

Definition, Concepts and Zeroth Law of Thermodynamics

1.1. Definition

One excellent definition of thermodynamics is that it is the science of Energy and Entropy. But since these terms have not yet been defined, an alternative definition in terms with which we are already familiar is : Thermodynamics is the science that deals with the relations between heat, work and those properties of systems that bear relation to heat and work. General laws of energy transformations concerning all types of systems, mechanical, electrical and chemical may fall within the purview of this science. It is a science based on a number of empirical laws formed by experimentation from which all predications concerning the physical behaviour of the system may be deduced by logical reasoning. The findings have been formalised into certain basic laws, which are known as Zeroth, First, Second and Third laws of thermodynamics.

1.2. Scope

The treatment of the subject-matter presented in this volume may be aptly described as the so called classical approach, as distinct from statistical approach. The matter is treated as continuous and the behaviour of the system is explained in terms of measurable properties without referring to the arrangement or behaviour of individual molecules. In the statistical or molecular thermodynamics, the behaviour of individual molecules is explained with significance. For most engineering applications of thermodynamics, the classical approach gives sufficiently precise and valid results and is perhaps the only approach at the preparatory level of a course in thermodynamics.

1.3. Thermodynamic System

The term system is defined in thermodynamics as a definite quantity of matter of fixed mass and identity which is bounded by a closed surface. This surface may be real surface, *i.e.* a vessel containing liquid forms a thermodynamic system with the walls of the vessel forming real boundary surface. The boundary surfaces may be imaginary, *i.e.* a mass of certain liquid flowing along a pipe. The boundary surfaces for this matter are formed by one's imagination. The boundary surfaces may not be constant in shape and volume. Thus a mass of gas enclosed in a cylinder with one side open and fitted with a moving piston is also a thermodynamic system with all real boundary surfaces and one boundary surface can move as the gas expands and the volume changes. System may be as simple as a free body or as complex as an entire Chemical refinery or entire Thermal power plant. Even a vacuum, which is devoid of matter, may be the focus of interest and comprise a system.

Thermodynamics deals with the study of energy transformations within systems and transfers of energy across the boundaries of the systems. *Thus a thermodynamic system may be defined simply as region in space or control volume or quantity of matter upon which attention is focused for study.*

1.4. Surrounding

The space and matter external to a thermodynamic system is called the surrounding. Every thing external to the system is considered to be part of the system's *surroundings*. The system is distinguished from its surroundings by specified *boundary* which may be at rest or in motion. You will see that interactions between system and surroundings, which takes place across the boundary, play an important part in engineering thermodynamics. It is essential for the boundary to be delineated carefully before proceeding with any thermodynamic analysis. However since the same physical phenomena often can be analysed in terms of alternative choices of the system, boundary, surroundings, the choice of particular boundary defining a particular system is governed by the convenience it allows in the subsequent analysis. (Refer Fig. 1.1). All the matter and space external to the fixed real boundaries formed by the inner surfaces of the cylinder and the movable boundary formed by the bottom surface of piston, are called surroundings. Thus the metal walls of cylinder are also the surroundings for the thermodynamic system considered.

1.5. Universe

System and surroundings put together is called Universe.

1.6. Thermodynamic System Classified

(i) *Closed system*. Sometimes energy, but not mass, may cross the boundaries of the system. In such cases the system is known as a closed system. Though the term closed system is quite proper, but it is redundant and a better choice is *batch system* since the attention is focused on the same batch of matter within boundaries across which only energy transfers are possible (Refer Fig. 1.2). The gas contained in the cylinder is an example of the batch system or the closed system so long the mass of the gas in the cylinder remains same and only energy transfer in the form of mechanical work, heat etc. is possible. The term control mass is sometimes used in place of closed system.

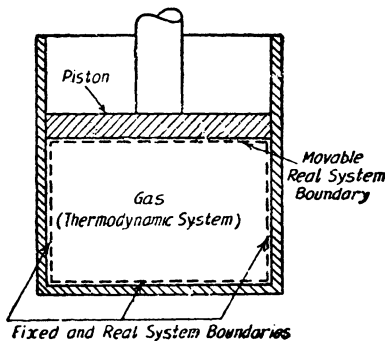
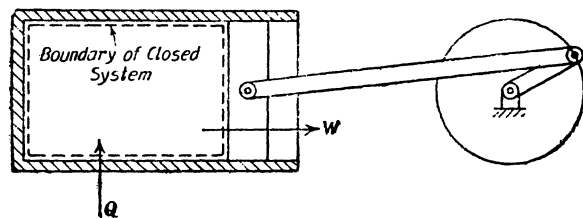


Fig. 1.1



Q = Heat transfer, W = Work transfer

Fig. 1.2

(ii) *Open system*. An open system is the one with transfer of energy and mass across its boundaries. This nomenclature although prevalent in mechanical engineering contains inherent contradiction in it; because, a system is a collection of matter, and by its very definition is always closed as long as the law of conservation of matter holds. Actually what is meant by an open system is situation wherein matter flows into and out of the control volume. A better name for such an illustration is *flow system*. The term open system is used interchangeably with control volume.

Refer Fig. 1.3 (a) for an open system or flow system, or control volume system.

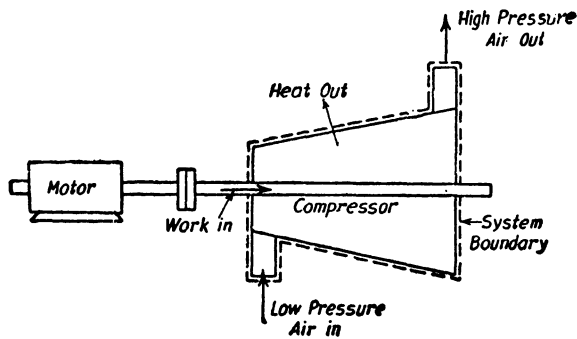


Fig. 1.3 (a)

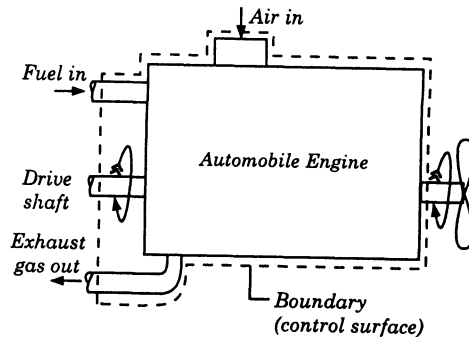


Fig. 1.3 (b)

There could be different types of substances flowing in and out of the control volume. Schematic diagram of an engine is shown in Fig. 1.3 (b). The dashed line defines a control volume that surrounds the engine. It may be observed that air, fuel, and exhaust gases cross the boundary. There are heat and work transfers at the system boundaries.

(iii) *Isolated system.* A thermodynamic system which is not influenced by the surrounding is defined as an isolated system. In this system neither mass, nor energy crosses the boundaries of the system.

Thus, this discussion throws light on an interesting situation of flow through pipe. If the attention is focused on control volume 1 with m as the mass of the working substance, and study is made of work and heat transfer across the control volume, the analysis can be made applying all the equations applicable to a flow system. Different batches of working substances flow through the control volume. (Refer Fig. 1.4).

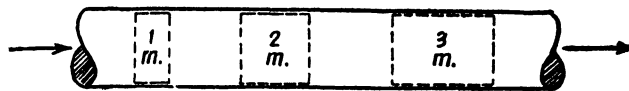


Fig. 1.4

If the attention is focused on the working substance and the work and heat transfer from this batch is analysed as it moves to position 2 and then to 3 having different volumes but same mass, the analysis can be made applying all the equations applicable to a closed system. The same batch of the working substance is studied for heat and work transfer.

Suppose the working substance contained in the entire length of pipe is considered as a system, and heat transfer determined. Obviously there will be same heat transfer whether the analysis is made on the basis of closed system, *i.e.* better worded as non-flow system, or flow system. But the analysis for work does not yield to this argument as there is work associated at entrance and exit due to interaction of the surrounding and thus the analysis of the system as non-flow system does not yield the same result as the analysis of the system as flow system. And obviously the flow system analysis is applicable.

1.7. Macroscopic and Microscopic Point of View

The characteristics of a thermodynamic system which may be explained by a few of the properties of the system which can be measured constitutes a point of view called macroscopic. For example, let us take a system as the contents of a cylinder of an internal combustion engine. At any instant the system has a certain volume, depending upon the position the piston occupies. This volume is easily measurable. Another quantity to explain the system is the pressure of the gas inside the cylinder. A pressure gauge can be fitted to measure the same. Similarly, temperature, chemical composition etc. may be described. Thus the system may be

explained by what may be called the large scale characteristics or properties of the system. These are called macroscopic co-ordinates and the description is called *macroscopic* description of the system. Different macroscopic co-ordinates may be described to explain different systems, but all the macroscopic co-ordinates have the following characteristics :

(i) No special assumption regarding the structure of matter is made in describing macroscopic co-ordinates.

(ii) They are readily measurable.

(iii) Only a few macroscopic co-ordinates are required to describe the system adequately.

For a *microscopic* point of view, the gas in the cylinder of the previous example will be assumed to contain a large number N of particles or molecules, all having the same mass and each having a velocity independent of the other. Thus each particle may be described by three cartesian co-ordinates x, y and z and three components of velocities u, v and w . Thus six variables describe one particle and the system will be described by $6N$ variables.

1.8. Concept of Continuum

The classical thermodynamics or the macroscopic point of view in dealing with the science of thermodynamics is explained with the help of continuum properties. It is desirable to define and clarify the concept of continuum.

Let us take the example of the density of a fluid at a point P . In order to find out the density of a fluid at point P , let us assume point P to be surrounded by a volume ΔV and the mass contained in this volume as Δm , as shown in Fig. 1.5 (a).

Let the graph be plotted between the ratio $\Delta m/\Delta V$ and ΔV , as shown in Fig. 1.5 (b). It may be observed that when the value of ΔV becomes smaller and smaller, or the volume about point P is shrunk, the ratio $\Delta m/\Delta V$ deviates very much from its asymptotic value at $\Delta V'$ and the value of $\Delta m/\Delta V$ is either very large or very small. This graph is explained by the fact that the volume ΔV contains very few particles and results in large fluctuations when particles pass into and out of the volume. The density at point P may be expressed as

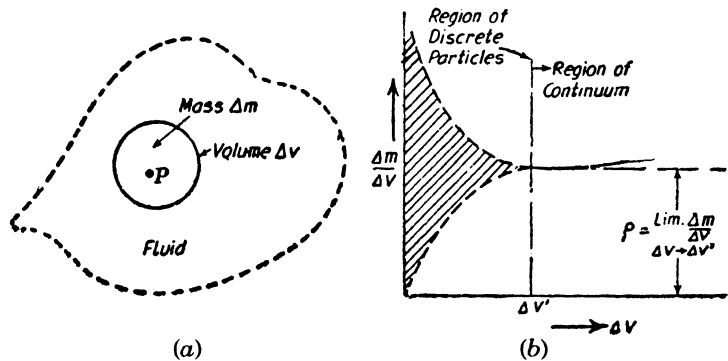


Fig. 1.5

large fluctuations when particles pass into and out of the volume. The density at point P may be expressed as

$$\rho = \lim_{\Delta V \rightarrow \Delta V'} \frac{\Delta m}{\Delta V} \quad \dots(1.1)$$

where $\Delta V'$ is the smallest volume about point P in which the fluid can be said to be continuum.

Normally for most engineering calculations, the continuum volume $\Delta V'$ is extremely smaller when compared to the dimensions of the installation. But it may not be always true. Let us take the example of ballistic missiles, supersonic rockets and earth satellites. At about 100,000 metres, the mean free path for a molecule, *i.e.* the average distance that a molecule has to move to collide with some other molecule is about 2.5 cm. This distance is approximately of the same dimension as the boundary layer thickness of the vehicle. But at 200,000 metres this mean molecular path may be as large as 3.5 metres, *i.e.* as large as the vehicle itself. Thus in these situations the fluid can no longer be treated as continuum and the microscopic point of view has to be adopted to study the discrete particles and their behaviour. In high vacuum

technology the mean free path of the molecules may be as large as the dimensions of vessel and only microscopic point of low is considered.

1.9. State

The state of a system is its condition or configuration described in sufficient detail so that one state may be distinguished from all other states. The state may be identified by certain recordable macroscopic properties, some familiar such properties being pressure, temperature, density etc.

1.10. Properties

A property of a system is any observable characteristic of the system. The properties we shall deal with are measurable in terms of numbers and units of measurements and include such physical quantities as location, speed, pressure, density etc. A listing of a sufficient number of independent properties constitutes a complete definition of the state of a system. Such characteristics are also called state variable or thermodynamic co-ordinates of the system.

When all the properties of a system have definite values, the system is said to be in a definite state. Systems in identical state must have identical values of their corresponding properties. *Thus property of the system depends solely upon the state of the system and not upon how that state may have been reached.* Thus the knowledge of the history of the system is not required.

Mathematically this may be stated as follows :

“If X and Y are two properties of a system, then dX and dY are exact differentials.”

Note. In general, if the differential is of the form $Mdx + Ndy$, the test for exactness is

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

Problem 1.1. Explain if the following quantities can be used as properties :

- (i) $\int pdv$
- (ii) $\int vdp$
- (iii) $\int (pdv + vdp)$

Solution. (i) p is a function of v and they are connected by a line path on p and v planes. Thus, it is not an exact differential and thus not a property.

(ii) $\int vdp$ is also not a property on the same reasoning as in (i).

(iii) $\int (pdv + vdp) = \int d(pv) = pv$.

Thus, it is an exact differential and hence it is a property.

Problem 1.2. If $pv = RT$, (where p = pressure, v = specific volume, R = a constant and T = temperature), determine whether the following quantities

- (i) $\int \left(\frac{dT}{T} - \frac{vdp}{T}\right)$ and
- (ii) $\int \left(\frac{dT}{T} + \frac{pdv}{v}\right)$ can be used as properties.

Solution. Each of the differential is of the form $Mdx + Ndy$, therefore, apply the test

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

Thus $\left[\frac{\partial(1/T)}{\partial p}\right]_T = \left[\frac{\partial(-v/T)}{\partial T}\right]_p = \left[\frac{\partial(-R/p)}{\partial T}\right]_p$

or

$$0 = 0$$

Thus $\left(\frac{dT}{T} - \frac{vdp}{T}\right)$ is exact and may be written as $\frac{dT}{T} - \frac{vdp}{T} = ds$, where s is a point

function and hence a property.

Again for $\frac{dT}{T} + \frac{pdv}{T}$, we have

$$\left[\frac{\partial(1/T)}{\partial v} \right]_T = \left[\frac{\partial(p/v)}{\partial T} \right]_v = \left[\frac{\partial(RT/v^2)}{\partial T} \right]_v \quad \text{or} \quad 0 = \frac{R}{v^2}$$

The differential is not exact and thus $\int \left(\frac{dT}{T} + \frac{pdv}{v} \right)$ is not a point function. It is not a property.

1.11. Intensive and Extensive Properties

An intensive property of a system is one whose value does not depend upon the mass of the system. Such properties include temperature, pressure, density and specific volume. Whatever may be mass of the system these are the same for the entire system.

An extensive property of the system is dependent on the mass of the system. It is in fact, proportional to the mass of the system. Volume is an extensive property. Many more extensive properties will be discussed later.

1.12. Specific Weight, w_s

It is also known as the weight density. It is the weight per unit volume. It may be expressed in kgf/m^3 in MKS units and Newton/m^3 in SI units and depends upon both the density of the substance and the value of the gravitational acceleration g . Let w represent the weight corresponding to certain mass m ; then according to Newtons Second Law of motion, the density

$$w = \frac{1}{g_c} \cdot mg$$

where g_c is the constant of proportionality and its numerical value may be the value for standard acceleration due to gravity in MKS units and **1 in SI units**.

If both sides of this equation are divided by volume, then

$$w_s = \rho \frac{g}{g_c} \quad [\because m = \rho V] \quad \dots(1.2)$$

where w_s is the weight density and ρ is the mass density. For example, if the density of water at 22°C is 1000 kg/m^3 , its specific weight is

$$w_s = \frac{1000}{g_c} \cdot g.$$

Specific weight is the same numerically as the density in MKS units only if the local acceleration g is numerically equal to the dimensional constant g_c . It may be noted that w_s is the force per unit volume whereas ρ is the mass per unit volume. The unit of g are m/sec^2 . The units of g_c are kgm/kgf sec^2 in MKS units, and kgm/N sec^2 in SI units. In MKS units, $g_c = 9.81 \text{ kgm/kgf sec}^2$ and in SI units $g_c = 1 \text{ kgm/N sec}^2$. Thus in SI units $w_s = \rho g \text{ N/m}^3$, where ρ is expressed in kg/m^3 .

1.13. Specific Volume

It is defined as the volume per unit mass and may be expressed in m^3/kg . The specific volume v is related to the mass of a body m and its volume V as follows :

$$v = V/m$$

It is reciprocal of the density, i.e. $v = 1/\rho$.

