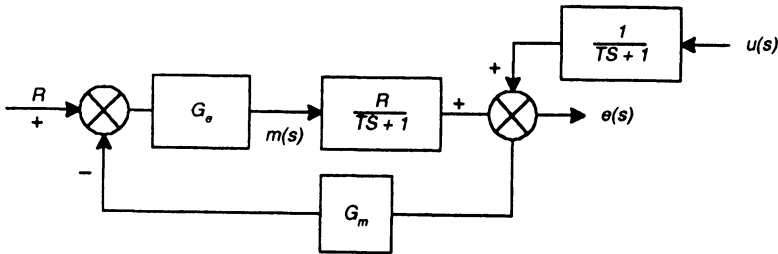


Fig. 1.26

$$\frac{H_2(s)}{Q(s)} = \frac{Q_1(s)}{Q(s)} \cdot \frac{H_2(s)}{Q_1(s)};$$

We know,  $\frac{Q_1(s)}{Q(s)} = \frac{1}{(T_1s + 1)}; \frac{H_2(s)}{Q_1(s)} = \frac{R_2}{(T_2s + 1)}$

So  $\frac{H_2(s)}{Q(s)} = \left( \frac{1}{(T_1s + 1)} \right) \frac{R_2}{(T_2s + 1)}$

In general for non-interacting systems,

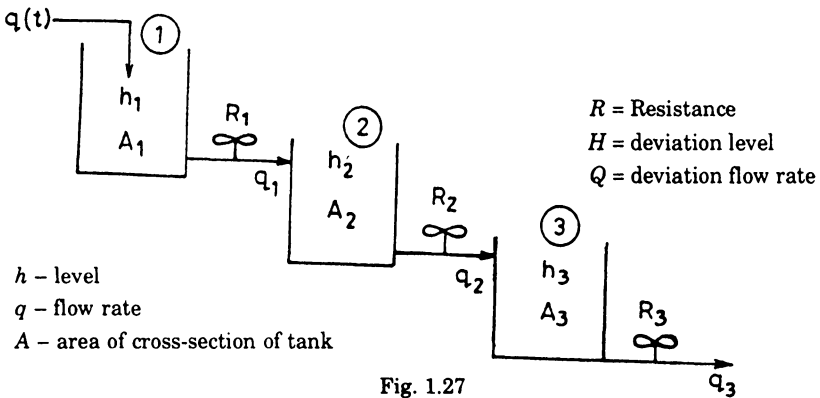


Fig. 1.27

$$\frac{X_n(s)}{X_0(s)} = \prod_{i=1}^n \frac{k_i}{(T_i s + 1)} \quad \dots(3)$$

**1.46. Interacting systems :** An example is arranging the tanks as shown in Fig. 1.28, side by side.

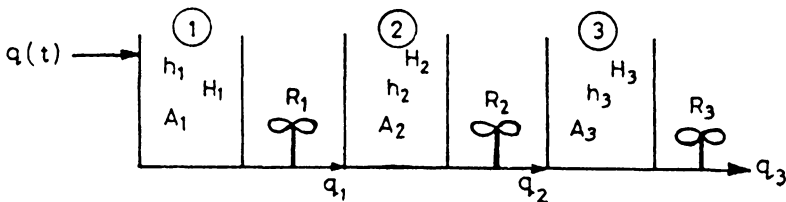


Fig. 1.28

In this arrangement, the flow into the second tank depends on level difference in first and second tanks.

$$q_1 = \frac{1}{R_1} (h_1 - h_2)$$

$$Q - Q_1 = A_1 dH_1/dt ;$$

$$Q_1 - Q_2 = A_2 dH_2/dt$$

in deviation variables,

$$Q_1 = \frac{1}{R_1} (H_1 - H_2) ;$$

$$Q_2 = H_2/R_2$$

Taking Laplace transforms for the above equations,

$$Q(s) - Q_1(s) = sA_1H_1 \quad \dots(4)$$

$$Q_1(s) - Q_2(s) = sA_2H_2 \quad \dots(5)$$

$$R_1Q_1(s) = H_1(s) - H_2(s) \quad \dots(6)$$

$$R_2Q_2(s) = H_2(s) \quad \dots(7)$$

Eliminating  $Q_1(s)$ ,  $Q_2(s)$  and  $H_1(s)$  in equations (4) to (7) among them, gives

$$\frac{H_2(s)}{Q(s)} = \frac{R_2}{T_1T_2 s^2 + (T_1 + T_2 + A_1R_2) s + 1} \quad \dots(8)$$

