1

Simple Stresses and Strains

1.1. DEFINITIONS

Strength. The strength of a material may be defined as the maximum resistance which a material can offer to the externally applied forces. The stength of a material depends upon a number of factor *e.g.*, type of loading, temperature, internal structure etc. It has been established beyond doubts that the actual strengh of the material is much below the theoretical cohesive strength of the material.

Stress. When some external forces are applied to a body, then the body offers internal resistance to these forces. The magnitude of the internal resisting force is numerically equal to the applied forces. The internal resisting force per unit area is called '*stress*'.

However, we name this subject as "strength of Materials", but at not stage we try to determine the strength of the material, we always calculate the stress in the material.

In order to understand the concept of stress, consider a body under the action of a number of forces as shown in Fig. 1.1 (*a*). If an imaginary cut is made by passing a cross-section 1.1

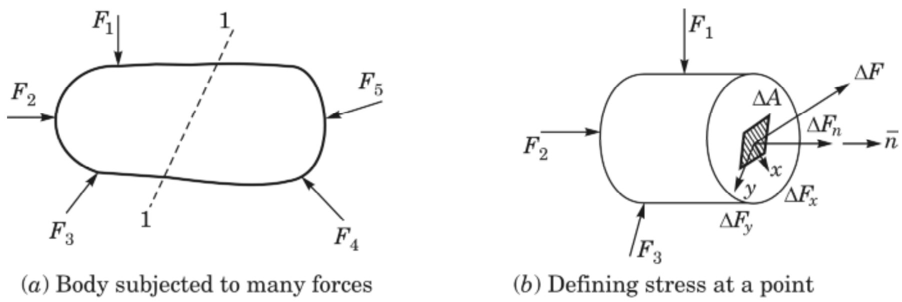

Fig. 1.1. Understanding stress.

and the left portion is drawn separately as shown in Fig. 1.1 (*b*), now consider the elemantary force ΔF acting on the elementary area ΔA. They by definition:

Stress, $$\sigma = \underset{\Delta A \to 0}{\text{Limit}} \frac{\Delta F}{\Delta A} = \frac{dF}{dA} \qquad ...(1.1)$$

The dimensional units of stress are N/m^2 (Newton per metre squared) and is always supposed to act at a point.

Normal stress. In Fig. 1.1 (*b*), the force ΔF can be resolved into components such that one of them is along the outward drawn normal to the area ΔA (since there can be only one normal at a point) and the other components lie in the plane of the area ΔA. Let ΔF_n be the normal component, then normal stress,

$$\sigma_n = \underset{\Delta A \to 0}{\text{Limit}} \frac{\Delta F_n}{\Delta A} = \frac{dF_n}{dA} \quad \text{...(1.2)}$$

The normal stress may be **tensile** or **compressive** depending upon the forces acting on the material to the either of the pull or push type respectively. Tensile and compressive stresses together are called *direct stresses*.

Shear stress. The force ΔF may be resolved into indinete number of components in the plane containing area ΔA, because there are infinite number of directions in the plane containing area ΔA which are perpendicular to the unit normal $\overline{n}$. However, if we restrict our studies to three-dimensional co-ordinate system, then we are left with only two directions x and y perpendicular to each other as shown in Fig. 1.1 (*b*). Then the shear stresses are defined as :

$$\tau_x = \underset{\Delta A \to 0}{\text{Limit}} \frac{\Delta F_x}{\Delta A} = \frac{dF_x}{dA} \quad \text{...(1.3}a\text{)}$$

$$\tau_y = \underset{\Delta A \to 0}{\text{Limit}} \frac{\Delta F_y}{\Delta A} = \frac{dF_y}{dA} \quad \text{...(1.3}b\text{)}$$

Conventional or engineering stress. It is defined as the ratio of load P to the original area of cross-section A_0. Thus,

$$\sigma = \frac{P}{A_0} \quad \text{...(1.4)}$$

True stress. It is defined as the ration of load P to the instantaneous area of cross-section A. Thus,

$$\overline{\sigma} = \frac{P}{A} = \frac{P}{A_0} \cdot \frac{A_0}{A} = \sigma \frac{A_0}{A}$$

Foe volume constancy, $Al = A_0 l_0$ where $l = l_0(1 + \varepsilon)$

where ε = engineering strain

$$\therefore \quad A = \frac{A_0}{1 + \varepsilon}$$

$$\overline{\sigma} = \sigma (1 + \varepsilon) \quad \text{...(1.5)}$$

Strain. It is defined as the change in length per unit length. The strain may be tensile or compressive depending upon whether the length increases (under tensile load) or decreases (under compressive load). It is a dimensionless quantity.

Conventional or engineering strain. It is defined as the change in length per unit original length. By definition

$$\varepsilon = \frac{l - l_0}{l_0} = \int_{l_0}^{l} \frac{dl}{l_0} = \frac{1}{l_0} \int_{l_0}^{l} dl \quad \text{...(1.6)}$$

where l = changed or deformed length

l_0 = original length, and dl = change in length.

Natural strain. It defined as the change in length per unit instantaneous length. By definition, the natural strain,

$$\overline{\varepsilon} = \int_{l_0}^{l} \frac{dl}{l} = \ln \frac{l}{l_0} = \ln (1 + \varepsilon) \quad \text{...(1.7)}$$

Normal strain. It is the strain produced under the action of direct or normal stresses.

Shear strain. It is the strain produced under the action of shear stresses. The shear strain is measured by the change in the angle. Thus in Fig. 1.2, if dl is the change in the length of face CD under the action of shear force F, then by definition, Shear strain, $\gamma = \tan \phi$

For small strains,

$\tan \phi \approx \phi$, thus, $\phi = \dfrac{dl}{l}$ ...(1.8)

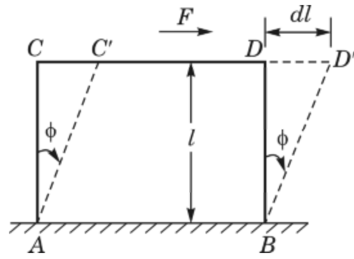

Fig. 1.2. Shear strain.

Gauge length. It is that portion of the test specimen over which extension or deformation is measured.

Percentage elongation. It is the change in length per unit original length of the test specimen expressed as a percentage, *i.e.*,

$$\text{Percentage elongation} = \frac{dl}{l} \times 100$$

Percentage reduction of area. It is defined as the change in area per unit original area expressed as a percentage, *i.e.*,

$$\text{Percentage reduction of area} = \left(\frac{A_0 - A}{A_0}\right) \times 100$$

Poisson's ratio. When a material is subject to longitudinal deformation then the lateral dimensions also change. The ratio of the lateral strain to longitudinal strain is a constant quantity called the Poisson's ratio and is designated by ν or $1/m$.

$$\nu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} \quad ...(1.9)$$

Superficial strain. It is defined as the change in area of cross-section per unit original area *i.e,* superficial strain,

$$\varepsilon_s = \frac{\mathrm{A} - \mathrm{A}_0}{\mathrm{A}_0} = \frac{dA}{A_0} \quad ...(1.10)$$

Volumetric strain. If a uniform stress is applied on all the three faces of a body, then all the three dimensions of the body will change resulting in change in volume. Thus, volumetric strain,

$$\varepsilon_v = \frac{V - V_0}{V} = \frac{dV}{V_0} \quad ...(1.11)$$

where V = Final volum, and V_0 = Original volume

Hooke's law. This law states that within elastic (proportional) limits, strain is proportional to stress.

Modulus of elasticity. Within elastic limits the ratio of normal stress to normal strain is a constant quantity and is defined as the Young's modulus of elasticity, *i.e.,*

$$E = \frac{\sigma}{\varepsilon} = \frac{Pl_0}{A_0 dl} \quad ...(1.12)$$

Modulus of rigidity. It is defined as the ratio of shearing stress to shearing strain, *i.e.*

$$G = \frac{\tau}{\gamma} \quad ...(1.13)$$

Bulk modulus. It is defined as the ratio of uniform stress intensity to volumetric strain, within the elastic limits and is denoted by K. Thus

$$K = \frac{\sigma}{\varepsilon_v} \quad ...(1.14)$$

Proof stress. It is the maximum stress which can be applied to a material without allowing the material to fail.

Factor of safety. Because of uncertainties of loading conditions, we introduce a factor of safety, defined as the ratio of the maximum stress to the allowable or working stress. The maximum stress is generally taken as the yeild stress for ductile materials. This is also called the '*factor of ignorance*'.

Free Body Diagram. The free body diagram of an element of a member in equilibrium is the diagram of only that member or element, as if made free from the rest, with all the internal and external forces acting on it.

1.2. STRESS-STRAIN DIAGRAM

1.2.1. Ductile Materials. Fig. 1.3 shows the stress-strain diagram for a ductile material like mild steel. The curve starts from the origin *O* showing thereby that there is no initial stress or strain in the test specimen. Upto point '*a*' Hoole's law is obeyed and stress is proportional to strain. Therefore, *oa* is a straight line and point *a* is called the limit of proportionality and the stress at point *a* is called the *proportional limit stress*, σ_p. The portion of the diagram between *ab* is not a straight line but upto point *b*, the material remains elastic,

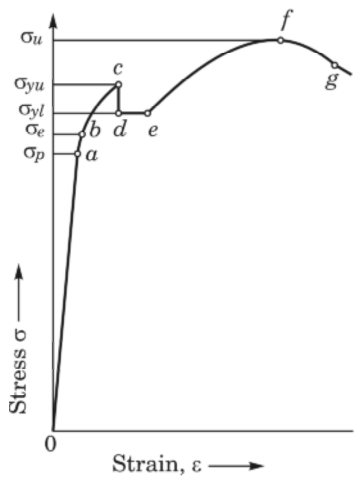

Fig. 1.3. Typical stress-strain diagram for a ductile material.