1.14. Pressure

The pressure exerted by a system is the force exerted normal to unit area of the boundary. When a fluid is contained within a vessel, the pressure exerted on the vessel is equal

to the mean change of momentum of the molecules exerted perpendicular to the confining boundary per unit area and per unit time.

$$p = \lim_{\Delta A \rightarrow \Delta A'} \frac{\Delta F_n}{\Delta A} \quad \dots(1.3)$$

where ΔF_n is the force normal to the area ΔA .

$\Delta A'$ is the smallest possible area capable of maintaining a continuum.

A fluid is defined as substance in which the shear stresses are zero, whenever it is at rest relative to its container. When the fluid is at rest, only normal stress exist ; when fluid is in motion shear or tangential stress exist in additional to the compressive stress.

In an ideal fluid, no shear stresses exist even if there is relative motion within the fluid. When a fluid is in equilibrium, the pressure at any point is the same in all directions. When the fluid is in equilibrium, the pressure exerted by it is called static pressure. When a fluid is not in equilibrium, the pressure may vary according to direction. One exception is the ideal or inviscid fluid. In such fluids, the shear forces are absent and thus pressure is independent of direction. This is true whether the ideal fluid is in motion or at rest.

Consider a homogeneous fluid of density ρ in static equilibrium. A pressure difference exists between two points which are separated by distance h in the vertical directions. The weight of the cylinder of fluid can be equated to the difference between the forces due to pressure at two ends of the cylinder so that fundamental relationship is

$$\Delta p A = w_s h A$$

or
$$\Delta p = w_s h = \rho \frac{gh}{g_c} \quad \dots(1.4 a)$$

In SI units $g_c = 1$, therefore $\Delta p = \rho gh \quad \dots(1.4 b)$

As for equilibrium in the horizontal direction, it can be shown that the pressure of fluid at rest does not vary in horizontal directions. Therefore, the hydrostatic pressure is same as all points in a horizontal plane and varies only with depth.

The standard atmospheric pressure is defined as the pressure produced by a column of mercury 760 mm high, the mercury density being 13.5951 g/cm³ and the acceleration due to gravity being its standard value of 980.665 cm/sec². The standard atmospheric pressure is 1.0332 kgf/cm² and is denoted by *atm*. In SI units it is expressed in *N/m²* or Pascal abbreviated as Pa. Pascal being small normally kilo-pascal (kPa) or Mega-pascal (MPa) is used.

1 kgf/cm² is also called technical atmosphere and denoted by *ata*. It is worth noting that :

1 atm = 101.325 kPa ; 1 atm \approx 760 mm Hg ; 1 bar (= 10⁵ N/m²) \approx 750 mm Hg ; 1 ata = 736 mm Hg ; 1 mm Hg is also denoted by 1 torr.

Thermodynamic investigations are more concerned with the values of absolute pressure rather than those of gauge pressure. Conversion from gauge pressure to absolute pressure is done by the relation

$$P_{abs} = P_{gauge} + P_{atmospheric} \quad \dots(1.5)$$

(Note. 1 kgf/cm² abs may be denoted as 1 ata.)

Fig. 1.6 shows schematically this relationship. The datum for absolute pressure is a perfect vacuum, datum for gauge pressure is atmospheric pressure.

In dealing with fluid flow problems, various type of pressures are commonly used. Static pressure is the pressure sensed by a measuring device, if it were moving with the same velocity as the fluid stream. Impact pressure is the force per unit area perpendicular to the direction of flow when the fluid is brought reversibly to rest. For a constant density fluid, the impact pressure is given by

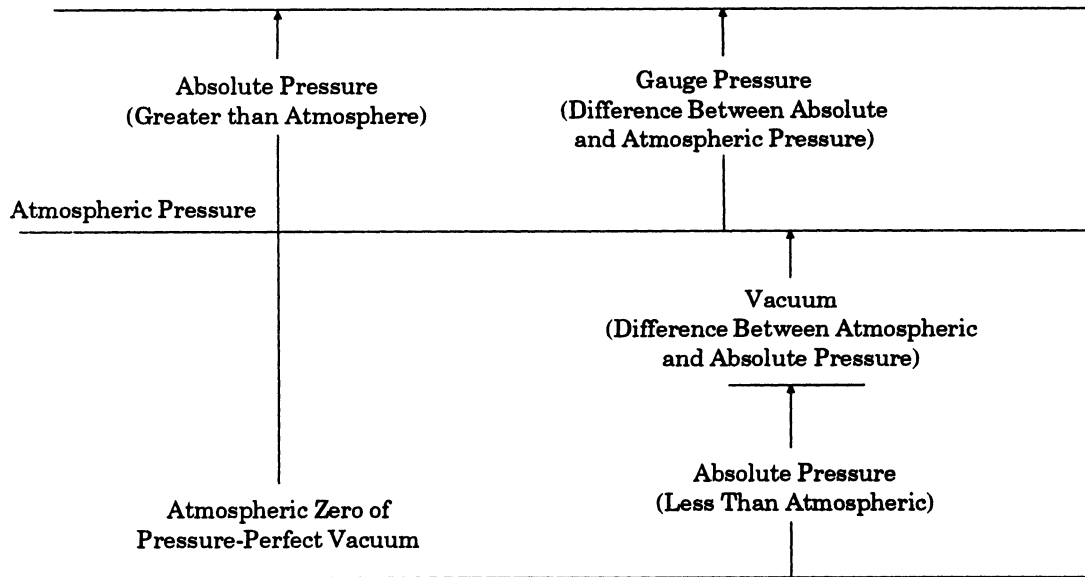


Fig. 1.6. Pressure relationships.

Table 1.1. Conversion Factors for Pressure

	bar	dyne/cm ²	kgf/cm ² ata	N/m ² or Pa	mm Hg (21°C) or torr	mm H ₂ O (21°C)	atm
bar =	1	10 ⁶	1.01972	10 ⁵	750.062	10197.2	0.986923
1 dyne/cm ² =	10 ⁻⁶	1	1.01972 × 10 ⁻⁶	0.1	750.062 × 10 ⁻⁶	10197.2 × 10 ⁻⁶	0.986923 × 10 ⁻⁶
1 kgf/cm ² =	0.980665	0.980665 × 10 ⁶	1	0.980665 × 10 ⁵	735.559	10000	0.967838
Pa or N/m ² =	10 ⁻⁵	10	10.1972 × 10 ⁻⁶	1	750.062 × 10 ⁻⁵	10197.2 × 10 ⁻⁵	0.986923 × 10 ⁻⁵
1 mm Hg = (21°C)	1.333223 × 10 ⁻³	1.333223 × 10 ³	1.3595 × 10 ⁻³	1.333223 × 10 ²	1	13.5951	1.31578 × 10 ⁻³
1 mm H ₂ O = (21°C)	98.0665 × 10 ⁻⁶	98.0665	10 ⁻⁴	9.80665	0.073556	1	96.7838 × 10 ⁻⁶
1 atm =	1.01325	1.01325 × 10 ⁶	1.03323	1.01325 × 10 ⁵	760	10332.3	1

$$P_{\text{impact}} = \frac{\rho V^2}{2g_c} + P_{\text{static}} \text{ and } g_c = 1 \text{ in SI Units} \quad \dots(1.6)$$

where ρ is the fluid density, V is the velocity. The difference between the impact pressure and static pressure is due to the velocity and this difference is, therefore, called *velocity pressure*, $\rho V^2/2g_c$. Fig. 1.7 shows the different types of pressures.

$$\left(\text{in MKS units } g_c = 9.81 \frac{\text{kgm}}{\text{kgf-sec}^2}; \text{ In SI unit } g_c = 1 \frac{\text{kgm}}{\text{N-sec}^2} \right)$$

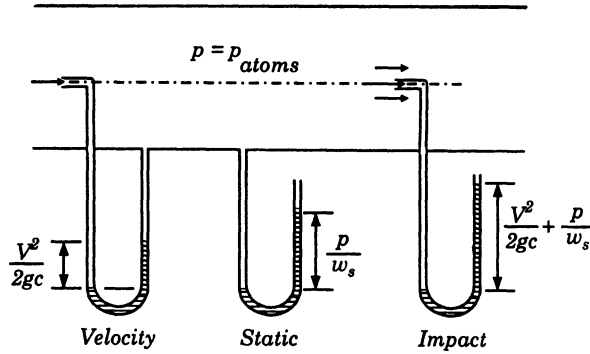


Fig. 1.7. Types of Pressure.

In fluid flow through a duct, static conditions prevail at the walls of the duct. The velocity at the wall is zero and, therefore, the pressure measured at the wall is the static pressure. If the fluid particles move parallel to the centre line of the duct, the static pressure is uniform across any section of the duct.

Measurement of Pressure ; Manometer

To measure pressure slightly different from atmospheric, a manometer is normally used and the pressure is determined according to the hydrostatic formula. The manometer liquid may be mercury, water, alcohol, etc. A typical manometer is shown in Fig. 1.8 (a). Since the

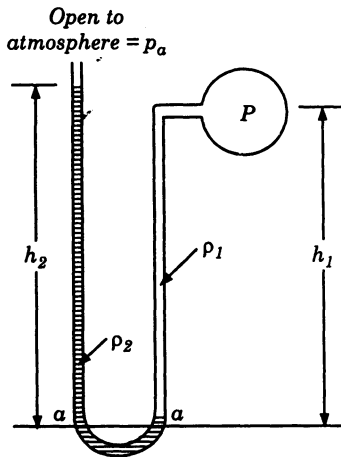


Fig. 1.8. (a) Measuring Pressure by means of manometer.

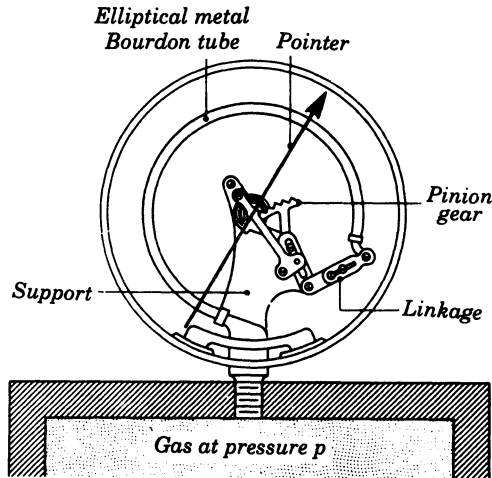


Fig. 1.8. (b)

manometer fluid is in equilibrium, the pressure along a horizontal line *aa* is same for either branch of the manometer. Then,

$$p + \rho_1 h_1 \frac{g}{g_c} = p_a + \rho_2 h_2 \frac{g}{g_c}$$

where *p* is the absolute pressure in the bulb, *p_a* is the atmospheric pressure exerted on the liquid free surface and ρ_1, ρ_2 are the densities of the liquid in the bulb and the manometer respectively.

$$\rho_1 \ll \rho_2$$

i.e. ρ_1 is insignificantly small as compared to ρ_2 , then

$$p - p_a = \rho_2 h_2 \frac{g}{g_c}$$

$$= \rho_2 h_2 g$$

$$\because g_c = 1 \text{ in SI units}$$

A Bourdon Tube. Refer Fig. 1.8 (b). A Bourdon tube gauge comprises a curved tube having an elliptical cross section with one end attached to the pressure to be measured and the other end connected to a pointer by a mechanism. When fluid under pressure fills the tube, the elliptical section tends to become circular and the tube straightens. This motion is transmitted by the mechanism to the pointer. By calibration of the deflection of the pointer for known pressures, a graduated scale can be determined from which any applied pressure can be read in suitable units. Because of its construction, the Bourdon tube measures the pressure relative to the pressure of surroundings existing at the instrument. Accordingly, the dial reads zero when the inside and outside of the tube are at the same pressure.

Problem 1.3. Mercury of density 13.59508 g/cm^3 is used as monometric fluid. What gauge pressure in bar is exerted by a column of mercury of 760 mm ?

Solution. The basic formula to use is, $p = \rho gh$

$$\text{Also, } \rho = \left(\frac{13.59508 \text{ g}}{1 \text{ cm}^3} \right) \left(\frac{1 \text{ kg}}{10^3 \text{ g}} \right) \left(\frac{10^6 \text{ cm}^3}{1 \text{ m}^3} \right) = 13.59508 \times 10^3 \text{ kg/m}^3$$

$$g = 9.80665 \text{ m/sec}^2; h = 760 \text{ mm.} = 0.76 \text{ m}$$

$$\text{Therefore } p = (13.59508 \times 10^3) (9.80665) (0.76) = 101325 P_a = 1.01325 \text{ bar}$$

Problem 1.4. A gauge fitted to a steam condenser indicates 700 mm of mercury when the barometer records 760 mm of mercury. Calculate the absolute pressure in the condenser in kgf/cm^2 , bar and kPa.

Solution. Absolute pressure in the condenser

$$= \text{Atmospheric pressure in the mm of Hg-Vacuum gauge reading in mm of Hg}$$

$$= 760 - 700 = 60 \text{ mm of mercury} = 60 \times 1.3595 \times 10^{-3} = \mathbf{0.08157 \text{ kgf/cm}^2}$$

$$\text{Also } 60 \text{ mm} = 60 \times 1.333223 \times 10^{-3} = \mathbf{0.08 \text{ bar}} = \mathbf{0.08 \times 10^5 \text{ N/m}^2} = \mathbf{8 \text{ kPa.}}$$

Problem 1.5. The forced draught fan supplies air to furnace of the boiler at draught of 30 mm of water. Calculate the absolute pressure of air supply if the barometer reads 760 mm mercury, in kgf/cm^2 , bar and kPa.

Solution. Since it is a forced draught, the absolute pressure is above atmospheric pressure.

$$\text{Absolute pressure} = \text{Barometer reading} + \text{Forced draught reading}$$

$$\text{We have, } 1 \text{ mm of water column} = 0.073556 \text{ mm Hg}$$

$$\text{Therefore, } 30 \text{ mm of water column} = 30 \times 0.073556 \text{ mm Hg} = 2.20668 \text{ mm Hg.}$$

$$\text{Thus absolute pressure} = 760 + 2.20668 = 762.20668 = \frac{762.20668}{735.559} \text{ kgf/cm}^2$$

$$= \mathbf{1.03623 \text{ kgf/cm}^2} = 0.981 \times 1.03623 \text{ bar} = \mathbf{1.01654 \text{ bar}}$$

$$= \mathbf{1.01654 \times 10^2 \text{ kPa.}}$$

Problem 1.6. The pressure in a gas pipe is measured by a mercury manometer. One leg of manometer is open to atmosphere. If the difference in the height of mercury column in the two legs is 450 mm, determine the gas pressure in the pipe in kPa, bar and in atm. The barometer reading is 755 mm Hg. The local acceleration due to gravity is 9.81 m/sec^2 and the mass density of mercury is 13595 kg/m^3 .

Solution. Refer Fig. 1.9. Neglecting the density of gas as compared to that of mercury, the pressure of gas at plane $a - a$ is given by

$$p = p_a + \rho gh$$

Also $p_a = \rho g h_0$

where ρ is density of mercury

h_0 is the barometric height

g is the local acceleration due to gravity

Thus $p = \rho g h_0 + \rho g h = \rho g(h_0 + h)$

$$= 13595 \text{ kg/m}^3 \times 9.81 \text{ m/sec}^2 \times (0.755 + 0.450) \text{ m}$$

$$= 160.707 \times 10^3 \text{ N/m}^2 = \mathbf{160.707 \text{ kPa}}$$

$$= \mathbf{1.60707 \text{ bar}}$$

$$= 1.60707 \times 0.986923 \text{ atm} = \mathbf{1.58605 \text{ atm.}}$$

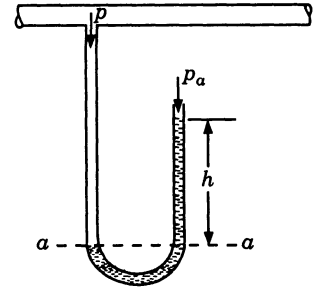


Fig. 1.9

Problem 1.7. The vessel is shown in Fig. 1.10 with pressure gauges indicating the readings. Gauge C reads 400 kPa (4 bar), gauge A reads 600 kPa (6 bar). If the barometer reads 760 mm Hg, determine the reading on gauge B, and absolute pressures in each region.

Solution. Refer Fig. 1.10.

$$760 \text{ mm Hg} = 760 \times 0.1333223 \text{ kPa} = 101.325 \text{ kPa}$$

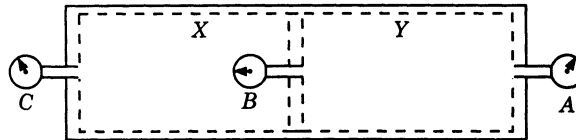


Fig. 1.10

Let the pressure in one region be Y kPa abs.

and the pressure in other region be X kPa abs.

Thus $Y - 101.325 = \text{reading on A} = 600 \text{ kPa}$

$$X - 101.325 = \text{reading on C} = 400 \text{ kPa}$$

Therefore $Y - X = \text{reading on B} = (600 + 101.325) - (400 + 101.325) = \mathbf{200 \text{ kPa.}}$

Absolute pressure in region $Y = 600 + 101.325 = \mathbf{701.325 \text{ kPa.}}$

Absolute pressure in region $X = 400 + 101.325 = \mathbf{501.325 \text{ kPa.}}$

Problem 1.8. A gas is contained in two cylinders A and B, connected by a piston of two different diameters, as shown in Fig. 1.11. The mass of the piston is 15 kg and the gas pressure inside cylinder A is 400 kPa (4 bar). Determine the pressure in cylinder B. Outside atmospheric air pressure is 100 kPa (1 bar).

