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CHAPTER

Definitions and Basic Concepts

1.1. (A) S.I. Units : Fundamental Units or S.I. Base Units. Quantity, unit, symbol and definition of the base units are given below :

(*i*) **Length : metre (m).** The metre is the length of the path travelled by light in vacuum during a time interval of $\frac{1}{299792458}$ of a second.

(*ii*) **Mass : kilogram (kg).** The kilogram is the unit of mass ; it is equal to the mass of the international prototype of the kilogram.

This international prototype is made of platinum-iridium and is kept at the International Bureau of Weights and Measures, Sevres, Paris, France.

(*iii*) **Time : seconds (s).** The second is the duration of 9192631770 periods of the radiation corresponding to the transition between the two hypertine levels of the ground state of the caesium-133 atom.

(*iv*) **Electric current : ampere (A).** The ampere is that constant current which, if maintained in two straight parallel conductors and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.

(*v*) **Thermodynamic Temperature : Kelvin (K).** The kelvin unit of thermodynamic temperature, is the fraction $\frac{1}{273.16}$ of the thermodynamic temperature of the triple point of water.

(*vi*) **Luminous intensity : candela (cd).** The candela is the luminous intensity, in a given direction of a source that emits monochromatic radiation frequency 540×10^{12} hertz and that has a radiant intensity in that direction as $\frac{1}{683}$ watts per steradian.

(*vii*) **Amount of substance : mole (mol).** The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12.

(B) **S.I. supplementary units :**

(*i*) **Plane angle : radian (rad).** The radian is the plane angle between two radii of a circle which cut off on the circumference an arc equal in length to the radius.

(*ii*) **Solid angle : steradian (sr).** The steradian is the solid angle which, having its vertex in the centre of the sphere, cuts off area of the surface of sphere equal to that of a square with sides of length equal to radius of the sphere.

(C) Examples of S.I. derived units expressed in terms of base units :

Quantity	*S.I. units name*	*Symbol*
area	square metre	m^2
volume	cubic metre	m^3
speed, velocity	metre per second	m/s
acceleration	metre per second squared	m/s^2
density, mass density	kilogram per cubic metre	kg/m^3
specific volume	cubic metre per kilogram	m^3/kg

(D) Examples of S.I. derived units with special names :

Quantity	*S.I. unit name*	*Symbol*	*Expression in terms of other units*	*Expression in terms of S.I. base units*
Force	Newton	N	—	$m\ kg\ s^{-2}$
Pressure, stress	Pascal	Pa	N/m^2	$m^{-1}\ kg\ s^{-2}$
Energy, work, Quantity of heat	Joule	J	Nm	$m^2\ kg\ s^{-2}$
Celsius Temperature	degree	°C	—	K

(E) Examples of S.I. derived units expressed by special names :

Quantity	*S.I. unit name*	*Symbol*	*Expression in terms of S.I. base units*
dynamic viscosity	pascal second	Pa s	$m^{-1}\ kg\ s^{-1}$
moment of force	metre newton	Nm	$m^2\ kg\ s^{-2}$
surface tension	Newton per metre	N/m	$kg\ s^{-2}$
heat capacity, entropy	Joule per Kelvin	J/K	$m^2\ kg\ s^{-2}\ K^{-1}$
Specific heat capacity, specific entropy	Joule per kilogram kelvin	J/(kg K)	$m^2\ s^{-2}\ K^{-1}$
Specific energy	Joule per kilogram	J/kg	$m^2\ s^{-2}$
Thermal conductivity	Watt per metre kelvin	W/(m K)	$m\ kg\ s^{-3}\ K^{-1}$

(F) S.I. prefixes :

Factor	*Prefix*	*Symbol*	*Factor*	*Prefix*	*Symbol*
10^{18}	exa	E	10^{-1}	deci	*d*
10^{15}	peta	P	10^{-2}	centi	*c*
10^{12}	tera	T	10^{-3}	milli	*m*
10^9	giga	G	10^{-6}	micro	μ
10^6	mega	M	10^{-9}	nano	*n*
10^3	kilo	k	10^{-12}	pico	*p*
10^2	hecto	h	10^{-15}	femto	*f*
10^1	deca	da	10^{-18}	atto	*a*

(G) Permitted Units :

Name	*Symbol*	*Value in S.I. units*
minute	min	1 min = 60 s (**Note.** s is also expressed as sec. in the text)
hour	*h*	1 h = 60 min = 3600 s
day	*d*	1 d = 24 h = 86400 s
degree	°	1° = (π/180) rad
minute	′	1′ = (1/60)° = (π/10800) rad
second	″	1″ = (1/60)′ = (π/648000) rad
litre	*l*	1 *l* = 1 dm^3 = 10^{-3} m^3
tonne	*t*	1 *t* = 10^3 kg

N.B. The information gives the base units in the International System (S.I.) and lists also a number of units derived from them, all of which form a coherent measurement system. In a coherent system, calculations involving a number of quantities may be made and the correct result obtained without the introduction of arbitrary constants.

(H) Temporarily accepted units :

Name	*Symbol*	*Value in S.I.*
nautical mile	—	1 nautical mile = 1852 m
knot	—	1 nautical mile per hour = (1852/3600) m/sec
are	a	1 a = 1 da m^2 = 10^2 m^2
hectare	ha	1 ha = 1 h m^2 = 10^4 m^2
barn	b	1 b = 100 fm^2 = 10^{-28} m^2
bar	bar	1 bar = 0.1 MPa = 10^5 Pa
standard atmosphere	atm	1 atm = 101325 Pa
carat	c	1 c = 200 mg
quintal	q	1 q = 100 kg

1.2. Machine. A machine, according to one of the definitions given in Oxford English Dictionary, is "*an apparatus for applying mechanical power, consisting of a number of interrelated parts, each having a definite function*", and this is the sense in which the word 'machine' is used here. It is very broad based definition and includes the concept of mechanical advantage as the sole object of a machine. A machine will be found to consist of an assembly of links or pieces. If motion is imparted suitably to one, all may receive motion, but their relationship will depend upon the nature of connections. If a force is applied suitably to one, a force may be obtained suitably from the other, but the relationships of the forces will depend upon the nature of connections.

A machine is an arrangement of parts for doing work, a device for applying power or changing its direction. In a machine, terms such as force, torque, work and power describe the predominant concepts.

Example. A beginner may be inclined to include petrol engine within the definition of a machine. Though, he is not far out in his thinking, a word of caution may be necessary. If it is confined to only this aspect that mechanical work, *i.e.* (Force × Displacement) is applied at the piston to obtain mechanical work at the crank shaft, *i.e.* (Torque × Angular displacement), in this case, the petrol engine can be called a machine. But the thermodynamics of combustion and all allied phenomenon compel us to call it an engine (Heat Engine). But machine tool in the workshop like lathe, shaper, planer etc. are machines. These satisfy both the definitions of mechanical advantage as sole object of machine and an apparatus for applying mechanical power.

1.3. The Science of Mechanics. That branch of science which deals with motions, time and forces is called Mechanics, and is made up of two parts, statics and dynamics. Statics deals with analysis of stationary systems, *i.e.* those in which time is not a factor and dynamics deals with systems which change with time. The dynamics is also made up of two major disciplines namely Kinematics and Kinetics.

(*a*) **Kinematics.** It is the study of motion, quite apart from forces which produce that motion. More particularly, kinematics is the study of position, displacement, rotation, speed, velocity and acceleration which arise in the design of mechanical systems, or kinematics of machine.

(*b*) **Kinetics.** It deals with the forces acting in mechanical system or on parts of the machine and also with the inertia forces due to mass and motion consideration.

Both these terms namely kinematics and kinetics will get elaborated precisely and illustratively subsequently.

1.4. Kinematics of Machine. As already stated, kinematics of machine deals with the relative motion of machine parts. Kinematic schemes of a machine can be investigated without considering the forces. Thus kinematics of machine can be dealt with as a separate subject.

When a machine is to be designed, and that is the ultimate object of all such studies, three distinct considerations are essential.

1. Determination of kinematic chain.
2. Determination of forces.
3. Proportioning of the parts.

Though all these steps are not independent and a systematic compromise is evolved it is convenient to consider the first two steps separately and apply to machine design, where third step is necessary.

1.5. Kinematic Link or Element. Kinematic link is a resistant body or an assembly of resistant bodies which go to make a part or parts of a machine connecting other parts *which have motion relative to it.* Kinematic link can be an assembly of parts forming one unit with no relative motion of the parts with respect to one another.

A kinematic link is assumed to be completely rigid. The machine components which do not fit this assumption of rigidity, such as springs, usually have no effect on the kinematics of device but do play a role in supplying forces. Such members are not called kinematic links. They are usually ignored during kinematic analysis, and their force effects are introduced during dynamic analysis. Some times, as with belt or chain, a machine member may passess one-way rigidity, such a member would be considered a link when in tension, but not under compression.

Example. In every machine there is one fixed link. This is essentially the frame of the machine. Let us consider any reciprocating engine schematically shown in Fig. 1.1 to explain the use of the term link.

Link 1 is the fixed link and includes the frame and all other stationary parts like cylinder, crankshaft bearing and cam-shaft bearing etc. Link 2 may include crankshaft, flywheel etc. all having a motion of rotation about the fixed axis, *i.e.* axis of the main bearings. Link 3 is the connecting rod, an intermediate floating link. Link 4 is the piston having reciprocating rectilinear translatory motion.

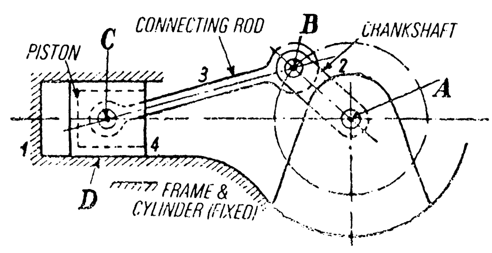


Fig. 1.1

All materials have elasticity, but in most case the deformation produced may be very small and can be neglected for kinematic analysis.

Example. In flexible links like ropes, chains and belts acting in tension, deformation produced on working load can be neglected for kinematic analysis, and so they are also treated as kinematic links. Similarly, fluid in compression transmitting motion, as in hydraulic press, will be treated as a kinematic link.

1.6. Structure. Structure is an assemblage of resistant bodies which are not kinematic links *because there is no relative motion between the links*. There is only straining action due to forces acting on them. Fig. 1.2 explains a very simple structure having three resistant bodies 1, 2 and 3 but no relative motion is possible.

Examples. Roof trusses, bridges, buildings etc.

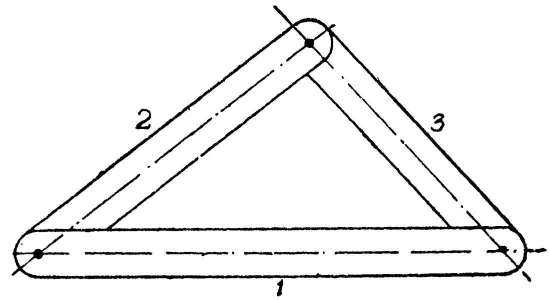


Fig. 1.2. Structure.

1.7. Difference Between a Machine and a Structure

(*i*) Machine serves to modify and transmit mechanical work.	(*i*) Structure serves to modify and transmit forces only.
(*ii*) Relative motion exists between its links or members.	(*ii*) No relative motion exists between its links or members.
Examples. Shaper, screw jack.	**Example.** Roof truss.

1.8. Kinematic Pair. A pair is a joint of two links that permits relative motion. The relative motion between the elements or links that form a pair is required to be completely constrained or successfully constrained. The degree of freedom of a kinematic pair is given by the number of independent co-ordinates required to completely specify the relative motion. These co-ordinates are called pair variables.

Example. Mechanism commonly used in petrol engine is shown in Fig. 1.1. This mechanism is called the *slider crank mechanism*, where *A* is pair (turning pair). Surface of the crankshaft is one element and surface of the bearing is another element. Similarly *B* and *C* are also turning pairs. But *D* is a pair which is sliding, formed by the surface of piston and surface of cylinder.

1.9. Types of Kinematic Pairs. Classification of kinematic pairs is based on the following considerations :

(*a*) Nature of relative motion between the elements.

(*b*) Nature of contact between the elements.

(*c*) Nature of the mechanical arrangement for complete or successful constraint between the elements.

(*a*) *Classification based on the nature of relative motion between elements.*

(*i*) *Sliding Pair.* Prismatic sliding pair is constituted by two elements so connected that one is constrained to have a sliding motion relative to the other. Refer Fig. 1.3. It is seen that relative motion between elements *A* and *B* can be expressed by a single co-ordinate *S* and thus it possesses one degree of freedom.

Example. Piston and cylinder in a slider crank mechanism form a sliding pair. There is sliding motion of the piston surface relative to cylinder surface (*see* Fig. 1.1). *D* is a sliding pair. The relative motion is not completely constrained. But the axis of gudgeon pin and the axis of the crank pin are maintained parallel to each other by the connecting rods so that the piston and the cylinder successfully form a constrained pair. It may be noted that piston does not rotate in the cylinder and degree of freedom is one. Fig. 1.3 shows a rectangular bar A sliding in a rectangular hole in a bearing B and this is another example of a sliding pair. It is a case of completely constrained motion.

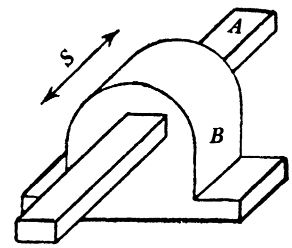


Fig. 1.3. Sliding Pairs.

(*ii*) *Turning Pair.* (Revolute pair). When connections of the two elements are such that only a constrained motion of rotation of one element with respect to the other is possible, the pair constitutes a turning pair. It is also called Hinged pair. It allows only relative motion of rotation which can be expressed by a single co-ordinate θ. Thus a turning pair or revolute pair has single degree of freedom.

Example. A and B in Fig. 1.1 of slider crank mechanism are examples of turning pairs. Fig. 1.4 shows a shaft A with two collars and a bearing B in which it rotates. This is a case of turning pair with completely constrained motion.

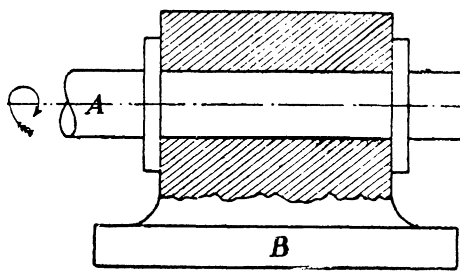


Fig. 1.4

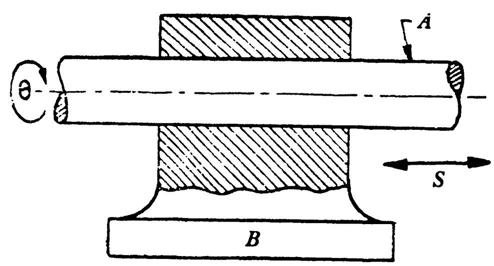


Fig. 1.5

(*iii*) *Cylindrical Pair.* Refer Fig. 1.5. If the collar from Fig. 1.4 is removed, the motion between links A and B can be both of turning and sliding. Thus it has two degrees of freedom namely that of rotation and translation parallel to the axis of rotation, between the links A and B. These motions have no relationship with each other. These relative motions of rotation and translation can be expressed by co-ordinates θ and S respectively. This motion may be completely constrained motion.

(*iv*) *Rolling Pair.* A wheel and the surfaces which it rolls form a rolling pair at the line of contact.

Example. In a belt drive, the pulley can be considered to be rolling on the belt and such a connection between the belt surface and the pulley surface constitutes a rolling pair. Fig. 1.6 shows a roller bearing. The rollers A are rolling on the surfaces of inner race B and the outer race C. This is a case of a rolling pair formed by the surface of the roller and the surface of the inner race or outer race at the surface of contact.

(*v*) *Spherical Pair.* A ball and a socket joint form a spherical pair. Fig. 1.7 shows the ball element A and the socket element B.

They form a spherical pair at the surface of contact. This pair has three degrees of freedom. The full description of the relative motion between links A and B needs three independent co-ordinates. Two co-ordinates namely α and β are needed to specify axis OM and the third co-ordinate θ describes the rotation about the axis OA.

(*vi*) *Helical pair or Screw Pair.* When contact surfaces are screw threads, the constraint is again complete. Though both rotational and sliding motion of A relative to B is obtained, a specified amount of rotation of A relative to B results in a strictly proportional amount of axial motion of A relative to B (*see* Fig. 1.8). Such a pair is called a screw pair.

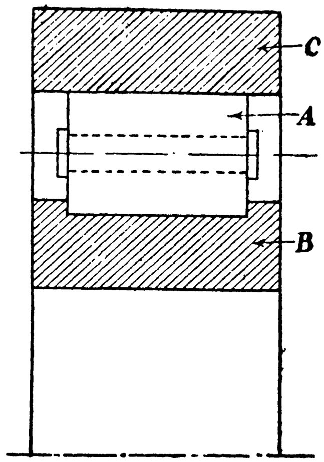
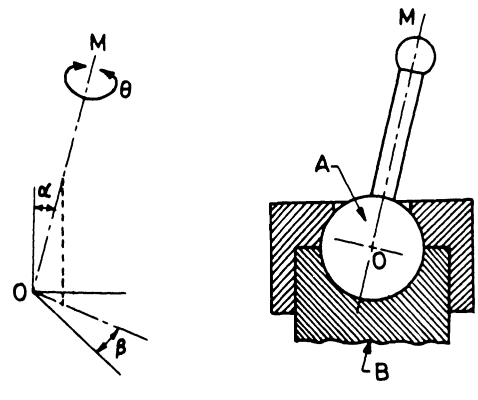


Fig. 1.6. Rolling Pair *AB* or *AC*.



(*a*) (*b*)

Fig. 1.7. Spherical Pair.

It has only one degree of freedom. The relative motion of link *A* with respect to *B* can be expressed by one co-ordinate either *S* or θ and $\frac{\Delta\theta}{2\pi} = \frac{\Delta S}{L}$, where *L* is the lead of the thread.

Example. Any nut and bolt. If bolt is kept fixed nut can have a simultaneous sliding as well as rotational motion and *vice versa.*

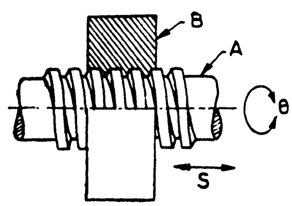


Fig. 1.8. Screw Pair or Helical Pair.

(*b*) *Classification based on the nature of contact between elements.*

(*i*) *Lower Pairs.* If a pair has a *surface contact* between the two elements while in motion, it is called a lower pair. The relative motion is purely turning or sliding. In Fig. 1.1 of a slider crank mechanism, *A*, *B*, *C* and *D* are all lower pairs.

(*ii*) *Higher Pairs.* The mechanism of slider crank can be modified as shown in Fig. 1.9 (*a*). This mechanism is kinematically equivalent to slider crank mechanism shown in Fig. 1.1. The piston has been replaced by a sphere, rigidly fastened to a connecting rod. The cylinder and sphere have line contact. The motion between the sphere and the cylinder is that of sliding and turning and, therefore, a complex motion. This arrangement would result in excessive wear and, therefore, not a good design. Kinematically it makes no difference whether it is a higher pair or a lower pair. A higher pair has generally theoretical point contact or line contact.

Example. Cam and followers [refer Fig. 1.9 (*b*)], tooth gears, ball bearings and roller bearings.

(*c*) *Classification of kinematic parts based on the nature of the mechanical constraint.*

(*i*) *Closed Pairs.* Elements of pairs held together mechanically constitute a closed pair. The pairs of Figs. 1.5, 1.6 and 1.8 are all closed pairs.

(*ii*) *Unclosed Pairs.* Elements of pairs not held together mechanically constitute unclosed pairs or open pairs.

Example. Cam and follower arrangement shown in Fig. 1.9 (*b*) constitutes an open pair at the line of contact between the cam and the flat footed follower. The contact is maintained by the force of gravity and the spring and without such external forces they are not mechanically held together.

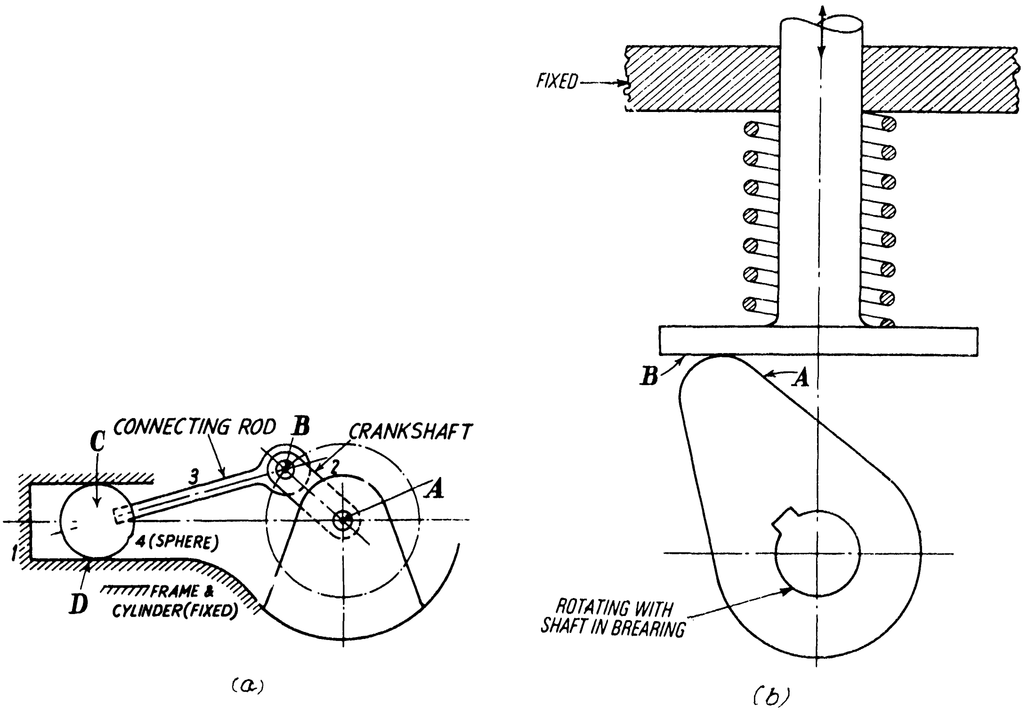


Fig. 1.9. Unclosed Pair.

1.10. Kinematic Chain. When number of links are connected in space such that, the relative motion of any point on a link with respect to any other point on the other link follows a law, the chain is called a kinematic chain. This type of motion is called completely constrained or successfully constrained motion. There can be the other type of motion, namely unconstrained motion. For a particular motion of any point on a link, there can be infinite number of motions of other links or points on other links. Such an arrangement is not called a kinematic chain. It may be noted that no link is fixed in a kinematic chain. It is a chain of links in space, with constrained motion. Thus the constrained kinematic chain has a single degree of freedom. *This is subject to the condition that the input motion is given to one link.*

Reader is advised to read advanced book on kinematics for multiple input motions and multiple degree of freedom. This topic is elaborated in brief under the subject of mobility in this book..

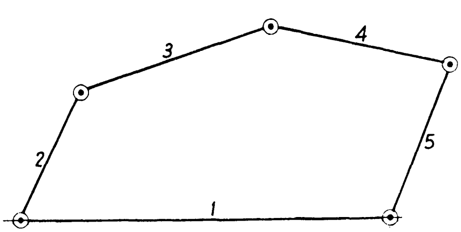


Fig. 1.10. Not a Kinematic Chain.