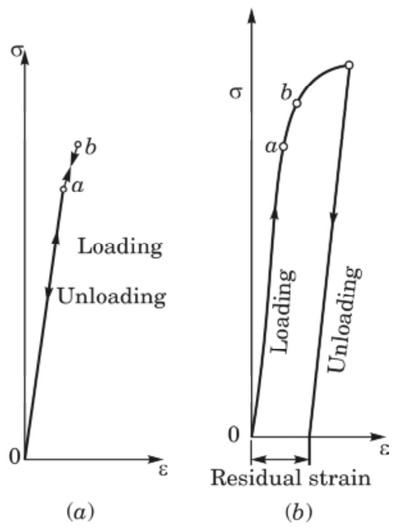

Fig. 1.4. Loading and unloading paths.

i.e. on removal of the load, no permanent set is formed and the path is retraced. The point *b* is called the *elastic limit* point and the stress corresponding to that is called the elastic limit stress, σ_e. In actual practice, the points *a* and *b* are so close to each other that it becomes difficult to differentiate between them. Beyond the point *b,* the material goes to the plastic stage until the *upper yeild point* '*c*' is reached. At this point the cross-sectional area of the material starts decreasing and the stress decreases to a lower value to a point *d*, called the *lower yeild point*. Corresponding to point *c*, the stress is known as upper yield point stress, σ_{yu} and corresponding to point *d*, the stress is known as lower yeild point stress, σ_{yl}. At point *d* the specimen elongates by a considerable amount without any increase in stress and upto point *e.* The portion *de* is called the yielding of the material at constant stress. From point *e* onwards, the strain hardening phenomena becomes predominant and the strength of the material increases thereby requiring more stress for deformation, until point *f* is reached. Point *f* is called the ultimate point and the stress corresponding to this point is called the *ultimate stress*, σ_u. It is the maximum stress to which the material can be subjected in a simple tensile test. At point *f* the necking of the material begins and the cross-sectional area starts decreasing at a rapid rate. Due to this local necking, the stress in the material goes on decreasing inspite of the fact that actual stress intensity goes on increasing. Ultimately the specimen breaks at point *g*, known as the breaking point, and the corresponding stress is called the nominal *breaking stress* based upon the original area of cross section. Whereas the true stress at fracture is the ratio of the breaking load to the reduced area of cross-section at the neck. The initial portions of the diagram are shown in Fig. 1.4 on exagerated scale.

Sometimes it is not possible to locate the yield point quite accurately in order determine the yield strength of the material. For such materials the yield point stress is defined at some particular value of the permanent set. It has been observed that if load is removed in the plastic range then the unloading path line is parallel to the straight portion of the stress-strain diagram as shown in Fig. 1.4 (*b*). The commonly used value of permanent set for determining the value of yield strength for mild steel is 0.2 per cent of the maximum strain as shown in Fig. 1.5.

1.2.2. Brittle Materials. The stress-strain diagram for a brittle material like cast iron is shown in Fig. 1.6. There is very little elongation and reduction in area of the specimen for such materials. The yeild point is not marked at all. The straight line portion of the diagram is also very small.

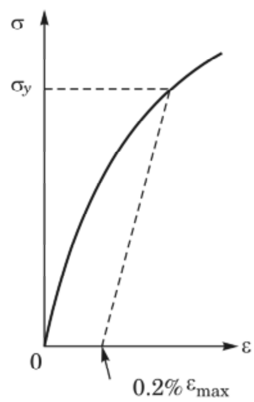

Fig. 1.5. Determining yield strength of brittle materials.

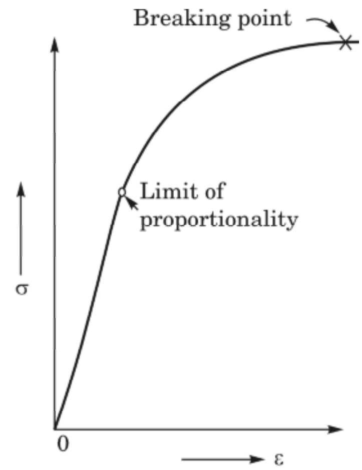

Fig. 1.6. Typical stress-strain diagram for a brittle material.

1.3. BAR OF VARYING CROSS-SECTION

Consider a bar of varying circular cross-section as shown in Fig. 1.7

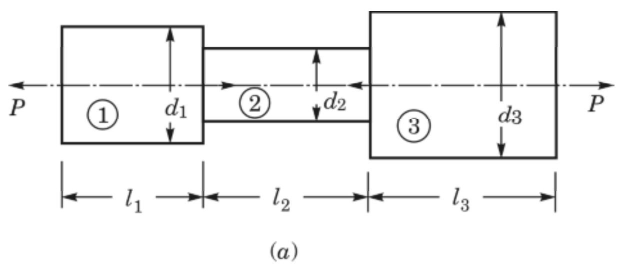

Fig. 1.7. Bar of varying cross-section.

and subject to axial load P througtout. The area of different cross-section is :

$$A_1 = \frac{\pi}{4} d_1^2, A_2 = \frac{\pi}{4} d_2^2, A_3 = \frac{\pi}{4} d_3^2$$

Let σ_1, σ_2 and σ_3 be the corresponding stresses, then,

$$\sigma_1 = \frac{P}{A_1}, \sigma_2 = \frac{P}{A_2}, \sigma_3 = \frac{P}{A_3}$$

The strains become, $\varepsilon_1 = \frac{\sigma_1}{E_1}, \varepsilon_2 = \frac{\sigma_2}{E_2}, \varepsilon_3 = \frac{\sigma_3}{E_3}$

The changes in lengths become, $\Delta l_1 = \varepsilon_1 l_1$, $\Delta l_2 = \varepsilon_2 l_2$, $\Delta l_3 = \varepsilon_3 l_3$.

Total change in length, $\Delta l = \Delta l_1 + \Delta l_2 + \Delta l_3 = P\left(\frac{l_1}{A_1 E_1} + \frac{l_2}{A_2 E_2} + \frac{l_3}{A_3 E_3}\right)$

or in general, we have, $$\Delta l = P \sum_{i=1}^{n} \frac{l_i}{A_i E_i} \quad \text{...(1.15)}$$

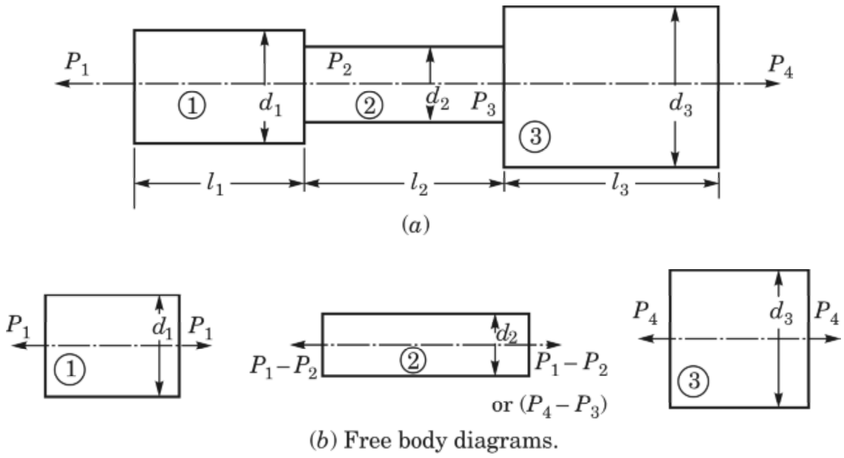

Fig. 1.8. Bar of varying cross-section.

If the loads in different sections of the bar are different as shown in Fig. 1.8 (*a*), then free body diagrams may be drawn for each section as shown in Fig. 1.8 (*b*), and the net forces acting in each section may be determined. Thus the stresses, strains and total elongation may be determined.

$$\sigma_1 = \frac{P_1}{A_1}, \sigma_2 = \frac{P_1 - P_2}{A_2}, \sigma_3 = \frac{P_4}{A_3}$$

$$\varepsilon_1 = \frac{\sigma_1}{E_1}, \varepsilon_2 = \frac{\sigma_2}{E_2}, \varepsilon_3 = \frac{\sigma_3}{E_3}$$

$$\Delta l_1 = \varepsilon_1 l_1, \Delta l_2 = \varepsilon_2 l_2, \Delta l_3 = \varepsilon_3 l_3$$

$$\Delta l = \Delta l_1 + \Delta l_2 + \Delta l_3$$

$$= \sum_{i=1}^{n} \frac{P_i l_i}{A_i E_i} \quad \text{...(1.16)}$$

Example 1.1. *A mild steel rod 20 mm diameter is subjected to an axial pull of 50 kN. Determine the tensile stress induced in the rod and the elongation if the unloaded length is 5 m. E = 210 GN/m*^{2}.

Solution. Given, d = 20 mm; P = 50 kN; l = 5 m

Area of cross-section of the rod,

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (20)^2 \times 10^{-6} = 314 \times 10^{-6} \text{ m}^2$$

Stress $$\sigma = \frac{P}{A} = \frac{50 \times 10^3}{314 \times 10^{-6}} = 159.155 \text{ MN/m}^2$$

Elongation, $$\delta = \frac{Pl}{AE} = \frac{50 \times 10^3 \times 5 \times 10^3}{314 \times 10^{-6} \times 210 \times 10^9} = 3.789 \text{ mm.}$$

Example 1.2. *A short hollow cast iron cylinder of wall thickenss 10 mm is to carry a compressive load of 600 kN. Determine the outside diameter of the cylinder if the ultimate crushing stress for the material is 540 MN/m². Use a factor of safety of 6.*

Solution. Let d_0 be the outside diameter of the cylinder in mm. Then area of cross-section of the cylinder is,

$$A = \frac{\pi}{4} \left\{ d_0^2 - (d_0 - 20)^2 \right\} \times 10^{-6} = \pi (d_0 - 10) \times 10^{-5} \text{ m}^2$$

Safe load $$= \pi (d_0 - 10) \times 10^{-5} \times \frac{540}{6} \times 10^6 = 900 \times \pi (d_0 - 10) = 600 \times 10^3$$

$\therefore$ d_0 = 222.2 mm.

Example 1.3. *A round bar as shown in Fig. 1.9 is subjected to an axial tensile load of 100 kN. What must be the diameter 'd' if the stress there is to be 100 MN/m²? Find also the total elongation. E = 200 GPa.*

Solution.

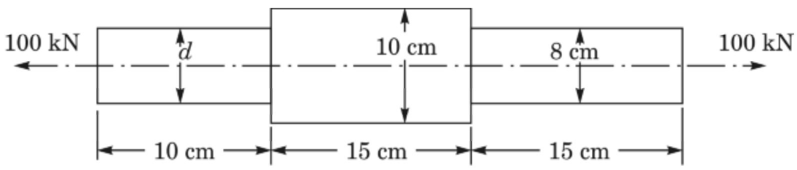

Fig. 1.9

Stress, $$\sigma = \frac{P}{(\pi/4)\, d^2}$$

$$100 \times 10^6 = \frac{100 \times 10^3}{(\pi/4)\, d^2}$$

$\therefore$ Diameter, $$d = \sqrt{\frac{4}{\pi \times 10^3}} = 0.03568 \text{ m} = 35.68 \text{ mm}$$

Total elongation, $$\Delta l = \frac{P}{E} \left[\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3} \right]$$

$$= \frac{100 \times 10^3}{200 \times 10^9} \left[\frac{0.10}{10^3} + \frac{0.15}{\frac{\pi}{4} \times (100)^2} + \frac{0.15}{\frac{\pi}{4} \times (80)^2} \right] \times \frac{1}{10^{-6}}$$

$$= \frac{10^{-4}}{2} [1 + 0.191 + 0.299] = \frac{1.490 \times 10^{-4}}{2} = 0.0745 \text{ mm.}$$

Example 1.4. *A steel bar 25 mm diameter is loaded as shown in Fig. 1.10. Determine the stresses in each part and the total elongation. E = 210 GPa.*

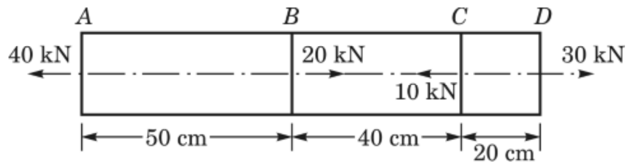

Fig. 1.10

Solution.