Solution. Refer Fig. 1.11

Force acting on the gas in cylinder B

$$F_B = \text{Force due to gas pressure on piston in cylinder A}$$

$$+ \text{Force due to weight of piston}$$

$$- \text{Force due to air pressure on piston of cylinder A}$$

$$= p_A \times \frac{\pi}{4} D_A^2 + m_{\text{piston}} \times g - p_a \times \frac{\pi}{4} (D_A^2 - D_B^2)$$

$$= 400 \times 1000 \times \frac{\pi}{4} \times \frac{200^2}{1000^2} + 15 \times 9.81 - 100 \times 1000$$

$$\times \frac{\pi}{4} \left(\frac{200^2 - 50^2}{1000^2} \right) \text{ Newtons}$$

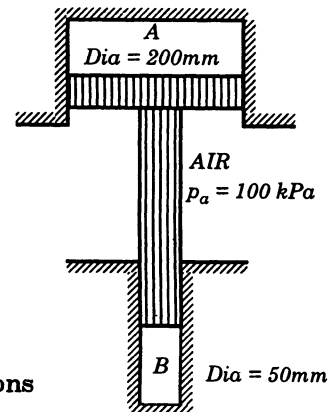


Fig. 1.11

$$= 12566.37 + 147.15 - 2945.243 = 9768.277 \text{ Newtons.}$$

Thus pressure in cylinder *B* is given by

$$p_B = \frac{F_B}{\pi/4 \times D_B^2} = \frac{9768.277}{\pi/4 \times 50^2/1000^2} = 4974943 \text{ N/m}^2 = \mathbf{4.974943 \text{ MPa.}}$$

Problem 1.9. A manometer contains a fluid having a density of 1200 kg/m^3 . The difference in height of two columns is 400 mm . What pressure difference is indicated. What would be the height difference if the same pressure difference is to be measured by a manometer having mercury of density 13600 kg/m^3 .

Solution. The pressure difference indicated is given by

$$p_1 = \rho g h = 1200 \times 9.81 \times \frac{400}{1000} = 4708.8 \text{ Pa} = 4.7088 \text{ kPa.}$$

$$\text{Again } p_1 = 4708.8 = \rho_g g h_g = 13600 \times 9.81 \times h_g \text{ or } h_g = \frac{4708.8}{13600 \times 9.81} = \mathbf{53.3 \text{ mm.}}$$

Problem 1.10. Given the vessel shown in Fig. 1.12 (a) with pressure gauges indicating reading of *A* = 300 kPa gauge and *B* = 100 kPa gauge. If the barometer reads 750 mm of Hg, determine the reading *C* and convert this value to absolute value.

Solution. The dial of a pressure gauge is marked to read the gauge pressure. The pressure gauge reads the difference in pressures between that inside the 'threaded for connection' and the region in which the dial is located.

Let the pressure in one region be *Y* kPa absolute and the pressure in other region *X* kPa absolute, and barometric pressure = 750 mm Hg = 100 kPa absolute.

$$\text{Thus } Y - 100 = \text{reading on } A = 300 \dots(i)$$

$$Y - X = \text{reading on } B = 100 \dots(ii)$$

$$X - 100 = \text{reading on } C \dots(iii)$$

From (i), $Y = 400 \text{ kPa}$ abs.

Substituting the value of *Y* in (ii), we get

$$400 - X = 100$$

$$\text{or } X = 400 - 100 = 300 \text{ kPa abs.}$$

Thus Substituting the value of *X* in (iii), we get

$$300 - 100 = \text{reading on } C = 200 \text{ kPa gauge}$$

The absolute pressure for reading on *C* = $200 + 100 = 300 \text{ kPa}$ abs.

Problem 1.11. Refer Fig. 1.12 (b). It shows a tank within a tank, each containing air. Pressure gauge *A* is located inside tank *B* and reads 1.5 bar . The U-tube manometer connected to tank *B* contains mercury. Relevant data are labelled on the diagram. Calculate the absolute pressure inside tank *A* and tank *B*, each in bars. The atmospheric pressure surrounding tank *B* is 101 kPa . The acceleration due to gravity is $g = 9.81 \text{ m/sec}^2$.

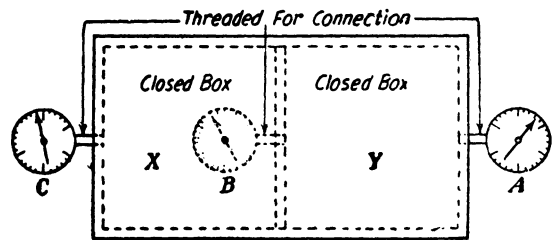
Solution. Refer Fig. 1.12 (b).

$$p_{\text{gauge, } B} = \rho_{\text{Hg}} g h$$

$$= (13.6 \text{ g/cm}^3) (9.81 \text{ m/sec}^2) (0.3 \text{ m}) \times \left(\frac{1 \text{ kg}}{10^3 \text{ g}} \right) \left(\frac{10^6 \text{ cm}^3}{1 \text{ m}^3} \right) \left(\frac{1 \text{ N}}{1 \text{ kgm/sec}^2} \right)$$

$$= 39996 \text{ N/m}^2 = 0.39996 \text{ bar}$$

$$\text{Therefore } p_{\text{abs } B} = p_{\text{atm}} + p_{\text{gauge } B} = 1.01 + 0.39996 = \mathbf{1.40996 \text{ bar}}$$



Box kept in open atmosphere

Barometer = 735.56 cm Hg

Fig. 1.12 (a)

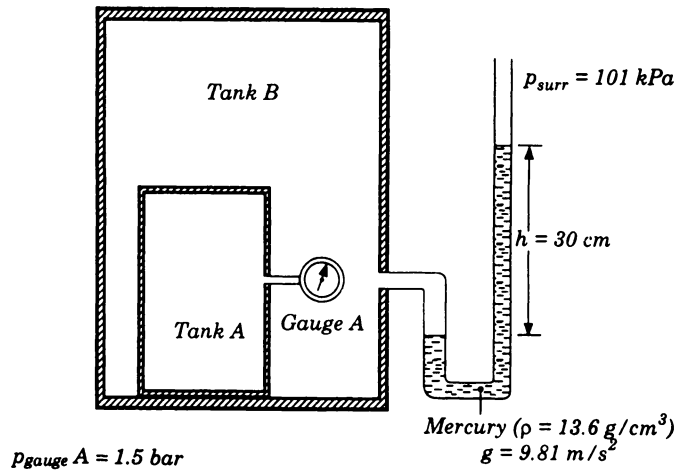


Fig. 1.12 (b)

Thus, $p_{abs} A = p_{abs} B + p_{gauge} A = 1.40996 + 1.5 = 1.90996 \text{ bar}$.

Problem 1.12. An inclined manometer with angle of inclination $\theta = 30^\circ$ is shown in Fig. 1.12 (c). It is fitted to measure the pressure of a gas. The fluid inside the manometer has a density of 0.75 g/cm^3 and the manometer reading is labelled on the diagram as $x = 0.3 \text{ m}$. If the atmospheric pressure is 101.25 kPa and the acceleration due to gravity is $g = 9.7 \text{ m/sec}^2$, determine the absolute pressure of the gas in kPa .

Solution. Refer Fig. 1.12 (c).

For the inclined manometer

$$\begin{aligned} h &= x \sin \theta \\ &= 0.3 \sin 30 \\ &= 0.3 (1/2) \\ &= 0.15 \text{ m.} \end{aligned}$$

Thus it may be observed that inclined manometer gives greater resolution as compared to the vertical one, i.e. $x > h$.

The gauge pressure is given by

$$\begin{aligned} p_{gauge} &= \rho gh = (0.75 \text{ g/cm}^3) \left(9.7 \frac{\text{m}}{\text{sec}^2} \right) (0.15 \text{ m}) \left(\frac{10^6 \text{ cm}^3/\text{m}^3}{10^3 \text{ g/kg}} \right) \left(\frac{1 \text{ N}}{\text{kg.m/sec}^2} \right) \left(\frac{1 \text{ kPa}}{10^3 \text{ N/m}^2} \right) \\ &= 1.09125 \text{ kPa} \end{aligned}$$

Thus, the absolute pressure is given by

$$p_{abs} = p_{gauge} + p_{atm} = 101.25 + 1.09125 = 102.34125 \text{ kPa.}$$

Problem 1.13. Convert (a) $3 \text{ kgf/cm}^2 \text{ abs.}$ to $\text{kgf/cm}^2 \text{ gauge}$. (b) 45 cm vacuum to cm Hg abs. and to $\text{kgf/cm}^2 \text{ abs. (ata)}$. (c) $0.5 \text{ kgf/cm}^2 \text{ abs.}$, i.e., 0.5 ata to cm Hg vacuum . (d) 25 cm Hg gauge to cm Hg abs. and to atm . (e) 1 ata to kPa .

Barometer may be assumed to be 760 mm Hg .

Solution. $1 \text{ kgf/cm}^2 = 735.559 \text{ mm Hg} \approx 735.6 \text{ mm Hg}$.

$$760 \text{ mm Hg} = 1.03323 \text{ kgf/cm}^2 \approx 1.033 \text{ kgf/cm}^2.$$

(a) Absolute pressure – Barometric pressure = Gauge pressure

$$3 - 1.033 = 1.967 \text{ kgf/cm}^2 \text{ gauge.}$$

(b) Barometric pressure – Absolute pressure = Vacuum pressure

$$76 \text{ cm Hg} - \text{Absolute pressure} = 45 \text{ cm Hg.}$$

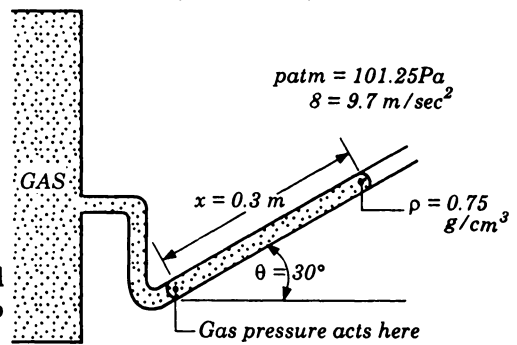


Fig. 1.12 (c)

Therefore, absolute pressure in cm Hg = **31 cm Hg abs.**

And absolute pressure in $\text{kgf/cm}^2 = 1.3595 \times 10^{-2} \times 31 = 0.4215 \text{ kgf/cm}^2 \text{ abs, i.e. } \mathbf{0.4215 \text{ ata.}}$

(c) Barometric pressure – Absolute pressure = Vacuum pressure.

$$1.033 \text{ kgf/cm}^2 - 0.5 \text{ kgf/cm}^2 = 0.533 \text{ kgf/cm}^2 \text{ Vacuum} = 0.533 \times 73.56 \text{ cm Hg vacuum} \\ = 39.20748 \text{ cm Hg vacuum.}$$

(d) Absolute pressure – Barometric pressure = Gauge pressure.

$$\text{Absolute pressure} - 76 \text{ cm Hg} = 25 \text{ cm Hg.}$$

Therefore, absolute pressure = $25 + 76 = \mathbf{101 \text{ cm Hg abs.}} = (101/76) \text{ atm} = \mathbf{1.33 \text{ atm.}}$

(e) $1 \text{ ata} = 1 \text{ kgf/cm}^2 = 0.981 \times 10^2 \text{ kPa} = \mathbf{98.1 \text{ kPa.}}$

1.15. Thermodynamic Co-ordinates

The thermodynamic properties used for locating thermodynamic state point are defined as thermodynamic co-ordinates.

1.16. Equilibrium

When an isolated system is left to itself, all its properties, like pressure, elastic stress etc. may change with time but eventually these changes will cease when the system will be said to be in *mechanical equilibrium*. This, however, does not mean that the pressure is the same at all the points. A vertical column of fluid will be in mechanical equilibrium with different pressures at different elevations of the fluid column.

In thermodynamic, it is imperative to take into account more than purely mechanical equilibrium of a system. If there are variation of temperature from point to point of an isolated system, these should cease so that the system comes to what is called *thermal equilibrium*.

Finally, suppose the system contains substances that can react chemically. All the chemical reactions will cease and eventually a chemical equilibrium will result. *Thus a system in mechanical, thermal and chemical equilibrium is said to be in thermodynamic equilibrium.*

1.17. Path

If the thermodynamic system passes through a series of states, it is said to describe a path.

1.18. Process

Whenever a change of state takes place, process is the cause. The thermodynamic process consists of enough information about the thermodynamic co-ordinates at successive state points in thermodynamic equilibrium to be able to plot a path of the change of state on the thermodynamic plane.

1.19. Cyclic Process

If we start with a particular set of thermodynamic co-ordinates of a system describing its state point and then carry the process through a complete cycle ending up with the initial state point, the system is said to undergo a cyclic process. Thus all the thermodynamic properties are the same at the start and at the end of cyclic process.

1.20. Quasi-static Process

If a process is carried out in such a way that at every instant the system departs only infinitesimally from the thermodynamic equilibrium state, such a process is defined as Quasi-static process, *i.e.* a process closely approximating a succession of equilibrium state is known as quasi-static process. Only a quasi-static process can be represented on a thermodynamic plane (Refer Fig. 1.13). *AB* is a quasi-static process and at the successive states, for example, at 1, 2, 3 etc. the system is very nearly in thermodynamic equilibrium.

1.21. Zeroth Law of Thermodynamics

In previous discussion the term temperature has been mentioned without precisely defining it. And precise definition is also hard to give, though it is a property of the system. Normally equality of temperature is defined. Two isolated systems in thermodynamic equilibrium may be taken ; they can be brought into contact with each other (contact excludes transmission of forces) and observation of thermodynamic co-ordinates taken. If there is no change in the thermodynamic co-ordinates of each system, thermal equilibrium exists between them, and these are in thermal equilibrium. Both the isolated systems have thus a thermodynamic property common and equal. This property is termed as temperature.

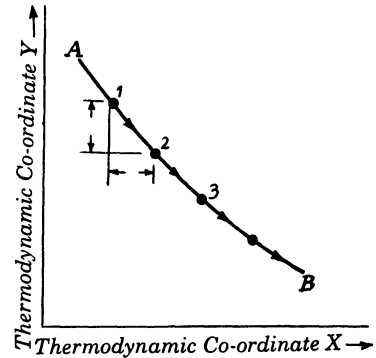


Fig. 1.13

Let an isolated system X be in thermal equilibrium with another isolated system Y. Also let the isolated system Y be in thermal equilibrium with the other isolated system Z. Then experimentally it is proved fact that the isolated system X is in thermal equilibrium with isolated system Z. This gives the postulate called as the Zeroth Law of Thermodynamics. *Zeroth Law of Thermodynamics postulates. "If two systems are both in thermal equilibrium with a third system, they are in thermal equilibrium with each other."*

This law gives us the basis for measuring the thermodynamic property called temperature. A reference system known as *thermometer* is brought in contact separately with two systems, and if the thermometer shows the same readings in both the cases, the two systems are at the same temperature.

1.22. Temperature Scales

Some measurable property of a body which changes with temperature is chosen, for example, a length of a column of mercury in an evacuated capillary tube. Two reference temperature such as ice point at atmospheric pressure and steam point at atmospheric pressure are chosen. Ice point is the equilibrium temperature of ice and air saturated water under one standard atmosphere. Similarly, the steam point is the equilibrium temperature of pure water and steam at one standard atmosphere. Common temperature scale adopted in this country is the celsius scale or centigrade scale. On this scale 0°C is assigned to ice point and 100°C is assigned to steam point. Another scale commonly employed in the British system of unit is Fahrenheit scale in which 32°F is assigned to ice point and 212°F is assigned to steam point. Let us define a third scale and name it as Excellent scale in which we assign 0°E to ice point and 1000°E to steam point. Let us work out the relationship between these three scales (Fig. 1.14).

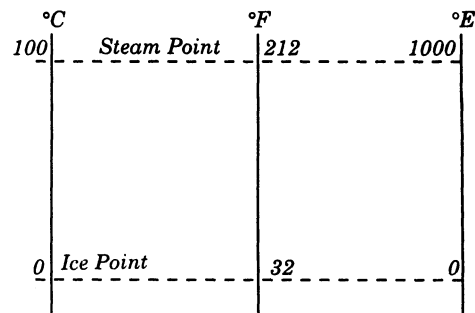


Fig. 1.14

Let the temperature t be linear functions of the property L (length) of the mercury column.