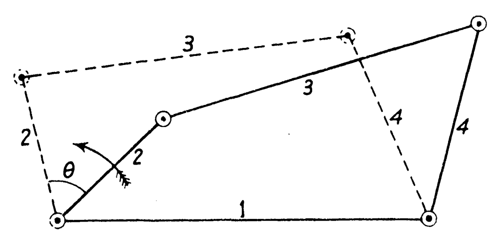
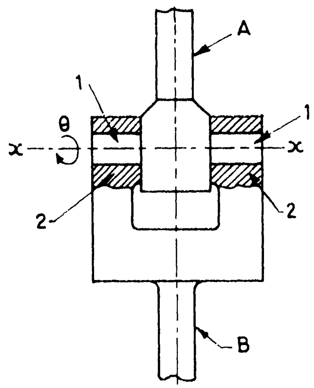
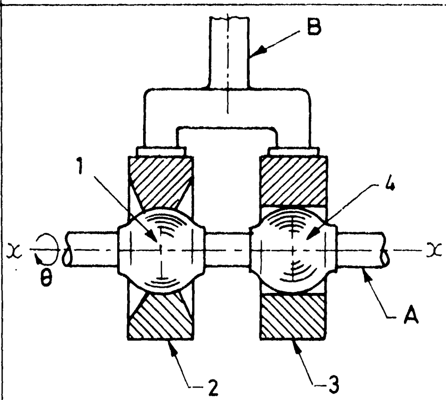
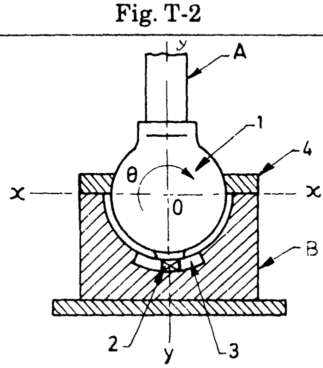
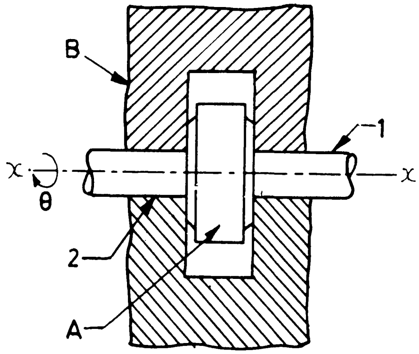
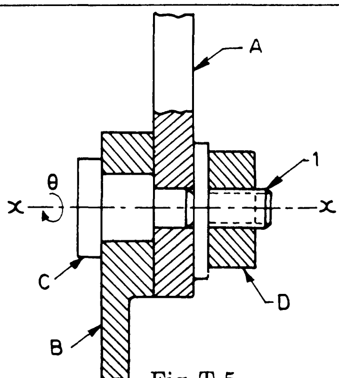
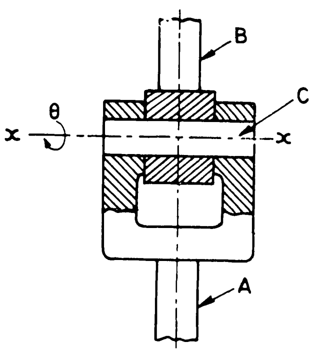
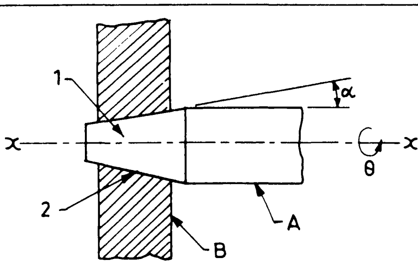


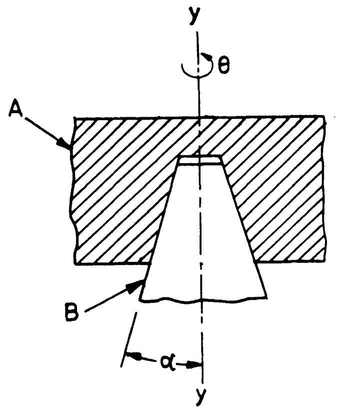
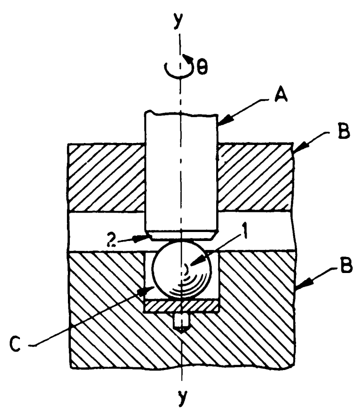
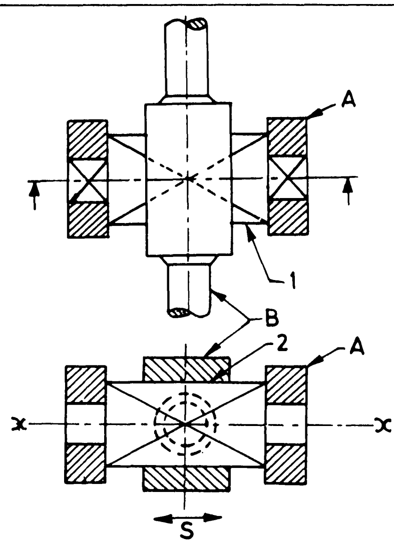
Fig. 1.11. Four Link Kinematic Chain.

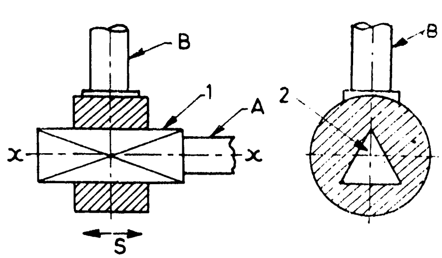
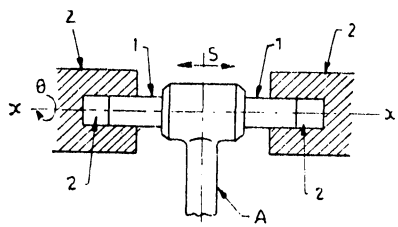
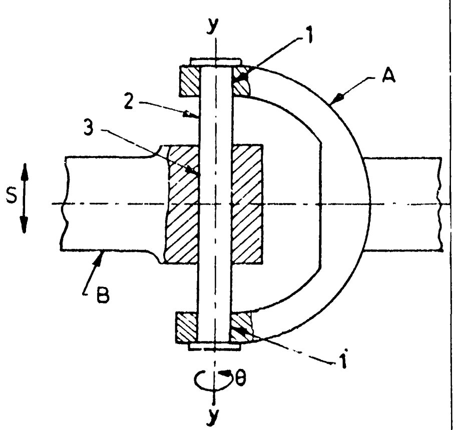
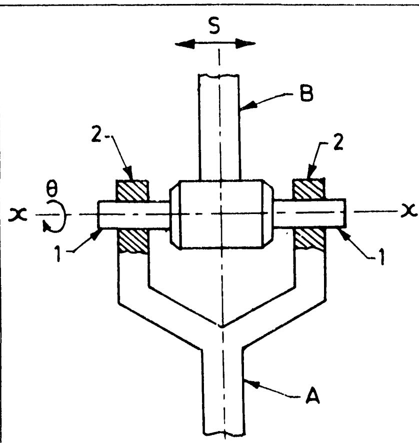
When any one link is fixed, the kinematic chain becomes a mechanism. Fig. 1.10 shows an arrangement of five links. Specific motion of one link or a point on it *i.e.* with one input, it leads to motion of other links or points on them which is unpredictable. An infinite number of positions of other links are possible. Thus, this arrangement is not a kinematic chain. But instead, if there are four links as shown in Fig. 1.11, and one input any motion to any point or link results in a motion relative to any other point or link which follows a definite law, or a definite constrained motion is imparted to the remaining links. Thus, arrangement satisfies the requirements of a kinematic chain.

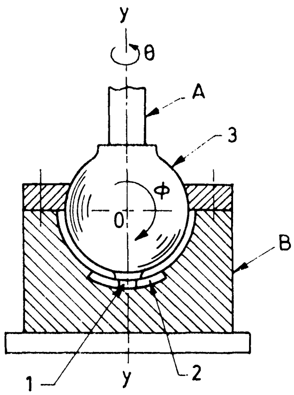
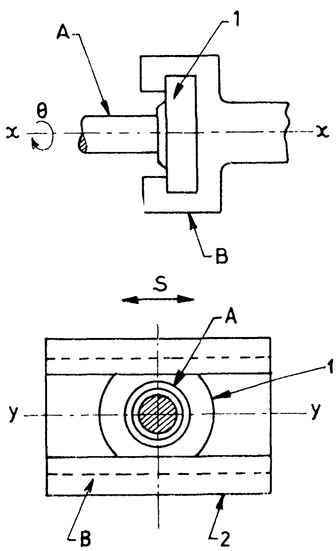
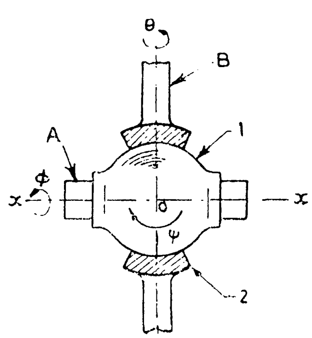
TABLE 1
Illustrative List of Kinematic Pairs and Movable Joints

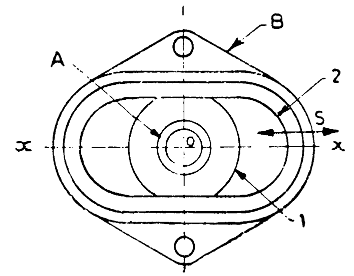
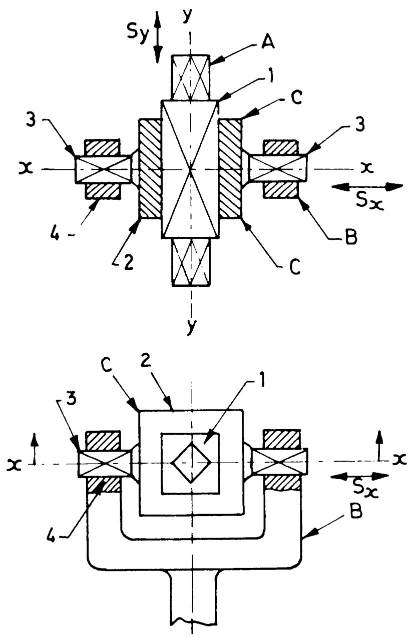
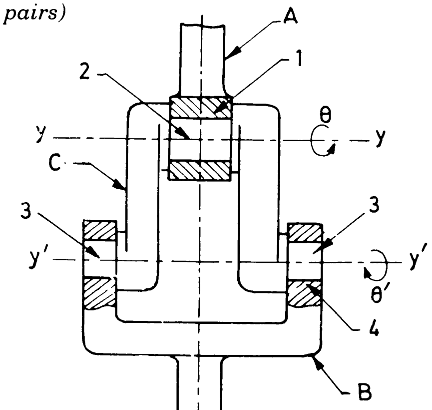
S. No.	*Kinematic pair sketch*	*Description*	*Degree of Freedom etc.*
(1)	(2)	(3)	(4)
1.	 Fig. T-1	Link *A* has two round cylindrical trunnions 1 fitting into cylindrical holes 2 of link *B*. There is turning motion θ of link *A* with respect to link *B* about axis *x-x*.	Single degree freedom turning pair or revolute pair.
2.	 Fig. T-2	Link *A* has two spherical journals 1 and 4. Journal 1 fits into spherical surface 2 of link *B* and journal 4 contacts cylindrical surface 3 of link *B*. There is only turning motion θ of link *A* with respect to link *B* about axis *x-x*.	Single degree freedom revolute pair. Surface contact between 1 and 2, and line contact between 3 and 4.
3.	 Fig. T-3	Link *A* has a spherical pivot and it contacts spherical surface of link *B*. There is a rectangular lug 2 in the pivot with parallel annular side surfaces that fit in and slide along the cylindrical groove of link *B*. There is only turning motion of link *A* with respect to link *B* about common axis perpendicular to plane *xoy* and passing through point *O*.	Single degree freedom turning or revolute pair with spherical joint.

(1)	(2)	(3)	(4)
4.	 Fig. T-4	Link A is fixed on shaft (or pin) 1 and that fits into holes 2, in link B. There is only turning motion θ on link A with respect to link B, about axis x-x.	Single degree freedom turning kinematic pair with cylindrical shaft.
5.	 Fig. T-5	Link A rotates freely on two step shaft C. Also link B rotates freely on two step shaft C. The shaft has a threaded end 1. Nut D holds link A and B on stepped shafts C. Thus link A and B accomplish a turning motion with respect to each other about common axis x-x.	Single degree freedom and surface sliding contact between the two links of the turning pair.
6.	 Fig. T-6	Link B turns freely on intermediate shafts C. The intermediate shaft C is fixed in link A. There is relative turning motion θ between links A and B about axis x-x.	Single degree freedom turning kinematic pair with fixed intermediate shaft. Surface contact is there between link A and B.
7.	 Fig. T-7	Link A has a tapered journal 1 with angle of taper α. Journal 1 fits into tapered hole 2 in link B which has same angle of taper. There is turning motion θ between links A and B about common axis x-x.	Single degree freedom turning kine matic pair with tapered journal.

(1)	(2)	(3)	(4)
8.	 Fig. T-8	Link A has internal conical surface with angle or taper α. Link B having the same taper fits into link A. There is rotary motion θ of link A with respect to link B.	Single degree freedom turning kinematic pair with a conical pivot.
9.	 Fig. T-9	Link A is rotating about axis y-y and has a flat end 2 resting on ball 1. The ball has freedom of movement in socket C of link B. There is turning motion θ about axis y-y of link A with respect to link B.	Single degree freedom turning kinematic pair with ball step bearing.
10.	 Fig. T-10	Link A has a sliding member 1 of rectangular cross-section and it fits in rectangular hole 2 of link B. There is sliding translatory motion S along common axis x-x of link A with respect to link B.	Single degree freedom sliding kinematic pair with a rectangular sliding member.

(1)	(2)	(3)	(4)
11.	 Fig. T-11	Like *A* has a sliding member 1 of triangular cross-section and it fits in a triangular hole 2 of like *B*. There is sliding translatory motion *S* along common axis *x-x* of link *A* with respect to link *B*.	Single degree freedom sliding kinematic pair with triangular sliding member.
12.	 Fig. T-12	Link *A* has two cylindrical trunnions 1 fitting into two cylindrical holes 2 of link *B*. There is translational sliding motion *S* of link *A* and *B* along axis *x-x* and also rotational motion θ about axis *x-x* of link *A* with respect to link *B*.	Two degree freedom cylindrical kinematic pair with round trunnions, one degree freedom of sliding accompanied by second degree of freedom of turning.
13.	 Fig. T-13	Link *A* has two cylindrical holes 1 in which round cylindrical shaft is mounted. This shaft slides along hole 3 of link *B*. There is translational sliding motion *S* of link *A* with respect to *B* about axis *y-y* and also rotational motion about axis *y-y*.	Two degree freedom cylindrical kinematic pair with intermediate shaft. One degree of freedom of sliding and other degree of freedom of turning.
14.	 Fig. T-14	Link *B* has two cylindrical trunnions 1. These fit into round cylindrical eyes 2 of link *A*. There is sliding motion *S* of link *A* with respect to link *B* along common axis *x-x* and also turning motion θ of link *A* with respect to link *B* about axis *x-x*.	Two degree freedom cylindrical kinematic pair with two round eyes.

(1)	(2)	(3)	(4)
15.	 Fig. T-15	Link A has a ball end with sphere at surface 3 and round cylindrical pin 1 sliding along circular groove 2 of width equal to diameter of the pin. There is turning motion θ about axis y-y of link A with respect to link B and also another turning motion ϕ about axis passing through O of the sphere and perpendicular to the groove 2.	Two degree freedom spherical kinematic pair with a pin and a groove.
16.	 Fig. T-16	Link A has round cylindrical disc 1. It slides along T-slot guide 2 of link B. There is translational motion S of link A with respect to link B along axis y-y and also rotational motion θ of link A with respect to link B about axis x-x.	Two degree freedom kinematic pair with a T-slot guide.
17.		Link A has barrel shaped spherical head 1. It fits into collar 2 of circular cross-section and spherical ends. Collar 2 is a part of link B. There are three turning motions θ, ϕ and ψ about three perpendicular axes x-x, y-y and z-z intersecting at O of link A with respect to link B. There is also one translational sliding motion S of link A with respect to link B along axis x-x.	Four degree freedom kinematic pair with barrel shaped head.

(1)	(2)	(3)	(4)
	Fig. T-17		
18.	Fig. T-18	Link A has prismatic sliding member 1 moving along square guide 2 of link B. Link C has prismatic sliding member 3 moving along guide 4 of link B. There is translational sliding motion S_x of link A with respect to link B along axis x-x and also translational sliding motion S_y of link A with respect to link B along axis y-y.	Two degree freedom **joint** with two prismatic sliding members.
19.	*Movable Joint (These are not kinematic pairs)* Fig. T-19	Link A has eye 1 turning about 2 of link C. Link C has trunnions 3 turning in holes 4 of link B. Link A and B have two turning motions about axes y-y and $y'y'$.	Two degree freedom **joint** or double pendulum.

The foregoing definitions can lead us to the formation of some equations. We have seen that the simplest of the kinematic chains, *i.e.* the four link chain has four kinematic links. Each link has two turning pairs. So in a four link kinematic chain the relationship works out to be the following :

$$n = 2f_1 - 4 \text{ or } f_1 - 4 = 0 \qquad ...(1.1)$$

where n = number of links, f_1 = number of pairs.

Alternatively, the relationship between the number of links and the number of joints can be reduced to

$$n = 2f_1 - 4$$

$$= \frac{2}{3}(f_1 + 2) + \frac{4}{3}(f_1 - 4) = \frac{2}{3}(f_1 + 2) + 0$$

i.e. $$n = \frac{2}{3}(f_1 + 2) \qquad ...(1.2a)$$

or $$3n - 2f_1 - 4 = 0 \qquad ...(1.2b)$$

where n = number of links, f_1 = number of joints.

There is a limitation to the use of these equations. These can be applied only to kinematic chains constituted by lower pairs. *If they are applied to chains constituted by higher pairs, each higher pair must be taken equivalent to two lower pairs and an additional link.*

While these equations are useful and have wide application, the beginner is likely to be misled without proper recognition of the pairs and the constraints involved. Thus much reliance on these equations is not advised. Judgment by inspection to decide whether kinematic constraint exists or not may prove helpful. Mobility with more than one input is discussed later, under Art. 1.24—Degrees of Freedom.

1.11. Mechanism. If one of the links of a constrained kinematic chain is fixed, the result is a mechanism. If a different link of the same chain is made the fixed link, the result is a different mechanism. Beginner is often apt to confuse the term *mechanism* with the term *machine*. Primary function of a mechanism is to transmit or to modify motion whereas that of a machine is to obtain mechanical advantage, *i.e.* besides motion, the forces are also involved. If, therefore, the chain is considered purely from the view point of motion modified or transmitted it should be referred to as mechanism. If on the other hand, the chain is considered as an agent for applying or modifying mechanical work, it should be referred to as machine. The equations (1.1) and (1.2) are also applicable to mechanism. Mechanism is only a kinematic chain with one link fixed. Slider crank arrangement of Fig. 1.1 is a mechanism.

1.12. Difference Between Machine and Mechanism

Mechanism	*Machine*
(*i*) Mechanism transmits and modifies motion.	(*i*) Machine modifies mechanical work.
(*ii*) A mechanism is the skeleton outline of the machine to produce definite motion between various links.	(*ii*) Machine may have many mechanisms for transmitting mechanical work or power.
(*iii*) When kinematic chain is analysed as mechanism no special consideration need be given to the forms and the cross-sectional proportions of the links except in so far as the assembly locations are involved.	(*iii*) As to the machine, cross-sectional and proportion requirement to give strength, stiffness, clearance etc., make it imperative to consider these links in all their details.

Examples. Clock work, type-writer (Though these items seem to lie on the border line between machine and mechanism according to above definitions, yet they are suitably termed as mechanisms, since, the force is not greater than necessary to overcome friction of mating parts to produce relative movement).

Example. Shaper in workshop receives mechanical power which is suitably converted to do the work of cutting metal.

1.13. Skeleton Outline of a Machine or Kinematic Representation of a Machine. For kinematic analysis a mechanism may be suitably abbreviated to give all geometrical information required for determining the relative motion of the links. Fig. 1.12 gives skeleton outline for an internal combustion engine.

It will be seen that everything necessary for finding the relative motion of the main links has been shown, namely the length OA of the crank, the length AB of the connecting rod, O the location of the axis of the main bearing and OB the path of point B, *i.e.* the axis of the gudgeon pin.

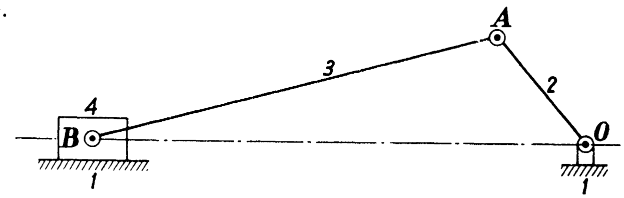
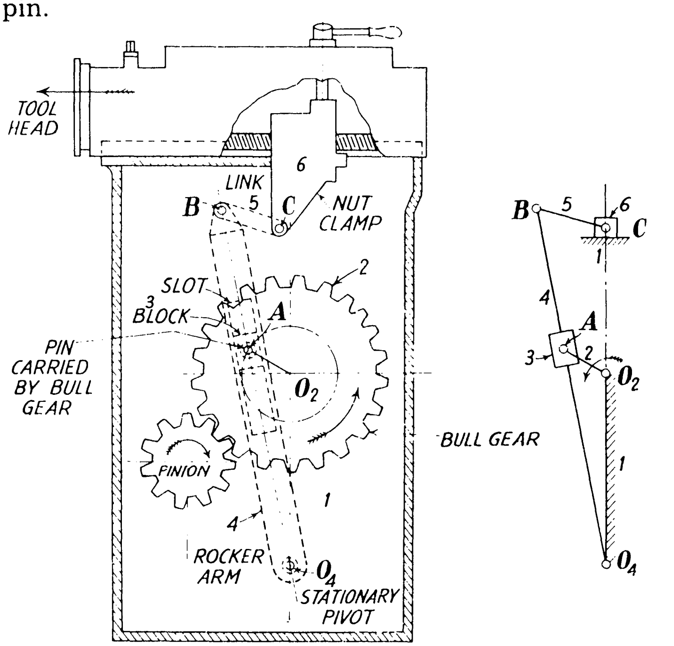


Fig. 1.12. Skeleton outline of a slider crank chain.

Turning pairs may be represented by a circle and preferably a dot at the centre and a link having only two turning pairs by a straight line joining the axes of two turning pairs. Fixed link may be shown by cross hatching and line for the link may not be drawn. A sliding link may be shown by a rectangle pivoted on the axis of the pairing element of the link with which it has a turning connection.

Another example of a little more complex mechanism is shown in Fig. 1.13 (*a*) and its skeleton outline or configuration diagram is shown in Fig. 1.13 (*b*). This is the shaper mechanism.



(*a*) Shaper mechanism Fig. 1.13 (*b*) Skeleton outline.

In the mechanism, the pinion drives the Bull gear 2, which carries a pin A whose path will be circular as the Bull gear rotates. On pin A, a sliding block 3 is pivoted. Due to rotation of gear 2 and sliding of block 3 in the slot provided in the rocker arm 4, the rocker arm is caused to oscillate. Since the link 6 is successfully constrained to have only reciprocating motion, but rocker arm 4 moves in the arc of a circle, an intermediate link 5 is necessary.

Analysing as before, it is obvious that the main driving link 2, the Bull gear, serves the purpose of a crank and is shown in the skeleton outline in Fig. 1.13 (*b*) as such. Link 6 is represented by a rectangle pivoted at the end of the intermediate link 5 and constrained to move horizontally by fixed link 1.

Simplest representation of the links 3 and 4 is again a rectangle sliding on a line as shown. Thus Fig. 1.13 (*b*) is kinematically the same as the machine shown in Fig. 1.13 (*a*).

For the study of relative motions, such skeleton outline presentation is very helpful and avoids a lot of confusion to the beginner.

1.14. Expansion of Pairs (Limit and disguise of Revolute Pairs). The beginner is many a time confronted with the problem of recognising kinematic chain, by the mere fact that the mechanism he is called upto to analyse, has its pairs expanded and the appearance of the mechanism is changed beyond recognition, *though the character of motion which is the property of kinematic chain remains unaltered.*

Example. The four bar mechanism shown in Fig. 1.14 has a basic and most elementary kinematic chain. If link *AB* is replaced by an eccentric such that the eccentricity is equal to the length of link 2 or *AB* we have a mechanism as shown in Fig. 1.15. It is obvious that the relative motions of two links are unaffected. Similarly Figs. 1.16 and 1.17 show arrangements, which are kinematically equivalent to that of Fig. 1.14. The arrangement shown in Fig. 1.18 looks radically different from the original configuration diagram but kinematically it is equivalent to the same four-bar mechanism of Fig. 1.14. No relative motion is altered by such expansions. But in appearance they hardly resemble the original four-bar mechanism.

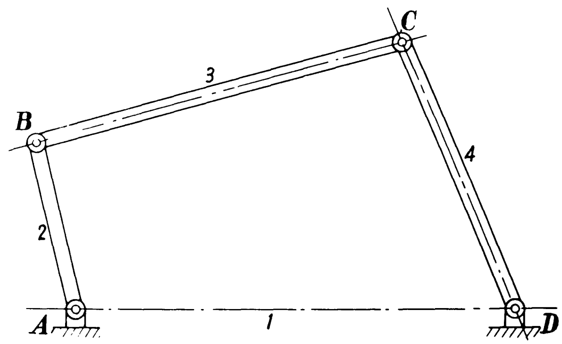


Fig. 1.14. Four-bar kinematic chain mechanism.

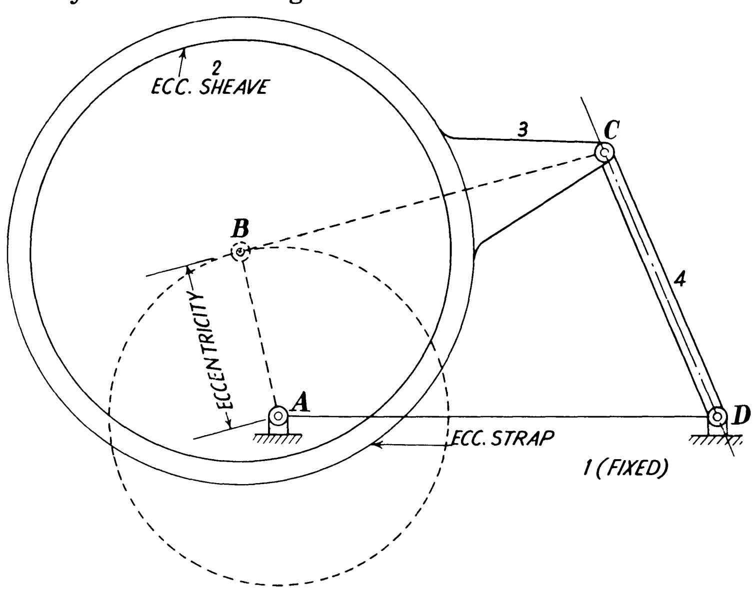


Fig. 1.15. Expansion of the four-bar mechanism.