Area of cross-section,

$$A = \frac{\pi d^2}{4} = \frac{\pi}{4} \times (25)^2 \times 10^{-6} = 490.87 \times 10^{-6} \text{ m}^2.$$

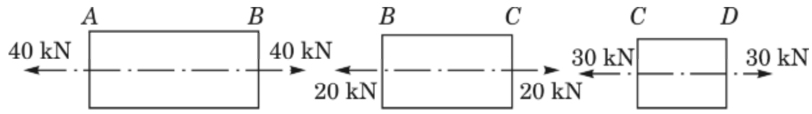

Fig. 1.11. Free body diagrams.

The free body diagrams for each portion have been shown in stresses in various parts are:

$$\sigma_{AB} = \frac{40 \times 10^3}{490.87 \times 10^{-6}} = 81.488 \text{ MN/m}^2$$

$$\sigma_{BC} = \frac{20 \times 10^3}{490.87 \times 10^{-6}} = 40.744 \text{ MN/m}^2$$

$$\sigma_{CD} = \frac{30 \times 10^3}{490.87 \times 10^{-6}} = 61.116 \text{ MN/m}^2$$

Total elongation,

$$\Delta l = \frac{1}{AE} \sum P_i \, l_i$$

$$= \frac{10^3}{490.87 \times 10^{-6} \times 210 \times 10^9} [40 \times 0.5 + 20 \times 0.4 + 30 \times 0.2]$$

$$= \frac{10^3 \times 34 \times 10^3}{490.87 \times 10^{-6} \times 210 \times 10^9} = 0.3298 \text{ mm.}$$

Example 1.5. *A steel bar as shown in Fig. 1.12 (a) consists of two parts AB and BC having areas of cross-section of 4 cm² and 5 cm² respectively. It is rigidly fixed at end A and end C is at a distance of 1 mm from the other rigid horizontal support. A load of 100 kN is applied vertically downward at B. Determine the reactions produced by the rigid horizontal support and the stress in the parts AB and BC of the bar. E = 200 GPa.*

Solution. In the absence of horizontal rigid support, the portion *AB* of the bar would elongate by an amount,

$$\delta_{AB} = \frac{100 \times 10^3 \times 1.25 \times 10^3}{4 \times 10^{-4} \times 200 \times 10^9} = 1.5625 \text{ mm}$$

Whereas the lower portion *BC* of the bar would have remained unaltered. Due to the presence of the horizontal rigid support, the bar *AC* can move downward by 1 mm only. Since the extension of the bar *AC* is more than 1 mm, therefore, the bar is subjected to an upward reaction. Let the upward reaction be *P* in kN. The force in bar *BC* will be equal to *P* and compressive in nature, whereas, the force in *AB* will be 100-*P*, and tensile in nature, as shown in Fig. 1.2 (*b*).

Total elongation of the bar then becomes

$$= \frac{(100 - \mathrm{P}) \times 10^3 \times 1.25}{4 \times 10^{-4} \times 200 \times 10^9} - \frac{\mathrm{P} \times 10^3 \times 1.2}{5 \times 10^{-4} \times 200 \times 10^9}$$

$= (15.625 - 028125\, P)\ 10^{-4}$ m

The total elongation is limited to 1 mm,

$\therefore \quad (15.625 - 0.28125\, P)\ 10^{-4} = 1 \times 10^{-3}$

or $\qquad P = 20$ kN

Hence force in $\qquad AB = 80$ kN

and force in $\qquad BC = 20$ kN

$$\therefore \qquad \sigma_{AB} = \frac{80 \times 10^3}{4 \times 10^{-4}} = 200 \text{ MPa (tensile)}$$

$$\text{and} \qquad \sigma_{BC} = \frac{20 \times 10^3}{5 \times 10^{-4}} = 40 \text{ MPa (compressive)}$$

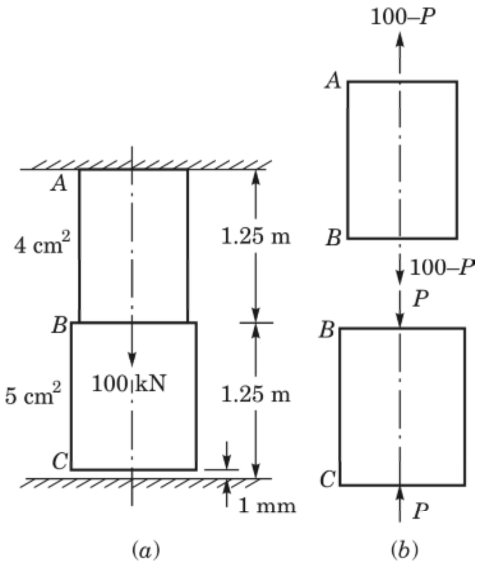

Fig. 1.12.

Example 1.6. *A bar of length 5 m is made of two materials as shown in Fig. 1.13. The first 3 m of its length is made of brass and is 7.5 cm² in cross-section and the remainder of its length is of steel and is 5 cm² in cross-section. Determine the total compression of the bar under a load of 20* kN. $E_{steel} = 210\ GN/m^2$, $E_{brass} = 84\ GN/m^2$.

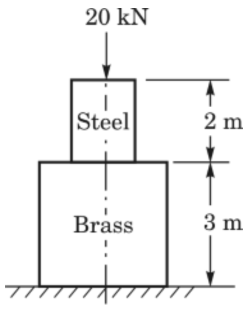

Fig. 1.13.

Solution. The load in brass and steel parts is the same and equal to 20 kN.

Compression of brass,

$$\delta l_b = \frac{20 \times 10^3 \times 3}{7.5 \times 10^{-4} \times 84 \times 10^9} = 0.095238 \times 10^{-2} \text{ m}$$

Compression of steel,

$$\delta l_s = \frac{20 \times 10^3 \times 2}{5 \times 10^{-4} \times 210 \times 10^9} = 0.038095 \times 10^{-2} \text{ m}$$

Total compression $= \delta l_b + \delta l_s = 0.133333 \times 10^{-2}$ m

$= 1.33333$ mm.

Example 1.7. *A prismatic bar as shown in Fig. 1.14 (a) carries an axial load 10 kN. Calculate the reaction at the supports assuming them rigid.*

Solution. Let R_A and R_C be the reactions at the supports A and C respectively. For the equilibrium of the bar these reactions must act towards the left so that

$$R_A + R_C = 10 \qquad \text{...(1)}$$

The free body diagrams for the portions AB and BC are shown in Fig. 1.14 (b). Thus

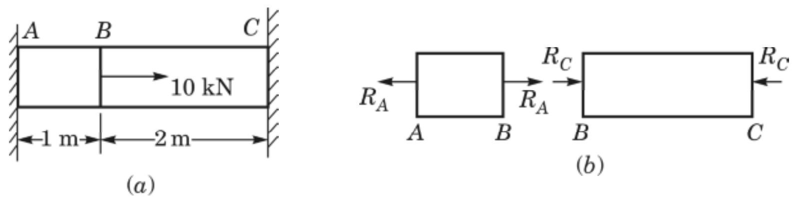

Fig. 1.14

$$\delta l_{AB} = \frac{R_A \times 1}{AE} \text{ (extension)}; \ \delta l_{BC} = \frac{R_C \times 2}{AE} \text{ (compression)}$$

Since the supports are rigid, therefore elongation of AB shall be equal to the compression of BC. Hence

$$\frac{R_A}{AE} = \frac{2R_C}{AE}$$

$$\therefore \quad R_A = 2R_C \qquad \text{...(2)}$$

Solving Eqs. (1) and (2), we get

$$R_C = \frac{10}{3} \text{ kN}; \ R_A = \frac{20}{3} \text{ kN.}$$

Example 1.8. *A load P is suspended from two rods as shown in Fig. 1.15. The rod AC is of steel, having a circular cross-section 30 mm in diameter, and an allowable stress of 160 MN/m²; the rod BC is of aluminimum having a diameter of 40 mm and an allowable stress of 60 MN/m². What is the maximum load P which can be suspended from these rods?*

Solution. Due to symmetry, the forces in the rods AC and BC shall be equal. Let F be the force in each rod, then

$$2F \cos 30° = P$$

$$\therefore \quad F = \frac{P}{2 \cos 30°} = \frac{P}{\sqrt{3}} N$$

Stress in steel rod

$$\therefore \quad P = 195.89 \text{ kN}$$

Stress in aluminium rod

$$= \frac{P}{\sqrt{3}} \times \frac{4}{\pi \times 900 \times 10^{-6}} = 160 \times 10^6$$

$$\therefore \quad P = 130.59 \text{ kN}$$

Hence safe load = 130.59 kN.

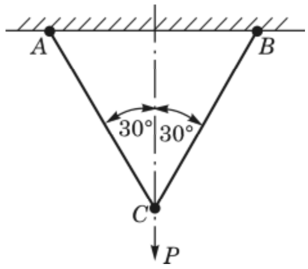

Fig. 1.15.

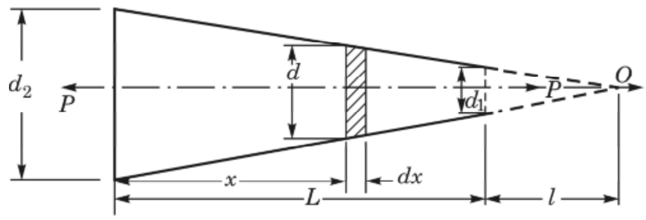

Fig. 1.16. Tapered bar under tension.