Thus $t = A \cdot L + B$

$$\left[\begin{array}{l} 100 = A \cdot L_S + B \\ 0 = A \cdot L_I + B \end{array} \right] \text{ For } ^\circ\text{C scale}$$

i.e. Solving them, we get $100 = A(L_S - L_I)$

or
$$A = \frac{100}{(L_S - L_I)} \text{ and } B = -A L_I = -\frac{100}{(L_S - L_I)} \cdot L_I$$

Thus
$$t^\circ C = \frac{100}{(L_S - L_I)} \cdot L - \frac{100}{(L_S - L_I)} = \frac{L - L_I}{(L_S - L_I)} \cdot 100 \quad \dots(1.7)$$

Similarly for Fahrenheit scale, we have,
$$t^\circ F = \frac{L - L_I}{(L_s - L_I)} \cdot 180 + 32 \quad \dots(1.8)$$

Equating Eqs. (1.7) and (1.8) for $\frac{L - L_I}{L_S - L_I}$, we have,
$$t^\circ C = \left(\frac{5}{9}\right) (t^\circ F - 32) \quad \dots(1.9)$$

and
$$t^\circ F = \left(\frac{9}{5}\right) \cdot t^\circ C + 32 \quad \dots(1.10)$$

Similarly for an arbitrary scale, *i.e.* Excellent scale

$$t^\circ E = \frac{L - L_I}{L_S - L_I} \times 1000$$

and
$$t^\circ C = t^\circ E \times 10 \quad \dots(1.11)$$

1.23. (a) Thermometers

Any body with at least one measurable property that changes as its temperature changes can be used as a thermometer. Such a property is called a **thermometric property**. The particular substance that exhibits changes in the thermometric properties is **thermometric substance**. A familiar device for temperature measurement is the liquid-in-glass thermometer as shown in Fig. 1.15 (a) which consists of glass capillary tube connected to a bulb filled with liquid such as mercury or alcohol and sealed at the other end. The space above the liquid is occupied by the vapour of liquid or an inert gas. With the increase in temperature, the liquid expands in volume and rises in the capillary. The length L of the liquid in the capillary depends on the temperature. Thus liquid is the **thermometric substance** and L is the **thermometric property**. This type of thermometer is very common but not useful for extreme accuracy. There are other methods of temperature measurements. Commonly used properties of materials employed in temperature sensing devices or thermometers are given below :

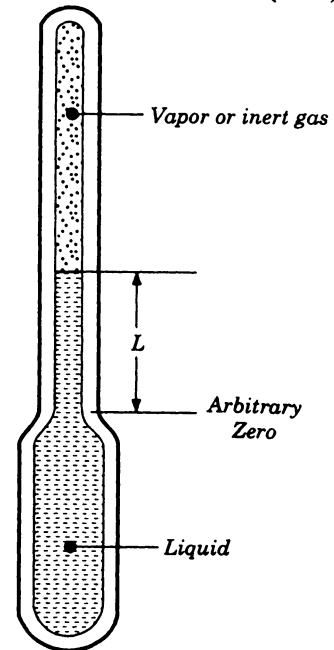


Fig. 1.15 (a)

Table 1.2. Types of Thermometers

S. No.	Thermometer	Thermometric property
1.	Mercury or alcohol in glass thermometer	Length
2.	Constant volume gas thermometer	Pressure
3.	Constant pressure gas thermometer	Volume
4.	Thermocouple	Electromotive force of two dissimilar solids <i>i.e.</i> thermal e.m.f.
5.	Electric resistance thermometer	Electrical resistance of solids
6.	Radiation pyrometer	Intensity of thermal radiation (at high temperatures)
7.	Optical pyrometer	Monochromatic radiation

Constant-Volume Gas Thermometer. The gas thermometer shown in Fig. 1.15 (b) is so exceptional in terms of precision and accuracy that it has been adopted internationally as the standard instrument for calibrating other thermometers.

The thermometric substance is the gas, normally hydrogen or helium, and the thermometric property is the pressure exerted by the gas. The gas is contained in the bulb treated as system with system boundaries shown in Fig. 1.15 (b), and the pressure exerted by it measured by open tube mercury manometer. With increase in temperature, the gas expands and forces the mercury up in the open tube. The constant volume of the gas is maintained by raising or lowering the mercury reservoir. The gas thermometer is used as standard worldwide by bureaus of standards and research laboratories. But due to gas thermometer needing elaborate apparatus and being large, and slowly responding and demanding painstaking experimental procedures, smaller, more rapidly responding thermometers are used for most temperature measurements, and these are calibrated directly or indirectly against gas thermometers.

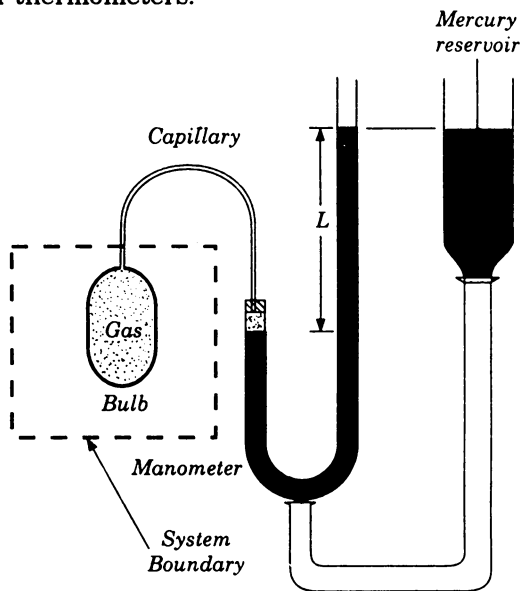


Fig. 1.15 (b)

Sensors, called **thermocouples** are based on the principle of **Seebeck effect**. It comprises a couple formed of two dissimilar metals joined ; an electromotive force (e.m.f.) that is primarily a function of temperature will exist in a circuit. If the junctions of couple are at different temperatures, a current will flow and e.m.f. will be generated. The e.m.f. is the thermometric property measured by microvoltmeter. It is a function of the difference in temperatures of the two junctions.

One junction is kept at ice point, the temperature of other junction is unknown. In certain thermocouples, one thermocouple wire is platinum of specified purity and the other is an alloy of platinum and rhodium. Thermocouples also utilise copper and constantan (an alloy of copper and nickel), or iron and constantan and several other pairs of materials. An illustrative diagram is shown in Fig. 1.15 (c). These thermometers are calibrated. The results of such calibrations can be given by

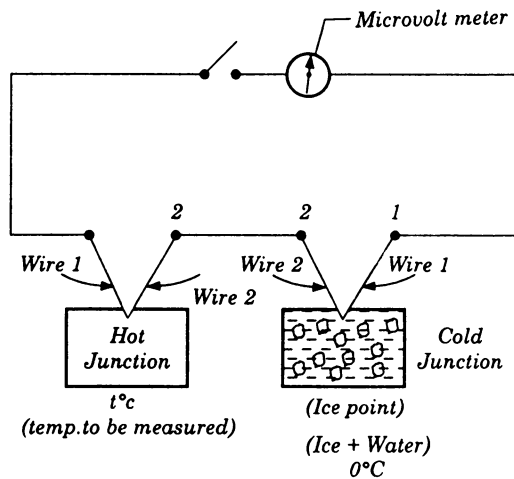


Fig. 1.15 (c)

$$e.m.f. = a + bt + ct^2 + dt^3 \quad \dots(1.12a)$$

Electrical-resistance sensors are another important class of temperature measurements devices. These sensors are based on the fact that electrical resistance of various materials, changes in a predictable manner with temperature. The materials used for this purpose are normally conductors such as platinum, nickel or copper, or semiconductors. Devices using

conductors are known as **resistance temperature detectors** with resistance as the thermometric property, and semiconductor types are called **thermistors**. A variety of instruments measure temperature by radiation. They are known as **radiation pyrometers** using thermal radiation as thermometric property and **optical pyrometers** using monochromatic radiation as thermometric property. These types of thermometers differ from those previously described in that these do not actually come in contact with the body whose temperature is to be measured which is an advantage when dealing with moving objects or bodies at extremely high temperatures.

1.23. (b) Gas Temperature Scale and Kelvin Scale

Numerical value assigned to a **Standard fixed point** defines temperature scales. The standard fixed point which is easily reproducible by international agreement is **triple point of water** : the state of equilibrium between steam, ice and liquid water discussed later. The temperature at this point is defined as 273.16 Kelvin, abbreviated as 273.16°K, as a matter of convenience. This makes the temperature interval from ice point = 273.15°K to steam point = 373.15°K equal to 100°K. The ice point is the state of equilibrium between ice and air-saturated water at pressure of 1 atm. The steam point is the state of equilibrium between steam and liquid water at pressure of 1 atm. Thus it is in agreement over the interval with the Celcius scale already explained.

Let p stand for pressure in constant volume gas thermometer in thermal equilibrium with, say, a bath. The bath temperature can be assigned the value by linear relation

$$T = ap \quad \dots(1.12b)$$

where a is arbitrary constant. The linear relationship is an arbitrary choice. Other relationship could also be chosen.

The value of a may be found out by putting the thermometer in another bath kept at the triple point of water and the pressure measured. Call this pressure p_{tp} of the confined gas.

Thus, we get
$$a = \frac{273.16}{p_{tp}} \quad \dots(1.12c)$$

And the temperature of original bath can be expressed as

$$T = 273.16 \left(\frac{p}{p_{tp}} \right) \quad \dots(1.12d)$$

However, since the values of both pressures p and p_{tp} , depend in part on the amount of the gas in the bulb, the value assigned to the bath temperature varies with the amount of gas in the thermometer. The problem is overcome in precision thermometry by repeating the measurements in original and the reference bath several times with less gas in the bulb in each successive attempts. For each trial the ratio p/p_{tp} is calculated and plotted versus the corresponding reference pressure p_{tp} of the gas at triple point temperature. Such points when plotted and extrapolated to ordinate where $p_{tp} = 0$ give revealing results as shown in Fig. 1.15 (d) for constant volume thermometer with number of different gases. At each non-zero value of the reference pressure, the p/p_{tp} values differ with the gas employed in the thermometer. But when pressure decreases, the p/p_{tp} values from thermometers with different gases approach one another and in the limit when pressure tends to zero, **the same value is obtained for each gas.**

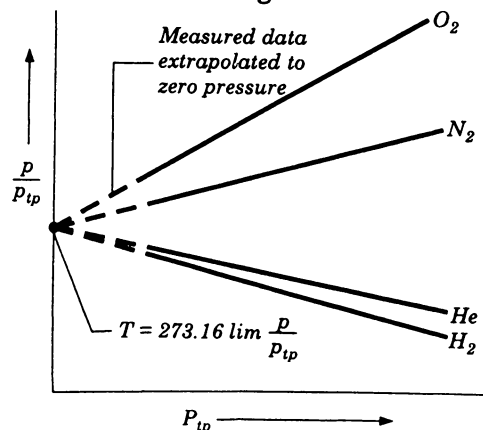


Fig. 1.15 (d)

Thus, the temperature scale is defined as $T = 273.16 \lim \frac{p}{p_{tp}}$.. (1.12e)

where “lim” means that both p and p_{tp} tend to zero.

At this stage it is sufficient to mention that this scale is independent of the properties of any one gas, but does depend on properties of gases in general. For example, it is imperative that gas does not condense. The lowest temperature measurable with such gas thermometer is 1°K with Helium. Also at high temperatures, the gases dissociate. Thus it is desirable to have temperature values measured which do not depend on the properties of the substance. Such a scale is called **Thermodynamic Temperature Scale** based on second Law of thermodynamics explained in subsequent chapters.

Problem 1.14. A constant volume gas thermometer containing nitrogen is brought into contact with a system of unknown temperature and then into contact with a system maintained at the triple point of water. The mercury column attached to the device has reading of 59.2 and 2.28 cm respectively for the two systems. If the barometric pressure is 960 m bar (96.0 kPa) what is the unknown temperature in kelvin, if $g = 9.806 \text{ m/sec}^2$. Specific gravity of mercury may be taken as 13.6.

Solution. The unknown temperature is determined by

$$T = 273.16 \frac{p}{p_{tp}}$$

$$p_{gauge} \text{ (unknown temperature)} = \rho g h_{unknown}$$

$$= 13.6 \frac{\text{g}}{\text{cm}^3} \times 9.806 \frac{\text{m}}{\text{sec}^2} \times 59.2 \text{ cm}$$

$$= 13.6 \frac{\text{kg} \times 10^6}{1000 \text{ m}^3} \times 9.806 \frac{\text{m}}{\text{sec}^2} \times 59.2 \frac{\text{m}}{100}$$

$$= 78950 \frac{\text{kg}}{\text{m sec}^2} = 78950 \text{ Pa}$$

$$= \mathbf{78.95 \text{ kPa} = 789.5 \text{ m bar}}$$

$$p_{gauge} \text{ (triple point temperature)} = \rho g h_{tp}$$

$$= 13.6 \times 10^3 \frac{\text{kg}}{\text{m}^3} \times 9.806 \frac{\text{m}}{\text{sec}^2} \times 2.28 \frac{\text{m}}{100}$$

$$= 3040.6 \frac{\text{kg}}{\text{m sec}^2} = 3040.6 \text{ Pa}$$

$$= \mathbf{3.0406 \text{ kPa} = 30.406 \text{ m bar}}$$

Therefore, the absolute pressures in the two cases are :

$$p_{abs} \text{ (unknown temperature)} = 789.5 + 960 = \mathbf{1749.5 \text{ m bar} = 174.95 \text{ kPa}}$$

$$p_{abs} \text{ (triple point temperature)} = 30.406 + 960 = \mathbf{990.406 \text{ m bar} = 99.046 \text{ kPa}}$$

Thus unknown absolute temperature is given by

$$T = 273.16 \frac{p}{p_{tp}} = 273.16 \frac{1749.5}{990.406}$$

$$= \mathbf{482.52^\circ\text{K} \approx 210^\circ\text{C}}$$

Problem 1.15. A constant volume gas thermometer (Fig. 1.15 (e)) immersed in boiling water at $t_1 = 100^\circ\text{C}$ reads $Z_1 = 30.0 \text{ cm Hg}$. Later the thermometer is immersed in a radiator fluid at temperature of 150°C ; that time, barometric pressure is 76.0 cm Hg. Determine the height Z_2 of the mercury column, in cm. Assume the gas in the thermometer as perfect gas.

Solution. For perfect gas maintained at constant volume $p/T = \text{constant}$, where the gas pressure p is the sum of the atmospheric pressure and the head pressure and the gas temperature is in °K. Thus,

$$(Z_0 + Z_2)/T_2 = (Z_0 + Z_1)/T_1$$

or $(76 + Z_2)/(150 + 273) = (76 + 30)/(100 + 273)$

or
$$Z_2 = \frac{(76 + 30)(150 + 273)}{(100 + 273)} - 76$$

$$= 42.2 \text{ cm.}$$

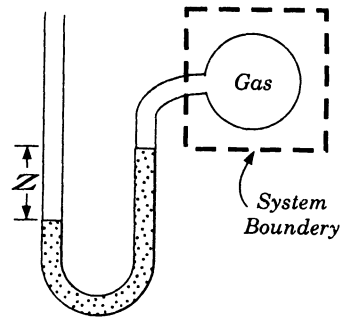


Fig. 1.15 (e)

Additional Comments. It has been shown that different curves are obtained for different gases, but as the pressure tends to zero, these curves indicate the same ratio $\frac{T}{T_{tp}} = \frac{p}{p_{tp}}$ and therefore the same temperature T . This temperature is known as **ideal gas temperature**. Extending the discussion further, if the pressures are measured at triple point (p_{tp}) and ice point p_i , the value of temperature at ice point namely T_i is given by

$$T_i = 273.16 \lim_{p_{tp} \rightarrow 0} \frac{p_i}{p_{tp}} \quad \dots(1.12f)$$

and this is equal to 273.15 as shown in Fig. 1.15 (e). Similarly the temperature at steam point T_s can be obtained in terms of pressure at ice point p_s . Thus,

$$T_s = 273.16 \lim_{p_{tp} \rightarrow 0} \frac{p_s}{p_{tp}} \quad \dots(1.12g)$$

and this is equal to 373.15 as shown in Fig. 1.15 (f).

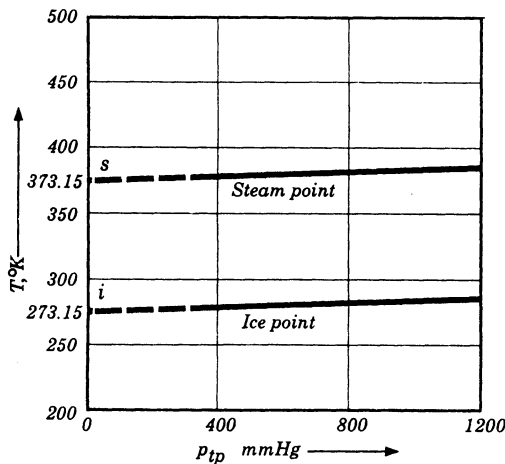


Fig. 1.15 (f)

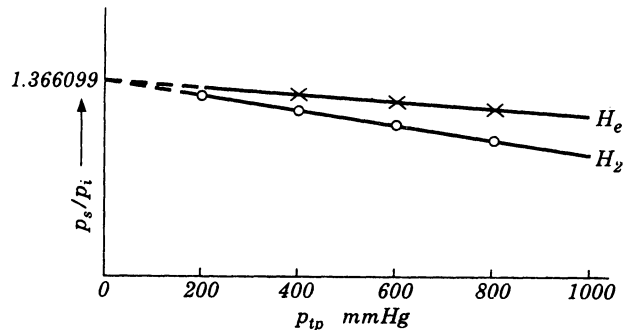


Fig. 1.15 (g)

Alternatively, if plots of p_s/p_i versus p_{tp} are made with different gases in the constant-volume gas thermometer bulb, straight line curves are obtained as shown in Fig. 1.15 (g) for Helium and Hydrogen. When these curves are extrapolated to p_{tp} equal to zero, they converge

and the ratio p_s/p_i tends to 1.366099 which is constant so that

$$\frac{T_s}{T_i} = \lim_{p_{tp} \rightarrow 0} \frac{p_s}{p_i} = 1.366099.$$

This value may be considered as **Universal constant of nature**.