**1.47. Problem :** Feed at a rate of  $q = 40$  lit/minute and a composition  $c_e = 0.30$  gm mole/litre is entering a continuous stirred tank reactor containing a constant volume of 2000 litres of reacting material. Assume a first order reaction with rate constant

$$K = 0.01 \left( \frac{\text{gm mole}}{\text{lit. min}} / \frac{\text{gm. mole}}{\text{lit}} \right)$$

(a) Calculate the exit composition  
(c) at the end of 10 min. if the feed composition is suddenly changed to 0.5 gm. mole/lit. at a time  $t = 0$ .

(b) How will you derive the overall dynamics of the system if both  $q$  and  $c$  are subject to change in the reactor ?

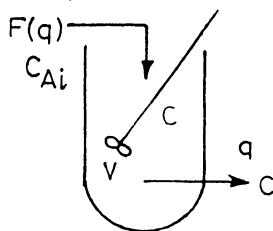


Fig. 1.29

**Solution.**  $F(q)$  — Feed rate lit/min.

$C_{Ai}$  — inlet composition,  $\frac{\text{gm. mole}}{\text{litre}}$

$V$  = Volume of reactor, litres = 2000

Using 1.21, 
$$\frac{C'(s)}{C_{Ai}'(s)} = \frac{q/(q + KV)}{\frac{Vs}{(q + kV)} + 1} = \frac{R}{Ts + 1}$$

at steady state, 
$$C = \frac{q C_{Ai}}{(q + KV)} = \frac{40 \times 0.30}{40 + 2000 \times 0.01} = 0.2$$

$$R = \frac{q}{(q + KV)} = \frac{40}{40 + 0.01 \times 2000} = 0.67$$

$$T = \frac{V}{q + KV} = \frac{2000}{40 + 0.01 \times 2000} = 33.3 \text{ min.}$$

$$C_{Ai}'(s) = \frac{(0.5 - 0.3)}{s}; C'(s) = \frac{0.2}{s} \frac{R}{(Ts + 1)}$$

$$C'(t) = 0.2 \times 0.67 (1 - e^{-t/33.3})$$

at  $t = 10 \text{ min}$ ,  $C'(t) = 0.0348$

$$C(t) = 0.20 + 0.0348 = 0.2348 \quad \text{Ans.}$$

(b) Mass balance equation is :

$$qC_{Ai} - qC - KVC = V dC/dt \quad \dots(9)$$

Using Taylor's series expansion,

$$f(x, y) = f(x_s, y_s) + \left. \frac{\partial f}{\partial x} \right|_{x_s, y_s} (x - x_s) + \left. \frac{\partial f}{\partial y} \right|_{x_s, y_s} (y - y_s)$$

$$FC_{Ai} = F_s C_{Ais} + F_s C_{Ai}' + C_{Ais} F'$$

$$FC = F_s C_s + F_s C' + C_s F'$$

$$F = q$$

Equation (9) becomes,

$$F_s C_{Ais} + F_s C_{Ai}' + C_{Ais} F' - F_s C_s - F_s C' - C_s F' - KVC = V \frac{dc}{dt} \quad \dots(10)$$

Eqn. (9) at steady state is :

$$F_s C_{Ais} - F_s C_s - KVC_c = \frac{VdC_s}{dt} \quad \dots(11)$$

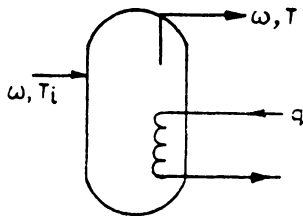
Eqn. (10) - Eq. (11) :

$$F_s C_{Ai}' + C_{Ais} F' - F_s C' - C_s F' - KVC + KVC_s = \frac{Vd}{dT} (C_{A0} - C_s)$$