1.4. EXTENSION OF A TAPERED BAR

Consider a bar of length L and tapering from diameter d_2 to d_1 and subjected to axial load P as shown in Fig. 1.16. The diameter of the bar at a distance x from the end having diameter d_2 is,

$$d = d_2 - \left(\frac{d_2 - d_1}{L}\right) x$$

Area of cross-section, $A = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left[d_2 - \left(\frac{d_2 - d_1}{L}\right) x\right]^2$

Stress in the bar, $\sigma = \frac{P}{A}$

Elongation of the elementary length dx of the bar,

$$\delta u = \frac{P\,dx}{AE} = \frac{\sigma\,dx}{E}$$

But $\sigma A = \sigma_1 A_1 = \sigma_2 A_2 = P$

$\therefore$ $$\delta u = \frac{\sigma_1 A_1}{A} \cdot \frac{dx}{E}$$

Also $$\frac{A_1}{A} = \frac{d_1^2}{d^2} \text{ and } \frac{d_1}{d} = \frac{1}{L+l-x}$$

$\therefore$ $$\delta u = \frac{\sigma_1}{E} \times \frac{l_2}{(L+l-x)^2}\, dx$$

Total elongation, $$\int \delta u = \int_0^L \frac{\sigma_1}{E} \times \frac{l^2}{(L+l-x)^2}\, dx$$

$$u = \frac{\sigma_1\, l^2}{E} \left| \frac{1}{L+l-x} \right|_0^L = \frac{\sigma_1\, l^2}{E} \left[\frac{1}{l} - \frac{1}{L+l} \right]$$

$$= \frac{\sigma_1 l^2 . L}{El\,(L+l)} = \frac{\sigma_1 . Ll}{E\,(L+l)}$$

$$= \frac{P}{A_1} \times \frac{L}{E} \times \frac{d_1}{d_2} \qquad \left[\therefore \quad \frac{l}{L+l} = \frac{d_1}{d_2} \right]$$

$$= \frac{4\, PL}{\pi d_1\, d_2 E} \qquad \text{...(1.17}a\text{)}$$

If $d = d_1 = d_2$,

Then $$u = \frac{4\, PL}{\pi d^2\, E} \qquad \text{...(1.17}b\text{)}$$

For a square tapering bar, $$u = \frac{PL}{d_1\, d_2\, E} \qquad \text{...(1.18)}$$

Example 1.9. *A steel rod, circular in cross-section, tapers from 2.5 cm diameter to 1.25 cm diameter inalength of 50 cm. Find how much of this length will increase under a pull of 25 kN if E = 210 GPa.*

Solution. Given $d_1 = 1.25$ cm, $L = 50$ cm, $d_2 = 2.5$ cm $P = 25$ kN

Extension of a tapering circular bar is gicen by Eq. (1.70*a*)

$$u = \frac{4\, PL}{\pi\, d_1\, d_2\, E} = \frac{4 \times 25 \times 10^3 \times 0.5}{\pi \times 1.25 \times 2.5 \times 10^{-4} \times 210 \times 10^9} = 0.2425 \text{ mm.}$$

Example 1.10. *A tension bar is found to toper uniformly from (D – a) cm diameter to (D + a) cm. Prove that the error involved in using the mean diameter to calculate Young's madulus is* $\left(\frac{10a}{D}\right)^2$ *per cent.*

Solution. Given $d_1 = D - a$; $d_2 = D + a$

Let L = Length of the bar and P = Load applied.

$$\text{Mean diameter} = \frac{d_1 + d_2}{2} = D \text{ and Mean stress, } \sigma = \frac{4P}{\pi\, D^2}$$

If u = Extension of the bar then strain, $\varepsilon = \dfrac{u}{L}$

Young's modulus, $$E = \frac{4\,PL}{\pi\,D^2 u} \qquad ...(1)$$

Now for a tapering round bar from Eq. (1.17*a*), we have

$$u = \frac{4\,PL}{\pi d_1 d_2 E}\;;\; E = \frac{4\,PL}{\pi d_1 d_2 u}$$

or $$E = \frac{4\,PL}{\pi\,(D-a)\,(D+a)u} = \frac{4\,PL}{\pi\,(D^2-a^2)u} \qquad ...(2)$$

$\therefore$ Error in Young's modulus from Eqs. (1) and (2) becomes,

$$= \frac{4\,PL}{\pi u}\left[\frac{1}{D^2-a^2} - \frac{1}{D^2}\right] = \frac{4\,PL}{\pi u}\left[\frac{a^2}{D^2\,(D^2-a^2)}\right]$$

$$\text{Percentage error} = \frac{4\,PL}{\pi u}\left[\frac{a^2}{D^2\,(D^2-a^2)}\right] \times \frac{\pi u\,(D^2-a^2)}{4\,PL} \times 100 = \frac{a^2}{D^2} \times 100 = \left(\frac{10\,a}{D}\right)^2$$

Example 1.11. *A mild steel plate 20 mm thick and 20 cm wide at the top, tapers uniformly to 10 mm thickness and 15 cm width over a length of 2 m. Find the elongation under a pull of 15 kN.*

E = 210 GPa.

Solution. Consider an elementary strip of length dx of the plate at a distance x cm from the lower end as shown in Fig. 1.17.

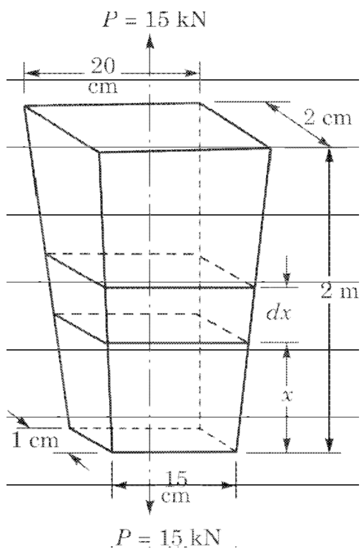

Fig. 1.17.

Width of strip $$= 15 + \left(\frac{20-15}{200}\right)x = \left(15 + \frac{x}{40}\right) \text{ cm}$$

Thickness of strip $$= 1 + \left(\frac{2-1}{200}\right)x = \left(1 + \frac{x}{200}\right) \text{ cm}$$

Area of the strip, $$A = \left(15 + \frac{x}{40}\right) \times \left(1 + \frac{x}{200}\right) \text{ cm}^2$$

Stress in the strip, $$\sigma = \frac{15 \times 10^7}{\left(15 + \frac{x}{40}\right)\left(1 + \frac{x}{200}\right)} \text{ N/m}^2$$

Extension of this strip, $$\delta u = \frac{\sigma\, dx}{E}$$

$$= \frac{15 \times 10^7}{\left(15 + \frac{x}{40}\right)\left(1 + \frac{x}{200}\right)} \cdot \frac{dx}{E}$$

Total extension, $$u = \frac{15 \times 10^7}{E}\int_0^{200} \frac{dx}{\left(15 + \frac{x}{40}\right)\left(1 + \frac{x}{200}\right)}$$

$$= \frac{15 \times 10^7}{E} \times \frac{1}{5}\int_0^{200} \frac{dx}{\left(3 + \frac{x}{200}\right)\left(1 + \frac{x}{200}\right)}$$

$$= \frac{3 \times 10^7}{E}\int_0^{200}\left(-\frac{1}{2}\right)\left[\frac{1}{\left(3 + \frac{x}{200}\right)} - \frac{1}{\left(1 + \frac{x}{200}\right)}\right] dx$$

$$= \frac{-15\,10^6}{E} \left| 200 \ln\left(3 + \frac{x}{200}\right) - 200 \ln\left(1 + \frac{x}{200}\right) \right|_0^{200}$$

$$= \frac{-3 \times 10^9}{E} [\ln 4 - \ln 3 - \ln 2 + \ln 1]$$

$$= \frac{-3 \times 10^9}{210 \times 10^9} [1.3863 - 1.0986 - 0.6931 + 0]$$

$$= \frac{3 \times 0.4054}{210} = 0.00579 \text{ cm.}$$

1.5. BAR OF UNIFORM STRENGTH

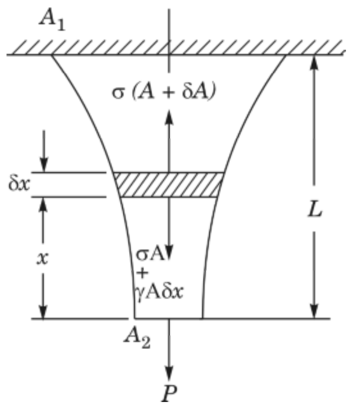

Fig. 1.18. Bar of uniform strength.

Consider a bar which is acted upon by tensile load P as shown in Fig. 1.18. Consider an elementary strip of the bar between cross-sections x and $x + \delta x$ from the lower end. Let the areas of cross-section at x and $x + \delta x$ be A and $A + \delta A$ respectively. Let σ be the stress in the bar through-out.

Total force acting on the strip upwards = $\sigma(A + \delta A)$

Total force acting on the strip downwards = $\sigma A + \gamma \delta x A$

where γ = specific weight of the bar.

For the equilibrium of the strip,

$$\sigma(A + \delta A) = \sigma A + \gamma A \delta x$$

$$\therefore \quad \frac{dA}{A} = \frac{\gamma}{\sigma} \delta x$$

In the limit, we get $\quad \dfrac{dA}{A} = \dfrac{\gamma}{\sigma} dx$

Integrating, we get

$$\int_{A_2}^{A} \frac{dA}{A} = \frac{\gamma}{\sigma} \int_0^x dx$$

$$\ln \frac{A}{A_2} = \frac{\gamma}{\sigma} x$$

$$\therefore \quad A = A_2 e^{\frac{\gamma}{\sigma} x} \qquad ...(1.19)$$

where $\quad A_2$ = area of cross-section at the bottom.

If $\; A_1$ = area of cross-section at the top, where $x = L$

Then $$A_1 = A_2 e^{\frac{\gamma}{\sigma} L} \qquad ...(1.20)$$

Example 1.12. *A vertical tie bar of 5 cm diameter is subjected to 45 MPa stress. If the stress in the bar is to constant at all cross-sections, find the diameter of the section at a point 5 m above the section whose diameter is 5 cm.*

Density of bar material is 7470 kg/m^3.

Solution. For a bar of uniform strength,

$$A = A_2 e^{\frac{\gamma}{\sigma} x}$$

$$d^2 = d_2^2 e^{\frac{\gamma}{\sigma} x}$$

$$= 5^2 \times e^{\frac{7470 \times 9.81 \times 5}{45 \times 10^6}} = 25.20425$$

$$d = 5.0204 \text{ cm.}$$

1.6. EXTENSION OF A BAR UNDER ITS OWN WEIGHT

1.6.1. Bar of uniform area

Consider a bar of uniform cross-section of area A and length L. Consider an elementary strip of this bar between cross-sections x and $x + dx$ as shown in Fig. 1.19. The downward force acting on this strip is due to the weight of the bar that lies below this element and is equal to γAx.