A further elaboration to prove that a slider crank mechanism is only a particular modification of the basic four-bar mechanism can be easily shown by changing the radius of

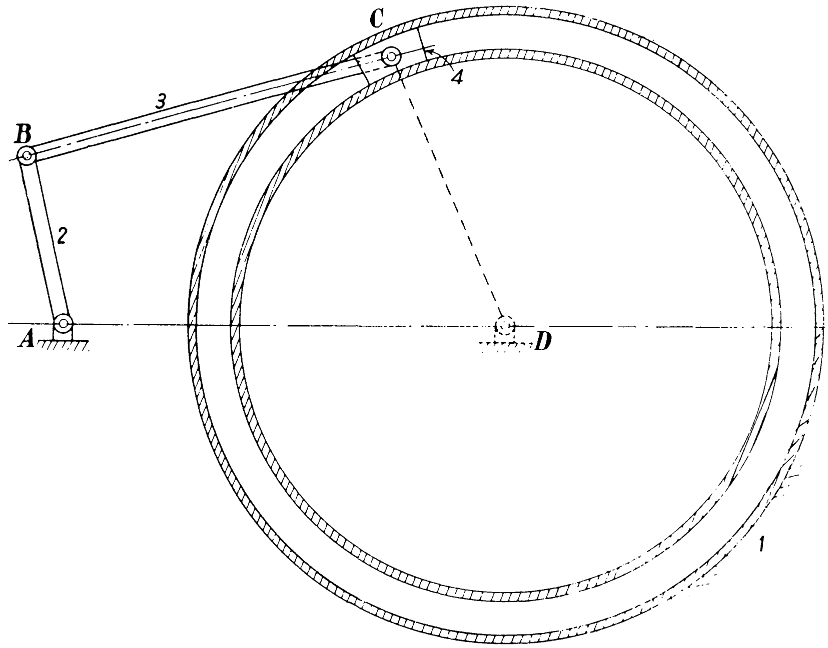


Fig. 1.16. Expansion of the four-bar kinematic chain.

the arc from DC in Fig. 1.17 to infinity. This is shown by Fig. 1.19 which is nothing but simple slider crank mechanism.

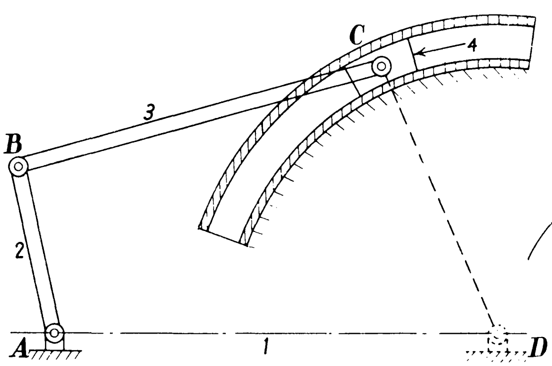


Fig. 1.17. Expansion of the four-bar kinematic chain.

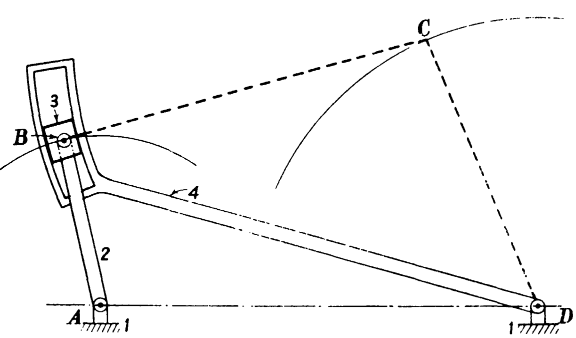
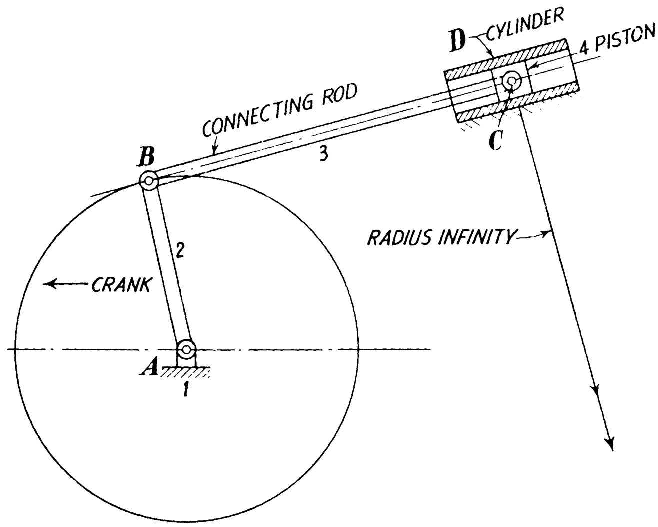


Fig. 1.18

1.15. Inversions of Mechanism. We have already stated that a mechanism has one link fixed. Inversions are the different mechanisms obtained by fixing different links in a kinematic chain. *Thus as many inversion are possible as the number of links. The motion of a link in a kinematic chain relative to some other links is the property of the chain and not of the mechanism. It is immaterial which link is kept fixed. Inversion has no effect on the relative motion. Thus it is important to observe that relative motion of links is not changed in any manner through the process of inversion.*

1.16. Four-bar Chain or Quadric Cycle Chain. It has already been stated in the definition of a kinematic chain that a minimum of four kinematic pairs are required, arranged in such a way that they transmit motion according to a definite law. A chain consisting of four links 1, 2, 3 and 4, all of them having turning pairs at the ends, forms a Four-bar chain or a quadric cycle chain. Simplest and the basic complete kinematic chain has four links (*see* Fig. 1.14) and four turning pairs.



Slider crank
Fig. 1.19. A particular case of four-bar chain.

1.17. (*a*) **Application of Quadric Cycle Chain.** (*a*) *Oscillatory motion* (Crank and Lever Mechanism). By properly proportioning the lengths of the links we can obtain an oscillatory motion (*see* Fig. 1.20).

It can be easily seen from the figure that O_2A, the crank, is able to rotate completely but the follower O_4B, can only oscillate from B_1 to B_2, B_1B_2 being the maximum arc of oscillation. Since BO_4P is one link pivoted at point O_4, the end P of the link has a travel equal to P_1P_2.

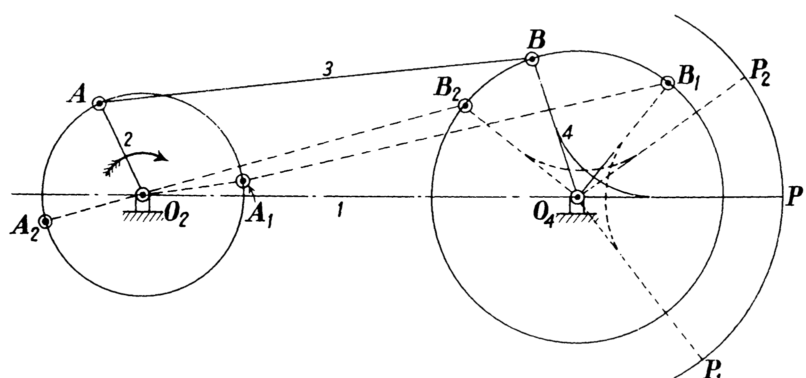


Fig. 1.20. Crank and Lever Mechanism.

(*b*) *Complete rotation of the crank and the follower* (Double Crank Mechanism ; *see* Fig. 1.21). This shows the mechanism for a Drag Link Quick Return Motion. Crank moves through an angle β on the working stroke and through an angle α on the return stroke. Thus time of working stroke is β/α times more or the return is, β/α time quicker.

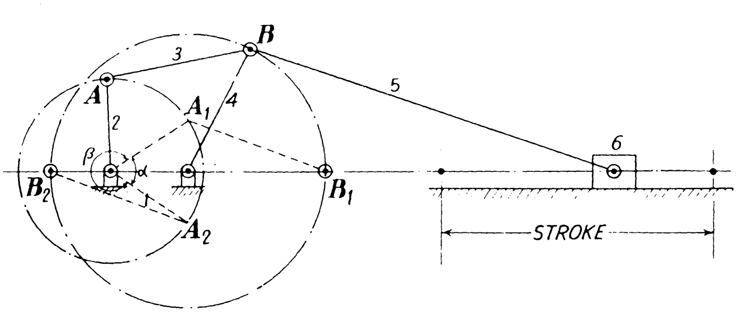


Fig. 1.21. Drag Link Quick Return Motion.

Shortest link is always the stationary link. Sum of the shortest and the longest links of the four links 1, 2, 3 and 4, are less than the sum of the other two. (*Necessary condition for Drag Link Quick return mechanism*).

(*c*) *Coupled wheels of a locomotive* (Double Crank ; *See* Fig. 1.22). Rotary motion from one wheel to the other wheel can be transmitted by this kinematic scheme. Both the links 2 and 4 work as cranks, and the coupling rod is the link 3. Though the link joining the turning pair A and turning pair D is treated here as a fixed link for the purposes of relative motion of the links of the kinematic chain under consideration, actually it has translatory motion relative to rails which are fixed.

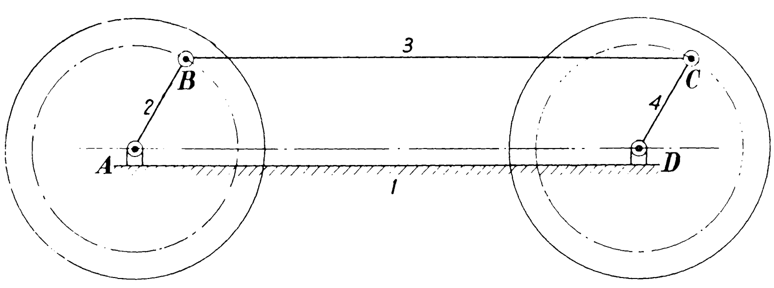
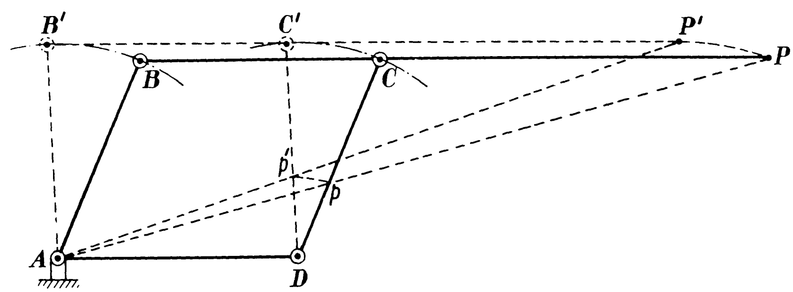


Fig. 1.22. Double Crank Mechanism.

(*d*) *Pantograph* (Double Lever Mechanism). When it is desired to duplicate some motion exactly but to a reduced or enlarged scale, some short of copying device in the form of Pantograph is used. It is basically a quadric cycle chain in the form of a parallelogram (*see* Fig. 1.23). From the similar triangles ApD and CpP, it is clear that p and P will follow similar path. Ratio of the motion of p to the motion of P will be $\frac{Ap}{AP}$ or $\frac{BC}{BP}$.



Double Lever Mechanism
Fig. 1.23. Pantograph.

Proof. $\angle ApD = \angle CpP$; $\angle ADp = \angle PCp$; $\angle pAD = \angle CPp$.

Therefore, the triangles ApD and CPp will be similar whatever be the configuration. Another position when point B occupies position B' is shown in dotted lines. ApP will always be in straight line.

One might guess that a particular four-bar would become one of these above types of mechanism depending upon some relationship involving the link lengths. The *Grashof Criteria* gives this relationship. Grashof's Law states that *the sum of the shortest and the longest links of a planar four-bar linkage cannot be greater than the sum of the remaining two links if there is to be continuous relative rotation between two links.*

1.17. Grashof's Law. A very important consideration while designing a mechanism to be driven by a motor, obviously, is to ensure that the input link can make complete revolution. Mechanisms in which no link makes complete revolution would not be useful in such applications. For four bar chain there is a simple test of whether this is the case.

Grashof's Law states that, for a planer four-bar linkage, the sum of the shortest and the longest link length cannot be greater than the sum of the remaining two link lengths, if there is a continuous relative motion between two members.

(*b*) **Different mechanism formed by inversions of the four bar chain.** A. Let the longest link be l, the shortest link be s and the remaining two p and q. Applying Grashof's law *i.e.* $l + s < p + q$ four possibilities exist. Fig. 1.24 (*a*) and (*b*) show two different crankrocker mechanisms. In each, the shortest link is the crank, the fixed link being either adjacent link. Fig. 1.24 (*c*) shows one double crank (drag link) when the fixed link is the shortest link. Fig.

1.24 (*d*) shows double-rocker mechanism (coupler rotates) when link opposite the shortest link is the fixed link.

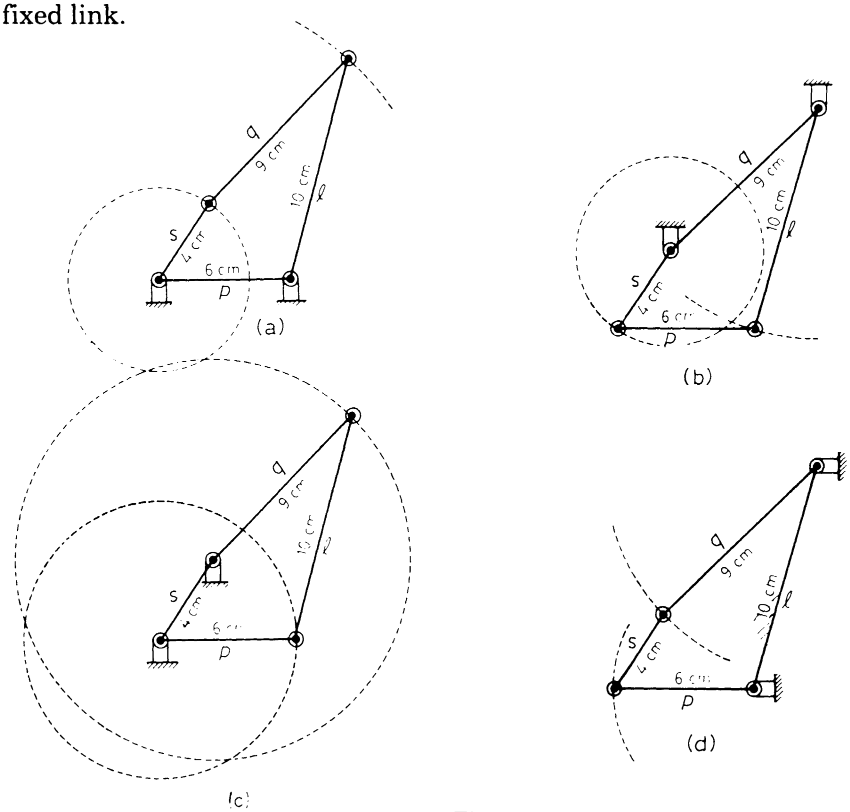


Fig. 1.24

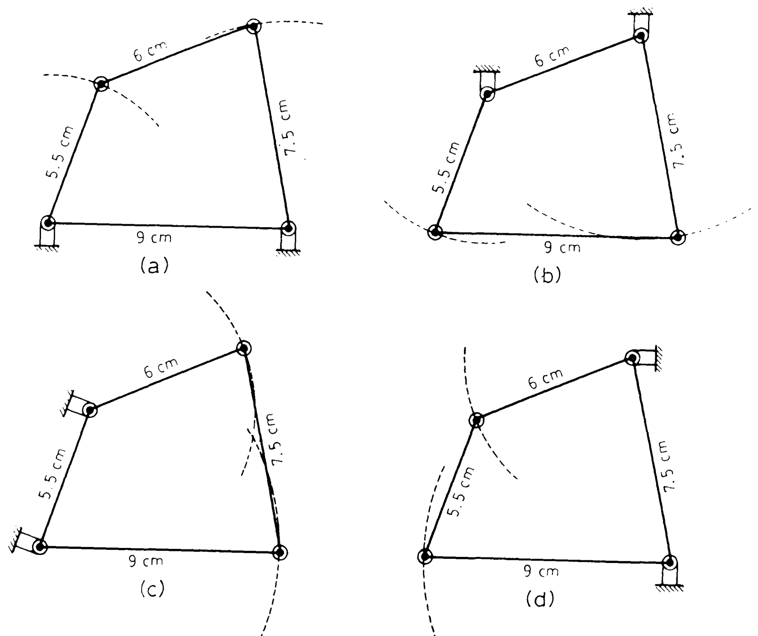


Fig. 1.25

B. If $l + s > p + q$, four triple rocker mechanism will be formed depending upon which one link is fixed. Refer Fig. 1.25.

C. If $l + s = p + q$, four possible mechanisms of type A can be formed but there are difficulties of dead centres or change points. At the dead centres or change points the centre line of all links becomes co-linear and additional guidance is necessary.

D. The parallelogram mechanisms is special case of type A and all the four invertions would be double-cranks if dead centres are given guidance.

Thus, if $s + l \leq p + q$ for four bar linkage which is the simplest-possible pin jointed mechanism for single degree freedom controlled motion.

The linkage is Grashoff and at least one link will be capable of making a full revolution with respect to ground plane. If the in-equality is not true, the linkage is non-Grashoff and no link will be capable of complete revolution relative to any other link.

1.18. Special Cases of Four Bar Chains. Fig. 1.26 (a) and (b) shows the *parallelogram* and *anti-parallelogram* configuration respectively of special-case Grashoff. The parallelogram linkage is quite useful as it exactly duplicates the rotary motion of the driver crank to the driven crank. One common use is to couple the two wind shield wiper output rockers across the width of the wind shield on an automobile. The coupler of the parallelogram linkage is in curvilinear translation remaining at the same angle while all points in it describe identical circular paths.

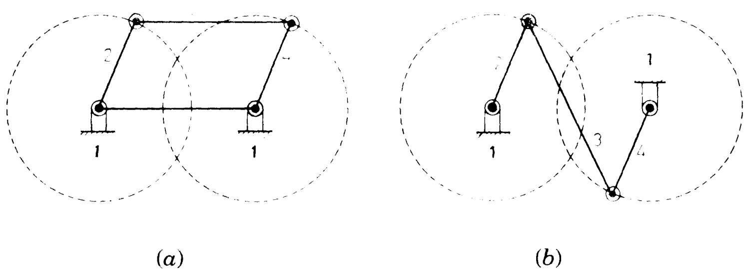


Fig. 1.26

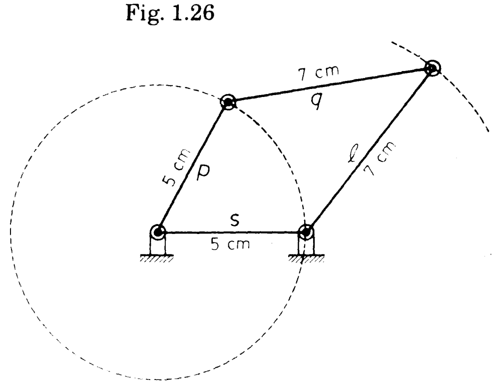


Fig. 1.27

The anti-parallelogram linkage is also a double crank but the output crank has an angular velocity different from the input link.

Fig. 1.27 shows a Deltoid or Kite form linkage which is a crank-rocker. In this case two adjacent links are equal. It comprises two pairs of equal links. It satisfies the Grashoff criterion $s + l = p + q$ but here $s = p$ and $l = q$.

Table 1.1 gives the summary of the Grashoff and Non-Grashoff and special Grashoff four bar chains.

Table 1.1

S. No.	*s + l vs p + q*	*Inversion*	*Designation*	*Code*	*Also known as*
1.	<	$L_1 = s$ = fixed link	Grashoff crank-crank-crank	GCCC	double-crank
2.	<	$L_2 = s$ = input link	Grashoff crank-rocker-rocker	GCRR	crank-rocker
3.	<	$L_3 = s$ = coupler	Grashoff rocker-crank-rocker	GRCR	double rocker
4.	<	$L_4 = s$ = output	Grashoff rocker-rocker-crank	GRRC	rocker-crank
5.	>	$L_1 = l$ = fixed link	Non-Grashoff 1 rocker-rocker-rocker	RRR1	triple rocker
6.	>	$L_2 = l$ = input link	Rocker-rocker-rocker 2 (Non-Grashoff)	RRR2	triple rocker
7.	>	$L_3 = l$ = coupler	Rocker-rocker-rocker-3 (Non-Grashoff)	RRR3	triple rocker
8.	>	$L_4 = l$ = output link	Rocker-rocker-rocker 4	RRR4	triple rocker
9.	=	$L_1 = s$ = fixed link	Change point crank-crank-crank	SCCC	SC double crank
10.	=	$L_2 = s$ = input link	Change point crank-rocker-rocker	SCRR	SC crank rocker
11.	=	$L_3 = s$ = coupler	Change point rocker-crank-rocker	SRCR	SC double rocker
12.	=	$L_4 = s$ = output link	Change point rocker-rocker-crank	SRRC	SC rocker-crank
13.	=	Two equal pairs	Double change point	S2X	parallelogram or Deltoid
14.	=	$L_1 = L_2 = L_3 = L_4$	Triple change point	S3X	Square

1.19. Tasks Performed by Four Bar Chain. The four bar mechanism can accomplish different tasks by properly proportioning the links and fixing them in positions. Fig. 1.28 shows an application in the form of Level Luffing crane. It may be noted that this four-bar generates approximate Straight Line motion of the path of the tracer point E. Fig. 1.29 shows a lawn sprinkler which can be adjusted for different oscillation ranges of the sprinkler head. By the clamping screw the length and the angle of output link can be varied in its function. It also comprises four links. Similarly a different task can be obtained as shown in Fig. 1.30. It is the Ford automotive hood linkage design. This controls the motion of the link 3 *i.e.* coupler relative to the body.

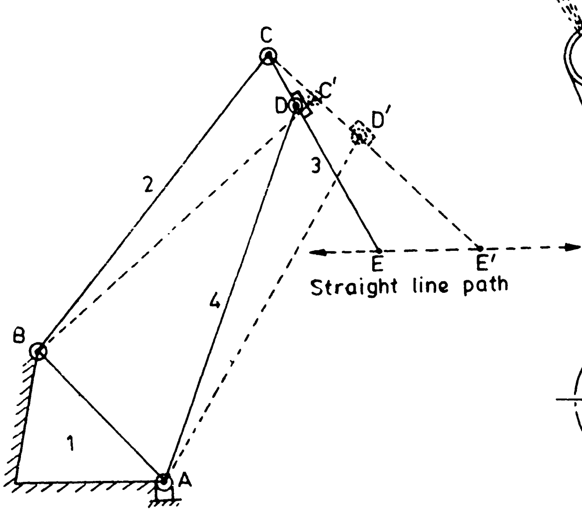


Fig. 1.28

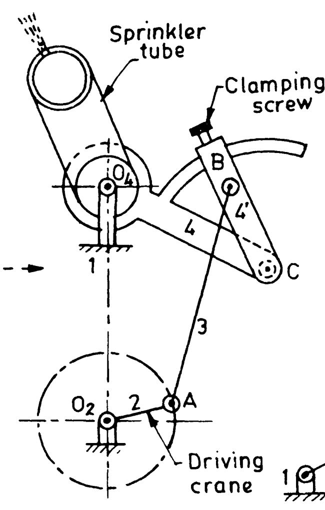


Fig. 1.29

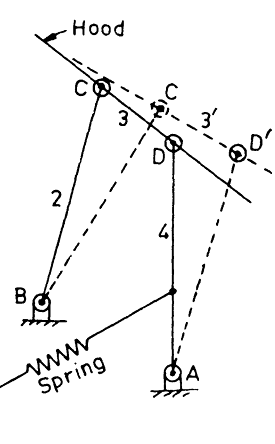


Fig. 1.30

Thus a linkage can be classified according to the task performed namely (*i*) A path generation where path of tracer point is of interest Fig. 1.28, (*ii*) A function generator in which relative motion between links generally with respect to fixed link is of interest, Fig. 1.29, (*iii*) A motion generator where the entire motion of the coupler is of interest, Fig. 1.30.

Problem 1.1. *In a drag link mechanism the length of the links of the basic four-bar mechanism are 5 cm, 8 cm, 9 cm and 7 cm respectively, and the stroke of the slider is 14 cm. Lay out the mechanism, and determine the ratio of the time of the forward stroke to the time of the return stroke and indicate the sense of rotation of the crank. Assume uniform angular velocity of the crank.*

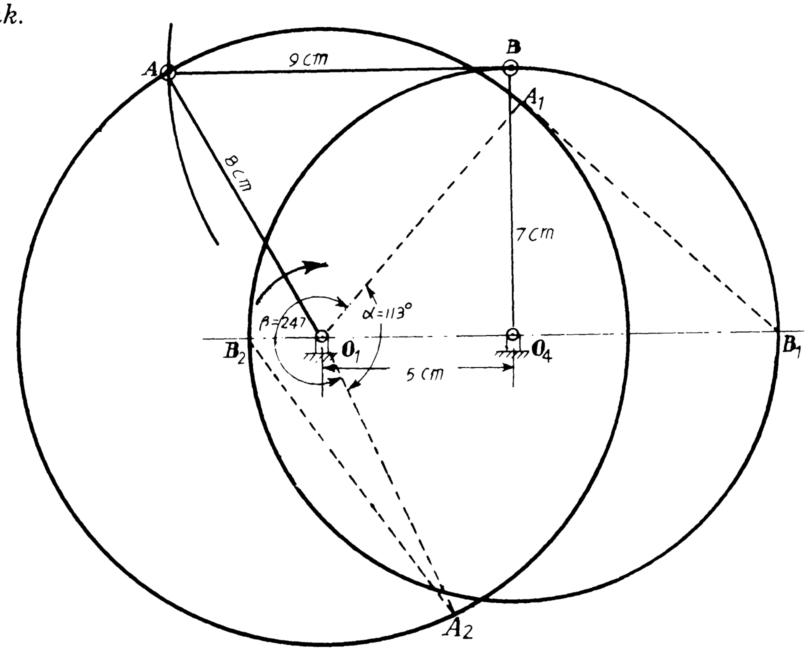


Fig. 1.31. Drag Link Mechanism.