Collecting common terms after taking Laplace transforms,

$$C'(s) = \frac{C_{Ai}(s) F_s}{(sV + F_s + KV)} + \frac{F'(s) (C_{Ais} - C_s)}{(sV + F_s + KV)} \quad \text{Ans.}$$



**1.48. Process degrees of freedom :**

$T$  — temperature  
 $\omega$  — water flow rate

Fig. 1.30

**Examples :** Water heater shown in Fig. 1.30.

Variables :     Inputs :  $\omega, T_i, q \rightarrow 3$   
                      Outputs :  $\omega, T \rightarrow 2$   
                      Total             $\rightarrow \underline{5}$

Number of equations are :

Material balance  $\rightarrow 1$   
 Energy balance  $\rightarrow 1$   
                                   $\underline{2}$

$$n_p = n_v - n_e = 5 - 2 = 3 \text{ Ans.}$$

These 3 can be placed as : one at  $\omega$ , one at  $T_i$  and one at  $q$  to control temperature. If more than 3 controllers are used then interaction occurs. Heat exchanger : shown in 1.42.

Inputs :      $\omega_t, T_{ti}, \omega_s, T_{si} \rightarrow 4$   
 Outputs :    $\omega_t, T_{to}, \omega_s, T_{so} \rightarrow 4$   
                  Total                             $\rightarrow \underline{8}$

Equations :

Material balance  $\rightarrow 2$      (one shell side, one tube side)  
 Energy balance  $\rightarrow 2$      (one shell side, one tube side)  
                  Total             $\rightarrow \underline{4}$

$n_p = 8 - 4 = 4$ . These four can be placed as one at  $\omega_t$ , one at  $T_{ti}$ , one at  $\omega_s$  and one at  $T_{si}$ .

**1.49. Parts of a control system.** For a CSTR with exothermic chemical reaction shown in Fig. 1.31, using cooling water to maintain constant temperature in the reactor, the various parts are :

The control system consists of

- (1) Process (exothermic CSTR)
- (2) Measuring instrument (temperature measuring instrument)
- (3) Automatic controller

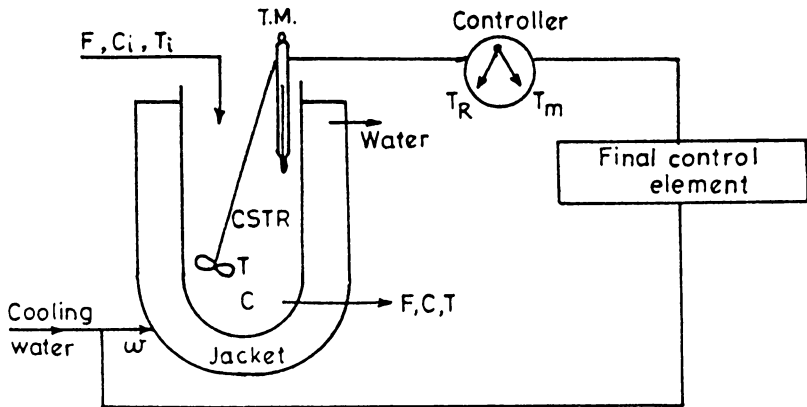


Fig. 1.31

- (4) Final control element (a valve to change flow rate of cooling water)  
 (5) Transmission (electrical or pneumatic).

### 1.50. Problem

A thermometer is immersed in a liquid which is heated at such a rate that its temperature is increasing at the rate of  $0.05$  degree centigrade per second. If both the thermometer and liquid are initially at  $20^\circ\text{C}$ , what rate of passage of liquid over the thermometer bulb is required if the error in the thermometer reading after ten minutes (long time) is to be not more than  $1^\circ\text{C}$ ? The mass of mercury in the bulb is  $10$  gms and the heat transfer coefficient to the bulb is given by the equation,  $h = 735 u^{0.8}$  where  $h$  is the heat transfer coefficient in  $\text{watt}/(\text{m}^2 \cdot ^\circ\text{C})$  and  $u$  is the linear velocity of the liquid in meters/second. The surface area of the bulb is  $0.01 \text{ m}^2$  and the specific heat of mercury is  $1.38 \text{ kJ}/(\text{kg} \cdot \text{deg C})$ .