In order that this elementary strip is in a state of equilibrium, a force equal to γAx must act upwards.

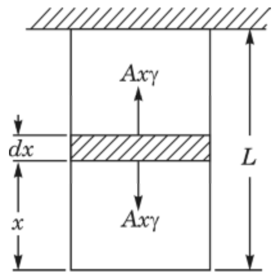

Fig. 1.19. Uniform bar under its own weight.

The stress in the strip, $\sigma = \dfrac{\gamma Ax}{A} = \gamma x$

Strain in the strip, $\varepsilon = \dfrac{\sigma}{E} = \dfrac{\gamma x}{E}$

Extension of the strip, $= \varepsilon\, dx = \dfrac{\gamma x}{E}\, dx$

Total extension of the bar $= \displaystyle\int_0^L \frac{\gamma x}{E}\, dx = \frac{\gamma L^2}{2E}$

If, W = Total weight of the bar $= \gamma AL$

$\therefore$ Total extension $= \dfrac{WL}{2AE}$ ...(1.21)

It may be observed that the total extension produced by the self-weight of the bar is equal to that produced by a load of half its weight applied at the lower end. Therefore, if the weight of the bar is to be taken into account for calculating extension, half of the total weight to the bar may be applied at the lower end.

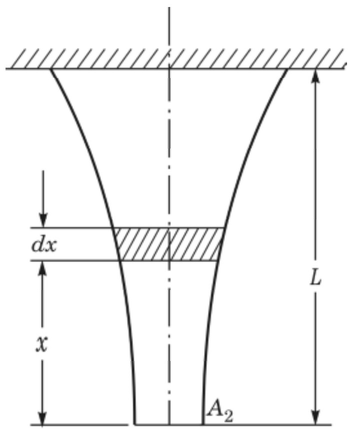

Fig. 1.20. Non-uniform bar under its own weight.

1.6.2. Bar of varying cross-section

Consider a bar of varying cross-section as shown in Fig. 1.20. Consider an elementary strip of the bar at a distance x from lower end and of length dx.

Total force acting on this strip downwards is equal to the weight of the bar upto x.

Let A be the area of cross-section at x.

$\therefore$ Weight of the bar upto $x = \displaystyle\int_0^x \gamma A dx$

Stress in the strip $= \dfrac{\gamma \int_0^x A dx}{A}$.

$\therefore$ Extension of the bar $= \dfrac{\displaystyle\int_0^L \left[\gamma \int_0^x A dx\right] dx}{AE}$ ...(1.22)

1.6.3. Conical Bar

Consider a conical bar shown in Fig. 1.21, whose base radius is r and height h. The radius of an elementary strip at a distance x from the base is

$$= \left(\frac{h-x}{h}\right) r$$

Load acting on the elementary strip,

$$\Delta P = \frac{1}{3} \cdot \pi \left[\left(\frac{h-x}{h}\right) r\right]^2 (h-x)\, \rho g$$

where ρ = density of the bar material.

Area of the elementary strip,

$$\Delta A = \pi \left[\left(\frac{h-x}{h}\right) r\right]^2$$

Stress in the elementary strip,

$$\sigma = \frac{\Delta P}{\Delta A} = \frac{1}{3}(h-x)\, \rho g$$

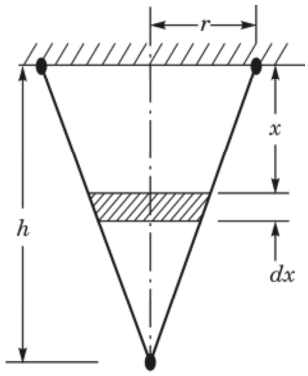

Fig. 1.21. Conical bar.

Extension of $dx = \frac{\sigma dx}{E} = \frac{(h-x)\rho g}{3E}\, dx$

Extension of bar, $\delta = \frac{\rho g}{3E} \int_0^h (h-x)\, dx = \frac{\rho g}{6E} \left| -(h-x)^2 \right|_0^h$

$$= \frac{\rho g h^2}{6E} \qquad ...(1.23)$$

Weight of the bar, $W = \frac{1}{3} \pi r^2 h\, \rho g$

Area of base, $A = \pi r^2$

$\therefore$ $W = \frac{1}{3} A h\, \rho g$

or $h \rho g = \frac{3W}{A}$

$\therefore$ $\delta = \frac{Wh}{2AE} \qquad ...(1.24)$

Example 1.13. *A metal bar 5 cm × 5 cm section is subjected to an axial compressive load of 500 kN. The contraction on a 20 cm gauge length is found to be 0.5 mm and the increase in thickness 0.045 cm. Find the value of Young's modulus and Poisson's ratio.*

Solution. Given P = 500 kN; l = 20 cm

Δl = 0.05 cm; Δt = 0.0045 cm

Area of cross-section, $A = 5 \times 5 = 25 \text{ cm}^2$

Longitudinal strain, $\varepsilon = \frac{\Delta l}{l} = \frac{0.05}{20} = 0.0025$ (compressive)

Stress, $\sigma = \frac{P}{A} = \frac{500 \times 10^3}{25 \times 10^{-4}} = 200$ MPa (compressive)

Young's modulus, $E = \frac{\sigma}{\varepsilon} = \frac{200 \times 10^6}{0.0025} = 80$ GPa

Lateral strain $= \frac{\Delta t}{t} = \frac{0.0045}{5} = 0.0009$ (tensile)

Poisson's ratio, $\nu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{0.0009}{0.0025} = 0.36$

Example 1.14. *A bar of 20 mm diameter is subjected to an axial tensile load of 120 kN, under which 200 mm gauge length of this bar elongates by an amount of 3.5 × 10⁻⁴ m. Determine the modulus of elasticity of the bar material. If ν = 0.3, determine its change in diameter.*

Solution. Modulus of elasticity, $E = \frac{P}{A} \cdot \frac{l}{\delta l}$

$$= \frac{120 \times 10^3}{(\pi/4)\left(20 \times 10^{-3}\right)^2} \times \frac{200 \times 10^{-3}}{3.5 \times 10^{-4}} = 218.27 \text{ GN/m}^2$$

$$\nu = \frac{\delta d / d}{\delta l / l}$$

Poisson's ratio,
$$0.3 = \frac{\delta d / 20}{\left(3.5 \times 10^{-4}\right)/\left(200 \times 10^{-3}\right)}$$

$$\delta d = \frac{0.3 \times 20 \times 3.5 \times 10^{-1}}{200} = 0.0105 \text{ mm.}$$

Example 1.15. *The following data was recorded in a tensile test:–*

Diameter of specimen = 12 mm; Gauge length = 60 mm

Minimum diameter after fracture = 6.5 mm

Determine:

(*a*) *Modulus of elasticity,* (*b*) *Ultimate tensile stress,*

(*c*) *Upper and lower yield point stress,* (*d*) *Percentage reduction of area,*

(*e*) *Percentage elongation, and* (*f*) *Nominal and actual stress at fracture.*

Load (kN)	*2.5*	*5.0*	*7.5*	*10*	*12.5*	*15*	*17.5*	*20*	*22.5*
Extension ($m \times 10^{-6}$)	*5.6*	*12*	*18*	*24.5*	*31.5*	*38.5*	*45*	*53*	*59.5*
Load (kN)	*25*	*27.5*	*30*	*32*	*33.5*	*31*	*32*	*31.5*	*32*
Extension ($m \times 10^{-6}$)	*66.5*	*74*	*81*	*90*	*110*	*225*	*450*	*675*	*1000*
Load (kN)	*34.5*	*36*	*37*	*39*	*39.5*	*40*	*39.5*	*36*	*28*
Extension ($m \times 10^{-6}$)	*1700*	*1950*	*2500*	*3650*	*5600*	*7850*	*11200*	*13450*	*14550*

Solution. The load-extension diagram for the elastic range is shown in Fig. 1.22.

(*a*) Modulus of elasticity,
$$E = \frac{\sigma}{\varepsilon} = \text{Slope of the curve} \times \frac{l}{A}$$

$$= 0.3731 \times 10^9 \times \frac{60 \times 10^{-3}}{(\pi/4) \times 144 \times 10^{-6}} = 197.94 \text{ GN/m}^2$$

(*b*) Ultimate tensile stress
$$= \frac{\text{Maximum load}}{\text{Area of cross-section}}$$

$$= \frac{40 \times 10^3}{(\pi/4) \times 144 \times 10^{-6}} = 353.68 \text{ MN/m}^2$$

(*c*) Upper yield point stress,
$$\sigma_{yu} = \frac{33.5 \times 10^3}{\pi/4 \times 144 \times 10^{-6}} = 296.21 \text{ MN/m}^2$$

Lower yield point stress, $\sigma_{yl} = \dfrac{31 \times 10^3}{\pi/4 \times 144 \times 10^{-6}} = 274.10 \text{ MN/m}^2$

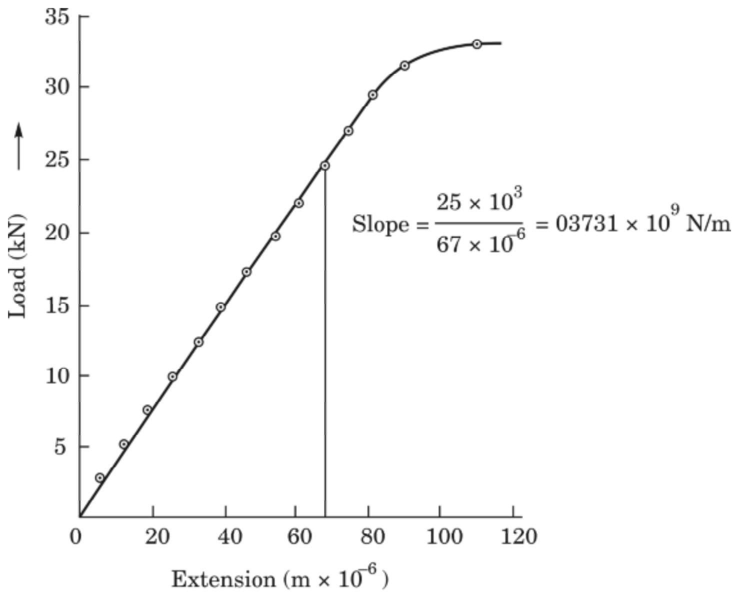

Fig. 1.22. Load extension diagram.