Thus, we have got two equations namely

$$\frac{T_s}{T_i} = 1.366099 \quad \dots(1.12h)$$

and $T_s - T_i = 100 \quad \dots(1.12i)$

Also 100 divisions are chosen between steam point and ice point as in kelvin and celsius scales.

Solving these two equations, we get

$$1.366099 T_i - T_i = 100$$

or $T_i = \frac{100}{0.366099} = 273.15^\circ\text{K}.$

and $T_s = T_i + 100 = 273.15 + 100 = 373.15^\circ\text{K}.$

These values do not assume any numerical value of triple point, which on the ideal temperature scale discussed earlier is 0.01°K above ice point and this becomes 273.16°K . Thus the value of 273.16 assigned arbitrarily is not so arbitrary.

1.22. (c) The International Practical Temperature Scale

In 1968, the International Committee on weights and Measures adopted a revised International Temperature Scale, IPTS-68 which is described below. This scale is an improvement in previous scales and is made to conform more closely to the thermodynamic temperature scale. It is based on a number of fixed points easily reproducible. These points are given definite numerical values of temperature.

It may be again explained that the **triple point** represents an equilibrium state between solid, liquid and vapour phases of a substance. **Normal boiling point** is the temperature at which the substance boils at standard atmospheric pressure of 1.01325 bar or 760 mm Hg. **Normal freezing point** is the solidification or the melting point temperature of substance at standard atmospheric pressure of 1.01325 bar or 760 mm Hg.

The primary fixed point temperatures in degrees celsius are as follows :

1. Triple point (equilibrium between solid, liquid and vapour phases) of equilibrium-hydrogen - 259.34°C
2. Boiling point (equilibrium between liquid and vapour phases) of equilibrium-Hydrogen at $\frac{25}{76}$ atm (0.3333 bar) pressure - 256.108°C
3. Normal boiling point (at 1 atm *i.e.* 1.01325 bar) of equilibrium-hydrogen - 252.87°C
4. Normal boiling point of Neon - 246.048°C
5. Triple point of oxygen - 218.789°C
6. Normal boiling point of oxygen - 182.962°C
7. Triple point of water 0.01°C
8. Normal boiling point of water 100°C
9. Normal freezing point (equilibrium between solid and liquid phases) of Antimony 357.57°C
10. Normal freezing point (equilibrium between solid and liquid phases) of Zinc 419.58°C

11. Normal freezing point (equilibrium between solid and liquid phase) of silver 961.93°C
12. Normal freezing point of gold 1064.43°C

The means available for measurements and interpolation lead to divisions of temperature scale in four ranges :

1. The range from -259.34°C to 0°C is based on a platinum resistance thermometer with temperature expressed in terms of $1/20$ of degree reference function equation. This range is sub-divided into four parts. In each, the difference between measured resistance ratios of a particular thermometer and the reference function at the fixed points are used to determine the constants in a specified polynomial interpolation equation.

2. The range from 0°C to 630.74°C (the normal freezing point of antimony, a secondary fixed point) is also based on a platinum resistance thermometer, with constants in a polynomial interpolating equation determined by calibration at three fixed points in the range.

3. The range from 630.74°C to 1064.43°C is based on a standard platinum vs rhodium-platinum thermocouple, and a three-term equation expressing e.m.f. as a function of temperature. The constants are determined by a platinum resistance thermometer measurements at antimony point and by calibration at the two primary fixed points in the range.

4. The range above 1064.43°C is based on measurements of the intensity of visible-spectrum radiation compared with that of the same wavelength at the gold point, and on Plank's equation for black body radiation.

Problem 1.16. Over a specified temperature range, the electrical resistance thermometer holds the relationship as :

$$R = R_0 [1 + a (t - t_0)]$$

where R_0 is resistance in ohms, measured at reference temperature t_0 in $^{\circ}\text{C}$ and a is material constant with units of $(^{\circ}\text{C})^{-1}$. The test results are :

$$R = 51.39 \text{ ohms at } t = 0^{\circ}\text{C}, R = 51.72 \text{ ohms at } t = 91^{\circ}\text{C}.$$

Determine the reading of the resistance for 70°C , 50°C and 100°C on this thermometer.

Solution. Let $t_0 = 0^{\circ}\text{C}$, then from the given data at $t = 0^{\circ}\text{C}$

$$51.39 = R_0 [1 + a (0 - 0)]$$

Therefore, $R_0 = 51.39$ ohms.

$$\text{Again } 51.72 = 51.39 [1 + a (91 - 0)]$$

$$\text{Therefore, } a = \frac{\left[\frac{51.72}{51.39} - 1 \right]}{91} = \frac{0.33}{51.39 \times 91} = 7.05657 \times 10^{-5}/^{\circ}\text{C}$$

$$\text{Thus } R = 51.39 [1 + (7.05657 \times 10^{-5}) t]$$

where t is expressed here in $^{\circ}\text{C}$

$$= 51.39 [1 + (7.05657 \times 10^{-5}) \times 70] = \mathbf{51.644 \text{ ohm at } 70^{\circ}\text{C}}$$

$$\text{And } R = 51.39 [1 + (7.05657 \times 10^{-5}) \times 50] = 51.57 \text{ ohm at } 50^{\circ}\text{C}$$

$$\text{And } R = 51.39 [1 + (7.05657 \times 10^{-5}) \times 100] = 51.75 \text{ ohm at } 100^{\circ}\text{C}.$$

Problem 1.17. The relationship of resistance R in ohm and temperature T in kelvin for a thermistor is given by :

$$R = R_0 e^{\left[\beta \left(\frac{1}{T} - \frac{1}{T_0} \right) \right]}$$

where R_0 is the resistance in ohms measured at temp. $T_0^{\circ}\text{K}$
and β is the material constant with units of $^{\circ}\text{K}$

For a particular case $R_0 = 2.2 \text{ ohm at } T_0 = 310^\circ\text{K}$

And $R = 0.31 \text{ ohm at } T = 422^\circ\text{K}$.

Determine the material constant β and hence find the resistance at 373°K and 273°K .

Solution. The equation for the thermistor is given by

$$R = R_0 e^{\left[\beta \left(\frac{1}{T} - \frac{1}{T_0} \right) \right]}$$

where $R_0 = 2.2 \text{ ohm}$ and $T_0 = 310^\circ\text{K}$

Since $R = 0.31 \text{ ohm at } T = 422^\circ\text{K}$

$$0.31 = 2.2 e^{\left[\beta \left(\frac{1}{422} - \frac{1}{310} \right) \right]}$$

or $0.31 = 2.2 e^{\left[-8.5613 \times 10^{-4} \beta \right]}$

Therefore, $\beta = \frac{\log e^{\left(\frac{0.31}{2.2} \right)}}{-8.5613 \times 10^{-4}} = \frac{-1.95964}{-8.5613 \times 10^{-4}} = 2289.95^\circ\text{K}$

Thus resistance at $T = 373^\circ\text{K}$

$$\begin{aligned} R_{373} &= 2.2 e^{\left[2289.95 \left(\frac{1}{373} - \frac{1}{310} \right) \right]} \\ &= 2.2 e^{-1.247} = \mathbf{0.632 \text{ ohm.}} \end{aligned}$$

and at $T = 273^\circ\text{K}$

$$\begin{aligned} R_{273} &= 2.2 e^{\left[2289.95 \left(\frac{1}{273} - \frac{1}{310} \right) \right]} \\ &= 2.2 e^{1.0007228} = \mathbf{5.9845 \text{ ohm.}} \end{aligned}$$

Problem 1.18. Define a new temperature scale, say $^\circ\text{N}$ in which the boiling and freezing points of water, (steam and ice points) are 400°N and 100°N respectively, and co-relate this with the Centigrade scale.

The $^\circ\text{N}$ reading on the scale is a certain number of degree on a corresponding absolute temperature. What is this absolute temperature at 0°N ?

Solution. $400 = A \cdot L_S + B$; $100 = A \cdot L_I + B$

or $A = \frac{300}{L_S - L_I}$ and $B = 100 - \frac{300}{L_S - L_I} \cdot L_I$

Thus $t^\circ\text{N} = \frac{300}{L_S - L_I} L + 100 - \frac{300}{L_S - L_I} = \frac{L - L_I}{L_S - L_I} \cdot 300 + 100$... (i)

Also we have $t^\circ\text{C} = \frac{L - L_I}{L_S - L_I} \cdot 100$... (ii)

One comparing Eqs. (i) and (ii), we get $t^\circ\text{N} = 3t^\circ\text{C} + 100$

Since $\frac{T_S}{T_I} = \lim_{P_I \rightarrow 0} \frac{P_S}{P_I} \approx 1.366099$ and $T_S - T_I = 300$

Therefore, $1.366099T_I - T_I = 300$

or $T_I = \frac{300}{0.366099} = 819.45^\circ\text{N abs} \approx 820^\circ\text{N abs}$

and $T_S = 819.45 + 300 = 1119.45^\circ \text{N abs} \approx 1120^\circ \text{N abs}$.

$$T^\circ \text{N abs} = (t^\circ \text{N} - 100) + 819.45 = t^\circ \text{N} + 719.45$$

Thus $T^\circ \text{N abs at } 0^\circ \text{N} = 719.45^\circ \text{N abs} \approx 720^\circ \text{N}$.

Problem 1.19. Define a new temperature scale, say $^\circ \text{F}$ in which the steam point and ice point are 212°F and 32°F . The $^\circ \text{F}$ reading on this scale is a number of degrees on a corresponding absolute temperature scale (also called Rankine scale). What is this absolute temperature at 0°F and 100°F ?

Solution.
$$\frac{T_S}{T_I} = \lim_{P_I \rightarrow 0} \frac{P_S}{P_I} = 1.366099 ; T_S - T_I = 180.$$

Therefore, $1.366099 T_I - T_I = 180$

and
$$T_I = \frac{180}{0.366099} = 491.67^\circ \text{F abs or } 491.67^\circ \text{R},$$

and $T_S = 491.67 + 180 = 671.67^\circ \text{F abs or } 671.67^\circ \text{R}$

$$T^\circ \text{R} = (t^\circ \text{F} - 32) + 491.67 = t^\circ \text{F} + 459.67$$

Thus $T^\circ \text{R} = 0^\circ \text{F} + 459.67 = 459.67^\circ \text{R} \approx 459.7^\circ \text{R}$

$$T^\circ \text{R} = 100^\circ \text{F} + 459.67 = 559.67^\circ \text{R} \approx 559.7^\circ \text{R}.$$

Problem 1.20. A Centigrade and a Fahrenheit thermometer are both immersed in a fluid and the numerical reading recorded on both the thermometers is the same. Determine the temperature of the fluid expressed as $^\circ \text{K}$ and $^\circ \text{R}$ and also find that identical numerical value shown by thermometer.

Solution. Since $t^\circ \text{C} = (T^\circ \text{K}) - 273.15$, $t^\circ \text{F} = (T^\circ \text{R}) - 459.67$

Thus for $t^\circ \text{C} = t^\circ \text{F}$ numerically, we have

$$(T^\circ \text{K}) - 273.15 = (T^\circ \text{R}) - 459.67$$

i.e., $(T^\circ \text{R}) - (T^\circ \text{K}) = 459.67 - 273.15$ numerically

where $T^\circ \text{R}$ is a figure on Rankine scale and $T^\circ \text{K}$ is the corresponding figure on Kelvin scale. Numerical figure on Rankine will be 1.8 times more than the numerical figure on the Kelvin scale.

$$\frac{T^\circ \text{R}}{T^\circ \text{K}} = 1.8$$

i.e. $(T^\circ \text{R}) = 1.8 (T^\circ \text{K})$ numerically

Thus $1.8 (T^\circ \text{K}) - T^\circ \text{K} = 459.67 - 273.15$

or $(T^\circ \text{K}) = \frac{186.52}{0.8} = 233.15^\circ \text{K} = -40^\circ \text{C}$

and $(T^\circ \text{R}) = 1.8 (T^\circ \text{K}) = 1.8 \times 233.15$ numerically $= 419.67 = -40^\circ \text{F}$.

Problem 1.21. Fahrenheit and Centigrade thermometers are both immersed in fluid. Fahrenheit reading is numerically twice that of the Centigrade reading. What is temperature of the fluid expressed as $^\circ \text{R}$ and $^\circ \text{K}$?

Solution. Since $t^\circ \text{C} = T^\circ \text{K} - 273.15$; $t^\circ \text{F} = T^\circ \text{R} - 459.67$

By data $t^\circ \text{F} = 2t^\circ \text{C}$

$\therefore T^\circ \text{R} - 459.67 = 2(T^\circ \text{K} - 273.15)$. Also $T^\circ \text{R} = 1.8T^\circ \text{K}$ numerically.

Thus $2T^\circ \text{K} - 1.8T^\circ \text{K} = 546.30 - 459.67$ or $0.2T^\circ \text{K} = 86.63$

or $T^\circ \text{K} = \frac{86.63}{0.2} = 433.15^\circ \text{K}$

or $433.15^\circ \text{K} = 1.8 \times 433.15^\circ \text{R} = 779.67^\circ \text{R}$.

Problem 1.22. A constant gas thermometer immersed in boiling water at $t_1 = 100^\circ\text{C}$ reads $h_1 = 30.0$ cm Hg. Then the thermometer is immersed in other fluid at a temperature of 150°C . The barometer pressure is 76.0 cm Hg (i) Determine the height h_2 of the mercury column in cm.

(ii) If the thermometer is dipped in another fluid and height of mercury column is 40 cm, determine the temperature $^\circ\text{C}$ of fluid.

(iii) At what temperature $^\circ\text{C}$ will the height of mercury column be exactly zero in the thermometer.

(iv) If the height of mercury column is -9.5 cm, what is fluid temperature $^\circ\text{C}$ and comment.

(v) Derive the working equation relating manometric height h in cm to temperature t in $^\circ\text{C}$ for the gas thermometer.

Solution. The unit of pressure is advisably cm Hg, for this problem. It is important to note that gas in the thermometer has to be perfect so that $p/T = \text{constant}$ at constant volume. The atmospheric pressure denoted by h_0 is 76 cm Hg. For convenience 273.16 or 273.15 is taken as 273 for conversion to absolute temperature.

$$\text{Thus} \quad \frac{h_0 + h_2}{T_2} = \frac{h_0 + h_1}{T_1} \quad \text{or} \quad \frac{76 + h_2}{150 + 273} = \frac{76 + 30}{100 + 273}$$

$$\text{or} \quad h_2 = \left(\frac{76 + 30}{100 + 273} \right) (150 + 273) - 76 = \mathbf{44.21 \text{ cm}}$$

$$\text{(ii)} \quad \frac{h_0 + h_2}{T_2} = \frac{h_0 + h_1}{T_1} \quad \text{or} \quad \frac{76 + 40}{T_2} = \frac{76 + 30}{100 + 273}$$

$$\text{or} \quad T_2 = \frac{(76 + 40)(100 + 273)}{76 + 30} = \mathbf{408.2^\circ\text{K}} \quad \text{or} \quad t_2 = 408.2 - 273 = \mathbf{135.2^\circ\text{C}}$$

$$\text{(iii)} \quad \frac{h_0 + h_2}{T_2} = \frac{h_0 + h_1}{T_1} \quad \text{or} \quad \frac{76 + 0}{T_2} = \frac{76 + 30}{100 + 273}$$

$$\text{or} \quad T_2 = 76 \times \frac{100 + 273}{76 + 30} = \mathbf{267.4^\circ\text{K}} \quad \text{or} \quad t_2 = \mathbf{5.6^\circ\text{C}}$$

$$\text{(iv)} \quad \frac{h_0 + h_2}{T_2} = \frac{h_0 + h_1}{T_1} \quad \text{or} \quad \frac{76 - 9.5}{T_2} = \frac{76 + 30}{100 + 273}$$

$$\text{or} \quad T_2 = (76 - 9.5) \times \frac{100 + 273}{76 + 30} = \mathbf{234.4^\circ\text{K}} \quad \text{or} \quad t_2 = \mathbf{-39^\circ\text{C}}$$

$$\text{(v) We have} \quad \frac{76 + h}{t + 273} = \frac{76 + 30}{100 + 273}$$

$$\text{Thus} \quad h = \left(\frac{76 + 30}{100 + 273} \right) (t + 273) - 76 = 1.582 + 0.2842 t$$

$$\text{or} \quad h = \mathbf{1.582 + 0.2842 t.}$$

Note. Finally, when making engineering calculations it is common to round off the last numbers or the decimals and write 273 for 273.15 and 460 for 459.67.