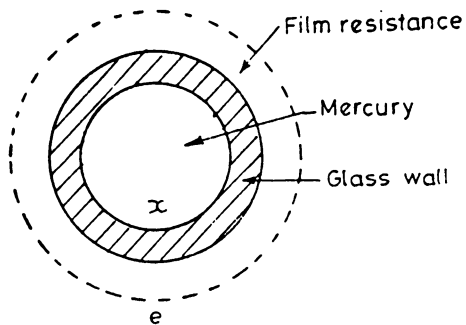


Fig. 1.32

**Solution.**  $e$  = surrounding temperature ;  $h$  = heat transfer coefficient ;  $f$  = temperature read by thermometer ;  $A$  = area of heat transfer ;  $c$  = specific heat of liquid in the bulb ;  $t$  = time ;  $m$  = mass of mercury.

By applying unsteady state heat balance for the bulb, (Fig. 1.32).

Input rate of heat to bulb – output heat rate from bulb  
= Rate of heat accumulation in bulb.

$$\text{Input heat rate} = hAe$$

$$\text{Output heat rate} = hAf$$

$$\text{Rate of heat accumulation} = mc \, df/dt$$

$$hA(e - f) = mc \, df/dt$$

at steady state ( $s$ ) ;  $hA(e_s - f_s) = mc \, df_s/dt$  in deviation form,

$$hA(E - F) = cm \, dF/dt$$

Taking Laplace transforms on both sides of the equation,

$$[E(s) - F(s)] hA = mc \, sF(s)$$

Collecting common terms gives,

$$\frac{F(s)}{E(s)} = \frac{1}{[mc/(hA)]s + 1} = \frac{1}{Ts + 1} \text{ where } T \text{ is time constant.}$$

$$m = 10 \text{ gms ; } c = 1.38 \times 0.2388 = 0.329544 \frac{\text{cals}}{(\text{gm} \cdot ^\circ\text{C})}$$

$$h = 735 \times 0.238 \, u^{0.8} = 175.6 u^{0.8} \text{ cals}/(\text{m}^2 \cdot \text{sec} \cdot ^\circ\text{C})$$

$$A = 0.01 \text{ m}^2$$

where  $0.238 \text{ cal}/(\text{m}^2 \cdot \text{sec} \cdot ^\circ\text{C}) = 1 \text{ watt}/(\text{m}^2 \cdot ^\circ\text{C})$

$$T = \frac{mc}{hA} = \frac{10 \times 0.329544}{175.6 \, u^{0.8} \times 0.01} = \frac{1.877}{u^{0.8}} \text{ seconds}$$

For linear change of error, the steady state error is  $KT$  where  $K = 0.05 \text{ deg.C/sec}$  and  $T$  is time constant in seconds.

$$\text{Equating } KT = 1 ; \frac{1.877}{u^{0.8}} \times 0.05 = 1$$

$$u^{0.8} = 1.877 \times 0.05 = 0.094$$

$$u = 0.052 \text{ m/sec} \quad \text{Ans.}$$

**1.51. U-Tube Manometer Oscillation.** A. Analyse the dynamic behaviour of a U-tube mercury manometer and develop its transfer function. Explain the terms (and obtain the expressions for them) given below : (a) over shoot (b) decay ratio (c) rise time (d) period of oscillation.

These terms are associated with the response curve of a second order underdamped system the input to which is subjected to a step change.

A U-tube manometer with a tube of 8 m. m i.d and a mercury column of 1.5 m is used to record the pressure in a process. At steady state the manometer reads 25 cms. When the pressure of the process suddenly changes by 20% determine,

(a) The peak reading (maximum possible) of the manometer

(b) The time that the manometer takes to first pass through the new steady state pressure.

The specific gravity, and viscosity of mercury are 13.6 and 1.6 CP respectively.