(*d*) Percentage reduction in area $= \dfrac{(12)^2 - (6.5)^2}{(12)^2} \times 100 = \left(\dfrac{144 - 42.25}{144}\right) \times 100 = 70.66\%$

(*e*) Percentage elongation $= \left[\dfrac{(60 + 14.55) - 60}{60}\right] \times 100 = 24.25\%$

(*f*) Nominal stress at fracture $= \dfrac{28 \times 10^3}{(\pi / 4) \times 144 \times 10^{-6}} = 247.57 \text{ MN/m}^2$

Actual stress at fracture $= \dfrac{28 \times 10^3}{(\pi / 4) \times 42.25 \times 10^{-6}} = 843.80 \text{ MN/m}^2$.

Example 1.16. *A 70 cm length of aluminium alloy bar is suspended from the ceiling so as to provide a clearance Δ = 0.03 cm between it and a 25 cm length of steel as shown in Fig. 1.23.*

$$A_{al} = 12.5 \text{ cm}^2, \quad E_{al} = 70 \text{ GN/m}^2,$$
$$A_s = 25 \text{ cm}^2, \quad E_s = 210 \text{ GN/m}^2.$$

Determine the stress in the aluminium and in the steel due to a 300 kN load applied 50 cm from the ceiling.

Solution. Elongation of *AB* under 300 kN load $= \dfrac{300 \times 10^3 \times 50 \times 10^{-2}}{12.5 \times 10^{-4} \times 70 \times 10^9}$

$$= 1.71428 \times 10^{-3} \text{ m} = 0.171428 \text{ cm}$$

This elongation is more than the clearance 0.03 cm between the bars. Hence bars DE and BC will be subjected to compression whereas AB will remain in tension. Let P be the compressive force in BC and DE then the tensile force in AB will be $(300 - P)$ kN. Then

$$\delta_{AB} - \delta_{BC} - \delta_{DE} = \Delta$$

or
$$\frac{(300 - P) \times 10^3 \times 50 \times 10^2}{12.5 \times 10^{-4} \times 70 \times 10^9} - \frac{P \times 10^3 \times 20 \times 10^{-2}}{12.5 \times 10^{-4} \times 70 \times 10^9} - \frac{P \times 10^3 \times 25 \times 10^{-2}}{25 \times 10^{-4} \times 210 \times 10^9} = 0.03 \times 10^{-2}$$

or $(300 - 1.4P) \times 0.0571428 \times 10^{-4} - 0.0047619 \times 10^{-4} \times P = 0.03 \times 10^{-2}$

or $(17.14284 - 0.084761 \times P)\, 10^{-4} = 0.03 \times 10^{-2}$

$$P = 166.854 \text{ kN}$$

$$\sigma_{AB} = \frac{(300 - 166.854) \times 10^3}{12.5 \times 10^{-4}} = 106.52 \text{ MN/m}^2 \text{ (tensile)}$$

$$\sigma_{BC} = \frac{166.854 \times 10^3}{12.5 \times 10^{-4}} = 133.48 \text{ MN/m}^2 \text{ (compressive)}$$

$$\sigma_{DE} = \frac{166.854 \times 10^3}{25 \times 10^{-4}} = 66.74 \text{ MN/m}^2 \text{ (compressive)}$$

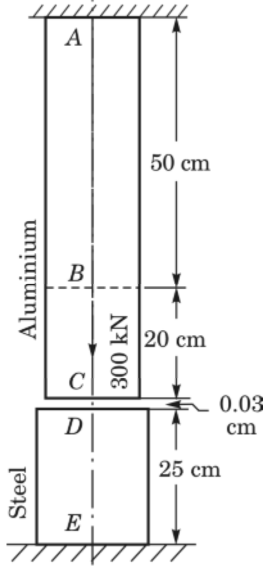

Fig. 1.23

1.7. ADVANCED SOLVED PROBLEMS

Problem 1.1. *The cylinder head an I.C. engine is attached to the cylinder by means of six steel bolts of 15 mm diameter. The maximum gas pressure is 1.5 MPa. Determine the stress developed in each bolt it the diameter of the cylinder is 200 mm.*

Solution. Resisting force of bolts = Gas force in the cylinder

$$6 \times \frac{\pi}{4} \times 15^2 \times \sigma = \frac{\pi}{4} \times 200^2 \times 1.5$$

Tensile stress in bolts, $\sigma = 44.44$ MPa.

Problem 1.2. *Find the total elongation of a steel bar as shown in Fig. 1.24 subjected to an axial load of 200 kN. E = 210 GPa.*

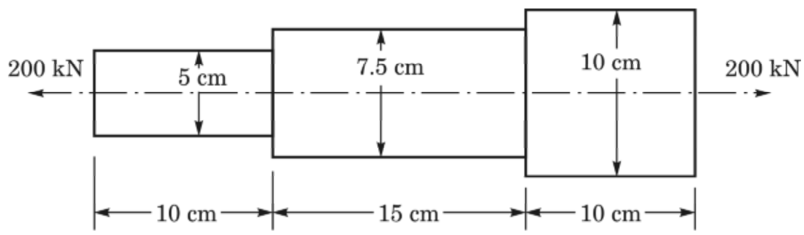

Fig. 1.24

Solution. Elongation, $\delta = \frac{P}{E} \sum \frac{l_i}{A_i} = \frac{200 \times 10^3 \times 4}{\pi \times 210 \times 10^9} \left[\frac{0.1}{(0.05)^2} + \frac{0.15}{(0.075)^2} + \frac{0.1}{(0.1)^2} \right]$

$= 0.093$ mm

Problem 1.3. *A steel bar of 30 mm diameter is loaded as shown in Fig. 1.25. Determine the stress in each portion and the total elongation. E = 210 GPa.*

Solution. The forces in the various parts are shown below in Fig. 1.26.

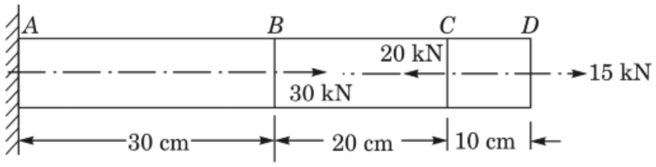

Fig. 1.25

Elongation, $\delta = \dfrac{1}{AE}\sum P_i l_i = \dfrac{4 \times 10^3}{\pi \times 30^2 \times 210 \times 10^3}[25 \times 0.3 - 5 \times 0.2 + 15 \times 0.1] = 0.0539$ mm

$$\sigma_{AB} = \frac{25 \times 10^3 \times 4}{\pi \times 30^2} = 35.37 \text{ MPa}$$

$$\sigma_{BC} = \frac{-5 \times 10^3 \times 4}{\pi \times 30^2} = -7.073 \text{ MPa}$$

$$\sigma_{CD} = \frac{15 \times 10^3 \times 4}{\pi \times 30^2} = 21.221 \text{ MPa.}$$

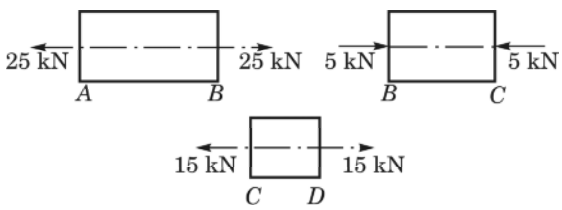

Fig. 1.26

Problem 1.4. *The ultimate shear stress for mild steel is 400 MPa. Find the force required to punch a 20 mm dia hole in a mild steel plate 12.5 mm thick. What is the compressive stress in the punch of dia 20 mm?*

Solution. Area sheared, $A = \pi dt = \pi \times 20 \times 12.5 = 785.4 \text{ mm}^2$

Punching force, $P = \sigma_u A = 400 \times 785.4 = 314.159$ kN

Compressive stress in punch $= \dfrac{4P}{\pi d^2} = \dfrac{4 \times 314.159 \times 10^3}{\pi \times 20^2} = 1000$ MPa.

Problem 1.5. *A column 2 m long tapers uniformly from 10 cm × 10 cm to 8 cm × 8 cm cross-section. Determine the load under which the column will shorten by 4 mm. E = 200 GPa. Assume that buckling is avoided.*

Solution. The tapering column is shown in Fig. 1.27.

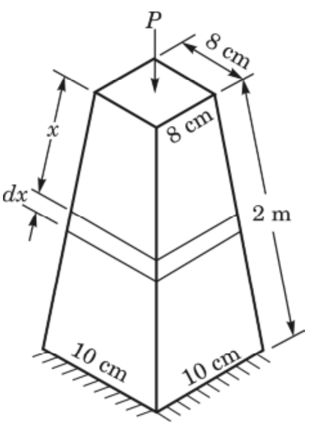

Fig. 1.27

Side length at a distance x from top $= \left(8 + \dfrac{x}{100}\right)^2$ cm

Area, $A = \left(8 + \dfrac{x}{100}\right)^2 \text{ cm}^2$

Stress, $\sigma = \dfrac{P}{A} = \dfrac{P}{\left(8 + \dfrac{x}{100}\right)^2}$ N/cm²

Extension of element $dx = \dfrac{\sigma dx}{E} = \dfrac{P dx}{\left(8 + \dfrac{x}{100}\right)^2 \times 2 \times 10^7}$ cm

Total extension $= \dfrac{P}{2 \times 10^7}\displaystyle\int_0^{200} \dfrac{dx}{\left(8 + \dfrac{x}{100}\right)^2} = \dfrac{P \times 100}{2 \times 10^7}\displaystyle\int_0^{200}\left(8 + \dfrac{x}{100}\right)^{-2} \times \dfrac{1}{100}\, dx$

$$= \frac{P}{2 \times 10^5} \left| \frac{\left(8 + \frac{x}{100}\right)^{-1}}{-1} \right|_0^{200} = \frac{P}{2 \times 10^5} \left[-\frac{1}{10} + \frac{1}{8} \right] = \frac{P}{8 \times 10^6} \text{ cm}$$

$$\frac{P}{8 \times 10^6} = 0.4$$

$$P = 3200 \text{ kN}$$

Problem 1.6. *A mild steel test piece was tested in tension and the following readings were obtained:*

Diameter of specimen = 20 mm, Length of specimen = 20 cm
Extension under 30 kN load = 0.08 mm, Load at yield point = 150 kN
Maximum load = 225 kN, Length of the specimen after fracture = 25 cm.

Calculate the values of (a) Young's modulus, (b) yield point stress, (c) ultimate strength and (d) percentage elongation.