Scale $\frac{1}{2}$ Full size.

Solution. O_1O_4BA is the required drag link mechanism with O_1O_4 = 5 cm as the fixed link being the shortest link.

Longest link + Shortest link < Sum of the other two

$$9 \quad + \quad 5 \quad < \quad 8 + 7$$

Therefore, the proportion of the links do satisfy the basic requirements for the drag link mechanism, O_1A is kept as the driving crank equal to 8 cm and O_4B is the driven crank to which the slider is attached through a connecting rod. One end of connecting rod forms a turning pair at B and the other end a turning pair with the slider moving horizontally. Thus slider will have a stroke equal to twice the length of link O_4B *i.e.* 2 × 7 = 14 cm equal to B_1B_2 in the figure. Driving crank is assumed to rotate in the clockwise directions. For two extreme positions of B, *i.e.* B_1 and B_2, the positions of the driving cranks are shown by A_1 and A_2 respectively. Dotted lines show the construction. From the figure $\alpha = 113°$, $\beta = 247°$.

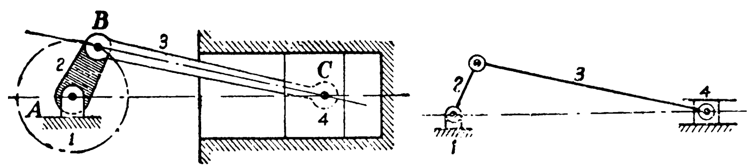
Therefore, for uniform angular velocity of O_1A

$$\frac{\text{Time of forward stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{247}{113} = \mathbf{2.185.}$$

1.20. Slider Crank Mechanism. It has already been proved in article 1.14 (Fig. 1.19) ; that a slider crank mechanism is a four link mechanism having three turning pairs A (1 and 2), B (2 and 3), C (3 and 4) and a fourth sliding pair D (4 and 1). The purpose of this mechanism is to convert rotary motion to reciprocating motion and *vice versa*. Simplest form of a slider crank mechanism of a reciprocating internal combustion engine is illustrated by Fig. 1.1. A (1 and 2), B (2 and 3) and C (3 and 4) are turning pairs and D (4 and 1) is a sliding pair. Frame forms link 1, crank is link 2, connecting rod is link 3 and the piston forms link 4.

1.21. Inversions of Slider Crank Chain

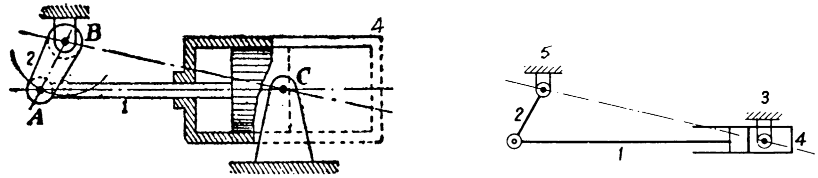
(*i*) *First inversion.* In the first inversion, link 1 is kept fixed (*see* Fig. 1.32). This represents the ordinary reciprocating steam engine or internal combustion engine if the driver is link 4, *i.e.* the piston, and it represents a pump, if the driver is the link 2, *i.e.* the crank.



(*a*) (*b*)

First Inversion
Slider Crank
Fig. 1.32

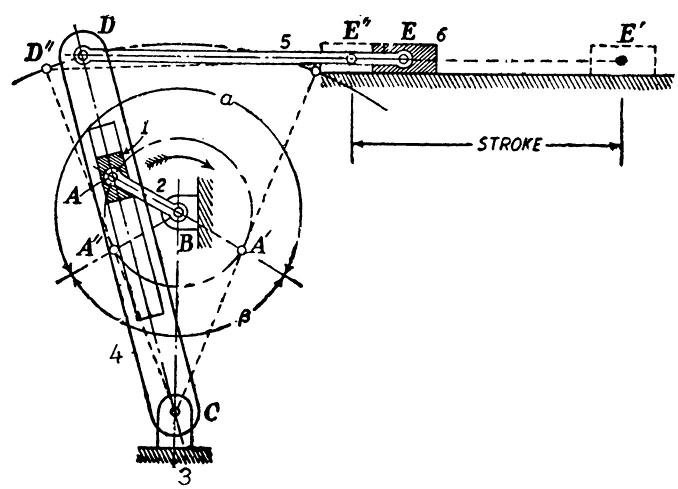
(*ii*) *Second inversion.* Second inversion is obtained by fixing the connecting rod or link 3. Some applications of this inversion are the oscillating cylinder engine (Fig. 1.33) and the quick return mechanism of a shaper machine (Fig. 1.34). This is called crank and slotted lever mechanism, or crank-shaper quirck return mechanism.



(*a*) Second Inversion (*b*) Oscillating Cylinder Engine

Fig. 1.33

In this type of mechanism, the link 1 is a slider which slides in a slotted lever which is link 4 and link 3 is fixed. Link 2 which is the crank rotates causing the slider 1 to slide in the slotted lever 4 and cause the lever to oscillate about pivot point C. Link 4 is extended to point D and at that point a ram E is linked to the lever by a link DE. This link DE or 5 carries a tool attached to the end E and the tool E or link 6 is constrained to slide. E carries a cutting tool. It will be easily seen from the figure that the stroke of the cutting tool starts from point E'' when the link 2 is at right angles to lever 4 or lever 4 is tangential to the crank radius circle at point A''. The end of the stroke is marked by point E' when again crank, after having rotated through an angle α, is again at right angles to the lever at position A'. Thus the cutting stroke is executed during the rotation of the



Second inversion
Fig. 1.34. Crank and slotted lever mechanism.

crank through angle α. Similarly the return stroke is executed when the crank rotates through angle (360° – α) or β as shown in the figure.

Therefore, when the crank rotates uniformly

$$\frac{\text{Time of cutting}}{\text{Time of return}} = \frac{\alpha}{\beta} = \frac{\alpha}{(360° - \alpha)} \qquad ...(1.3)$$

The necessity of such a type of motion arises from the fact that during the cutting stroke there is a limit of speed imposed to the cutting tool due to various reasons-like heat dissipation, life of tool bits etc., but no speed limit exists for the return stroke or the idle stroke.

Problem 1.2. *The driving crank 2 of a quick return mechanism shown in Fig. 1.35, runs at a uniform speed of 200 r.p.m. Find the effective stroke and the ratio of the cutting time to idle time or the ratio of cutting stroke time to the return stroke time. Crank is 7.5 cm. Slotted lever length is 45 cm.*

Solution. Dotted lines show the construction for finding the two end points of the stroke when the lever is tangential to the crank radius circle at points P and Q. Reflex angle QOP = α is the angle of cutting stroke and the obtuse angle QOP = β is the angle of return stroke. AB is the length of the effective stroke of the cutting tool which will be attached to the tool box 6.

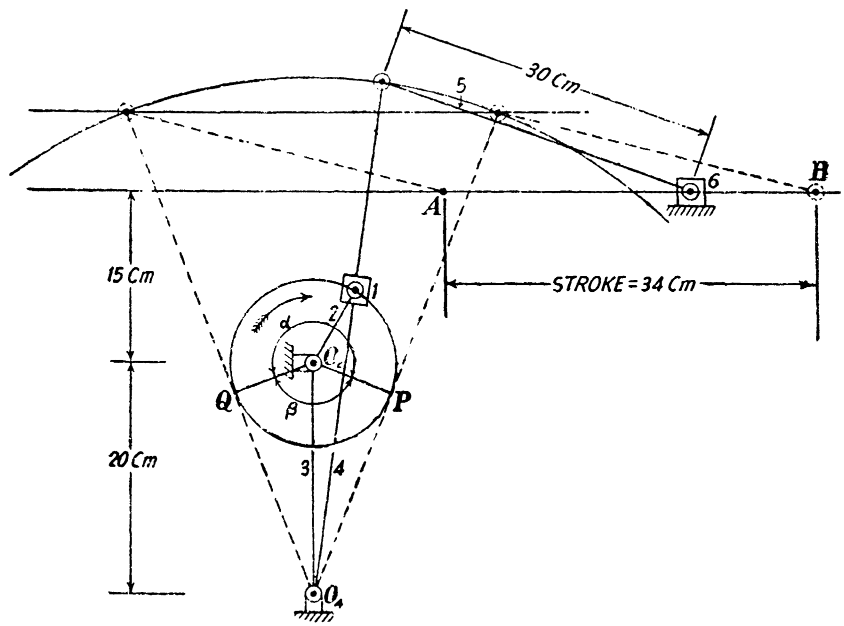


Fig. 1.35. Crank end slotted lever mechanism.

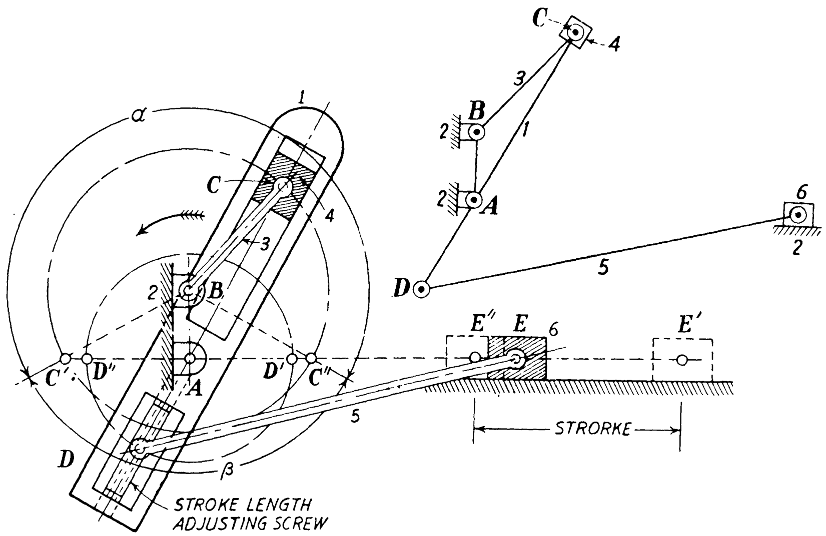
$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{252°}{138°} = \frac{\alpha}{\beta} = \frac{222}{138} = 1.6.$$

Length of the effective stroke = AB = 34 cm.

(*iii*) *Third inversion.* If crank, *i.e.* link 2, is kept fixed, a third inversion is obtained. An application of such a mechanism is Whitworth quick return motion, Fig. 1.36, a rotary engine, Fig. 1.37.

In Whitworth mechanism link 3 acts as the driver rotating at uniform angular speed. Whitworth mechanism is very much used in shapers and textile machinery, while the rotary engine is not much used now.

In the Whitworth quick return motion the link 3, *i.e.* BC rotates at a uniform speed. The slide block attached to the pin C slides along the slotted link 1 and causes this link to revolve about the pivot point A, with variable angular velocity. From the pin D on the slotted link, connecting rod, *i.e.* link 5, passes to a pin E on the arm which carries the tool box and is constrained to move along a line passing through A and perpendicular to AB. The extreme positions of the arm correspond to the positions. AD' and AD'', *i.e.* the poitions of the pin D at D' and D''. This will happen when the pin C occupies positions C' and C'' respectively. Thus for counter-clockwise rotation of the link 3 which acts as the driver, the angle turned by link 3



Third Inversion
Fig. 1.36. Whitworth return motion.

will be α for the return stroke it will be β. Since the link 3 or the driver rotates at uniform angular velocity, the ratio of cutting stroke time to return stroke time will be given by

$$\frac{\text{Cutting stroke time}}{\text{Return stroke time}} = \frac{\text{Reflex angle } \alpha}{\text{Angle } \beta} = \frac{\alpha}{(360° - \beta)} \qquad ...(1.4)$$

In the rotary engine mechanism (Fig. 1.37), the crank A_1B_2 *i.e.* link 2 is fixed, and the result is that the link 1 which is equivalent of the case and body of the engine referred to in the first inversion, rotates. Thus the only difference between first inversion, and this example of third inversion is that the crank rotates and the body is fixed in the first inversion, whereas here the crank is fixed and the body rotates.

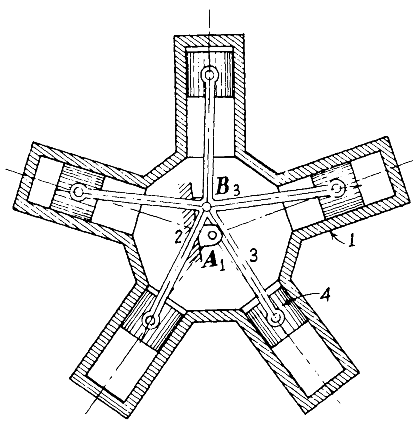
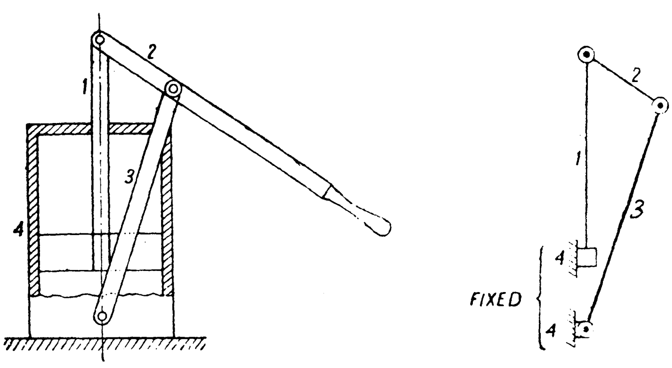


Fig. 1.37. Rotary engine.



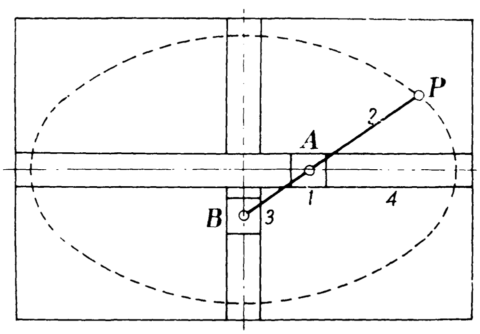
(*a*) Hand-pump (*b*) Skeleton outline of hand-pump
Fig. 1.38. Fourth Inversion.

(*iv*) *Fourth inversion.* By keeping the slider fixed a fourth inversion is obtained. Application of this inversion is very much limited. One example of this inversion is the hand pump shown in Fig. 1.38.

1.22. Double Slider Crank Chain. It is a four-bar kinematic chain containing two turning pairs and two translatory pairs such that two pairs of the same kind are adjacent. Fig. 1.39 shows the arrangement of a double slider crank chain.

Two slide blocks, links 1 and 3, slide along the slots in a frame, link 4, which is fixed, and the turning pairs formed at pins A and B are connected together by a link 2. Each of the slide block forms a sliding pair with the frame, *i.e.* link 4 and the turning pair with the link 2. Such a kinematic chain has three inversions.

(*i*) *First inversion.* Frame, *i.e.* link 4, is fixed and the slide blocks form sliding pairs with the frame and, turning pairs with the link 2 in Fig. 1.39. An application of such an inversion is the Elliptical Trammel (Fig. 1.40). A plate is taken and two slots at right angles are cut on it. In the slots, two sliding blocks are fitted. And these slide blocks are connected by a link. Any point except the mid-points of AB or points A and B on the link will trace an ellipse. The points A and B move in straight line. The mid-point of AB traces a circle.



First inversion

Fig. 1.39. Elliptical Trammel.

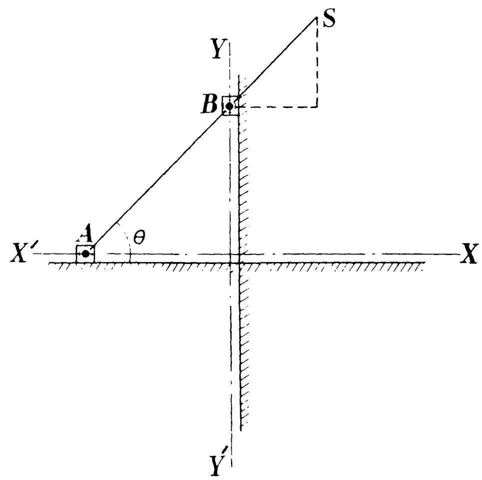


Fig. 1.40. Elliptical Trammel.

Proof. From Fig. 1.40. The co-ordinates of point S are given by

$$X = BS \cos \theta, \quad Y = AS \sin \theta$$

Therefore, $$\frac{X}{BS} = \cos \theta \quad \text{and} \quad \frac{Y}{AS} = \sin \theta$$

Thus $$\frac{X^2}{BS^2} + \frac{Y^2}{AS^2} = \cos^2 \theta + \sin^2 \theta$$

But $\cos^2 \theta + \sin^2 \theta = 1.$

Therefore, $$\frac{X^2}{BS^2} + \frac{Y^2}{AS^2} = 1 \qquad ...(1.5)$$

This equation is that of an ellipse. Thus the path traced by point S on the link will be an elliptical path, the semi-major and semi-minor axes of the ellipse being AS and BS respectively.

Path traced by mid-point of AB is a circle. In that case

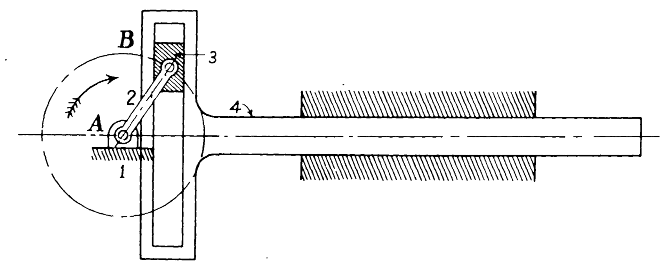
$$BS = AS, \quad X = BS \cos \theta, \quad Y = AS \sin \theta = BS \sin \theta$$

or $$\frac{X^2}{BS^2} + \frac{Y^2}{AS^2} = \cos^2 \theta + \sin^2 \theta$$

$$\therefore \quad \frac{X^2}{BS^2} + \frac{Y^2}{BS^2} = 1.$$

It is an equation of a circle with $BS = AS$ = radius of the circle.

(*ii*) *Second inversion*. In the second inversion, one of the two slide blocks, *i.e.* either link 1 or link 3, is kept fixed. In such an arrangement the whole frame, *i.e.* link 4 will reciprocate. An application of such an inversion in the Scotch Yoke mechanism is shown in Fig. 1.41. The fixed block 1 guides the frame and the link 2 is the driver. This inversion is used for converting rotary motion into a sliding or reciprocating motion.



Second Inversion
Fig. 1.41. Scotch Yoke.

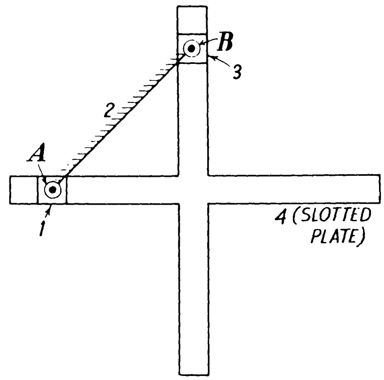
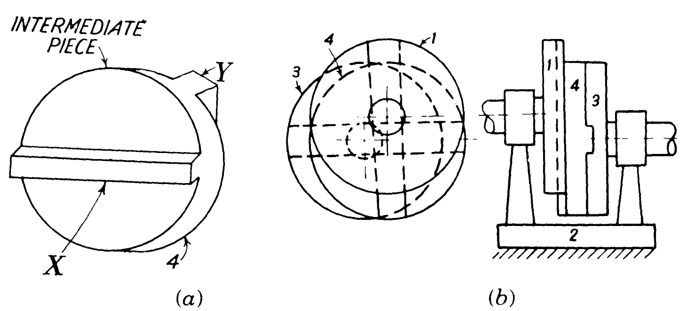


Fig. 1.42

(*iii*) *Third inversion*. Third possibility is to keep the link 2 fixed (*see* Fig. 1.42). In this case each of the side blocks can turn about the pins A and B. If one block is turned through a definite angle, the frame and the other block must turn through the same angle.

An application of the third inversion of the double slider crank mechanism is the Oldham's Coupling (Fig. 1.43).

The coupling is used for connecting two parallel shafts when the distance between the shafts is small. The two shafts to be connected have flanges rigidly fastened to the shafts, generally forged at the end. These flanges form links 1 and 3. An intermediate piece [shown in Fig. 1.43 (*b*)] is circular disc having two tongues X and Y which can slide-fit in the slots in the flanges. These tongues are formed at right angles to each other and on the opposite sides of the intermediate piece. The intermediate link forms the link 4 which can slide in the slots in the flanges. Frame and bearings form the link 2 which is fixed.



Third Inversion
Fig. 1.43. Oldham's coupling.

The rotary motion is given to the link 1, *i.e.* the driver shaft to which flange is fixed, the intermediate piece also rotates by the same angle through which the flange has rotated. And similarly the driven flanges or link 3 must rotate by the same angle. Thus the links 1, 3 and 4 have the same angular velocity at every instant.

The distance between the axes of the shafts is constant and, therefore the centre of intermediate piece will follow the path of a circle with diameter equal to the distance between the axes of the shafts.

Therefore, maximum sliding speed of each tongue of the intermediate piece in the slot will be given by peripheral velocity of the centre of the disc along its circular path.

Peripheral velocity of the disc

= Angular velocity of the shaft × Diameter of this circle

= Angular velocity of the shaft × Distance between the axes of the shafts ...(1.6)

Problem 1.3. *The distance between the two parallel shafts connected by Oldham's Coupling is 25 mm. The speed of the driving shaft is 250 rpm. Find the maximum speed of sliding of the tongue of the intermediate piece in the slot in the flange.*

Solution. Angular speed of the shaft, $\omega = \dfrac{2\pi \times N}{60}$

where N is the number of revolutions per min.

and ω is the angular velocity of shafts in radians/sec.

Therefore, $\omega = \dfrac{2\pi \times 250}{60} = 26.18$ radians/sec.

Maximum speed of sliding,

$$v = d \cdot \omega$$

where v is the velocity of sliding, d is the distance between the axes of parallel shafts.

ω is the angular velocity of the driving or the driven shaft.

Therefore, $v = 25 \times 26.18$

$= 654.5$ mm/sec = **0.6545 m/sec.**

Problem 1.4. *A four bar linkage has the following dimensions $O_1O_2 = 50$ mm, (distance between fixed centres of rotation) $O_1A = 62$ mm, $AB = 37$ mm, $O_2B = 68$ mm, O_1 and O_2 are fixed centres of rotation.*

Determine the type of four bar by Grashoff's criterion.

If the link AB is crank, discuss the Grashoff's criterion.

If the link AB is fixed, discuss the Grashoff's criterion.

Solution. Fig. 1.43 is drawn to scale for the problem.

$O_1O_2 = p = 50$ mm, $O_1A = q = 62$ mm

$AB = s = 37$ mm (shortest link)

$O_2B = l = 68$ mm (longest link)

And $s + l = 37 + 68 = 105$ mm ; $p + q = 50 + 62 = 112$ mm.

Therefore, $s + l < p + q$ and also the link opposite to the shortest link is fixed. Thus by Grashoff's criterion it is a double rocker four bar mechanism.

If AB is the crank, the possibilities of fixed links are either BO_2 or AO_1 *i.e.* the links adjacent to the crank. This by Grashoff's criterion gives crank rocker mechanism.

If AB is a fixed link, *i.e.* if the shortest link is fixed, it gives double crank mechanism (Drag link quick return mechanism).

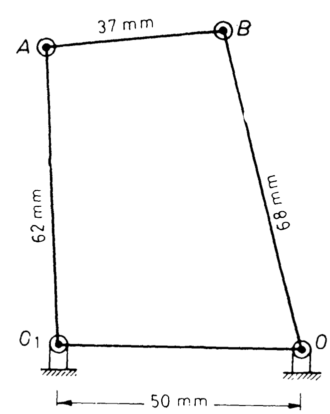


Fig. 1.43

Problem 1.5. *Fig. 1.44 shows a plane mechanism with link lengths given in some units. If slider A is driver, will the link CG revolve or oscillate. Justify the answer.*

Solution. The mechanism shown in Fig. 1.41 comprises three sub-chains.

(*i*) A four bar chain *CBED* in which the shortest link length $s = CD = 2$ units, and longest link length $= l = BE = 4$ units. Thus $s + l <$ sum of the other two lengths *i.e.*, $2 + 4 < 3 + 4$. It is a Grashoff's chain. And the shortest link CD is fixed. Therefore it is a double crank mechanism. (Drag link quick return mechanism).

(*ii*) The second sub-chain $CDFG$ is again a four bar chain with shortest link $s = CD = 2$ units, and longest link $l = FG = 4$ units and $s + l = 2 + 4$ = sum of other links 3 + 3.