**Solution.** We will assume the flow in the manometer is laminar and will consider fluid friction. We will neglect the density of gas above the manometer fluid. The various forces which make up the force balance equations are : (Fig. 1.33)

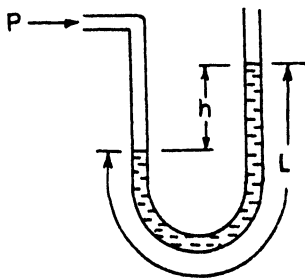


Fig. 1.33

$$F_a = m_a/g_c = \rho LA^a/g_c$$

where  $a = dh^2/dt^2$  (acceleration),  $L$  is length of column. Potential head,

$$Fh = 2\rho Ahg/g_c$$

Frictional resistance in laminar flow by Hagen-Poiseuille's law is,

$$R = 32\mu L/(g_c D^2)$$

$\rho$  = Liquid density ;  $P$  = Applied pressure ;  $R$  = Frictional resistance ;  $\mu$  = Viscosity of fluid.

Equating the forces.

Acceleration of fluid = Potential head - frictional head

$$\begin{aligned}\frac{\rho LA}{g_c} \frac{d^2h}{dt^2} &= A (P - 2h\rho) \frac{g}{g_c} - RA \frac{dh}{dt} \\ \frac{\rho LA}{g_c} \frac{d^2h}{dt^2} &= A (P - 2h\rho) \frac{g}{g_c} - \frac{32\mu LA}{g_c D^2} \frac{dh}{dt}\end{aligned}$$

dividing through out by  $\rho LA/g_c$  gives

$$\frac{d^2h}{dt^2} + \frac{32\mu}{\rho D^2} \frac{dh}{dt} + \frac{2g}{L} h = \frac{Pg}{L\rho}$$

for oscillations, 
$$\frac{d^2h}{dt^2} + \frac{32\mu}{\rho D^2} \frac{dh}{dt} + \frac{2g}{L} h = 0$$

Taking in deviation form and Laplace transforming gives

$$\frac{h'(s)}{P'(s)} = \frac{1/(2\rho)}{\left(\frac{L}{2g} s^2 + \frac{16\mu Ls}{\rho D^2 g} + 1\right)}$$

Standard form of second order transfer function is,

$$= \frac{R}{(\tau^2 s^2 + 2\xi \tau s + 1)}$$

where  $\tau^2 = L/2g$ ;  $\tau = \sqrt{L/(2g)}$

and  $\xi = \frac{8\mu}{\rho D^2} \sqrt{2L/g}$

for step change of  $P(s)$  :

(a) overshoot  $= \exp(-\pi \xi / \sqrt{1 - \xi^2})$

(b) decay ratio  $= \exp(-2\pi \xi / \sqrt{1 - \xi^2})$

(c) Rise time is the time required in the beginning to reach final steady state value.

(d) period of oscillation

$$(\omega) = \sqrt{1 - \xi^2} / \tau$$

B. In deviation form

$$\frac{h'(s)}{P'(s)} = \frac{1/2\rho}{\frac{L}{2g} s^2 + \left( \frac{16\mu L}{\rho D^2 g} \right) s + 1}$$

$$R = 1/2\rho = \frac{1}{2 \times 13.6} = 0.0368 \text{ cm}^3/\text{gm}$$

$$\tau = \sqrt{L/2g} = \sqrt{\frac{150}{2 \times 981}} = 0.28 \text{ sec.}$$

$$\xi = \frac{8\mu}{\rho D^2} \sqrt{2L/g} = \frac{8 \times 1.6 \times \sqrt{2 \times 150/981}}{100 \times 13.6 \times (0.8 \times 0.8)} = 0.0088$$

$h_s$  (steady state level) = 25 cms

(a) Overshoot  $= e^{-\pi \xi / \sqrt{1 - \xi^2}} = e^{-0.0277} = 0.9727$

Peak reading  $= 25 + 25 \times 0.2 + 25 \times 0.2 \times 0.9727$   
 $= 34.86 \text{ cms}$