1.24. Energy

Simplest definition of Energy is "capacity for producing an effect". It can be classified as (i) stored energy which is contained within the system boundaries and (ii) energy in transition which crosses the system boundaries. Examples of stored energy are : (i) Potential energy, (ii) Kinetic energy, (iii) Internal energy etc. Examples of energy in transition are : (i) Heat (ii) Work (iii) Electrical energy etc.

1.25. Potential Energy

Potential energy is the energy contained in the system by virtue of its elevation with reference to an arbitrary chosen datum usually the sea level. Or alternatively it is equivalent to energy required to raise the system from an arbitrary datum usually the sea level.

If F = Force exerted by the system of mass m where the local acceleration due to gravity is g .

$$\text{We have} \quad F \propto m \cdot g = \frac{1}{g_c} \cdot m g = \frac{m}{g_c} \cdot g$$

If Z = Total distance through which system is raised.

$$\text{Potential energy (P.E.)} = \int_0^Z F dZ = \int_0^Z \frac{m}{g_c} \cdot g dZ = \frac{m}{g_c} \cdot Zg \quad \dots(1.14(A))$$

Note : [$g_c = 9.81$ in MKS Units, and $g_c = 1$ in S.I. units]

$$\text{Thus} \quad F \text{ (newton)} = m \text{ (kg)} \times g \text{ (m/sec}^2\text{)}$$

$$\text{and} \quad PE \text{ (Nm or J)} = m \text{ (kg)} \times z \text{ (m)} \times g \text{ (m/sec}^2\text{)} \quad \dots(1.14(B))$$

1.26. Kinetic Energy

Energy possessed by a system by virtue of its motion as a whole is called Kinetic Energy. Alternatively, it is the energy that could be recovered when the system is brought to stagnation state in the absence of friction.

$$\text{We have} \quad F = \frac{m}{g_c} \cdot \frac{dV}{dt}$$

$$\text{and} \quad \text{Work} = F dx = \frac{m}{g_c} \cdot \frac{dV}{dt} \cdot dx = \frac{m}{g_c} \cdot \frac{dx}{dt} \cdot dV$$

$$\text{or} \quad \text{Work} = \int_0^x F dx = \int_0^V \frac{m}{g_c} \cdot \frac{dx}{dt} dV = \text{Kinetic energy}$$

$$= \int_0^V \frac{m}{g_c} V dV = \left[\frac{M}{2g_c} V^2 \right]_0^V$$

$$\text{Thus} \quad K.E. = \frac{1}{2} \frac{m}{g_c} V^2 \quad \dots(1.15(a))$$

Note : [$g_c = 9.81$ in MKS Units and $g_c = 1$ in S.I. units]

$$K.E. \text{ (Nm or J)} = \frac{1}{2} m \text{ (kg)} V^2 \text{ m}^2/\text{sec}^2 \quad \dots(1.15(b))$$

Problem 1.23. An artificial satellite in the earth's atmosphere has a velocity of 900 m/sec relative to earth. Calculate the kinetic energy possessed by it (a) where acceleration due to gravity is 9.5 m/sec² and the gravitational force is 4500 N, (b) where acceleration due to gravity is 9 m/sec² and gravitational force is 4500 N.

Solution. Force $(N) = m \text{ (kg)} \times g \text{ (m/sec}^2\text{)}$

$$4500 = m \times 9.5 ; \text{ or } m = 4500 \times \frac{1}{9.5} = 473.684 \text{ kg}$$

$$\text{Again} \quad K.E. = \frac{1}{2} m V^2$$

$$\begin{aligned} \text{(a) i.e.} \quad K.E. &= \frac{1}{2} \times 473.684 \times (900)^2 = 19184.21 \times 10^4 \text{ Nm or Joules} \\ &= \mathbf{191.8421 \text{ MJ (Mega Joules).} \end{aligned}$$

(b) And for

$$g = 9 \text{ m/sec}^2$$

$$K.E. = \frac{1}{2} \cdot \frac{4500}{9} \times (900)^2 = 20250 \times 10^4 \text{ Nm or Joules} = \mathbf{202.5 \text{ MJ.}}$$

Problem 1.24. A 5 kg mass contains 12000 J of kinetic energy at datum, where $g = 9.8 \text{ m/sec}^2$. Determine the maximum height to which the mass will be replaced if all the kinetic energy is converted into potential energy.

Solution. $P.E. = mgZ = 5 \times 9.8 \times Z = K.E. = 12000 \text{ J}$

or $Z = \frac{12000}{5 \times 9.8} = \mathbf{244.9 \text{ metres.}}$

1.27. Work

In thermodynamics, work is defined as the energy in transition. A definition for work which would allow for all forms of work including mechanical work may be given as follows :

Work is transferred from the system during a given operation if the sole effect external to the system or the surrounding can be reduced to the rise of a weight. The different phrases used in the definition require explanation. When a system is considered for macroscopic study, the manner in which a process takes place within the system is of no consequence. Only the energy transfer across the system boundary is of significance. Therefore, the effect of the energy in transition should be felt external to the system boundary or on surrounding. And this effect could be reduced to only the rise of a weight.

In Thermodynamics, by convention, work done by the system is considered *positive* and the work done on the system is considered *negative*. When the system does positive work, its surroundings do an equal amount of negative work. Thus in any process $W_{system} + W_{surrounding} = 0$.

Also, it is relevant to mention that work is manifest at system boundary only during any interaction between the system and surroundings. Before interaction no work is present and after interaction no work is present. Only work transfer takes place.

Let us consider a system comprising a battery and a motor as shown in Fig. 1.16.

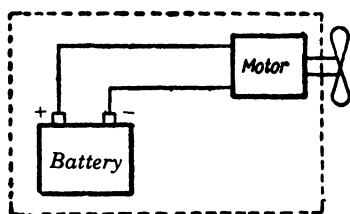


Fig. 1.16

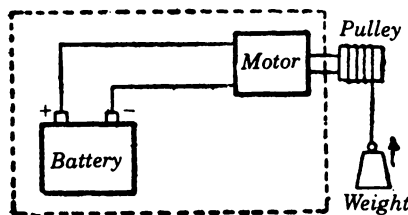


Fig. 1.17

The motor may drive a fan. Thus the effect of the work transfer from the system boundary is the driving of the fan. In order to satisfy the definition fully, it should be possible to reduce this effect to only the rise of weight. If the fan is replaced by a pulley weight arrangement shown in Fig. 1.17, the turning of motor will raise the weight and that the sole effect is the raising of weight. Thus, for system of Fig. 1.16, the sole effect external to the system can be reduced to the rise

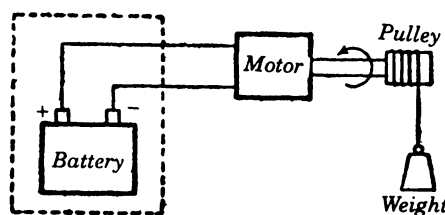


Fig. 1.18

of weight and the work is thus transferred from the system. Let us consider a system as shown in Fig. 1.18. The system boundaries have been changed to exclude the motor. In this case, the flow of electricity takes place across the boundaries. If the motor on bearings is idealised to have 100% efficiency, the sole effect of the transfer of electrical energy external to the system

is the raising of a weight. Thus flow of electrical energy across the boundary of the system shown in Fig. 1.18 constitutes work.

Problem 1.25. Fig 1.19 shows an electrical storage battery discharging through a resistor. Apply definition of the work and show that the effect external to the system can be reduced to the rise of a weight.

Solution. Consider the system shown in Fig. 1.19 comprising an electrical storage

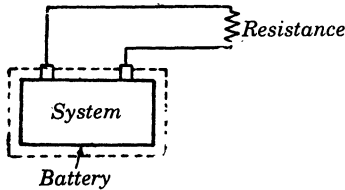


Fig. 1.19

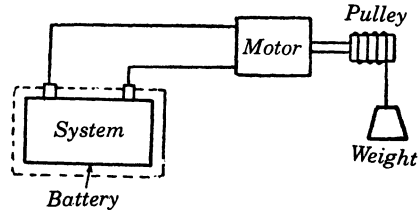


Fig. 1.20

battery. We suppose that current flows through the battery and the resistance becomes warmer. The system has, therefore, interacted with the surroundings. Can this interaction be classified as work? According to the definition, by Laws of Mechanics the answer is 'No', for no force has moved its point of application. Yet according to thermodynamic definition, the answer is 'Yes', as explained below.

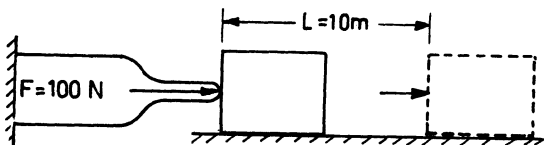
Imagine the surrounding of the system to be altered by the replacement of the resistance by an electric motor and a pulley on its shaft which winds up the string on which a weight is suspended as shown in Fig. 1.20. This can be arranged so that exactly the same current flows when the resistance was there. Thus taking care that all friction is eliminated and that lead wires do not warm up, sole effect external to the system has been the rise of a weight. Thus the system does work.

Problem 1.26. A horizontal force of 100 N is applied by the system on its surroundings at one point of its boundary. The surrounding comprise a block on a rough surface, which is pushed 10 metres horizontally along as the system changes shape. Determine work done by the system by the definition of mechanics and also by the definition of thermodynamics.

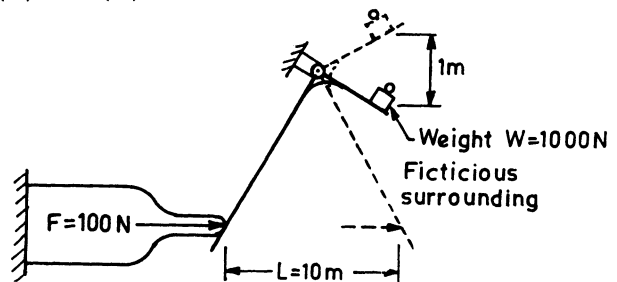
Solution. Measurement of work by thermodynamic definition may be explained thus. The work done by the system may be measured by mounting the standard weights which could have been raised through standard vertical height in a standard gravitational field. In metric system of units, the weight, is taken to comprise 1 kg of mass each, the standard height is 1 metre and the gravitational acceleration has the standard value of 9.80665 metres per sec. $\approx 9.81 \text{ m/sec}^2$. The gravitational force for each kg is, therefore, 1 N and the work unit is $N\cdot m$.

Refer to Fig. 1.21 showing the system exerting force of 100 N and displacing the actual surrounding by 10 m by its boundary change. By mechanics definition

$$\text{Work} = F.L. = 100 \text{ (N)} \times 10 \text{ (m)} = 1000 \text{ N}\cdot\text{m}.$$



(Actual Surrounding)
Fig. 1.21



(Fictitious Surrounding)
Fig. 1.22

Refer to Fig. 1.22, where the surrounding has been replaced by fictitious surrounding to suit the thermodynamic definition of work. The sole effect external to the system is the rise of weight = 1000 N through a distance equal to 1 metre. Thus work $W = 1000 \text{ N}\cdot\text{m}$ or J .

1.28. Work done during a Quasi-static or Quasi-Equilibrium Process

There are various ways of work done by the system like work done by rotating shafts, electrical work, work done by displacement of the boundary of the system as in piston cylinder arrangement, Prior to this no specific mention was made to how the system co-ordinates

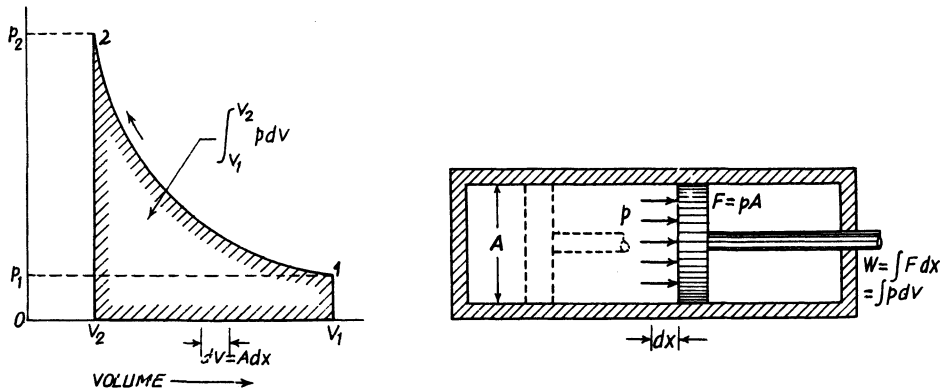


Fig. 1.23

changed. In this section a piston and cylinder arrangement is taken and the work done at the moving system boundaries during a quasi-static process is considered. A technically important phenomenon in many engineering processes is one in which work is done *by* the system by the expansion of gases in a piston and cylinder arrangement and work is done *on* the system by compression process. The piston moves along the cylinder and exerts a progressively varying pressure on the fluid and for a small displacement dx of the piston, the process may be considered quasi-static (Fig. 1.23).

Total force on the piston is $F = p \cdot A$, where p is the pressure of fluid and A is the area of the piston. Therefore,

$$\text{Work} \quad \delta W = p \cdot A \cdot dx \quad \dots(i)$$

$$\text{Therefore,} \quad \delta W = p \cdot dV \text{ where } V \text{ is the volume.} \quad \dots(ii)$$

The work done at the moving system boundaries can be calculated by integrating equation (ii). But the integration can be done only if the relationship between p and V is known during the process. This relationship may be known in the form of a graph as shown in Fig. 1.23 or by an equation. Consider the graph in Fig. 1.23 which shows quasi-static compression process. Thus each point on the process 1-2 represents a definite state and these states will correspond to the actual states of the system only if there is infinitesimal departure from the thermodynamic equilibrium. Thus

$$\text{Work done on the system} \quad {}_1W_2 = \int_1^2 \delta W = \int_{V_1}^{V_2} p dV \quad \dots(1.16)$$

And it is clear that the work done during this quasi-static process is the area under curve shown hatched in Fig. 1.23.

If the process was carried out from V_2 to V_1 , *i.e.* from 2 to 1, it would be an expansion process and the work will be done *by* the system in that case. Work done by the system is considered positive and work done *on* the system is considered negative. It is very important to note that the system considered is a closed non-flow one. Thus $\int p dV$ is the work-done during a quasi-static process for only a non-flow system. For unit mass it can be written as $\int p dv$, where v is the specific volume.

1.29. pdV Work, a Path Function [Fig. 1.24 (a), (b), (c) and (d)]

Since many problems in thermodynamics involve the use of the equation

$$\delta W = p \cdot dV$$

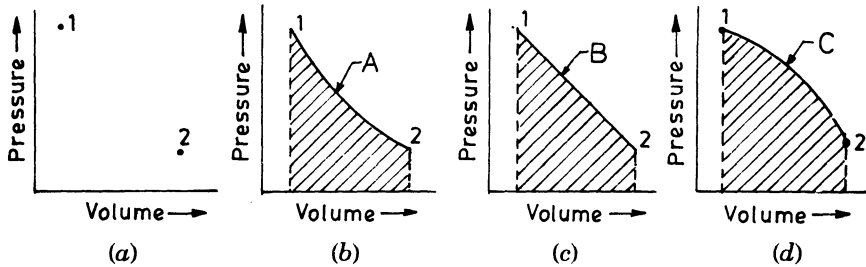


Fig. 1.24

it is convenient to represent equation $\delta W = p \cdot dV$ graphically. Let us consider state 1 and state 2 as shown on the p - V plane in Fig. 1.24 (a). It is possible to go from state 1 to state 2 by many different quasi-static paths as shown in Fig. 1.24 (b),

(c) and (d). And since, ${}_1W_2 = \int_1^2 p dV = \text{area under the curve}$,

it is clear that amount of work done in each case is dependent on the path followed and not merely the end state. Thus work is called a path function, or in mathematical parlance, δW is an inexact differential. In general quasi-static process may be represented by an equation of the form $pV^n = \text{constant}$, where the index n may have a value from $-\infty$ to ∞ . Such a process, is often called polytropic process. Fig. 1.25 shows a few typical cases of the polytropic process. For $n = 0$, we have $pV^0 = p = \text{constant}$ (Isobaric process) for $n = 1$, we have $pV = \text{constant}$ (Isothermal process) and for $n = \infty$, we have $pV^\infty = c$ or $p^{1/\alpha} V = c_1$, or $p^\circ V = V = \text{constant}$ (Isometric process). In evaluating the integral of equation 1.16, we may note that we are concerned in finding area under the curve of Fig. 1.24. Thus we must identify the following two classes of problems.

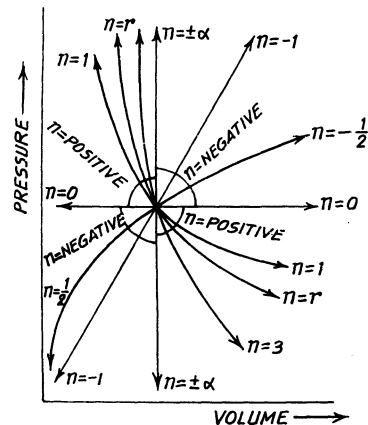


Fig. 1.25

(i) The relationship between p and V is given in terms of experimental data or graphical form as for example, a trace on oscilloscope or on any indicator. The integral of equation 1.16 by graphical or numerical integration.