Hence this sub-chain satisfies Grashoff's chain and as shortest link is fixed, this represents a double crank mechanism. But there would be difficulties of dead centres or change points.

Also links DE, EF and DF comprise a three link loop and hence it is a structure and not a kinematic chain.

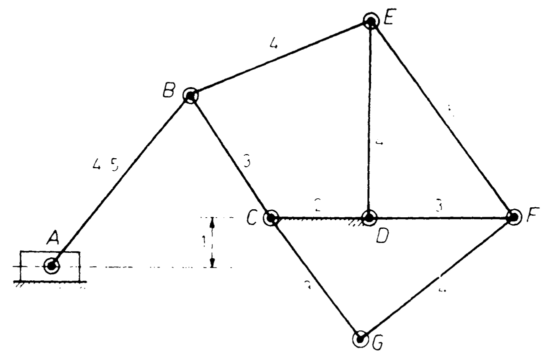


Fig. 1.44

(*iii*) The third sub-chain is the slider-crank chain ABC. Since length AB = 4.5 units > length BC (3 units) + off-set 1 unit, the crank CB can revolve fully.

Thus, $CDEB$ and $CDFG$ are both double crank mechanisms, and link BG can have full revolution when motion is given to slider A.

Problem 1.6. *A four bar linkage has the following dimension : Shortest link = 37 mm. The longest link = 81 mm, other two links are 50 mm and 62 mm respectively. Determine the type of four bar by Grashoff's criterion.*

Solution. s = 37 mm, l = 81 mm, p = 50 mm and q = 62 mm.

Thus 37 + 81 = 118 m > 50 + 62 = 112 mm or $s + l > p + q$. Therefore, by Grashoff's criterion, it gives four double rocker mechanisms depending on which link is fixed.

Problem 1.7. *Some four bar linkages are shown in Fig. 1.45 where the numbers indicate the respective link lengths in cm. Identify the nature of each mechanism whether (i) double crank (ii) crank and rocker (iii) double rocker. Give reasons in brief.*

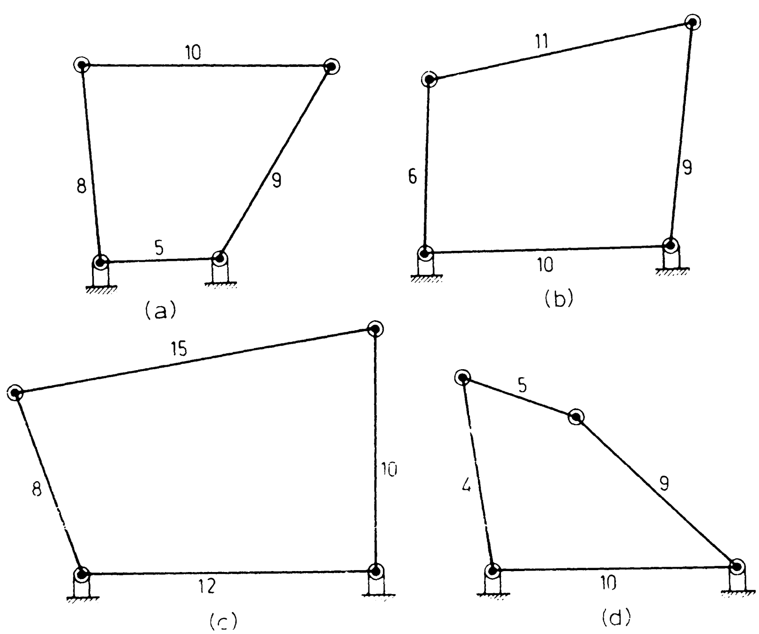


Fig. 1.45

Solution. Chain at Fig. 1.45 (*a*) satisfies Grashoff's law

$$s + l = 10 + 5 = 15 \text{ cm}$$
$$p + q = 9 + 8 = 17 \text{ cm}$$

Thus, $s + l < p + q$.

And the shortest link is the fixed link and therefore it is a double crank mechanism. It is Drag Link Quick Return Mechanism.

Chain at Fig. 1.45 (*b*) satisfies Grashoff's law

$$l + s = 11 + 6 = 17 \text{ cm}$$
$$p + q = 10 + 9 = 19 \text{ cm}$$

Since, neither the shortest link, nor the link opposite the shortest link is fixed, the mechanism is of the type "Crank- Rocker" type.

Chain at Fig. 1.45 (*c*) does not satisfy Grashoff law

$$l + s = 15 + 8 = 23 \text{ cm}$$
$$p + q = 12 + 20 = 22 \text{ cm}$$

Thus, $l + s > p + q$.

Hence the mechanism is "Rocker-Rocker type".

The guiding law to decide the category of mechanism is Grashoff law. According to this law if l and s are the lengths of longest link and the shortest link and p and q are the lengths of the remaining two links, then, if

$$s + l \le p + q$$

The chain belongs to category of Grashoff chain.

1.23. Binary, Ternary, Quaternary Links. In our discussion so far, we have discussed only binary kinematic links, *i.e.* a body with two elements of pairs. One with three elements is called a ternary link and one with four elements is defined as quaternary link. Fig. 1.46 shows examples of binary, ternary and quaternary links ; 5, 6, 2, 3 are examples of binary link ; 1 is a ternary link and 4 is a quaternary link. Thus, the joint which is restricted to apply to the connections of two links by closed pair, is called a binary joint ; if three links are joined at the same connection, it is called a ternary joint. If four links are joined at the same connection, it is a quaternary joint. An ordinary joint of two links is called a binary joint.

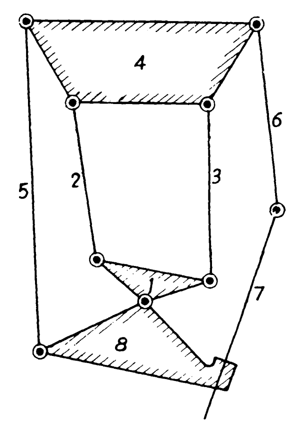


Fig. 1.46

To determine whether a given chain is a structure or kinematic chain or an unconstrained chain poses a problem, as the number of links and kinds of links go on increasing. A rule called the *Criterion of Constraint* is enunciated by A.W. Klein. This algebraic equation applies to chains having plane motion

$$J + \frac{1}{2}H = \frac{3}{2}L - 2 \qquad \ldots(1.7)$$

where J represents the number of binary joints in the chain.

H represents the number of higher pairs.

L represents the number of links in the chain.

A ternary joint is equivalent to two binary joints.

A quaternary joint is equivalent to three binary joints.

In equation (1.7) if,

Left-hand side > Right-hand side → Chain is locked.

Left-hand side = Right-hand side → Chain is constrained.

Left-hand side < Right-hand side → Chain is unconstrained.

Equation (1.7) is very useful one, but the author recommends that judgment by inspection should precede the use of the formula.

1.24. Compound Chain. A basic kinematic chain, as we have discussed before, consists of four kinematic links. If the number of links is more than four and the rule of the criterion of constraint enunciated by Klein also applies, the chain is defined as a compound kinematic chain. Many examples of compound kinematic chains will be found in this chapter as well as subsequent chapters. Shaper quick return mechanism, Whitworth quick return mechanism, Drag link mechanism etc., discussed in this chapter, are all defined as compound kinematic chains.

Problem 1.7. *A Six links chain shown in Fig. 1.47 has 5 binary joints, 1 ternary joint. Prove that it is constrained kinematic chain.*

Solution. The number of joints is, therefore, equal to 7 binary joints. Therefore, $J = 7$, $L = 6$ and $H = 0$ because there is no higher pair. Thus we have

$$J + \frac{1}{2}H = \frac{3}{4}L - 2.$$

Therefore, $$7 + 0 = \frac{3}{2} \times 6 - 2$$

$$7 = 7.$$

Thus Left-hand side = Right-hand side.

Therefore, the chain is constrained and it is a kinematic chain.

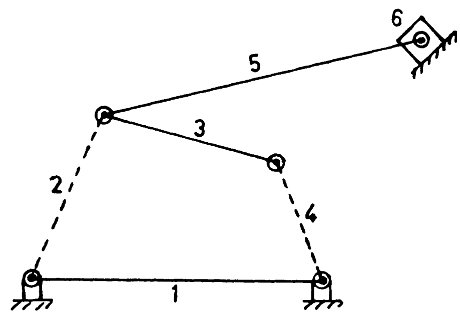


Fig. 1.47

Problem 1.8. *An eight-link chain is shown in Fig. 1.46. Prove that it is a constrained kinematic chain.*

Solution. It has 10 binary joints. We have

$$J + \frac{1}{2}H = \frac{3}{2}L - 2.$$

Therefore, $$10 + 0 = \frac{3}{2} \times 8 - 2$$

$$10 = 10.$$

Thus Left-hand side = Right-hand side.

Therefore, it is a constrained kinematic chain.

Problem 1.8. *Fig. 1.48 shows a type-writer mechanism. Apply the rule of the criterion of constraint and state whether it is kinematically sound.*

Solution. The links have been numbered starting with link 1, the frame as the fixed link. Link 2 has a turning pair with link 3. Link 3 is having a turning pair with link 4. Link 4 is having a turning pair with fixed link 1. Thus, links 1, 2, 3 and 4 form a four-bar mechanism. Similarly links 1, 4, 5 and 6 form another four-bar mechanism. Thus it is a case of two four bar mechanisms in series, O_2, O_4, A, B, D, E and O_6 form 7 binary joints and it has six links.

We have $$J + \frac{1}{2}H = \frac{3}{2}L - 2$$

where $J = 7$, $H = 0$ and $L = 6$

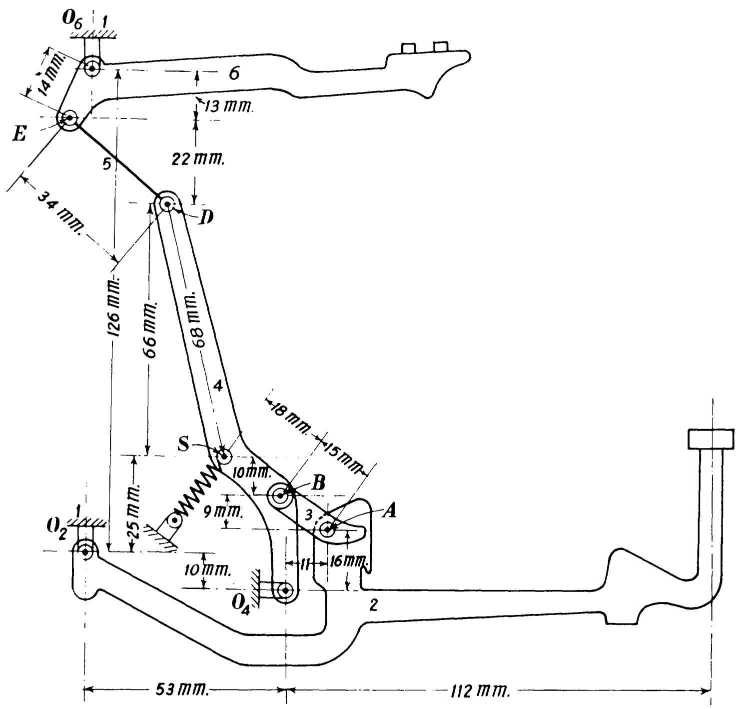


Fig. 1.48. Typewriter Mechanism.

Therefore, $7 = \frac{3}{2} \times 6 - 2$

Hence L.H.S. = R.H.S. **Thus it is kinematically sound.**

Problem 1.8. *Figs. 1.49, 1.50 and 1.51 are to be identified whether they are the mechanism kinematically possible ; and justify the statements.*

Solution. Fig. 1.49 forms two gears rotating in fixed bearings. They are in mesh. There is a lower turning pair between fixed bearing (link 1) and gear P (link 2) ; and also a lower turning pair between fixed bearing (link 1) and gear Q (link 3). There is a higher pair between gears P and Q which is equivalent to two lower pairs and an additional link. Thus it is equivalent to 4 links and 4 lower pairs. Therefore, it is kinematically sound.

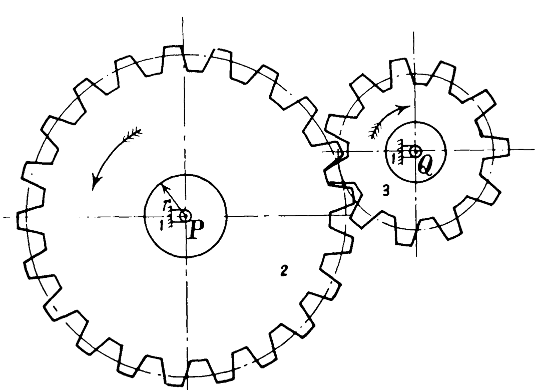


Fig. 1.49

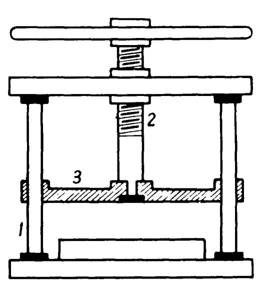


Fig. 1.50

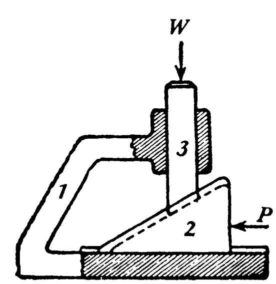


Fig. 1.51

Fig. 1.50 is a hand-press mechanism. There is a lower sliding pair between links 1 and 3, and lower turning pair between links 2 and 3. There is a screw pair between links 1 and 2. Thus by inspection we find that a kinematic chain is possible between one sliding pair, one turning pair and one screw pair. Though screw pair does not fall in the class of high pair, the complex motion, *i.e.* simultaneous turning and sliding is in conformity with the definition of higher pair. Thus this pair serves the purpose of two lower pairs and an additional link. Therefore, it is a sound mechanism.

Fig. 1.51 is also a mechanism which on inspection is kinematically sound. The wedge or the inclined plane works fundamentally on the same principle as the screw opened out and the same reasoning holds. Thus three sliding pairs and three links can form a kinematic chain.

Thus while applying the equation of the criterion of constraint judgement is warranted.

Problem 1.9. *For the crank and rocker mechanism shown in Fig. 1.52, plot the path followed by point R.*

Solution. Draw the four-bar mechanism to a convenient scale. Locate point R for the rotation of crank angle by 30°. Thus the pin joints A and B in different configurations obtained help us to find point R by striking arcs from these points and radii equal to AR and BR on the chosen scale to which the mechanism is drawn. Intersection gives us point R. Repeat the process for 12 such configurations at intervals of 30° of crank rotation. Draw a smooth curve through all such 12 intersection points to locate the path of point R.

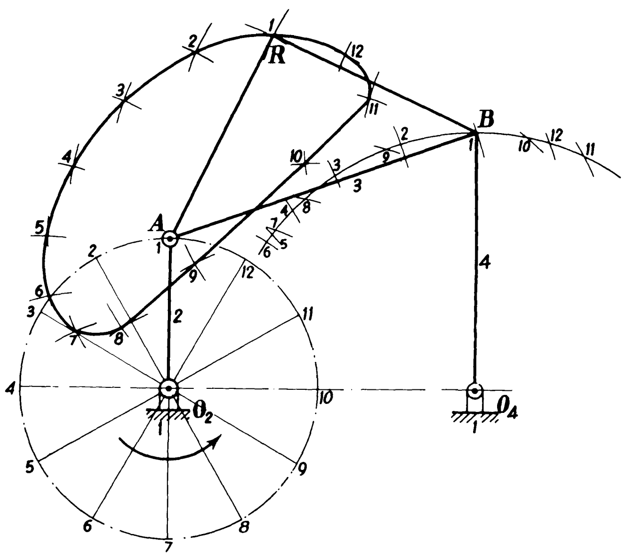


Fig. 1.52

Problem 1.10. *In a four bar chain it is required to give an oscillatory motion to the follower for continuous rotation of the crank. For the length of 55 mm for the crank and 47.5 mm for the follower, determine the theoretical minimum and maximum length for coupler.*

Explain why these theoretical limits have to be less than maximum and more than minimum for the mechanism to have a practical use.

Solution. Theoretical maximum length will be equal to

$$O_1O_2 + O_2B - O_1A = 50 + 47.5 - 35 = \mathbf{62.5\ mm.}$$

But to avoid a dead centre the practical length would be slightly less than 62.5, for example 62 mm as shown in Fig. 1.53.

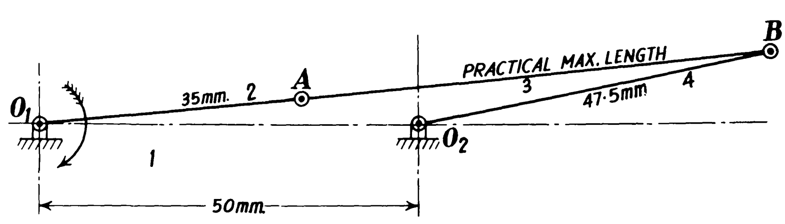


Fig. 1.53

Theoretical minimum length similarly will be equal to

$$O_1O_2 + O_1A - O_2B = 50 + 35 - 47.5 = \mathbf{37.5\ mm.}$$

But to avoid a dead centre, the practical length would be slightly greater than 37.5 mm, for example, 38.5 mm as shown in Fig. 1.54.

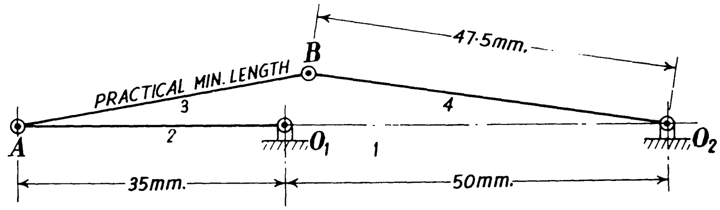


Fig. 1.54

Problem 1.11. *In Fig. 1.55 is shown Watt-Straight Line Mechanism. Show that the position of the path traced by point O is very nearly straight, if*

$$\frac{RO}{QO} = \frac{PQ}{RS}.$$

Solution. Mark a point O on link RQ such that

$$\frac{RO}{QO} = 1$$

because $\qquad \dfrac{PQ}{RS} = 1.$

Find out the path of point O by turning SR at intervals of 15° and locating point O on link RQ in the new configurations thus obtained. The path traced is $CABD$. It is obvious from the figure that portion AB is very nearly a straight line. Thus if this mechanism is used for obtaining straight line mechanism, the portion AB will be the desired working range.

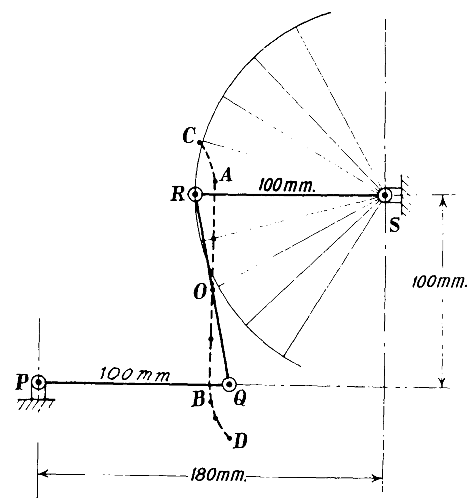


Fig. 1.55. Watt-Straight line motion.

Problem 1.12. *A four-bar mechanism, called film moving mechanism, produces, intermittent motion of a film strip in a movie projector. Crank rotates, causing the coupler end R to move into a film slot, pull the frame down one frame, then leave the slot and up to be in a position to engage another slot.*

(*a*) *Determine the speed of the film in frames per second if the crank rotates at 1080 r.p.m.*

(*b*) *Determine also the ratio of the time during which the film dwells to the time during which the film moves.*

Solution. Path of point R is plotted for 15 equal intervals of 30° rotation. The number of frames per second

$$= \frac{\text{r.p.m.}}{60} = \frac{1080}{60} = 18 \text{ frames/second}$$

Approximately for 5.25 intervals of 30° each the film moves. Therefore, time during which film moves

$$= \frac{5.25}{18 \times 12} = \mathbf{0.0243\ second.}$$

and the time during which film dwells

$$= \frac{6.75}{18 \times 12} = \mathbf{0.03125\ second.}$$

[**Hint.** A template of the link PQR could be prepared from a stiff-paper as shown in the figure. This will be a considerable help in plotting the path of point R.]

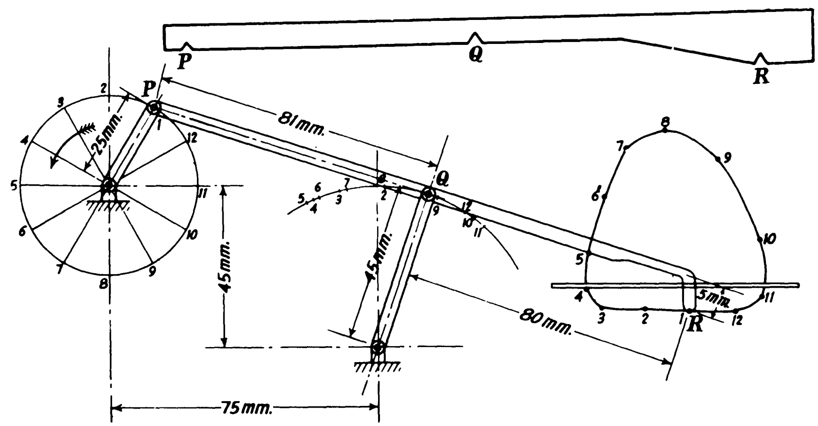


Fig. 1.56. Film moving mechanism.

Problem 1.13. *A slider crank mechanism has a crank length of 6.75 cm and the connecting rod 18.75 cm long. Draw the locus of point on connecting rod 7.5 cm from the crank pin.*

(*a*) *When the slider moves horizontally and the crank shaft bearing axis and the gudgeon pin axis lie in the same horizontal plane.*

(*b*) *When the slider moves horizontally in a plane off set by 3 cm above the horizontal plane through the axis of crank shaft bearing. If the crank rotates with uniform angular velocity in the clockwise direction determine the ratio of the time for the forward stroke to the time for the return stroke in both the cases.*

Solution. Locus of point P which is 7.5 cm from the crank pin B has been drawn in Fig. 1.57 for case (*a*) and in Fig. 1.58 for case (*b*). Construction is quite obvious from the figures.

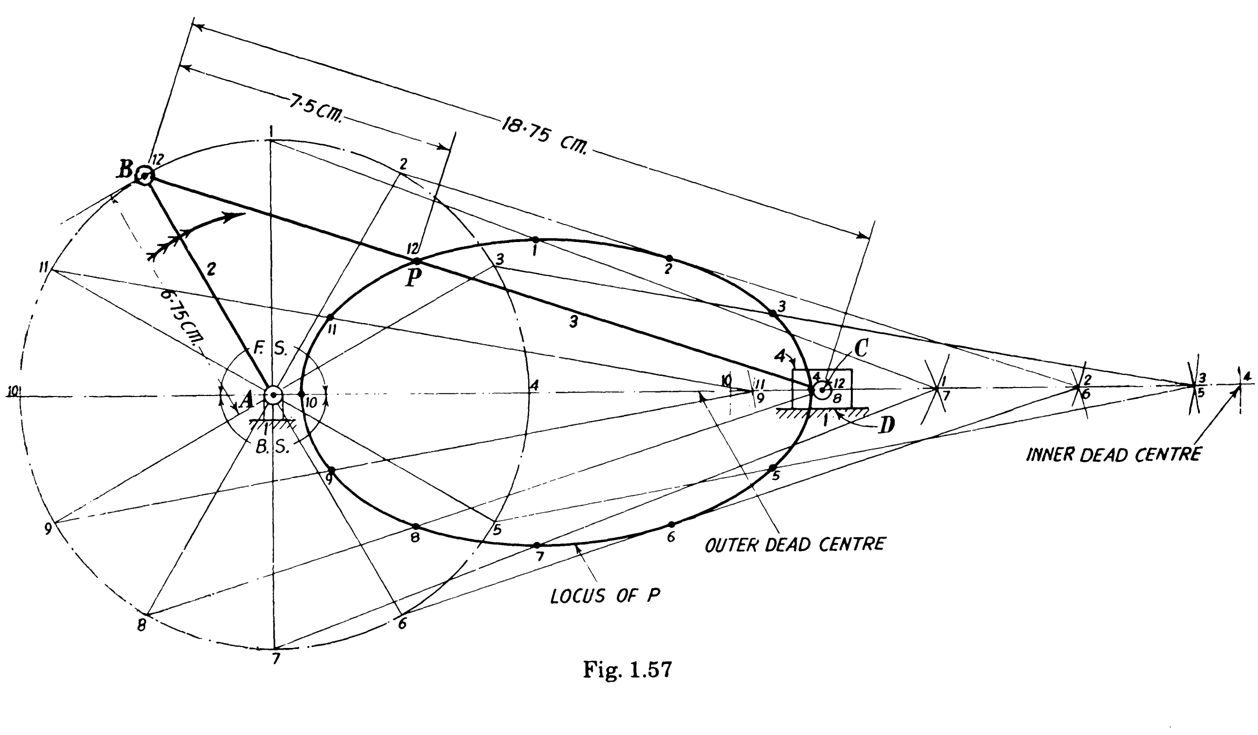


Fig. 1.57

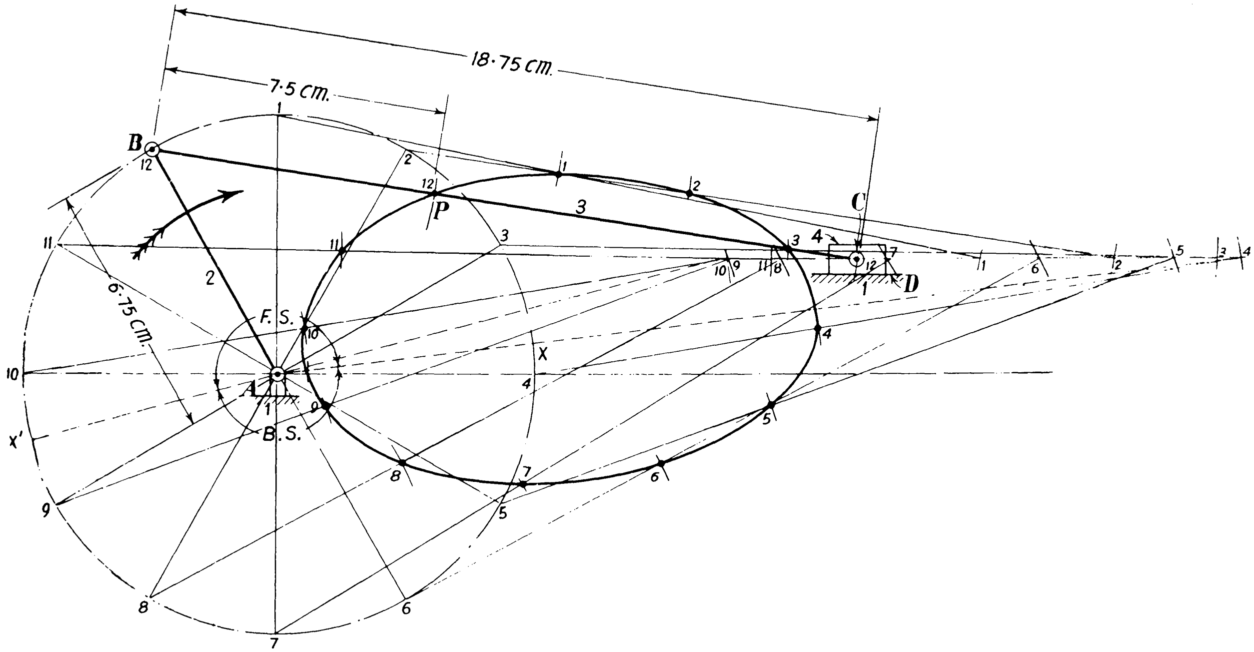


Fig. 1.58

For case (*a*) the ratio of the time of forward stroke to the time for the return stroke is unity.