Solution.

$$(a)\ E = \frac{Pl_0}{A\delta} = \frac{30 \times 10^3 \times 200 \times 4}{\pi \times 20^2 \times 0.08} = 238.73 \text{ GPa}$$

$$(b)\ \sigma_y = \frac{P_y}{A} = \frac{150 \times 10^3 \times 4}{\pi \times 20^2} = 477.465 \text{ MPa}$$

$$(c)\ \sigma_u = \frac{P_u}{A} = \frac{225 \times 10^3 \times 4}{\pi \times 20^2} = 716.197 \text{ MPa}$$

$$(d)\ \%\text{ elongation} = \left(\frac{l - l_0}{l_0}\right) \times 100 = \left(\frac{25 - 20}{20}\right) \times 100 = 25\%$$

Problem 1.7. *The upper part of the arrangement shown in Fig. 1.28 is of steel, whereas its lower part is of cast iron. The axial load P shortens the overall length by 0.2 mm. Determine the magnitude of load P. For steel, E = 210 GN/m² and for C.I., E = 102 GN/m².*

Solution.

$$\delta = P \sum \frac{l_i}{A_i E_i}$$

$$0.02 = P \sum \left(\frac{25}{5 \times 5 \times 210 \times 10^5} + \frac{30}{7.5 \times 7.5 \times 102 \times 10^5} \right)$$

$$P = 200.187 \text{ kN}$$

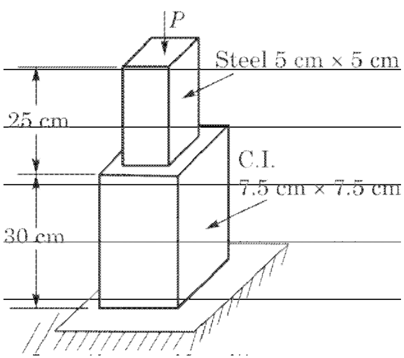

Fig. 1.28

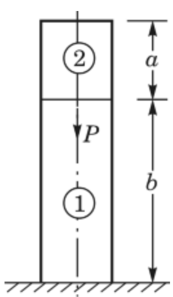

Fig. 1.29

Problem 1.8. *Determine by taking into account the weight of the bar, the displacement of the free end of the bar shown in Fig. 1.29, if its cross-sectional area is A, the modulus of elasticity E, and the specific weight of material γ.*

Solution. Compression of part 1 due to P, $\delta_1 = \dfrac{Pb}{AE}$

Compression of bar due to self weight, $\delta_2 = \dfrac{\gamma(a+b)^2}{2E}$

Displacement of free end, $\delta = \delta_1 + \delta_2 = \dfrac{Pb}{AE} + \dfrac{\gamma(a+b)^2}{2E}$.

Problem 1.9. *Determine the displacement of section x-x of the bar shown in Fig. 1.30, if its cross-section is A, modulus of elasticity E, and the specific weight of the material γ.*

Solution. Displacement of section x-x due to load P,

$$\delta_1 = \frac{Pa}{AE}$$

Displacement of section x-x due to self weight of bar of length b,

$$\delta_2 = \frac{\gamma A\, ab}{AE}$$

Displacement of section x-x due to self weight of bar of length 'a',

$$\delta_3 = \frac{1}{2}\frac{\gamma a^2}{E}$$

Total displacement, $\delta = \delta_1 + \delta_2 + \delta_3 = \dfrac{(P + \gamma\, Ab)a}{AE} + \dfrac{\gamma a^2}{2E}$.

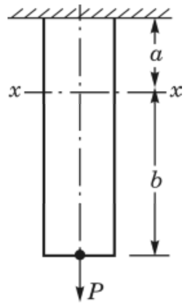

Fig. 1.30

Problem 1.10. *A stepped bar is loaded as shown in Fig. 1.31. Calculate stress in each part and total elongation. E = 200 GPa.*

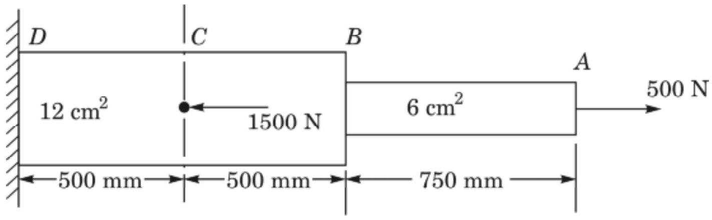

Fig. 1.31

Solution. The forces in the various parts are shown in Fig. 1.32.

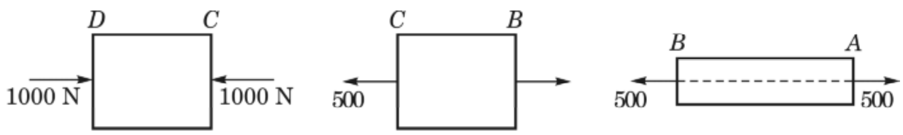

Fig. 1.32.

$$\sigma_{AB} = \frac{500}{6 \times 10^{-4}} = 833.3 \text{ kPa};\ \sigma_{BC} = \frac{500}{12 \times 10^{-4}} = 416.6 \text{ kPa};\ \sigma_{CD} = -\frac{1000}{12 \times 10^{-4}} = -833.3 \text{ kPa}$$

Elongation, $\delta l = \dfrac{1}{2 \times 10^5}\,[-0.8333 \times 500 + 0.4166 \times 500 + 0.8333 \times 750]$

$= 2.083 \times 10^{-3}$ mm.

Problem 1.11. *A 50 mm diameter steel bar 200 mm long was subjected to a tensile force. The length was found to increase by 0.08 mm and decrease in diameter was 0.006 mm. Determine the Poisson's ratio.*

Solution. Longitudinal strain, $\varepsilon_l = \dfrac{\delta l}{l} = \dfrac{0.08}{200} = 4 \times 10^{-4}$

Lateral strain, $\varepsilon_d = \dfrac{-\delta d}{d} = \dfrac{0.006}{50} = 1.2 \times 10^{-4}$

Poisson's ratio, $\nu = \dfrac{\varepsilon_d}{\varepsilon_l} = \dfrac{1.2 \times 10^{-4}}{4 \times 10^{-4}} = 0.3$

Problem 1.12. *Calculate the stresses in various parts of the mild steel member shown in Fig. 1.33. If E = 200 GPa, calculate the total extension of the member.*

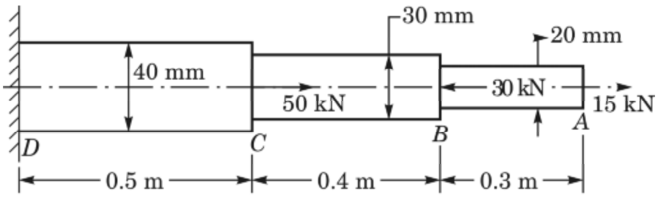

Fig. 1.33

Solution. The free body diagrams of various parts are shown in Fig. 1.34.

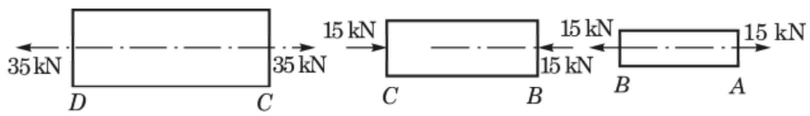

Fig. 1.34

$$\sigma_{AB} = \frac{15 \times 10^3 \times 4}{\pi \times 400} = 47.746 \text{ MPa}$$

$$\sigma_{BC} = \frac{-15 \times 10^3 \times 4}{\pi \times 900} = -21.220 \text{ MPa}$$

$$\sigma_{CD} = \frac{35 \times 10^3 \times 4}{\pi \times 1600} = 27.852 \text{ MPa}$$

$$\text{Total extension} = \frac{1}{E}\sum \frac{P_i l_i}{A_i} = \frac{1}{E}\sum \sigma_i l_i$$

$$= \frac{1}{2 \times 10^5}\,[47.746 \times 0.3 - 21.220 \times 0.4 + 27.852 \times 0.5]$$

$$= 98.81 \times 10^{-6} \text{ m or } 0.09881 \text{ mm}$$

MULTI-CHOICE QUESTIONS*

1.1. Strength of a material may be defined as:
(*a*) the maximum internal resistance offered by the material to the externally applied forces
(*b*) the capability of the material to absorb strain energy
(*c*) the maximum internal resistance offered by the material against deformation
(*d*) the capability of the material to resist bending.

1.2. Stress may be defined as:
(*a*) the load per unit area
(*b*) the internal resistance offered by the material per unit area
(*c*) the internal force acting on the material per unit area
(*d*) the internal resisting force per unit area.

*Answers to multi-choice questions are given at the end of the book.

1.3. Strength of a material depends upon its:
(*a*) cross-section (*b*) type of loading (*c*) internal structure (*d*) volume.

1.4. Strength of a material is a:
(*a*) fixed quantity (*b*) variable quantity
(*c*) changes with time (*d*) remains constant at all times.

1.5. Conventional stress is based upon:
(*a*) the instantaneous area of cross-section (*b*) the original area of cross-section
(*c*) the average area of cross-section (*d*) the final area of cross-section.

1.6. True stress is based upon:
(*a*) the original area of cross-section (*b*) the final area of cross-section
(*c*) the average area of cross-section (*d*) the instantaneous area of cross-section.

1.7. In a two-dimensional body, the number of independent stresses can be:
(*a*) 6 (*b*) 5 (*c*) 4 (*d*) 3.

1.8. In the SI system of units, the units of stress are:
(*a*) kgf/mm^2 (*b*) kg/mm^2 (*c*) N/m^2 (*d*) Pa.

1.9. Engineering stress is the same as the:
(*a*) true stress (*b*) conventional stress (*c*) average stress (*d*) final stress.

1.10. True stress is always:
(*a*) equal to the conventional stress (*b*) greater than the conventional stress
(*c*) lesser than the conventional stress (*d*) depends upon the type of loading.

1.11. Conventional strain may be defined as:
(*a*) the change in length per unit original length
(*b*) the change in length per unit instantaneous length
(*c*) the change in length per unit final length
(*d*) the change in length per unit average length.

1.12. Naturral strain may be defined as:
(*a*) the change in length per unit original length
(*b*) the change in length per unit final length
(*c*) the change in length per unit instantaneous length
(*d*) the change in length per unit average length.

1.13. The relationship between true stress $\overline{\sigma}$ and conventional stress σ is given by:

(*a*) $\frac{\overline{\sigma}}{\sigma} = 1 + \varepsilon$ (*b*) $\frac{\overline{\sigma}}{\sigma} = \frac{1}{1+\varepsilon}$ (*c*) $\frac{\overline{\sigma}}{\sigma} = (1+\varepsilon)^2$ (*d*) $\frac{\overline{\sigma}}{\sigma} = \frac{1}{(1+\varepsilon)^2}$

where ε is the conventional strain.