(b) for step response, the solution for  $\xi < 1$  is :

$$y(t) = 1 - \frac{e^{-\xi t/\tau}}{\sqrt{1 - \xi^2}} \sin \left[ \sqrt{1 - \xi^2} \frac{t}{\tau} + \tan^{-1} \frac{\sqrt{1 - \xi^2}}{\xi} \right]$$

new steady state value at first comes when  $y(t) = 0$

by trial and error  $t/\tau \cong 0.1$

Since  $\tau = 0.28 \text{ sec}$ , so  $t = 0.028 \text{ sec}$ . **Ans.**

**1.52.** A tank of 10 feet high and cross-sectional area of 5 feet  $\times$  5 feet has initial steady flow rate of  $q_i = 300$  gallons per minute. The inlet flow rate is suddenly increased to 400 gal/minute. Plot the level in the tank with time. The flow rate and level are related as :

<i>Input, gal / hr (<math>q_i</math>)</i>	<i>level, feet (<math>h</math>)</i>
0	0
5000	0.70
10,000	1.10
20,000	3.90
30,000	8.80

**Solution.** Both linear and non-linear relations are tried to test  $q_i$  and  $h$ .

$q_i$	$h$	<i>R by linear Relation, <math>\frac{h}{q}</math></i>	<i>(C) Non-linear <math>q = c\sqrt{h}</math></i>
0	0	0	0
5,000	0.70	$1.4 * 10^{-4}$	5976
10,000	1.10	$1.10 * 10^{-4}$	9535
20,000	3.90	$1.95 * 10^{-4}$	10127
30,000	8.80	$2.933 * 10^{-4}$	10113

$R$  is not constant but  $C$  is some what constant so non-linear relation is agreeing. Taking  $C$  average in the operating range of 10,000 to 30,000 gal/hr,  $C$  is  $(9535 + 10113)/2 = 9824$

$$\frac{h'(S)}{q_i'(s)} = \frac{R}{TS + 1} ; R = 2h_s^{1/2}/C$$

$$h'(t) = R | q_i' | (1 - e^{-t/T}) ; T = R \times A$$

$$h_s \text{ at } q_i = 18,000 \text{ is } C\sqrt{h_s} = 18,000$$

$$h_s = 3.357 \text{ feet}$$

$$R = 2 \times (3.357)^{1/2} / 9824 = 3.7301 \times 10^{-4}$$

$$T = 23.3 \times 10^{-4} \times 25 \times 60 = 3.495 \text{ MINS.}$$

$$h'(t) = 6000 \times 3.7301 \times 10^{-4} (1 - e^{-t/3.495})$$

$h'(t)$	$t, MIN$	$h(t) = h'(t) + h_s$
0	0	3.357
0.975	2	4.332
1.525	4	4.882
2.11	10	5.467
2.238	$\infty$	5.595

**1.53.** An isothermal CSTR has Reaction described by :

$$\frac{dC_A}{dt} + \left( \frac{1}{T} + K \right) C_A = \frac{1}{T} C_{A0}$$

(a) write the transfer function model for the CSTR

(b) Calculate the process time constant

(c) find process steady state gain ?

(a) Taking Laplace transforms on both sides.

$$SC_A(S) - C_A(0) + \left( K + \frac{1}{T} \right) C_A(S) = \frac{1}{T} C_{A0}(S)$$

$$\text{Collecting Common terms : } C_A(S) \left[ S + K + \frac{1}{T} \right] = \frac{1}{T} C_{A0}(S)$$

$$\frac{C_A(S)}{C_{A0}(S)} = \frac{1}{(ST + T + TK)} = \frac{1/(T + KT)}{S/(T + TK) + 1} = \frac{R_1}{T_1 S + 1}$$

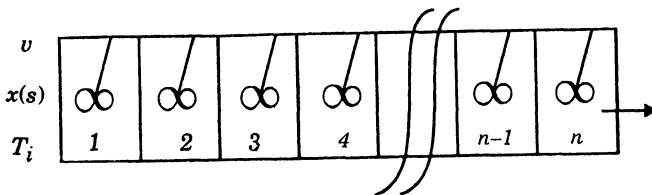
$$(b) \quad T_1 = \frac{1}{(T + TK)} ; \quad (c) \quad R_1 = 1/(T + TK)$$