For case (*b*) the crank pin has reached point X when the crank and connecting rod fall in one line on the forward stroke and that is the inner dead centre position for slider ; again X' is the position occupied by crank pin when crank and connecting rod fall in line and this gives the outer dead centre position for slider. Thus the reflex angle XAX' = 188° gives the angle turned by crank for forward stroke, and obtuse angle XAX' = 172° gives the angle turned by crank for return stroke. Therefore, the ratio of times

$$= \frac{188}{172} = \mathbf{1.039}.$$

Problem 1.14. *In an Oldham's coupling the distance between the driver and the driven shaft is 25 mm. Prove that the centre of the intermediate piece with crossed tongues describes a circle with 25 mm as a diameter. Also prove that the angular velocity of the centre of the cross about the centre of this circle is twice the angular velocity of the cross. Hence find out the linear velocity of the centre of the cross and maximum velocity of sliding of the tongue in the slot. The shaft rotates at 60 r.p.m.*

Solution. (*Refer* Fig. 1.59). The kinematic equivalent of the Oldham's coupling has been drawn to scale full size. The link 2 is fixed as the equivalent of the frame and the bearings. The cross tongue intermediate link is shown as link 4 by a cross having centres at C. The link 2 subtends a right angle at point C. The two sliders, *i.e.* links 1 and 3 are equivalent of the driver shaft and the driven shaft respectively.

$PQRS$ shows the initial position of the cross with centre at C. With AB, *i.e.* length of link 2 or the distance between the shaft as diameter, a circle is drawn. It will pass through point C. The centre of this circle is point O. Take any other points C', C'' and C''' on this circle at an angular interval of 30° each. These points also are the centre of the cross. For point C' the perpendiculars from B on AC', and from A on BC' meet at point C'. Same holds good for points C'' and C'''. Corresponding configuration of the cross is given by $P'Q'R'S'$ for centre at C',

$P''Q''R''S''$ for centre at C'' and $P'''Q'''R'''S'''$ for centre at C'''. It will be seen that for 30° angular displacement of the C about centre O, the angular displacement of the cross is only 15°. Also angular displacements of the sliders about centres A and B are only 15°. Thus the two shafts and the intermediate cross tongue disc, all have the same angular displacement of 15°. This is in conformity with the Principle of Oldham's coupling. But it is obvious that the centre of the disc has an angular displacement of 30° about this centre O of this circle. Thus the centre of the cross tongue piece moves about the centre of this circle with angular velocity twice that of the cross tongue piece.

Also OC, OC', OC'' and OC''' are radii of this circle with AB as its diameter. Therefore, if ω is the angular velocity of the shaft, 2ω is the angular velocity of centre C about O. Thus, the linear velocity of C is given by

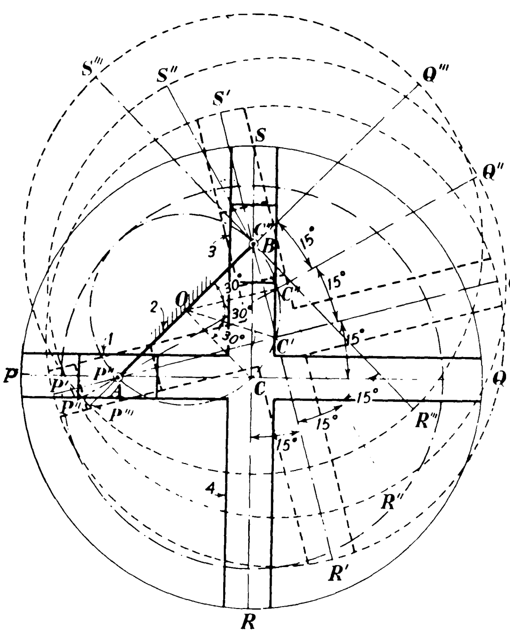


Fig. 1.59. Oldham's Coupling.

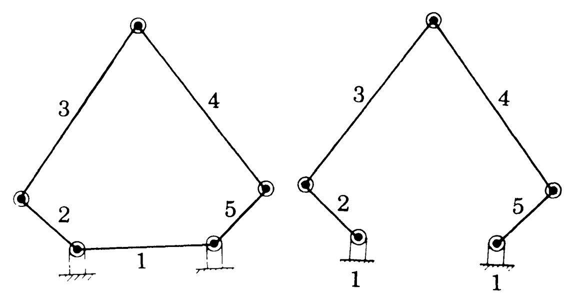
$$v_c = 2\omega.OC = 2\omega \, . \, \frac{AB}{2} = \omega \, . \, AB$$

$$= \frac{2\pi \times 60}{60} \times 25 = 157 \text{ mm/sec.}$$

In the configuration $P'''Q'''R'''S'''$ when centre of the cross C''' coincides with point B, this velocity v_c is also the maximum velocity of sliding of tongue in the slot, and is

$= 157$ mm/sec. $=$ **0.157 m/sec.**

1.24. Linkage of more than Four bars with Constraint. Refer Fig. 1.60. It has been seen that the simplest one degree freedom linkage is four bar mechanism. It is extremely versatile and useful device. Many quite complex motion control problems can be solved with just four bars and four pins. Adding one link and one joint to form a five bar mechanism as shown in Fig. 1.60 (*a*) will not work with one degree of freedom. However increasing degree of freedom from one to two, the linkage works satisfactorily. Topic on degrees of freedom is discussed in details later. It is interesting to note that by adding a pair of gears to tie two links together, the degree of freedom again reduced to one and the geared five bar mechanism with one degree of freedom is created. The geared five bar mechanism provides more complex motions than the four bar mechanism at the expense of the added link.



(*a*) Five bar linkage. (*b*) Geared five bar linkage.

Fig. 1.60

1.25. Six-Bar Chains. In case a four-bar chain does not provide the required performance of an application, one of the two single-degree of freedom six bar chain with seven turning or revolute pairs is considered. There are two types of Six-Bar Chains (*i*) the Watt Chain and (*i*) the Stephenson Chain.

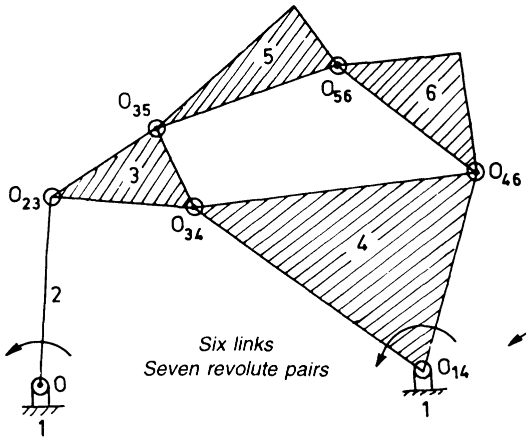


Fig. 1.61. Watt. I Six bar mechanism.

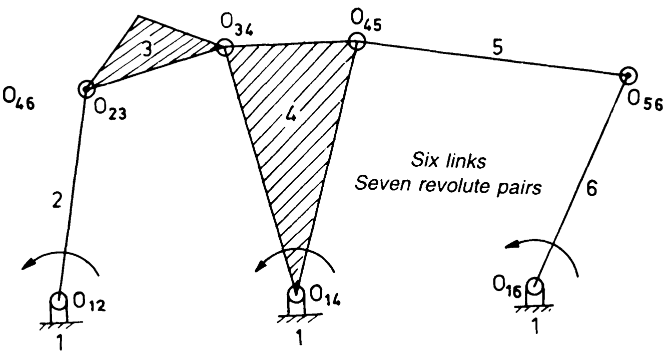


Fig. 1.62. Watt. II Six bar mechanism.

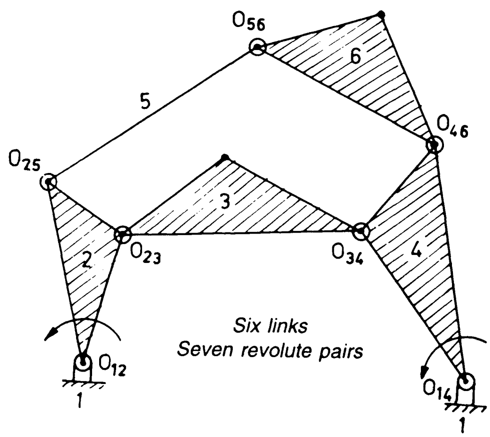


Fig. 1.63. Stephenson I Six bar mechanism.

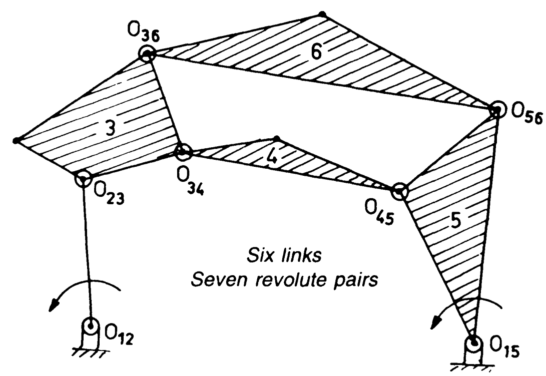


Fig. 1.64. Stephenson II Six bar mechanism.

(*i*) In Watt chain, the ternary links are adjacent. Refer Fig. 1.61 for Watt I six bar mechanism, and Fig. 1.62 for Watt II six bar mechanism.

(*ii*) In Stephenson chain, the ternary links are separated by binary links. Refer Fig. 1.63 for Stephenson I six bar mechanisms. Fig. 1.64 for Stephenson II six bar mechanism and Fig. 1.65 for Stephenson III six bar mechanism. It may be noted that in both these types of mechanisms, some triangular shaped links are truly ternary links while others are shown as triangular to indicate the possible path of tracer points on floating links.

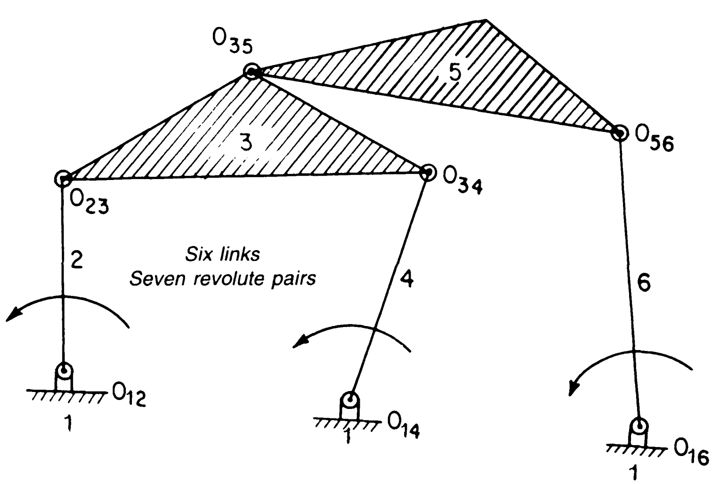


Fig. 1.65. Stephenson III Six bar mechanism.

Problem 1.15. *A six bar lift mechanisms is shown in Fig. 1.66. Indicate the task performed by it and the type of six bar mechanism.*

Solution. Refer Fig. 1.66. The same figure is drawn for another configuration shown in dotted lines. The mechanism can be identified as Stephenson I mechanism with six links and seven revolute pairs and also links 2 and 4 are ternary links separated by binary link 3. The motion of entire link 6 is of interest. Its task is motion generator.

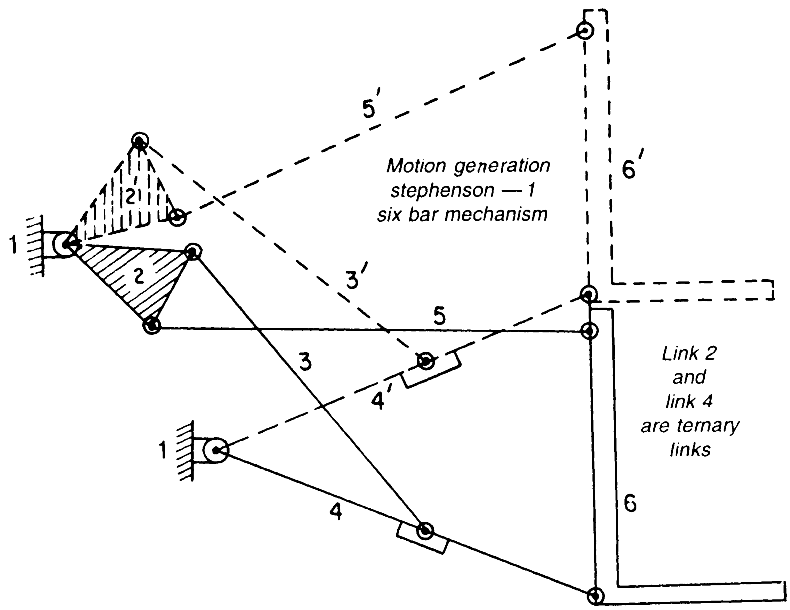


Fig. 1.66

1.26. Degrees of Freedom or Criteria of Constraint or Mobility of Mechanism. By degrees of freedom, it is meant the number of inputs needed to determine the configuration or position of all the links of the mechanism under study with respect to the fixed link. The study is restricted here to only planer motion in parallel planes. The connections that allow three dimensional relative motion are not considered. Refer Fig. 1.67. The location of point A of link P is determined by co-ordinates x_A and y_A or two translations x_A and y_A locate point A. The link P can be completely defined in plane XY if angle of line AB with respect to X axis is known. Thus three independent variables x_A, y_A θ *i.e.* two translations and one rotation are necessary to completely locate link P in plane. Thus an unconstrained rigid link in the plane has *three degrees of freedom*.

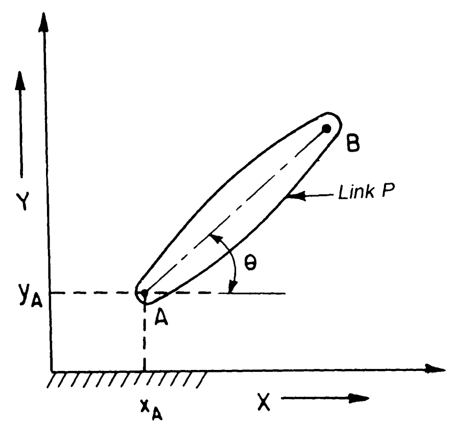


Fig. 1.67

Thus, if there is an assembly of n links, there would be a total of $3n$ degrees of freedom before these are joined together in the form of a linkage. And each pin joint or revolute pair will remove two degrees of freedom. For example if point A is a pin joint between link P and ground, the two independent variables x_A are y_A are fixed leaving one θ as the one remaining degree of freedom for link P.

Thus, if n = number of links of a mechanism with fixed link *i.e.* ground

f_1 = number of pin joints or revolute pairs or pairs that permit one degree of freedom.

Degrees of freedom of Mechanism = $F = 3(n-1) - 2f_1$ ...(1.8)

or F gives the number of input motion for mechanism to be driven. For example for $F = 1$ the mechanism can be driven by single input motion.

If $F = 2$, then two separate input motions are necessary to produce constrained motion for mechanism.

Equation (1.8) is called *Gruebler's Equation* or Kutzback criterion of constraint.

where $(n - 1)$ = number of moving links or mobile links.

If Kutzbach criterion yields $F = 0$ motion is impossible and the mechanism forms structure.

If $F = -1$ or less, then there are redundant constraints and it forms a statically in determinate structure. However in this textbook only cases of $F > 0$ and planar motions are considered.

Also, it may be observed that the slider link as link 4 in Fig. 1.1 is constrained with respect to the fixed link (ground) against moving in vertical direction as well as being constrained from rotating in plane. Thus slider joint also subtracts two degrees of freedom of relative motion, one translation and one rotation.

Thus f_1 = sum of the number of pin joints plus the number of slider joints.

Let us now formulate a relationship for degrees of freedom with higher pairs and other type of pairs.

Pin (revolute)

Fig. 1.68.

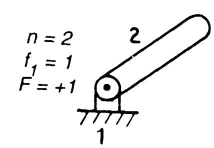


Fig. 1.68

Slider (Prismatic)

Fig. 1.69 (*a*) shows a prismatic slider only sliding constrained not to rotate. Fig. 1.69 (*b*) shows that it is kinematic link pivoted to fixed link or having a revolute pair with fixed link at radius of infinity. It is a lower pair with equivalent instantaneous velocity.

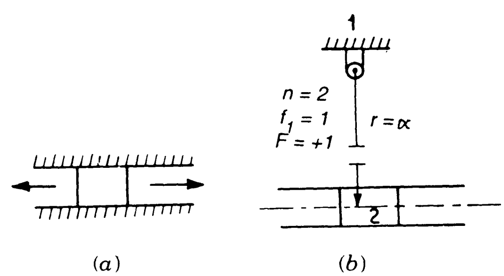


Fig. 1.69

Rolling Contact (no sliding). Refer Fig. 1.70 (*a*). It allows only one degree of freedom of relative motion due to absence of sliding. It only has relative motion θ. Thus it can be treated as f_1 type of pair. Fig. 1.70 (*b*) shows the lower pair equivalent and the equivalent instantaneous centre of the two links at the contact point.

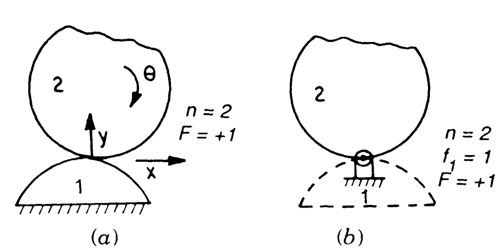


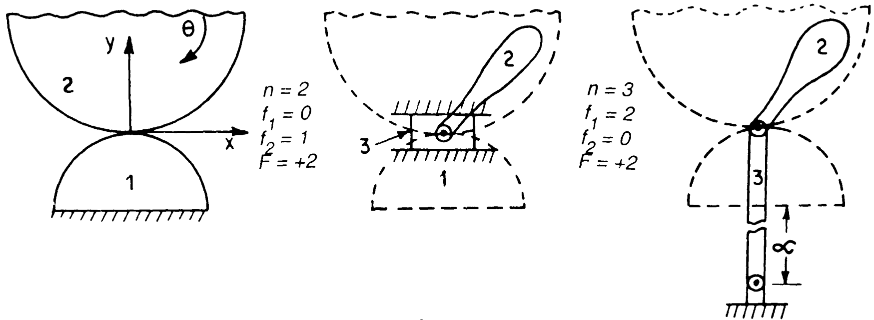
Fig. 1.70

Rolling contact with sliding. It constrains only one degree of freedom in y direction as shown in Fig. 1.71 (*a*). The lower pair combination with equivalent instantaneous velocity is shown in Fig. 1.71 (*b*) which is a slider and revolute pair combination. This gives two degrees of freedom of relative motion.

Thus Gruebler's Equation may be modified as

$$F = 3\,(n - 1) - 2\,f_1 - 1\,f_2 \qquad \text{...(1.9)}$$

where f_2 = number of roll-slide pairs (these permit *two degrees* of freedom of relative motion across the pair). Thus $F = 3\,(2-1) - 1 = +\,2$.



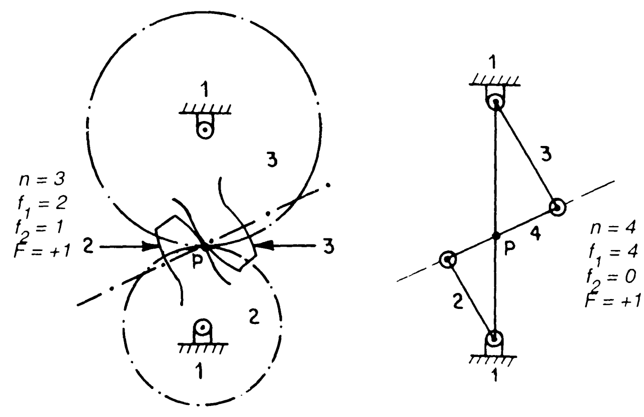
(*a*) (*b*) (*c*)

Fig. 1.71

Gear Contact Pair. Refer Fig. 1.72 (*a*) and (*b*). The bearings of the gears are revolute pairs and the tooth contact is roll-slide contact pair. Thus $f_1 = 2$ and $f_2 = 1$.

Thus $F = 3\,(3-1) - 2\,(2) - 1 = +\,1$.

The lower-pair linkage for instantaneous equivalence is a four-bar with fixed revolute pairs at the centre of the gears, and the moving revolute pairs at the centre of the curvature of tooth profiles is shown in Fig. 1.72 (*b*). The coupler of the four bar passes through the pitch point *P* along the line of action of gear mesh, perpendicular to the common tangent of the contacting tooth flank surfaces and to the two grounded links. Thus lower pair model of gear set will give the same degree of freedom



(*a*) (*b*)

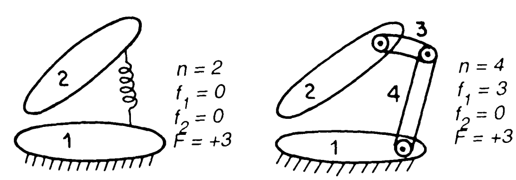
Fig. 1.72

$$F = 3\,(4-1) - 2\,(4) = +\,1.$$

Spring Connection. Refer Fig. 1.73 (*a*) for a spring connection. It does not kinematically constrain the relative motion between two links. The lower pair instantaneous velocity equivalent model is shown in Fig. 1.73 (*b*). There are two binary links and two lower revolute pairs. Thus

$$F = 3\,(4-1) - 2\,(3) = +\,3.$$

Thus there are three degrees of freedom for spring connection.



(*a*) (*b*)

Fig. 1.73

Belt and pulley (no sliding) or chain and Sprocket. Refer Fig. 1.74 (*a*) and (*b*).

A ternery link with three pin joints (making three revolute pairs) is the instantaneous-velocity-equivalent lower pair connection to the belt and pulley. This give an equivalent six link mechanism. Thus

$$F = 3\,(6-1) - 2\,(7) - 1\,(0) = +\,1.$$

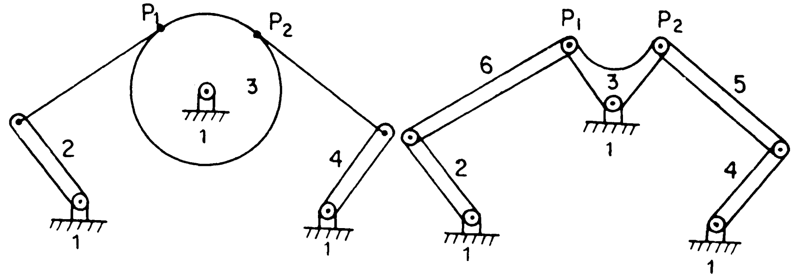


Fig. 1.74

Note. An earlier mobility criterion named after Grübler applies to mechanism with only one-degree of freedom-joints, where the over all mobility of the mechanism is unity. Putting $F_2 = 0$ and $F = 1$ in the equation, we find Grüblers criterion for plane mechanism with constrained motion :

$$3n - 2f_1 - 4 = 0$$

And this is the same equation as earlier derived *i.e.* equation (1.2*b*).