(ii) The relationship between p and V is such that analytical relationship can be fitted and we may integrate directly as for example $pV^n = c$.

1.30. Limitations of the Use $\int p dV$

For $\int p dV$ to represent work, following conditions must be satisfied :

- (i) The system is closed one and the process takes places in non-flow system.
- (ii) The process is Quasi-Static. Such a process is often referred to as reversible process since a quasi-static expansion process according to a particular p - v relationship between two states can be reversed as quasi-static compression process according to the same p - v relationship between the same state.
- (iii) The boundary of the system should move in order that work may be transferred.

All the conditions must be simultaneously satisfied to allow the use of equation $\int \delta w = \int p dV$, for unit mass.

Problem 1.27. A non flow quasi-static (reversible) process occurs for which $p = (-3V + 16)$ bar, where V is volume in m^3 . What is the work done when V changes from 2 to 6 m^3 ?

Solution.

$${}_1W_2 = \int_{V_1=2}^{V_2=6} p dV$$

$$p = (-3V + 16) \text{ bar} = (-3V + 16) \times 10^5 \text{ N/m}^2$$

$${}_1W_2 = \int_{V_1=2}^{V_2=6} (-3V + 16) \times 10^5 dV = \left[\frac{3V^2}{2} + 16V \right]_2^6 \times 10^5$$

$$= \left[-\frac{3}{2}(6^2 - 2^2) + 16(6 - 2) \right] \times 10^5 = \left[-\frac{3}{2} \times 32 + 64 \right] \times 10^5$$

$$= [-48 + 64] \times 10^5 = 1600000 \text{ Nm or J.}$$

Problem 1.28. A non-flow reversible (quasi-static) process can be written down by an equation

$$p = (V^2 + 8/V) \text{ bar.}$$

Determine the work done if volume changes from 1 to 3 m^3 .

Solution.

$$p = (V^2 + 8/V) 10^5 \text{ N/m}^2$$

Therefore

$${}_1W_2 = \int_{V_1=1}^{V_2=3} p dV = 10^5 \int_{V_1=1}^{V_2=3} (V^2 + 8/V) dV = 10^5 \left[\frac{V^3}{3} + 8 \log_e V \right]_{V_1=1}^{V_2=3}$$

$$= 10^5 \left[\frac{3^3 - 1^3}{3} + 8 \log_e \frac{3}{1} \right] = 10^5 (8.666 + 8.7889)$$

$$= 17.455 \times 10^5 \text{ Nm or Joules.}$$

Problem 1.29. Work done by substance in a reversible non-flow manner is in accordance with $V = \left(\frac{15}{p}\right) m^3$, where p is in bar. Evaluate the work done on or by the system as pressure increases from 10 to 100 bar. Indicate whether it is a Compression process or Expansion process.

Solution. We have, $V = \frac{15}{p} m^3$

i.e. $V_1 = \frac{150}{10} = 15 m^3$ and $V_2 = \frac{150}{100} = 1.5 m^3$

Also $p = \frac{150}{V} \text{ bar} = \frac{150}{V} \times 10^5 \text{ N/m}^2$

Thus

$${}_1W_2 = \int_{V_1=15}^{V_2=1.5} \frac{150}{V} \times 10^5 dV = 150 \times 10^5 \left[\log_e V \right]_{15}^{1.5}$$

$$= 150 \times 10^5 \log_e 0.1 = 150 \times 10^5 \times (-2.3026) = -3.454 \times 10^7 \text{ N-m.}$$

This work is done on the system. It is compression work.

Problem 1.30. Evaluate the non-flow work of a gas undergoing a reversible process in terms of p_1, V_1 and p_2, V_2 according to the following relationships:

- (i) $p = C$, i.e. Isobaric (ii) $V = C$, i.e. Isometric (iii) $pV = C$, i.e. Isothermal
 (iv) $PV^n = C$, i.e. polytropic (v) $PV^{1.4} = C$ (vi) $pV^3 = C$

Determine, the quantities of work if initial pressure and volume are 5 bar and 5 m^3 and final volume is 25 m^3 .

Solution. (i) $p = C$ or $p_2 = p_1 = C$

Therefore, ${}_1W_2 = \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} C dV = C \left[V \right]_{V_1}^{V_2} = C(V_2 - V_1)$... (i)

Thus putting the values, ${}_1W_2 = 5 \times 10^5 (25 - 5) = 100 \times 10^5 \text{ Nm or J}$.

(ii) $V = C$ or $V_1 = V_2 = C$. Therefore, ${}_1W_2 = \int_{V_1}^{V_2 - V_1} p dV = 0$.

Since the volume is same, there is no movement in system boundary at the point of application of force and thus no work is done on or by the system.

(iii) $pV = C$ or $p_1V_1 = p_2V_2 = C$ or $5 \times 10^5 \times 5\text{m}^3 = 25 \times 10^5 \text{ Nm} = C$.

Therefore, ${}_1W_2 = \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} \frac{C}{V} dV = p_1V_1 \int_{V_1}^{V_2} \frac{dV}{V} = p_1V_1 \left[\log_e V \right]_{V_1}^{V_2} = p_1V_1 \log_e \frac{V_2}{V_1}$... (ii)

Putting the values ${}_1W_2 = 25 \times 10^5 \log_e \frac{25}{5} = 25 \times 10^5 \times 1.6094 = 42.2 \times 10^5 \text{ Nm or J}$

(iv) $pV^n = C$ or $p_1V_1^n = p_2V_2^n = C$.

Therefore, ${}_1W_2 = \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} \frac{C}{V^n} dV = \int_{V_1}^{V_2} CV^{-n} dV = C \left[\frac{V^{-n+1}}{-n+1} \right]_{V_1}^{V_2}$
 $= C \left[\frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1} \right] = \frac{1}{-n+1} [p_2 V_2^n \cdot V_2^{-n+1} - p_1 V_1^n \cdot V_1^{-n+1}]$
 $= \frac{p_2 V_2 - p_1 V_1}{-n+1} = \frac{p_1 V_1 - p_2 V_2}{n-1}$... (iii)

(v) $pV^{1.4} = C$, here $n = 1.4$

Therefore, ${}_1W_2 = \frac{p_1 V_1 - p_2 V_2}{1.4 - 1}$... (iv)

And since $p_1 V_1^{1.4} = p_2 V_2^{1.4} = C$, we have

$$p_2 = p_1 \left(\frac{V_1}{V_2} \right)^{1.4} = 5 \times 10^5 \times \left(\frac{5}{25} \right)^{1.4} = 5 \times 10^5 \times 0.1051 = 0.5255 \times 10^5 \text{ N/m}^2$$

Therefore, ${}_1W_2 = \frac{5 \times 10^5 \times 5 - 0.5255 \times 10^5 \times 25}{1.4 - 1} = 10^5 \left[\frac{11.8625}{0.4} \right]$
 $= 29.656 \times 10^5 \text{ Nm or J}$

(vi) $pV^3 = C$, here $n = 3$

Therefore, ${}_1W_2 = \frac{p_1 V_1 - p_2 V_2}{3 - 1}$... (v)

And since $p_1 V_1^3 = p_2 V_2^3 = C$, we have

$$p_2 = p_1 \left(\frac{V_1}{V_2} \right)^3 = 5 \times 10^5 \times \left(\frac{5}{25} \right)^3 = 5 \times 10^5 \times 0.008 \text{ N/m}^2 = 0.04 \times 10^5 \text{ N/m}^2$$

Therefore ${}_1W_2 = \frac{5 \times 10^5 \times 5 - 0.04 \times 10^5 \times 25}{3 - 1} = \frac{24 \times 10^5}{2} = 12 \times 10^5 \text{ Nm or J}$

$$(vii) \quad p = \left(\frac{200}{V^2} + 2 \right), \text{ i.e. } p_1 = \left(\frac{200}{V_1^2} + 2 \right) \text{ bar} = \left(\frac{200}{V_1^2} + 2 \right) \times 10^5 \text{ N/m}^2$$

and
$$p_2 = \left(\frac{200}{V_2^2} + 2 \right) \text{ bar} = \left(\frac{200}{V_2^2} + 2 \right) \times 10^5 \text{ N/m}^2$$

$$\begin{aligned} \text{Therefore, } {}_1W_2 &= \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} \left(\frac{200}{V^2} + 2 \right) \times 10^5 dV = \left[\frac{-200}{V} + 2V \right]_{V_1}^{V_2} \times 10^5 \\ &= 10^5 \left\{ -\left[\frac{200}{V_2} - \frac{200}{V_1} \right] + 2(V_2 - V_1) \right\} = 10^5 \left\{ -\left[\frac{200}{25} - \frac{200}{5} \right] + 2(25 - 5) \right\} \\ &= 10^5(32 + 40) = 72 \times 10^5 \text{ Nm or J.} \end{aligned}$$

1.31. Further Examples of Work

(a) **Paddle Wheel Work.** Refer Fig. 1.26.

As the weight is lowered, the paddle wheel turns. The work is transferred across the system boundary in the fluid system. The volume of the system remains unaltered.

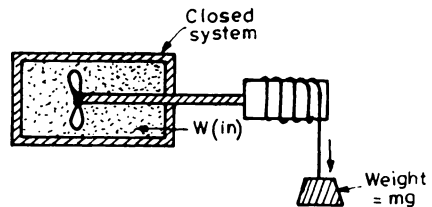


Fig. 1.26

$$\int p dV = 0$$

If the mass of the weight is m , and the it gets lowered by a height dz and T is the torque transmitted by the shaft in rotating through angle $d\theta$, we have

$$dW = mg dz = Td\theta$$

$$\text{Work transfer } W = \int_1^2 mg dz = \int_1^2 Td\theta.$$

(b) **Extension of Solid Bar.** Refer Fig. 1.27. The solid bar is fixed at $x = 0$, and a force equal to F is applied at the other end to have the bar under tension. Thus $F = fA$ where f = normal stress acting at the end of the bar and A is the cross-sectional area of the bar. The work done as the end of the bar elongates a distance dx given by $\delta W = -fA dx$. The negative sign shows that work is done on the bar when dx is positive.

The work done for change of length from x_1 to x_2 is given by,
$$W = - \int_{x_1}^{x_2} fA dx$$

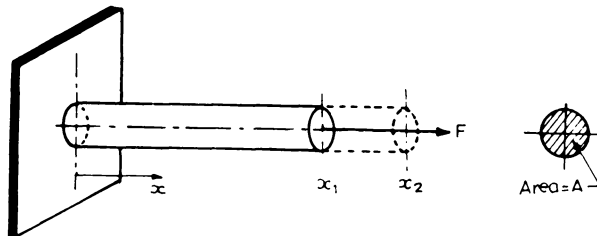


Fig. 1.27

This equation is counter part of equation already derived for expansion and compression.

Problem 1.31. A solid cylindrical bar (Refer Fig. 1.27) of diameter 5 mm is gradually and very slowly stretched from an initial length of 10 cm to a final length of 10.1 cm. The normal stress acting at the end of the bar follows the law $f = C(x - x_1)/x_1$ where x is the position of the end of the bar, x_1 is the initial length and C is the Youngs modulus. For $C = 2 \times 10^7 \text{ KN/m}^2$. Calculate the work done on the bar is N-m.

Solution. Work done is given by

$$\begin{aligned}
 W &= - \int_{x_1}^{x_2} fA \, dx = - \frac{CA}{x_1} \int_{x_1}^{x_2} (x - x_1) \, dx = - \frac{CA}{x_1} \left[\int_{x_1}^{x_2} x \, dx - x_1 \int_{x_1}^{x_2} dx \right] \\
 &= - \frac{CA}{x_1} \left[\frac{(x_2^2 - x_1^2)}{2} - x_1(x_2 - x_1) \right] = - \frac{C \pi d^2}{4x_1} \left[(x_2^2 - x_1^2) - x_1(x_2 - x_1) \right] \\
 &= - \frac{2 \times 10^7 \times 1000 \frac{\text{N}}{\text{m}^2} \times \pi (5 \times 10^{-3} \text{ m})^2}{4(0.1 \text{ m})} \cdot \left[\left(\frac{0.101^2 - 0.1^2}{2} \right) \text{m}^2 - (0.1)(0.101 - 0.1) \text{ m} \right] \\
 &= - 1.96 \text{ N-m [The -ve sign shows that work is transferred to the bar.]}
 \end{aligned}$$

(c) **Stretching of a Liquid Film.** Refer Fig. 1.28.

It shows a liquid film suspended on a wire frame. The thin liquid layer inside is suspended by two surfaces of the film by Surface Tension due to microscopic forces between molecules near the liquid-air interface. The force per unit length across such a line is surface tension. Thus force is given by, $F = 2l\tau$; where the factor 2 indicates two film surfaces act at the wire.

And work for displacement dx is given by, $\delta W = - 2l\tau \, dx$

Negative sign shows that work is done on the system when dx is positive.

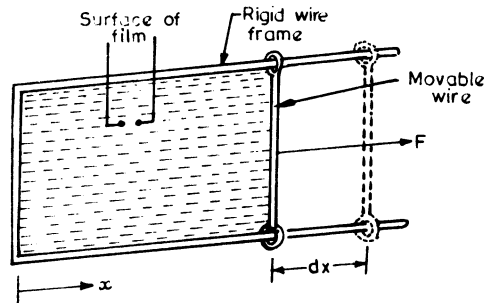


Fig. 1.28

For displacement dx , the change in total area of the surface in contact with wire is $dA = 2l \, dx$

Thus
$$\delta W = - \tau dA$$

or
$$W = - \int_{A_1}^{A_2} \tau dA \text{ where } \tau \text{ is the surface tension (N/m).}$$

Problem 1.32. A soap film is suspended on a $5 \text{ cm} \times 5 \text{ cm}$ wire frame as shown in Fig. 1.29. The movable wire is displaced by 1 cm by an applied force while the surface tension of the soap film remains constant at $25 \times 10^{-5} \text{ N/cm}$. Estimate the work done in stretching the film in N-m.

Solution. Refer Fig. 1.29. The film is a closed system with moving boundary only work done, and surface tension is constant acting on both sides of the film i.e. 2 sides,

$$\begin{aligned}
 W &= - \int_{A_1}^{A_2} \tau dA = - \int_{x_1}^{x_2} \tau 2l \, dx \\
 &= - \tau 2l (x_2 - x_1) \text{ for constant surface tension} \\
 &= - (25 \times 10^{-5} \text{ N}) \times 2 \times 5 \text{ cm} \times 1 \text{ cm}
 \end{aligned}$$

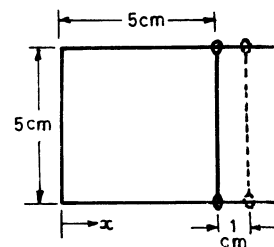


Fig. 1.29

$$= -250 \times 10^{-5} \text{ N-cm} = -2.5 \times 10^{-5} \text{ N-m or J.}$$

The negative sign shows work done on the film.

Problem 1.33. (Fig. 1.30) A gas in the cylinder and piston arrangement comprises the system. It expands from 1.5 m^3 to 2 m^3 while receiving 200 kJ of work from a paddle wheel. The pressure on the gas remains constant at 600 kPa . Determine the net work done by the system.

Solution. The work done by the piston is given by $\int p dV$ and this is the work done by the system.

$$\begin{aligned} W_{gas} &= \int_{1.5}^2 p dV = p \int_{1.5}^2 dV \\ & \quad (\because \text{pressure is constant}) \\ &= 6 \times 10^5 (2 - 1.5) = 3 \times 10^5 \text{ Nm or J} \end{aligned}$$

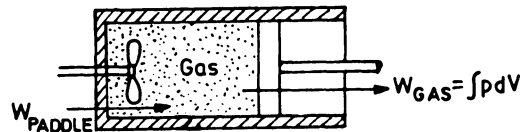


Fig. 1.30

The gas also receive 200 kJ from the paddle wheel and this is work done on the system. Thus, the net work by the system is,

$$W_{net} = 3 \times 10^5 - 2 \times 10^5 = 1 \times 10^5 \text{ Nm or 100 kJ.}$$

Problem 1.34. Show that for a Vander Waals gas whose equation of state is described by the equation, $(p + a/v^2)(v - b) = RT$ the isothermal work per unit mass is given by

$$RT \log_e \frac{v_2 - b}{v_1 - b} - a \left[\frac{1}{v_1} - \frac{1}{v_2} \right]$$

where v_1 and v_2 are the initial and final specific volumes.

Solution. We have $\left(p + \frac{a}{v^2} \right) (v - b) = RT$ or $p = \frac{RT}{v - b} - \frac{a}{v^2}$

Therefore,

$$\begin{aligned} {}_1w_2 &= \int_1^2 p dv = \int_1^2 \left(\frac{RT}{v - b} - \frac{a}{v^2} \right) dv = \int_1^2 \frac{RT}{v - b} dv - \int_1^2 \frac{a}{v^2} dv \\ &= RT \log_e \frac{v_2 - b}{v_1 - b} - a \left[\frac{1}{v_1} - \frac{1}{v_2} \right]. \end{aligned}$$

Problem 1.35. A gas is at a pressure of 3 bar in a cylinder with frictionless movable piston shown in Fig. 1.31. The spring force exerted through the piston is proportional to the volume of gas. Also an additional atmospheric pressure of 1 bar acts on the spring side of piston. Determine the work done by gas in expansion from 0.1 m^3 to 0.5 m^3 .