1.14. The relationship between natural strain $\overline{\varepsilon}$ and conventional strain ε is given by:

(*a*) $\overline{\varepsilon} = \frac{1}{\ln(1+\varepsilon)}$ (*b*) $\overline{\varepsilon}\ \ln(1+\varepsilon)$ (*c*) $\overline{\varepsilon} = \ln(1+\varepsilon)^2$ (*d*) $\overline{\varepsilon} = \frac{1}{2}\ln(1+\varepsilon)^2$.

1.15. Poisson's ratio may be defined as:
(*a*) the ratio of longitudinal strain to lateral strain
(*b*) the ratio of lateral strain to longitudinal strain
(*c*) the ratio of conventional strain to true strain
(*d*) the ratio of true strain to conventional strain.

1.16. Young's modulus is defined as:
(*a*) the ratio of conventional stress to conventional strain
(*b*) the ratio of true stress to conventional strain
(*c*) the ratio of conventional stress to natural strain
(*d*) the ratio of true stress to natural strain.

1.17. Hooke's law holds good upto:

(*a*) the elastic limit (*b*) the yield point

(*c*) the limit of proportionality (*d*) the ultimate point.

1.18. Elasticity is a property of the material due to which:

(*a*) it does not come back to its original position after the external forces are removed

(*b*) it comes back to its original position after the external forces are removed

(*c*) it exhibits stress strain curve

(*d*) it does not deform.

1.19. Ductility is a property of the material due to which it can be:

(*a*) drawn into wires (*b*) beaten up into sheets

(*c*) drawn into thinner sections (*d*) rolled into bars.

1.20. Within elastic limits, Hooke's law states that:

(*a*) stress is proportional to strain (*b*) strain is proportional to stress

(*c*) stress is inversely proportional to strain (*d*) strain is inversely proportional to stress.

1.21. Modulus of rigidity may be defined as the ratio of:

(*a*) shearing stress to longitudinal strain (*b*) shearing stress to shearing strain

(*c*) longitudinal stress to shearing strain (*d*) longitudinal stress to longitudinal strain.

1.22. The point in the stress-strain curve at which the cross-sectional area of the test specimen starts decreasing is called the:

(*a*) elastic limit (*b*) upper yield point

(*c*) lower yield point (*d*) ultimate stress point.

1.23. The point in the stress-strain curve at which the strain increases considerably without any increases in stress is called the:

(*a*) limit of proportionality (*b*) elastic limit

(*c*) upper yield point (*d*) lower yield point.

1.24. The necking in case of ductile materials begins at the:

(*a*) elastic limit point (*b*) upper yield point

(*c*) lower yield point (*d*) ultimate point.

1.25. There shall be no residual strain left in the material on unloading if load is removed at the instant of:

(*a*) limit of proportionality (*b*) elastic limit

(*c*) upper yield point (*d*) lower yield point.

1.26. The yield point of brittle materials can be ascertained by drawing a line parallel to the stress-strain curve at:

(*a*) 0.2 per cent of maximum strain (*b*) 2 per cent of maximum strain

(*c*) 5 per cent of maximum strain (*d*) 10 per cent of maximum strain.

1.27. Free body diagram is the diagram:

(*a*) having no forces acting on the body

(*b*) showing all the external forces acting on the body

(*c*) showing all the internal forces acting on the body

(*d*) showing all the external and internal forces acting on the body.

1.28. The extension produced by the self weight of the bar is equal to that produced by a load of:

(*a*) double of its weight applied at the lower end

(*b*) equal to its weight applied at the lower end

(*c*) half of its weight applied at the lower end

(*d*) half of its weight applied at its mid-length.

1.29. A vertical hanging bar of length l and weighing w N/unit length carries a load W at the bottom. The tensile force in the bar at a distance y from the support will be given by:

(a) W (b) $W + wl$ (c) $W + w(l - y)$ (d) $(W + w)\dfrac{y}{l}$.

1.30. The units of strain are:

(a) m/m (b) dimensionless (c) N/m (d) m/N.

1.31. If a piece of material neither expends nor contracts in volume when subjected to stresses then the Poisson's ratio must be

(a) zero (b) 0.25 (c) 0.33 (d) 0.5.

1.32. The tensile stress-strain diagram for cast iron shows:

(a) a linear relationship upto the point of fracture
(b) a pronounced yield point
(c) no yield point at all
(d) large deformation before fracture.

1.33. The actual fracture strength of materials is:

(a) sometimes less than (b) always less than
(c) always more than (d) equal to the ultimate strength of the material.

1.34. A measure of toughness of a material is its:

(a) ultimate strength (b) percentage elongation
(c) yield strength
(d) area under the stress-strain diagram upto fracture.

1.35. If the radius of a wire stretched by a load is doubled, then its modulus of elasticity will be:

(a) doubled (b) halved (c) remain unaffected (d) become four times.

1.36. Which of the following materials is more elastic?

(a) rubber (b) plastic (c) steel (d) brass.

EXERCISES

1.1. Calculate the stresses in the bar shown in Fig. 1.35. E_{cu} = 105 GPa, E_s = 200 GPa.

[**Ans.** 58.13 MPa, – 18.93 MPa]

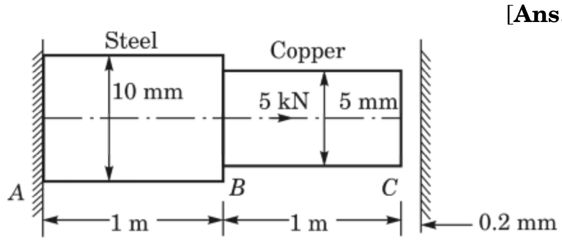

Fig. 1.35

1.2. Calculate the stresses in the bar and total elongation shown in Fig. 1.36. E = 200 GPa.

[**Ans.** 254.648 MPa, 254.648 MPa, 509.296 MPa, 6.68 MPa]

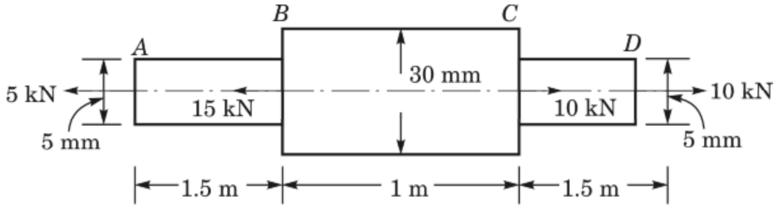

Fig. 1.36

1.3. A steel bar tapers uniformly from 20 mm diameter to 10 mm diameter over a length of 2 m. If E = 200 GPa, calculate the elongation under a force of 10 kN. [**Ans.** 0.637 mm]

1.4. A conical bar of base 500 mm and height 1000 mm hangs vertically. If density of bar is 7400 kg/m^3 and E = 200 GPa, find its elongation. [**Ans.** 60.495×10^{-6}]

1.5. A bar is loaded as shown in Fig. 1.37. Calculate the total elongation. Take E = 210 GPa. [**Ans.** 0.2121 mm]

1.6. A steel bar round in cross-section tapers from 20 mm diameter to 10 mm diameter over a length of 0.6 m. Calculate the increase in length under a pull of 2 kN. Take E = 210 GPa. [**Ans.** 0.03638 mm]

1.7. A bar is loaded as shown in Fig. 1.38. Determine the stresses in each part and the vertical displacement of points A and B. E = 200 GPa.

[**Ans.** σ_1 = 150 MPa, σ_2 = 100 MPa;
δ_A = 0.125 mm downwards,
δ_B = 0.375 mm upwards]

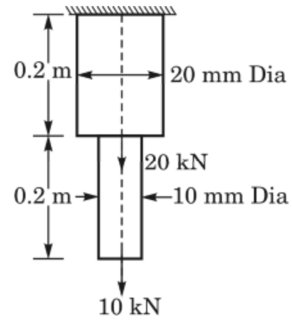

Fig. 1.37

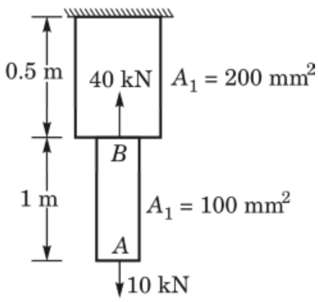

Fig. 1.38

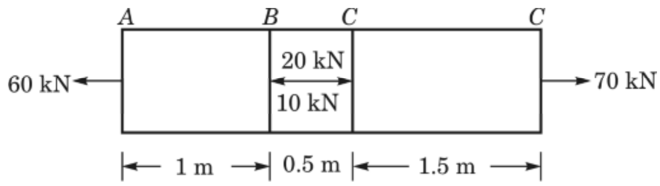

Fig. 1.39

1.8. Calculate the total elongation of the steel bar of area of cross-section 600 mm^2 loaded as shown in Fig. 1.39. E = 210 GPa. [**Ans.** 1.23 mm]

1.9. A steel tie rod 40 mm in diameter and 2 m long is subjected to a pull of 100 kN. To what length the rod should be bored centrally with a bore of 20 mm diameter so that the total extension will increase by 20 percent? E = 210 GPa. [**Ans.** 1.2 m]

1.10. A tie bar has enlarged ends of square section 60 mm × 60 mm as shown in Fig. 1.40. Find the size of the middle position, assuming circular, if the stress there is not to exceed 150 MPa and total elongation is 0.5 mm. Take E = 200 GPa. [**Ans.** 0.136]

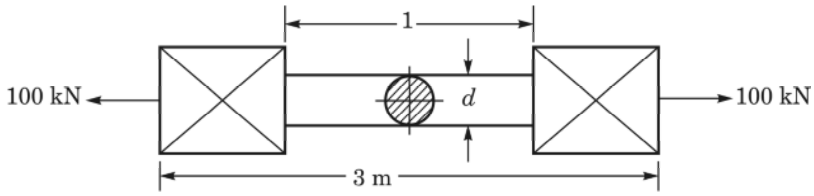

Fig. 1.40

1.11. Calculate the elongation of the bar loaded as shown in Fig. 1.41. Take E = 200 GPa.

[**Ans.** 1.000 mm]

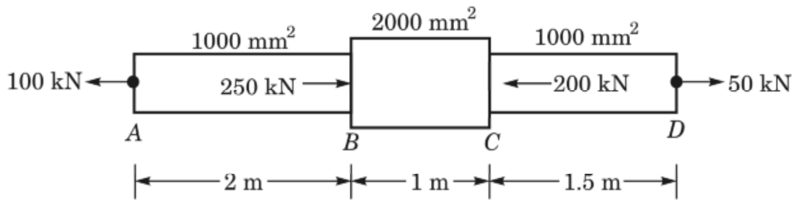

Fig. 1.41