Problem 1.16. *Determine the degrees of freedom of the mechanism shown in Fig. 1.75 (a).*

Solution. There are 7 links, 7 lower pairs, 1 roll-slide pair and 1 spring From Gruebler's Equation,

$$F = 3\,(n-1) - 2\,(f_1) - 1\,(f_2) = 3\,(7-1) - 2\,(7) - 1\,(1) = +\,3$$

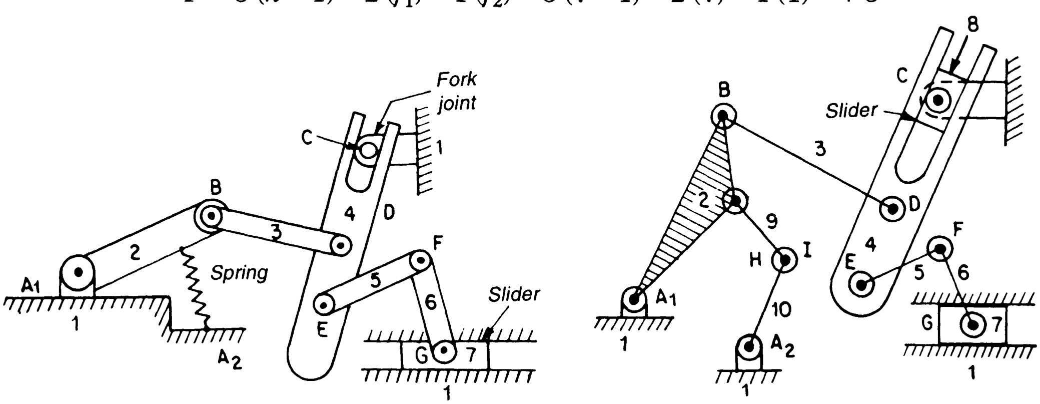


Fig. 1.75

This can be confirmed by converting this linkage into velocity equivalent lower pair connection as shown in Fig. 1.75 (*b*). The spring is substituted by two binary links ; fork joint or pin-in-slot joint or roll-slide contact pair is replaced by a pin and a slider. Thus, there are 10 links, and 12 lower pairs in this arrangement.

Thus, $F = 3\,(10-1) - 2\,(12) = +\,3$

Over Constrained Mechanism. Refer Fig. 1.76. There are 5 links and 6 lower pairs and Gruebler's Equation gives

$$F = 3\,(5-1) - 2\,(6) = 0$$

But from intution, it is definitely not a structure as indicated by zero degree of freedom. Due to parallelogram configuration

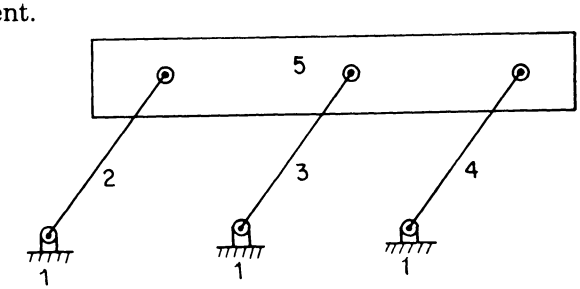


Fig. 1.76

the mechanism can move. This is a case of *over-constrained* mechanism and link 3 provides *redundent* constraint.

Redundent Degree of Freedom. Refer Fig. 1.77 for cam and follower mechanism. In this case the rotation of follower roller 4 does not affect the motion of output oscillating link 3. But if we apply Gruebler's Equation and consider the roller motion as rollslide, we get

$$F = 3\,(4-1) - 2\,(3) - 1\,(1) = +\,2$$

Again, if the roller is welded to link 3, the pair becomes roll-slide and motion of arm 3 is unaffected and links are reduced by one. Thus Gruebler's Equation gives

$$F = 3\,(3-1) - 2\,(2) - 1\,(1) = +\,1$$

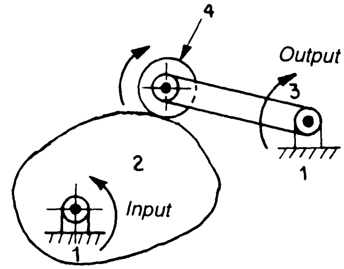


Fig. 1.77

Alternatively if roller is made to only roll and slipping is prevented, we get

$$F = 3\,(4-1) - 2\,(4) = +\,1.$$

Thus slipping is the redundent freedom between cam and follower.

Problem 1.17. *A kinematic mechanism is to be designed for two degrees of freedom so that two inputs could be incorporated to determine the relative position of all links. Assume all the pairs that allow one relative degree of freedom. Determine :*

(*a*) *the minimum number of binary links for the mechanism.*

(*b*) *if the minimum number mechanism is not workable what is the fewest number in the next complicated mechanism.*

Solution. (*a*) **By Gruebler's** Equation $F = 3\,(n-1) - 2f_1$

And for binary links $f_1 = n$

Therefore, $F = 3\,(n-1) - 2n = 2$ or $n = 5$.

(*b*) if mechanism with minimum number of binary links does not work, add a group of links with zero degree of freedom. Thus $F = 3n - 2f_1 = 0$, then $n = \frac{2}{3} f_1$. But n and f_1 have to be integers. Therefore $n_{min} = 2$ and $f_{1_{(min)}} = 3$. Thus total of 5 + 2 = 7 links are the fewest number in the next complicated linkage. It may be concluded that next higher would have 9 links.

Problem 1.18. *Determine if it is possible to have two degree freedom mechanism with 10 links and all lower pairs of one degree freedom.* (*a*) *What is the nearest number of links and pairs possible,* (*b*) *Determine the maximum number of elements that can occur on one link,* (*c*) *determine the number of links which can have the maximum number of elements.*

Solution. With all lower pairs of one degree freedom, Gruebler's equation is given by $F = 3\,(n-1) - 2f_1$

or $$f_1 = \tfrac{1}{2}\,[3\,(n-1) - F] = \tfrac{1}{2}\,[3\,(10-1) - 2] = 12\tfrac{1}{2}$$

And it is *Not Possible* being fraction.

(*a*) Nearest number of links possible is **n = 9** and then $f_1 = \frac{1}{2}\,[3\,(9-1) - 2\,] = 11$.

(*b*) $n - f_1 = 11 - 9 = 2$ and original number of elements of binary link = 2. Therefore maximum number = 2 + 2 = 4.

(*c*) 11 pairs have 22 elements. If n_2 is the number of links with 2 elements and n_4 is the number of links with four elements, we have

$$n_2 + n_4 = 9 \qquad \text{...}(i)$$

and $$2n_2 + 4n_4 = 22 \qquad \text{...}(ii)$$

or $$4n_4 + 2\,(9 - n_4) = 22 \quad \text{or} \quad 2n_4 = 4 \text{ i.e. } \mathbf{n_4} = 2$$

Problem 1.19. *For the mechanism shown in Fig 1.78 determine the number of inputs required so that the mechanism works kinematically sound.*

Solution. Refer Fig. 1.78. Total number of links are 14. The diagram is numbered for links.

1. Fixed link or ground 2. slider A, 3. AB 4. BCE, 5. CD 6. slider D, 7. DF 8. GEF, 9. GI, 10. GH, 11. HJ, 12. JI, 13. KI, 14. JK. Thus total number of links = 14 or n = 14.

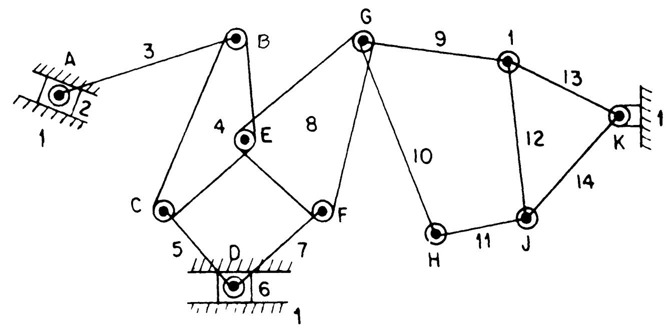


Fig. 1.78

Total number of lower pairs : 1—2, 2—3, 3—4, 4—5, 5—6, 6—1, 6—7, 7—8, 4—8, 8—9, 8—10, 10—11, 11—12, 11—14, 9—12, 9—13, 13—1, 14—1.

Thus total lower pairs = 18 or f_1 = 18.

Thus $F = 3(n-1) - 2(f_1) = 3(14-1) - 2(18)$

$= +3$ or **Three degrees Freedom or Three inputs required**

Also it may be observed that 12, 13 and 14 form rigid triangle and can be treated as one link. In that case n = 12.

Also lower pairs 9—13, 11—14, 13—1 and 14—1 will not exist but will be replaced by one lower pair 12—1. Thus total pairs will be 15 or f_1 = 15.

Thus $F = 3(n-1) - 2(f_1) = 3(12-1) - 2(15) = +$ **3**, confirmed.

Discussion. *IJK* is a rigid triangle and can rotate about the pivot point *K* as one kinematic link. Also *GHJI* is a four bar kinematic chain or linkage with only one degree of freedom. Thus two degrees of freedom have been established. Again, if we treat *IJK* and *GHJI* as fixed and the slide *D* is given motion, *DFG, DFE, DCE* and then *ABC* form four rigid triangles. Therefore total is three degrees of freedom. (confirmed)

Problem 1.20. *For the mechanism shown in Fig. 1.79, determine the number of inputs required so that the mechanism works satisfactory kinematically.*

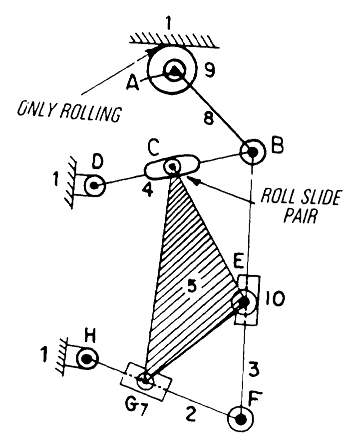


Fig. 1.79

Solution. Refer Fig. 1.79. We have n = 9 and f_1 = 11, *i.e.*, (1—2, 2—3, 3—4, 4—1, 5—10, 10—3, 5—7, 7— 2, 4—8, 8—9 and 9—1)

Also f_2 = 1 *i.e.* (5—4 Roll-slide pair)

Thus, $F = 3(n-1) - 2(f_1) - 1(f_2) = 3(9-1) - 2(11) - 1(1)$

$= 24 - 22 - 1 = +$ **1**

Thus only one input is required as the degree of freedom is one.

Discussion. It may be observed that *DBFH* is a four bar mechanism with only one degree of freedom. And when the configuration of the four bar mechanism *DBFH* is known, the positions of the rest of the mechanism are fully known. Thus it has only one degree of freedom.

Problem 1.21. *Examine the mechanism shown in Fig. 1.80 and indicate the cases where unique relation between the motions of the input link and the output link exists. Give reasons for the answers.*

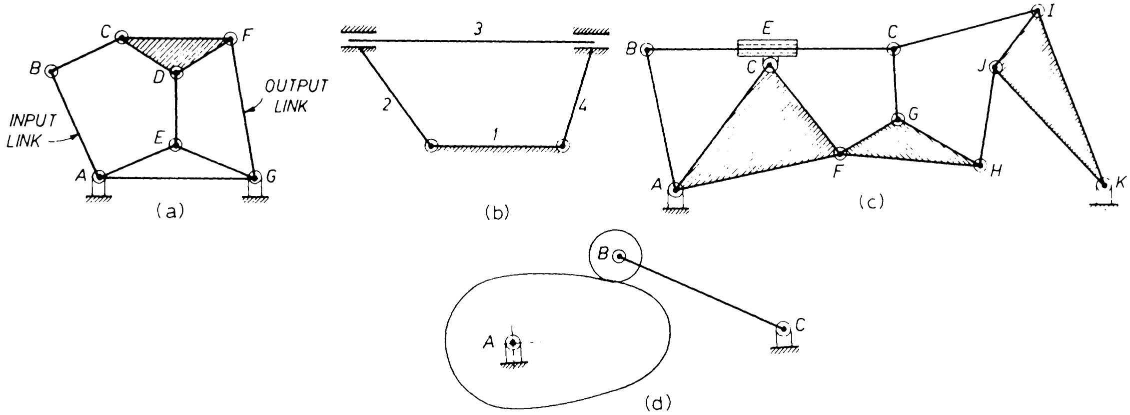


Fig. 1.80

Solution. Mechanism at Fig. 1.80 (*a*) is Stephenson's type. It satisfied the Grubler's criterion. This is obtained by putting F (degrees of freedom) equal to 1 in general equation for planar mechanisms.

$$F = 3(n - 1) - 2f_1 - 1f_2$$

where n = number of links and f_1 = number of lower pairs of degree of freedom = 1 and f_2 = number of higher pairs or roll-side pairs permitting two degrees of freedom.

With $n = 6, f_1 = 7$

$$\mathbf{F = 3(6 - 1) - 2(7) = 1.}$$

Mechanism at Fig. 1.80 (*b*). It represents a locked system. Though the mechanism has 4 links and 4 lower pairs, but link 3 is capable sliding without motion of the remaining links of the mechanism. The link 3 has redundant degree of freedom.

Mechanism at Fig. 1.80 (*c*). It does not have unique relation between input and output motions as the degree of freedom is 3. Also note that joint at A is a double joint connecting three links and is equivalent to two simple pairs.

$$n = 10, f_1 = 10 + 2 = 12$$

Therefore, $F = 3(n - 1) - 2f_1 = 3(10 - 1) - 2(12) = 3.$

Mechanisms at Fig. 1.80 (*d*). It has a roller carried at the end of output link which can be rotated without causing any motion in the remaining linkage. Thus, roller is a link with redundant degree of freedom. The roller can therefore be treated as fixed output link. Thus the mechanism has three links with two lower turning pairs and one higher pair

$$F = 3(n - 1) - 2f_1 - 1f_2 = 3(3 - 1) - 2(2) - 1 = 1.$$

Thus, there is unique relationship between input and output link.

TEST YOUR COMPREHENSION

(Tick Mark Correct Statements)

1. Kinematic link or element is
   (*a*) Any part of the machine (*b*) Only the fixed part of the machine
   (*c*) Any resistant body or assembly of resistant bodies which go to make part of a machine connecting other parts which have a motion relative to it.
   (*d*) None of the above.

2. In a reciprocating engine
   (a) Piston, gudgeon pin form two kinematic links
   (b) Piston, gudgeon pin form one kinematic link
   (c) Piston, gudgeon pin and connecting rod form one kinematic link
   (d) None of the above statements is true.
3. In a reciprocating engine
   (a) Crank shaft and flywheel form two kinematic links
   (b) Crank shaft and flywheel form one kinematic link
   (c) Crank shaft and flywheel do not form kinematic link
   (d) Flywheel and crank shaft separately form kinematic links.
4. Joint of two elements that permits relative motion which is completely constrained or successfully constrained is called
   (a) Mechanism (b) Machine (c) Structure (d) Kinematic pair.
5. Kinematic pairs are classified on the basis of
   (a) Nature of relative motion between the elements
   (b) Nature of contact between the elements
   (c) Nature of mechanical arrangement for constraint between elements
   (d) All of the above.
6. Classification of kinematic pairs based on relative motion between elements may give
   (a) Sliding pair (b) Turning pair (c) Rolling pair (d) All of the above.
7. A ball and socket joint forms
   (a) A turning pair (b) Rolling pair (c) Spherical pair (d) Sliding pair.
8. A bolt and nut forms
   (a) Turning pair (b) Rolling pair (c) Screw pair (d) Sliding pair.
9. In the kinematic pair, if the elements have surface contact when in motion, the pair is called
   (a) Higher pair (b) Lower pair (c) Closed pair (d) Unclosed pair.
10. In a kinematic pair, if the elements have line contact or point contact when in motion, the pair is called
    (a) Higher pair (b) Lower pair (c) Closed pair (d) Unclosed pair.
11. Various kinematic pairs are given below. Choose the lower pair
    (a) Ball bearings (b) Tooth gears in mesh
    (c) Cam and follower (d) Crank shaft and bearing.
12. Various kinematic pairs are given. Choose the higher pair
    (a) Roller bearing (b) Tooth gears in mesh
    (c) Cam and follower (d) All of the above
13. Choose the correct statement
    (a) Tooth gears in mesh constitute a higher kinematic pair
    (b) Belt on pulley drive constitute a higher kinematic pair
    (c) Chain and sprocket drive constitute a higher kinematic pair
    (d) All of the above.
14. The motion of a rotating shaft in a foot step bearing constitutes between the elements of kinematic pair
    (a) Successfully constrained motion (b) Completely constrained motion
    (c) Incompletely constrained motion (d) None of the above. It is not a kinematic pair.
15. The motion of a circular shaft with collars at each end rotating in a round hole constitutes between the elements of kinematic pair
    (a) Successfully constrained motion (b) Completely constrained motion
    (c) Incompletely constrained motion (d) None of the above. It is not a kinematic pair.

16. The motion of a circular shaft in a circular hole constitutes between the elements of kinematic pair
(*a*) Successfully constrained motion (*b*) Completely constrained motion
(*c*) Incompletely constrained motion (*d*) None of the above.

17. Kinematic chain
(*a*) Comprises a chain of links in space with constrained motion
(*b*) Comprises a chain of links with at least one link fixed and completely constrained motion
(*c*) Comprises chain of links with at least one link fixed and successfully constrained motion
(*d*) Comprises chain of links with incompletely constrained motion.

18. If J = number of binary joints in the kinematic chain
H = number of higher kinematic pairs
L = number of links in the kinematic links.
The equation for criterion of constraint is given by
(*a*) $J + \frac{H}{2} = \frac{3}{2}L - 2$ (*b*) $J + H = \frac{3}{2}L - 1$ (*c*) $J + H = \frac{3}{2}(L - 1)$ (*d*) $J + H = \frac{3}{2}(L + 1)$.

19. The equation for criterion of constraint for kinematic chain having plane motion is given by
$$2J + H = 3L - 4$$
If L.H.S. of equation is less than R.H.S.
(*a*) Chain is locked (*b*) Chain is completed constrained
(*c*) Chain is successfully constrained (*d*) Chain is incompletely constrained.

20. Same as 19, if L.H.S. of equation is greater then R.H.S.
(*a*) Chain is locked (*b*) Chain is completed constrained
(*c*) Chain is successfully constrained (*d*) Chain is incompletely constrained.

21. In a kinematic chain, a ternary joint is equivalent of
(*a*) One binary joint (*b*) Two binary joints
(*c*) Three binary joints (*d*) None of the above.

22. In a Kinematic chain, a quarternary joint is equivalent of
(*a*) Two binary joints (*b*) Three binary joints
(*c*) Four binary joints (*d*) None of the above.

23. If N kinematic links are connected at same joint in a kinematic chain, the joint is equivalent of
(*a*) $(N - 1)$ binary joints (*b*) $(N - 2)$ binary joints
(*c*) $(2N - 3)$ binary joints (*d*) $(2N - 1)$ binary joints.

24. Refer Fig. T-1. It shows
(*a*) Locked chain
(*b*) Constrained kinematic chain
(*c*) Unconstrained kinematic chain
(*d*) Successfully constrained kinematic chain.

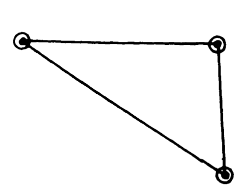


Fig. T-1

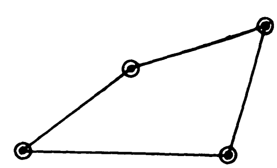


Fig. T-2

25. Refer Fig. T-2. It shows
(*a*) Locked chain
(*b*) Constrained kinematic chain
(*c*) Unconstrained kinematic chain
(*d*) Successfully constrained kinematic chain.

26. Refer Fig. T-3. It shows
(*a*) Locked chain
(*b*) Constrained kinematic chain
(*c*) Unconstrained kinematic chain
(*d*) Successfully constrained kinematic chain.

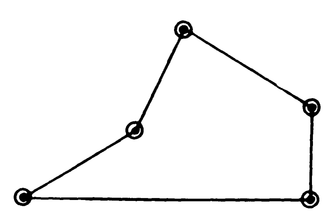


Fig. T-3

27. Choose the correct statement

(*a*) Mechanism transmits and modifies motion

(*b*) Mechanism is the skeleton outline of machine to produce definite motion between various links

(*c*) Machine modifies mechanical work

(*d*) All of the above.

28. Inversions are

(*a*) Different mechanisms obtained by fixing different links in a kinematic chain with the object of changing relative motions of links with respect to one another

(*b*) Different mechanisms obtained by fixing different links in a kinematic chain but keeping relative motions of links unchanged with respect to one another

(*c*) Different mechanisms obtained by fixing different links in kinematic chain to modify the mechanical advantage

(*d*) None of the above.

29. The kinematic chain having N links will have

(*a*) $(N - 1)$ inversions

(*b*) N inversions

(*c*) $(N - 2)$ inversions

(*d*) $(N - 3)$ inversions.

30. The necessary condition for Drag Link Quick Return mechanism is that

(*a*) Shortest link is fixed link. Sum of the shortest link and the longest link is less than the sum of other two links.

(*b*) Longest link is a fixed link. Sum of the shortest link and the longest link is greater than the sum of other two links.

(*c*) Shortest link is fixed link. Sum of the shortest link and the longest link is greater than the sum of other two links.

(*d*) Longest link is fixed link. Sum of the shortest link and the longest link is less than the sum of other two links.

31. Task performed by a four bar chain by different proportioning of links could be

(*a*) A path generation

(*b*) A motion generation

(*c*) A function generation

(*d*) Any one or all of the above.

32. Watt I type of Mechanism has

(*a*) Six kinematic links and six revolute kinematic pairs

(*b*) Six kinematic links and five revolute kinematic pairs

(*c*) Six kinematic links and seven revolute kinematic pairs

(*d*) Six kinematic links and eight revolute kinematic pairs.

33. Wall II mechanism has

(*a*) One degree of freedom and six kinematic links

(*b*) Two degrees of freedom and six kinematic links

(*c*) One degree of freedom and six kinematic revolute pairs

(*d*) Two degrees of freedom and seven kinematic revolute pairs.

34. Watt I mechanism

(*a*) The ternary kinematic links are adjacent

(*b*) The ternary kinematic links are separated by binary link

(*c*) There is no ternary link at all

(*d*) There is only one ternary link

35. Watt II and Stephenson II kinematic chains have

(*a*) One degree of freedom and two degrees of freedom respectively

(*b*) One degree of freedom and one degree of freedom respectively

(*c*) Seven revolute pairs and six revolute pairs respectively

(*d*) Six revolute pairs and seven revolute pairs respectively.

36. Unconstrained rigid link in a plane has

(*a*) One degree of freedom (*b*) Two degrees of freedom

(*c*) Three degrees of freedom (*d*) Zero degree of freedom.

37. Gruebler's Equation to determine degrees of freedom is written as :

(n = number of links, f_1 = number of lower pairs, f_2 = number of roll-slide pairs F = degree of freedom)

(*a*) $F = 3(n-1) - 2(f_1) - 1(f_2)$ (*b*) $F = 3(n-1) - 2(f_1) - 2(f_2)$

(*c*) $F = 3(n-1) - 1(f_1) - 2(f_2)$ (*d*) $F = 2(n-1) - 1(f_1) - 2(f_2)$

38. The Gruebler's Equation for two spur gears of involute tooth profile in mesh can be written as $F = 3(n-1) - 2(f_1) - 1(f_2)$ where

(*a*) $n = 3, f_1 = 2$ and $f_2 = 1$ (*b*) $n = 4, f_1 = 4$ and $f_2 = 0$

(*c*) $F = +1, n = 4$ and $f_1 = 4$ (*d*) All of the above

39. The locking toggle pliers shown in Fig. T-4 is

(*a*) An adjustible four-bar linkage (*b*) A mechanism with the task of Function Generator

(*c*) A mechanism used for force multiplication (*d*) All of the above.

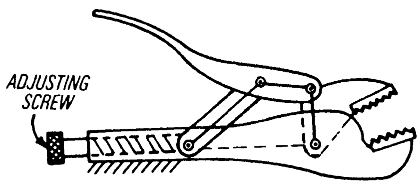


Fig. T-4

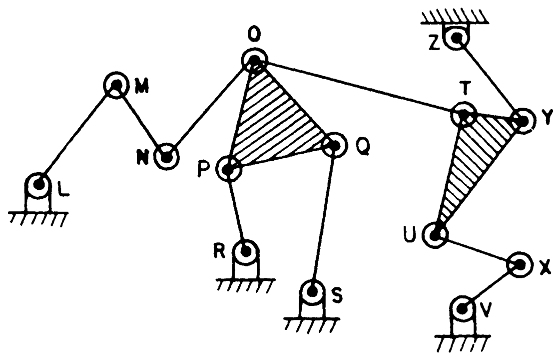


Fig. T-5

40. The mechanism shown in Fig. T-5 has

(*a*) One degree of freedom (*b*) Two degrees of freedom

(*c*) Three degrees of freedom (*d*) Four degrees of freedom.

41. The mechanism shown in Fig. T-5 has

(*a*) $n = 12, f_1 = 15, f_2 = 1$ (*b*) $n = 12, f_1 = 15, f_2 = 0$

(*c*) $n = 14, f_1 = 17, f_2 = 0$ (*d*) $n = 12, f_1 = 14, f_2 = 0$.