**1.54.** Develop Transfer function for Transportation Lag ?

$$n = \frac{V}{v} = \text{Total Volume/Element Volume}$$

$$L = \text{Length of pipe} ; \quad \frac{T_0(S)}{T_i(S)} = \frac{1}{(TS + 1)^n}$$

(Considering as  $n$  interacting systems)



$$\begin{aligned} &= \left[ \frac{1}{\left( \frac{L}{v} \right) \frac{S}{n} + 1} \right] ; \text{ as } n \rightarrow \infty \\ \frac{T_0(S)}{T_i(S)} &= \text{Lt}_{n \rightarrow \infty} \left[ \frac{1}{\left( \frac{L}{v} \right) \frac{S}{n} + 1} \right]^n = e^{-(L/v)S} \end{aligned}$$

### PROBLEMS

**1.1.** Solve  $y'' + ty' - y = 0$ ;  $y(0) = 0$ ;  $y'(0) = 0$ .

[Ans.  $y(t) = t$ ]

**1.2.** Solve  $y'''(t) + 4y(t) = 9t$ ;  $y(0) = 0$ ,  $y'(0) = 7$ .

[Ans.  $y(t) = 3t + 2 \sin 2t$ ]

- 1.3. Solve  $y^{11} + Z^1 = t, y'' - Z = e^t; y(0) = 3; y'(0) = -2; Z(0) = 0$ .  
 [Ans.  $y(t) = 2 + t^2/2 + e^{-t}/2 - 3 \sin t/2 + \cos t/2$ ]
- 1.4. Draw operational and block-diagram for a single tank to control liquid level.
- 1.5. Draw operational and block diagram to keep the speed of an automobile constant with varying load.
- 1.6. Draw operational and block diagram for ship-steering mechanism ?
- 1.7. Calculate the process degrees of freedom for a liquid-liquid heat exchanger. [Ans. 5]
- 1.8. Calculate the process degrees of freedom for an endothermic chemical reactor, having first order reaction.
- 1.9. Calculate the process degrees of freedom for a water-heater. [Ans. 3]
- 1.10. Derive the transfer function for the liquid level in a tank and inlet flow rate. [Ans.  $h'(s)/q_i'(s) = R/(Ts + 1)$ ]
- 1.11. Derive the transfer function of a mixer.  
 [Ans.  $C_0'(s)/C_i'(s) = 1/(T_s + 1)$  where  $T$  = volume of mixer/flow rate,  $C_0'$  and  $C_0'$  are the outlet and inlet concentrations respectively.]
- 1.12. A steady heat flow occurs in an electrically heated furnace with walls at 1800°F to a large steel casting at 1400°F. The surface area is 1.0 sq. ft. Calculate the thermal resistance. Assume emissivity is unity.  
 [Ans. 60 deg. sec./BTU]

## NOMENCLATURE

- $A$  — amplitude of sine wave or area of cross-section  
 $C_0$  — outlet concentration  
 $C_i$  — inlet concentration  
 $F$  — feed rate  
 $h_1, h_2, h$  — level  
 $i$  — current  
 $l$  — load variable  
 $m$  — manipulated variable or mass of solid  
 $n$  — moles of gas  
 $P_1, P_2$  — pressure  
 $q_t$  — inlet flow rate to tank  
 $q_0$  — outlet flow rate from tank  
 $q$  — liquid flow rate or heat flow rate  
 $R$  — resistance or gas constant or set point  
 $s$  — Laplacian variable  
 $T$  — time constant or temperature  
 $t$  — time  
 $t_i$  — inlet temperature  
 $T_1, T_2$  — temperatures or time constants



$T_s$  — steady state temperature

$V_1, V_2, V_3$  — hold up volumes

$v$  — electrical charge

$x$  — function, input

$y$  — function, output

$y'$  — first derivative of  $y$

$y''$  — second derivative of  $y$

$y'''$  — third derivative of  $y$

$\theta$  — value indicated by instrument

“1” — deviation variable

$s$  — steady state value

$\xi$  — damping coefficient in second order transfer function.