42. The mechanism shown in Fig. T-5 has

(*a*) LMNO as four bar linkage with one degree freedom

(*b*) RPQS as four bar linkage with one degree freedom

(*c*) OTYZUXV as Stephenson III linkage with one degree freedom

(*d*) All the above as true.

Answers

1. (*c*)	**2.** (*b*)	**3.** (*b*)	**4.** (*d*)	**5.** (*d*)
6. (*d*)	**7.** (*c*)	**8.** (*c*)	**9.** (*b*)	**10.** (*a*)
11. (*d*)	**12.** (*d*)	**13.** (*d*)	**14.** (*a*)	**15.** (*b*)
16. (*c*)	**17.** (*a*)	**18.** (*a*)	**19.** (*d*)	**20.** (*a*)
21. (*b*)	**22.** (*b*)	**23.** (*a*)	**24.** (*a*)	**25.** (*b*)
26. (*c*)	**27.** (*d*)	**28.** (*b*)	**29.** (*b*)	**30.** (*a*)
31. (*d*)	**32.** (*c*)	**33.** (*a*)	**34.** (*b*)	**35.** (*b*)
36. (*c*)	**37.** (*a*)	**38.** (*d*)	**39.** (*d*)	**40.** (*c*)
41. (*b*)	**42.** (*d*).			

TEST QUESTIONS

1. Explain giving examples the following terms :

(*a*) Kinematic link (*b*) Kinematic pair

(*c*) Kinematic chain (*d*) Mechanism.

2. Explain the difference between mechanism and machine.

3. Explain the difference between a lower kinematic pair and higher kinematic pair and give two examples of each.

4. Explain giving examples, different types of kinematic pairs.

5. Explain with the help of neat sketches a slider crank chain and its various inversions.

6. Explain with the help of neat sketch a quick return mechanism using four-bar chain.

7. Explain two inversions of slider crank chain giving quick return mechanism.

8. Explain the following :

(*a*) Elliptical Trammel

(*b*) Scotch Yoke mechanism

(*c*) Oldhams's coupling.

PRACTICE PROBLEMS

1.1. Define 'Structure and Machine'. State how these differ from each other.

1.2. Define a kinematic link or element. "Kinematic link may be a machine component but machine component may not be kinematic link." Explain the statement.

1.3. Define a kinematic pair. Explain the various types of kinematic pairs giving at least one distinguishing feature of each.

1.4. Define kinematic chain. How does it differ from a mechanism ?

1.5. Define inversion of mechanism. Explain with the help of suitable sketches the inversion of (*i*) Slider crank chain mechanism, (*ii*) double slider crank mechanism, (*iii*) quadric cycle chain mechanism.

1.6. "Slider crank mechanism is only a special case of a four-bar mechanism." Justify the statement.

1.7. In a crank and slotted lever quick return mechanism, the distance between the fixed centres is 15 cm and the driven crank is 8 cm long. Find the ratio of the time taken during the cutting and return strokes.

[**Ans.** 2.105]

1.8. Describe the construction of an Oldham's coupling and state the use of it.

The distance between the axes of parallel shafts connected by Oldham's coupling is 20 mm. The speed of rotation of the shafts 300 r.p.m. Determine the maximum velocity of sliding of each tongue in its slot.

[**Ans.** 0.627 m/sec]

1.9. Fig. P-1.1 shows to scale the mechanism of the Crossby engine indicator. Show that the pencil point *P* traces a path which is approximately parallel to the indicator piston. Determine the ratio of the displacement of the pencil to the displacement of the piston. Figure is drawn to scale.

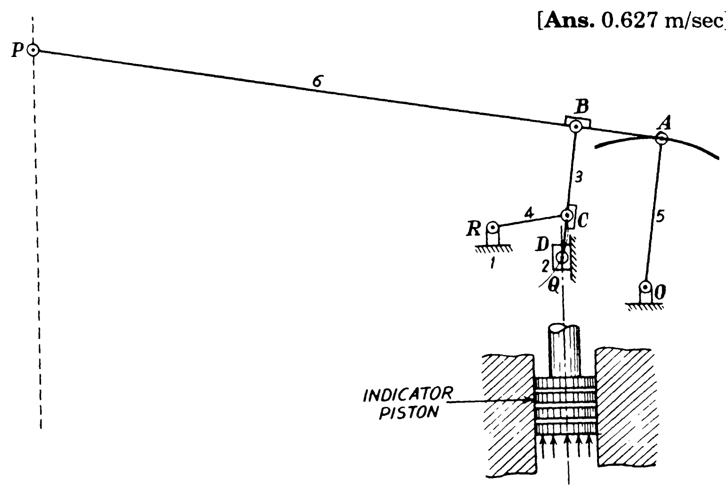


Fig. P-1.1

1.10. Fig. P-1.2 shows a Crossby indicator. Carry out the construction and plot the locus of point H. Also find the ratio of the displacement of H to the displacement of piston.

[**Hint.** Construction is shown in Fig. P-1.2.]

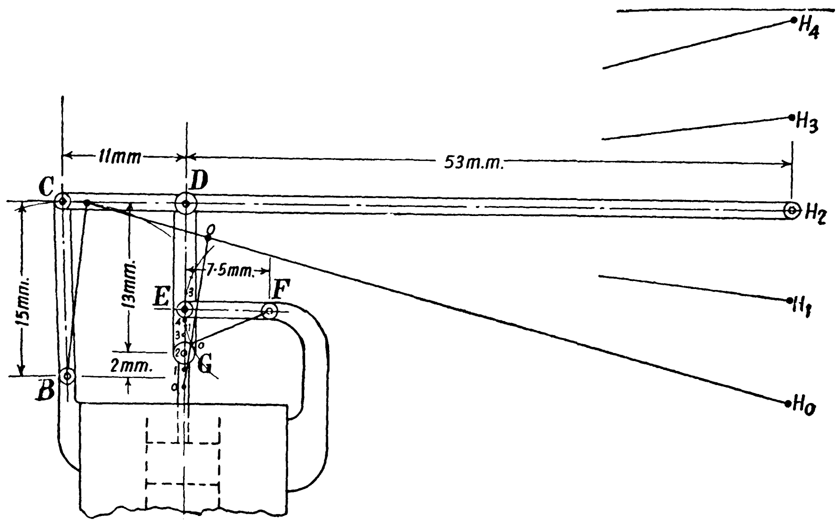


Fig. P-1.2. Indicator.

1.11. Draw to scale a Pantograph mechanism to magnification of 5.

1.12. Set out a Pantograph mechanism to a magnification of 10.

1.13. Design a quick return mechanism of the oscillating beam type shown in Fig. P-1.3. The working stroke is 15 cm and the time ratio of the working to return stroke is 2 : 1. The driving crank is 4 cm long.

[**Ans.** OC = 8 cm, CD = 15 cm, α = 240° ; β = 120°]

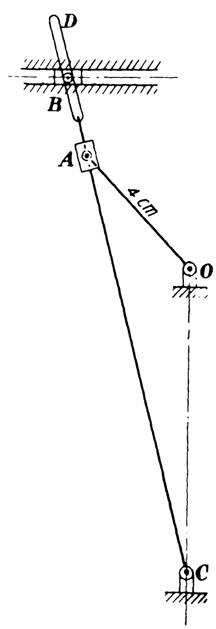


Fig. P-1.3. Scale : $\frac{1}{2}$ Full size.

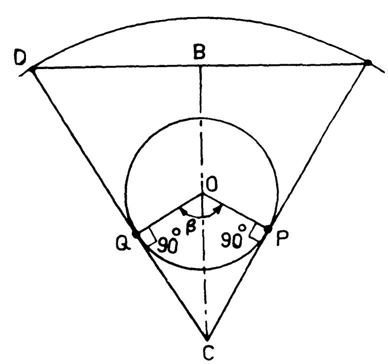


Fig. P-1.3 (a)

[**Hint.** $\frac{\text{Time of working stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{360 - \beta}{\beta} = 2.$

Therefore, $\beta = 120°.$

The extreme positions of the crank is shown in Fig. P-1.3 (*a*).

From right angle CQO, we have $\frac{QO}{OC} = \cos\frac{\beta}{2}$

Therefore, $OC = \frac{QO}{\cos\beta/2} = \frac{4}{\cos 60°} = \frac{4}{0.5} = 8$ cm.

The length of stroke = $2DB = 2CD \sin(90° - \beta/2)$ *i.e.* $15 = 2CD \sin 60$

or $CD = \frac{15}{2 \times \sin 60°} = \frac{15}{2 \times 0.5} =$ **15 cm.**

1.14. With is main distinguishing feature of Whithworth quick return mechanism when compared to oscillating beam quick return mechanism. Design a Whithworth quick return linkage shown in Fig. P-1.4 having the following particulars. Stroke return = 15 cm ; time ratio of working to return stroke 2 ; the driving link = 4 cm. Both cranks complete revolutions giving a compact mechanism for long stoke.

[**Ans.** OP = 7.5 cm, OA = 2 cm, α = 240°, β = 120°, $PF > OP$]

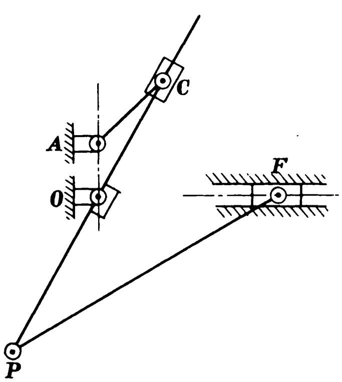


Fig. P-1.4

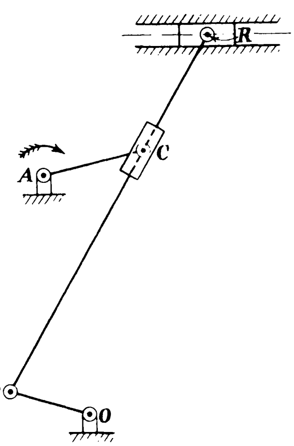


Fig. P-1.5

1.15. Quick return linkage of the Atlas shaper is given in Fig. P-1.5. The figure is drawn to scale : 3 cm = 1 m. Determine the time ratio of working to return strokes and the stroke of the tool R.

[**Ans.** Stroke 41 cm, Time ratio = 2.6]

1.16. A double slider mechanism is used to draw an ellipse with major axis equal to 20 cm and minor axis equal to 15 cm. Set out the mechanism and draw the locus of the points tracing the required ellipse.

1.17. A quick return mechanism is shown in Fig. P-1.6 on next page. Driving link OP turns about fixed axis O, counter-clockwise. The slotted bar S swings about fixed shaft T and has a gear quadrant 62.5 mm pitch radius cut on its lower end.

This quadrant drives rack R. Determine the stroke of the rack and the time ratio of the slow stroke to the fast stroke. [**Ans.** Stroke = 91.5 mm, Time ratio = 2.75]

1.18. Fig. P-1.7 on next page shows the linkage for Atlas chaper. The figure is drawn to scale. Determine the time ratio of the working to the idle stroke and also the length of the stroke. What is the difference between this mechanism and the oscillating beam shaper mechanism ?

[**Ans.** Stroke = 18 mm, Time ratio = 1.725]

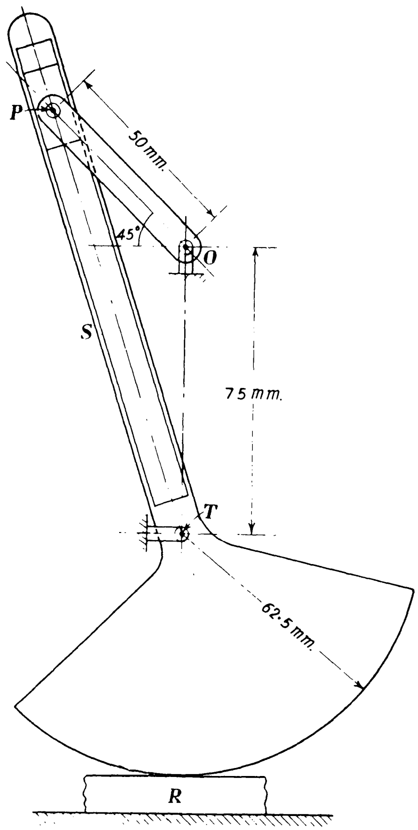


Fig. P-1.6. Scale : $\frac{1}{2}$ Full size.

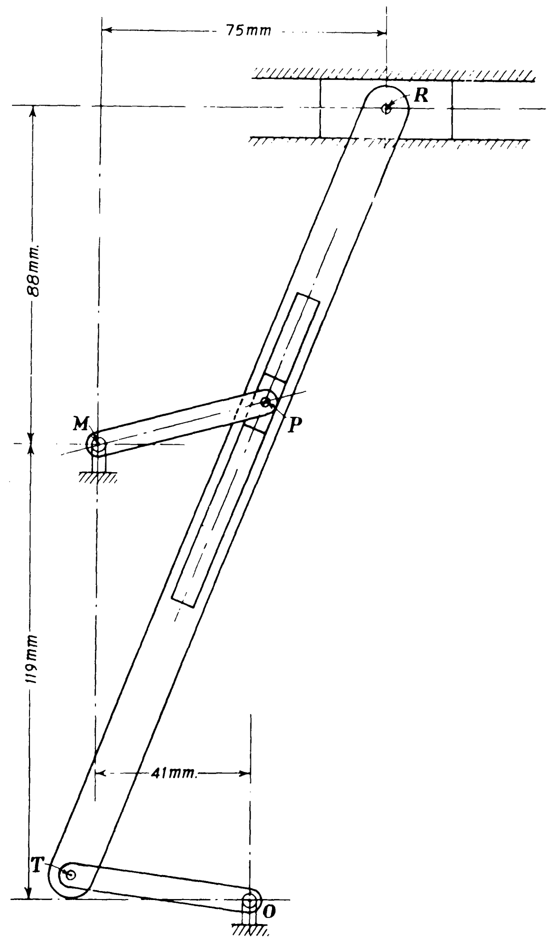


Fig. P-1.7

1.19. The distance between the axes of the two parallel shafts connected by Oldham's coupling is 1 cm. Determine the kinetic energy in the intermediate piece when the shafts rotate at 300 r.p.m. Particulars of the intermediate disc are the following :

Mass = 4 kg.

Radius of gyration about C.G. located at the geometric centre of the disc = 15 cm k_G.

[**Hint**. K. E. $= \frac{1}{2} I\omega^2 + \frac{1}{2} mr^2 = \frac{m}{2} (k_G^2 + r^2)\, \omega^2$]

[**Ans.** Peripheral velocity of the centre of disc $v = \omega.r = \left(\frac{2\pi \times 300}{60} \cdot \frac{1}{100} \right)$ m/sec.

K.E. = 5.14 N-m]

1.20. Fig. P-1.8 shows a Drag Link Quick Return Mechanism. Explain the condition to be satisfied by the given four bar mechanism for drag link quick return mechanism. Can a quick return mechanism be obtained by fixing any other link except O_1, O_2 ?

Determine the time ratio of the working stroke to the return stroke for uniform angular velocity of crank O_1A. Determine also the length of stroke of the slider. [**Ans.** Time ratio = 2.27, Stroke = 12 cm]

1.21. Fig. P-1.9 shows a mechanism, *BC* = *CE* = *FE*. Carry out two construction for assumed values of the lengths of the mechanism and plot the locus of point *F*. Show that point *F* will trace the path of an ellipse.

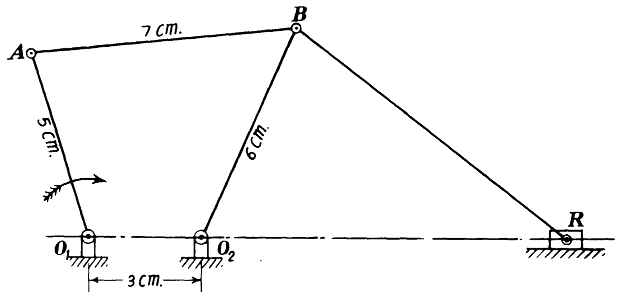


Fig. P-1.8

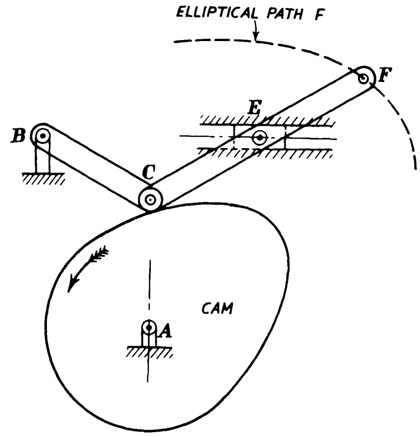


Fig. P-1.9

1.22. Fig. P-1.10 shows a toggle mechanism. Eccentric rotates uniformly at 5. r.p.m. Plot a graph of the time in seconds versus displacement of the slider for one revolution of the eccentric. By graphical differentiation plot graph of time in seconds versus velocity in metres/sec of the slider for one revolution.

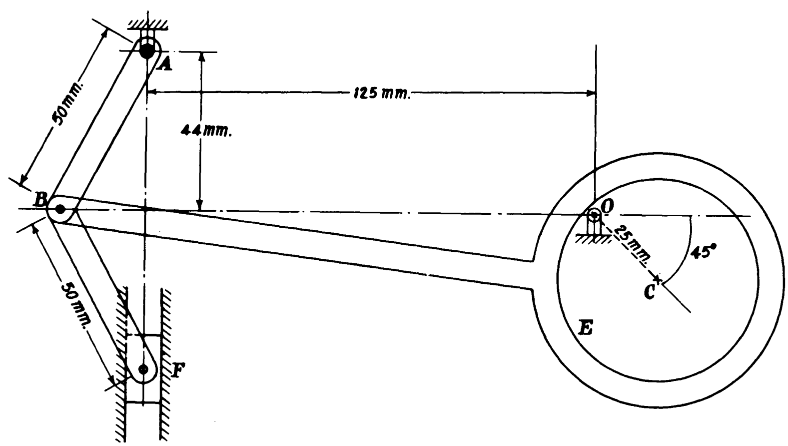


Fig. P-1.10

1.23. Fig. P-1.11 shows a web cutter, cutting a continuously moving web or strip of material. Plot the locii of points U and T, the knife edges for complete revolution of crank OP.

1.24. Fig. P-1.12 shows one of the kinematic arrangements for a lawn sprinkle oscillator. The figure is drawn to scale. It is required to find the angle of oscillation for the sprinkler tube for at least three positions for the clamping screw.

1.25. Fig. P-1.13 shows a drag link mechanism with its crank O_2A rotating at 120 r.p.m. Using time as abscissa and displacement of link 6 from the right hand dead centre position as ordinate, plot a displacement diagram. Differentiate this graphically to obtain the velocity diagram.

1.26. Plot the displacement diagram for link 4 of the off-set slider crank mechanism shown in Fig. P-1.14. The crank rotates at 1000 r.p.m. In plotting the diagram choose the scale of abscissa in seconds. By graphical differentiation, obtain the values of velocities of the slider for different configurations and plot the graph of velocity of slider versus time in seconds.

1.27. The linkage shown in Fig. P-1.15 is drawn to scale. It is required to determine approximately the dwell in the motion of the link 6 for complete revolution of crank or link 2. [**Ans.** Approx. 54°]

1.28. Fig. P-1.16 shows a mechanism for adjusting the pusher plate for a bulldozer. Trace the path of point B for any six positions of the piston in the hydraulic cylinder.

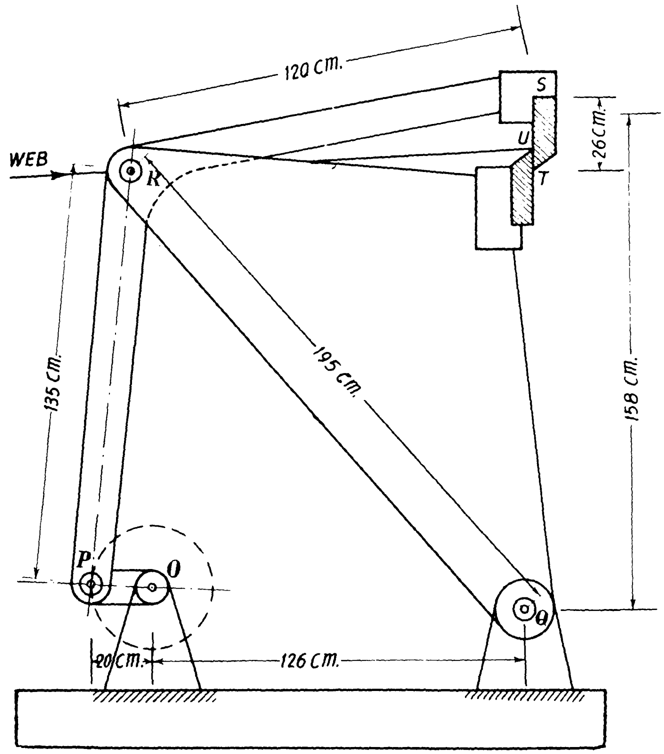


Fig. P-1.11

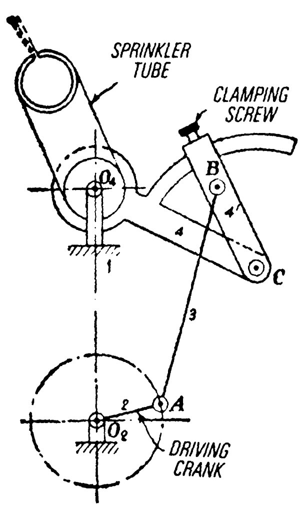


Fig. P-1.12

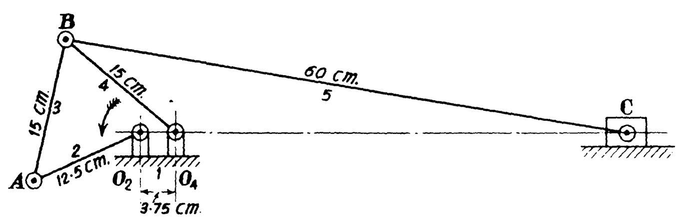


Fig. P-1.13

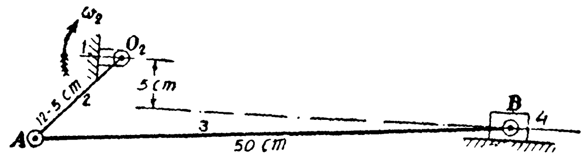


Fig. P-1.14

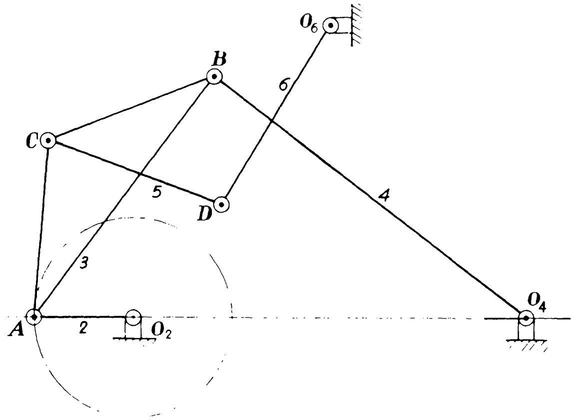


Fig. P-1.15

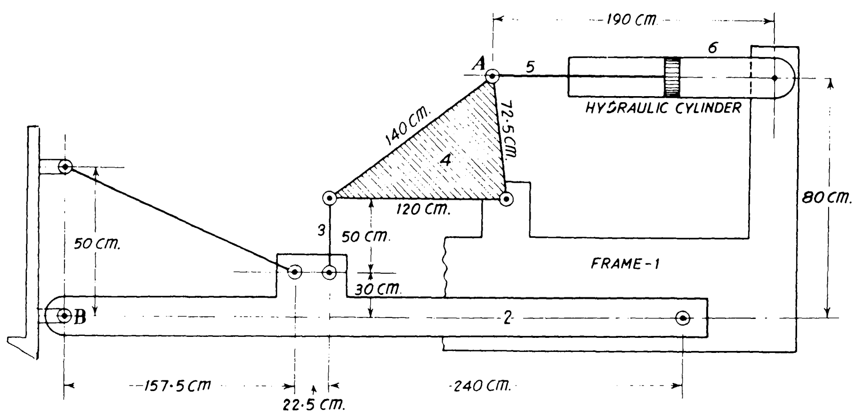


Fig. P-1.16

1.29. Fig. P-1.17 shows a mechanism for moving materials. The figure is drawn to scale. Plot the path of point C on the transport link 5 for one revolution of the crank link 2. Also show that the articles transported, four pieces shown in the figure, move by equal distances and along the same path.

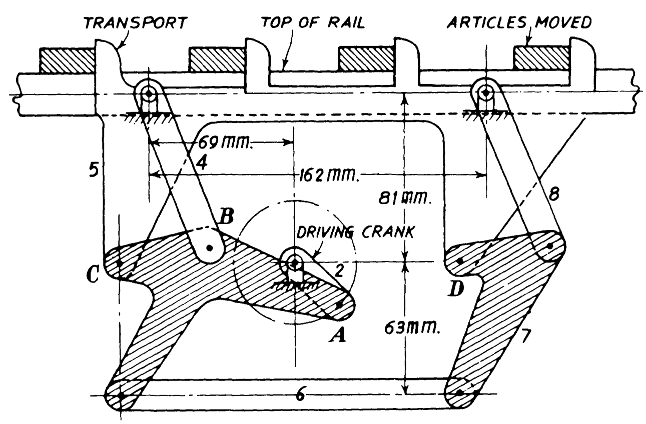


Fig. P-1.17