Solution. (Fig. 1.31). The pressure exerted by the spring.

$$\begin{aligned} p_S &= \text{Gas pressure} - \text{Atmospheric pressure} \\ &= 3 - 1 = 2 \text{ bar} \end{aligned}$$

Thus spring force is given by, $F_S = p_S \times 10^5 \times A$ where, $A = \text{Piston area in m}^2$.

This force gives spring displacement (compression) of

$$x_1 = \frac{0.1}{A} = \frac{\text{Volume}}{\text{Area}}$$

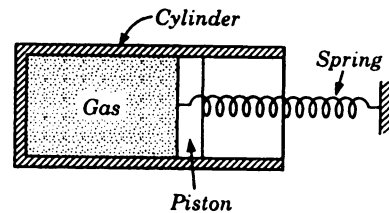


Fig. 1.31

And thus spring constant is given by, $K_S \text{ N/m} = \frac{2 \times 10^5 \times A}{0.1/A} = \frac{2 \times 10^5 \times A^2}{0.1}$

The gas, when it expands has to do work against the atmosphere and also against the spring. The work done against atmospheric pressure

$$W_{\text{against atmosphere}} = 1 \times 10^5 \times (0.5 - 0.1) = 0.4 \times 10^5 \text{ Nm}$$

The work done against the spring

$$W_{(\text{against spring})} = \int_{x_1}^{x_2} \frac{2 \times 10^5 \times A^2}{0.1} x dx = \frac{2 \times 10^5 \times A^2}{0.1} \left[\frac{x^2}{2} \right]_{x_1}^{x_2}$$

where $x_1 = \frac{0.1}{A}$ and $x_2 = \frac{0.5}{A}$

$$\text{Thus } W_{(\text{against spring})} = \frac{2 \times 10^5 \times A^2}{0.1} \times \frac{1}{2} \left[\frac{(0.5)^2}{A^2} - \frac{(0.1)^2}{A^2} \right] = \frac{2 \times 10^5 \times 0.24}{0.1 \times 2} = 2.4 \times 10^5 \text{ Nm.}$$

Thus total work done by gas = $2.4 \times 10^5 + 0.4 \times 10^5 = 2.8 \times 10^5 \text{ Nm or J.}$

Problem 1.36. Ammonia gas is compressed in a frictionless cylinder and piston arrangement. The initial pressure is 500 kPa and the final pressure is 1400 kPa. The pressure-volume data are the following for the process :

Pressure, kPa	Volume m^3
500	0.00125
650	0.00096
800	0.00078
950	0.00068
1100	0.00057
1250	0.00050
1400	0.00045

Determine the work done on the system during the process, considering the ammonia gas as the system.

Solution. The process is considered reversible and the work is done on the system.

$${}_1W_2 = \int_1^2 \delta W = \int_{V_1}^{V_2} p dV$$

and since the pressure-volume coordinates for this process can be plotted, the work done during this quasi-static process is the area under the curve. Refer Fig. 1.32. The area under the curve is 256 units (squares).

$$\begin{aligned} \text{Each unit} &= 0.00005 \text{ m}^3 \times 50 \times 10^3 \text{ N/m}^2 \\ &= 2.5 \text{ N-m.} \end{aligned}$$

$$\begin{aligned} \text{Thus work done on the system} &= 256 \times 2.5 \\ &= 640 \text{ N-m or Joules.} \end{aligned}$$

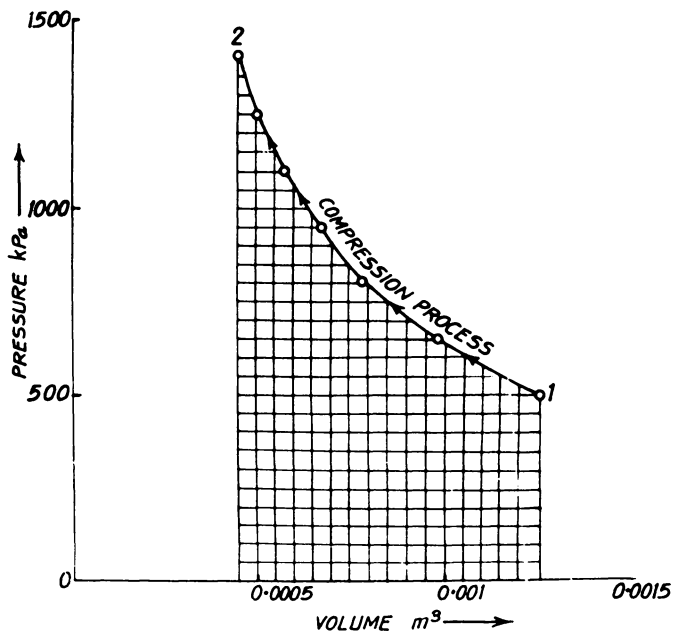


Fig. 1.32

1.31. Flow Work

(Fig. 1.33. A further form of energy transition as a work is invariably associated with the mere fact of the maintenance of the flow of fluid along a channel, or its introduction into any region against the pressure then existing in the fluid or region. The concept might be

developed by reference to Fig. 1.33. Here the boundaries for the system, i.e. the stream (of transverse A) are represented by the solid horizontal lines and section line XX' ; the other boundary of the steam to the left is not shown. A particular mass dm of the fluid is represented as about to pass along the channel pass the boundary XX' and into the region to the left of it. The mass dm has volume dV . Therefore, the specific volume of the fluid v

$$v = \frac{dV}{dm}$$

and absolute pressure p exists at section XX' . The moving piston which may be regarded as real or in fact as formed by the fluid to the right of the mass dm , is the agency by which flow is maintained.

For affecting the entry of the fluid of mass dm and volume dV into the region, a force pA must have acted through a distance dx , where

$$dx = \frac{dV}{A} \quad \dots(1.17)$$

Defining as flow work, mechanical work so required for affecting entry of the mass into the region, the amount of such work per unit mass of fluid entering becomes

$$w_{flow} \text{ (per unit mass)} = \frac{pAdx}{dm} \quad \dots(1.18)$$

$$= \frac{(pA)dV/A}{dm} = p \frac{dV}{dm} = pv \quad \dots(1.19)$$

Comments on Flow Work

- (i) The magnitude of the flow work per unit mass of fluid is uniquely expressible by the simple product of the two properties of the fluid, pressure and specific volume.
- (ii) This product may, however, be assigned energy significance only in association of the flow phenomenon and the flow energy significance that has been indicated.
- (iii) The item is not a measure of the energy that has been required for prior compression of the fluid to existing pressure, and is independent of any energy that may have been required to give the fluid a velocity. Instead it relates solely to the work that in some manner must be furnished for affecting the advance of the fluid against the restraining pressure during that advance.

1.32. Heat

When two systems original at different temperature are brought into contact such that the mass flow across the boundary does not take place, but energy transfer can take place, it is shown by common experience that these two systems come to the equilibrium and reach a temperature common to both, and that temperature is in between the two original temperatures of the systems. Thus the systems have influenced each other. The influence has passed from system at higher temperature to system at lower temperature, or we say that heat has transferred from system at higher temperature to system at lower temperature by virtue of temperature difference. In thermodynamics heat is defined as energy in transition flowing by virtue of temperature difference between two systems or between a system and the surrounding. Important characteristic of heat is that it is manifest only at the system boundaries. It is not contained in the system. It is a transient form of energy like work. In fact work and heat are interchangeable. If in a closed system with insulated boundaries, paddle work is introduced without changing the system boundaries, it is experimentally proved that the temperature of the system increases. Same change of temperature could be affected if heat instead of work is transferred to the system. Thus effect of heat on the system could be same as effect of work.

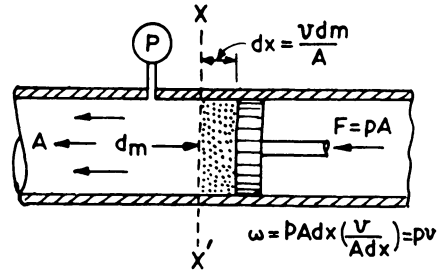


Fig. 1.33

Thus heat energy just as work is and has the units of work. Therefore there is energy transfer by heat by virtue of temperature difference, and also energy transfer by work which influences temperature of the system.

Heat transfer **into** the system is taken to be positive and heat transfer **from** the system as **negative**. The sign convention for heat transfer is just the **reverse** of that for work transfer. The value of a heat transfer depends on the details of a process and not just the end states. Thus like work, heat is not a property and its differential is written as δQ . Heat will be denoted by Q and its units in metric system are kcal. In SI units it is expressed in Joules abbreviated as J.

$$\text{Also} \quad 1 \text{ kcal} = 4186.8 \text{ J} = 4.1868 \text{ kJ} \\ 1 \text{ kJ} = 0.23885 \text{ kcal}$$

Heat flow rate is expressed as kcal/hr in MKS units and Watts or J/sec in SI units.

$$\text{Also} \quad 1 \text{ kcal/hr} = 1.163 \text{ J/sec} = 1.163 \text{ watt. (watt-sec)} \\ 1 \text{ watt} = 0.86 \text{ kcal/hr.}$$

1.33. Specific Heat

The heat required by a unit mass of a substance to raise temperature by one degree is called the specific heat of the substance. It is called the heat capacity of the substance.

If C_n = specific heat of substance, q = heat added to unit mass, dT = rise in temperature

$$\text{We have } C_n = \lim_{\Delta T \rightarrow 0} \frac{\Delta Q}{m \Delta T} = \lim_{\Delta T \rightarrow 0} \frac{\Delta q}{\Delta T} = \frac{\delta q}{dT} \text{ or } \delta q = C_n dT \text{ or } \delta Q = m C_n dT \dots(1.20)$$

$$\text{Therefore } {}_1Q_2 = \int_1^2 m C_n dT \dots(1.21)$$

Thus C_n is the specific heat when heat is transferred to the system undergoing a general process or polytropic process. Two special cases of significance are when heat is transferred during constant volume process and during constant pressure process.

$$\text{Specific heat } C_p = \left(\frac{\delta q}{\delta T} \right)_p \text{ at constant pressure} \dots(1.22)$$

$$\text{and specific heat } C_v = \left(\frac{\delta q}{\delta T} \right)_v \text{ at constant volume} \dots(1.23)$$

More elegant description of C_n , C_p and C_v will be given in the next chapter.

Unit for specific heat follow from the definition, *i.e.* kcal/kg °K in MKS units and kJ/kg °K in SI units. Also kJ/kg °K = 0.23885 kcal/kg °K

$$\text{and } \frac{\text{kcal}}{\text{kg } ^\circ\text{K}} = 4186.8 \frac{\text{J}}{\text{kg } ^\circ\text{K}} = 4.1868 \frac{\text{kJ}}{\text{kg } ^\circ\text{K}}$$

It may be interesting to note that numerical value of the specific heat in all units except SI units is the same.

$$\frac{BTU}{lb ^\circ F} = \frac{CHU}{lb ^\circ C} = \frac{\text{kcal}}{\text{kg } ^\circ C} = 4.1868 \frac{\text{kJ}}{\text{kg } ^\circ K}$$

Problem 1.37. 4 kg of substance receive 480 kJ of heat and undergo a temperature change from 100°C to 250°C. Determine the average specific heat of substance during the process.

$$\text{Solution. } {}_1Q_2 = m \int_{373}^{523} C_n dT = 4 C_n (523 - 373)$$

$$\text{i.e. } 480 = 3 \times C_n \times 150 \text{ or } C_n = \frac{480}{600} = \frac{0.8 \text{ kJ}}{\text{kg}} \text{ } ^\circ\text{K.}$$

Problem 1.38. For a system whose mass is 4.5 kg undergoes a process and the temperature changes from 50°C to 100°C. Assume that the specific heat of the system is a function of temperature only. Calculate the heat transfer during the process for the following relationship : $C_n = 0.3 + 20/(T + 150)$ kJ/kg °K.

Solution. ${}_1Q_2 = \int_{323}^{373} mC_n dT = 4.5 \int_{323}^{373} \left(0.3 + \frac{20}{T + 150} \right) dT$

$$= 4.5 \left[\int_{323}^{373} 0.3 dT + \int_{323}^{373} \frac{dT}{\frac{T}{20} + \frac{150}{20}} \right] = 4.5 \left\{ \int_{323}^{373} 0.3 dT + \int_{323}^{373} \frac{dT}{0.05T + 7.5} \right\}$$

$$= 4.5 \left\{ \left[0.3T \right]_{323}^{373} + \left[\frac{1}{0.05} \log_e (0.05T + 7.5) \right]_{323}^{373} \right\}$$

$$= 4.5 \left\{ [0.3 \times 50] + \left[\frac{1}{0.05} \log_e (18.65 + 7.5) - \log_e (16.15 + 7.5) \right] \right\}$$

$$= 4.5 \left\{ 15 + \frac{1}{0.05} \log_e \frac{26.15}{23.65} \right\} = 4.5 [15 + 20 \times 0.100486]$$

$$= 4.5 \times 17.0097 = 76.54 \text{ kJ.}$$

1.34. (a) Heat—a path function

The equation for heat transfer is given by ${}_1Q_2 = \int_{T_1}^{T_2} mC_n dT$... (1.24)

Integration of equation (1.24) will depend upon how temperature T_1 changes to T_2 . Fig. 1.34 shows two constant temperature lines on the p - V plane. It is clear that there are many ways for the system to change its temperature from T_1 to T_2 . A little thought will reveal that the specific heat C_n may thus have any value, depending upon the process undergone by the system. $C_n = C_p$ for constant pressure process and $C_n = C_v$ for constant volume process.

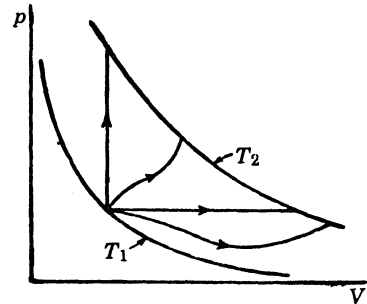


Fig. 1.34

Thus heat transferred from state point 1 to state point 2 can be written as ${}_1Q_2$ or Q_{1-2} but not as $(Q_2 - Q_1)$.

1.34. (b) Comparison of Heat and Work

There are many similarities between Heat and Work.

- (i) Heat and Work are both transient phenomenon. Systems never possess heat or work, but either or both cross system boundaries, when a system undergoes a change of state.
- (ii) Both heat and work are boundary phenomenon. Both are observed only at the boundaries of the system and both represent energy crossing the boundaries of the system.
- (iii) Both heat and work are path function and in-exact differentials.

1.35. Perfect Gas

A perfect gas strictly obeys all the gas laws under all condition of pressure and temperature. In this sense no gas which exists in nature is perfect. But this definition applies fairly well to some gases within the range of pressure and temperature met with in applied thermodynamics. Examples are, air and nitrogen. Thus these gases may be considered as perfect. These are also called ideal gases.

1.36. Law I-Boyle' Law

The volume of a given mass of perfect gas varies inversely as the absolute pressure when temperature is constant.

Let p be the absolute pressure of the gas

v be the specific volume of the gas at pressure p

T be the absolute temperature of the gas

$$\therefore v \propto \frac{1}{p} \text{ then } T \text{ is constant}$$

$$pv = \text{constant} = C \quad \dots(1.25)$$

where C is the constant of proportionality.

This law can be verified by means of the simple apparatus shown in Fig. 1.35. A glass tube C is sealed at one end and placed vertically as shown. A reservoir R of mercury is connected to the lower end of C by a flexible rubber tube. R can be raised or lowered to any position and the pressure of the air inside C is due to the column of mercury of height h together with that of the atmosphere at that time; this latter, b , can be obtained from the barometer. The volume of the air is represented by V the length in C occupied by the air. By raising and lowering R , a series of values of V and h can be obtained and when $(h + b)$ and V are plotted, we obtain a straight line passing through the origin. In performing this experiment considerable time must elapse between each reading because by compressing the air in C , we cause the temperature to rise, and we remember that Boyle's Law is true only if the temperature for the air remains constant.

Problem 1.39. Air in vessel having inside volume 10 cube metres at 3 bar is compressed to 3 cube metre without any change in temperature. Calculate the pressure of air after compression.

Solution. Let p_1 be the initial pressure ; p_2 be the final pressure

V_1 be the initial volume ; V_2 be the final volume

$$\therefore p_1 V_1 = C = p_2 V_2 \text{ or } 3 \times 10 = p_2 \times 3 \text{ or } p_2 = 10 \text{ bar.}$$

1.37. Law II-Charles' Law

The volume of a given mass of a gas varies directly as its absolute temperature, if the pressure remains unchanged.

Let p be the absolute pressure, v be the specific volume of gas,

T be the absolute temperature.

$$\text{Therefore } v \propto T \text{ when } p \text{ is constant or } \frac{v}{T} = \text{constant} \quad \dots(1.26)$$

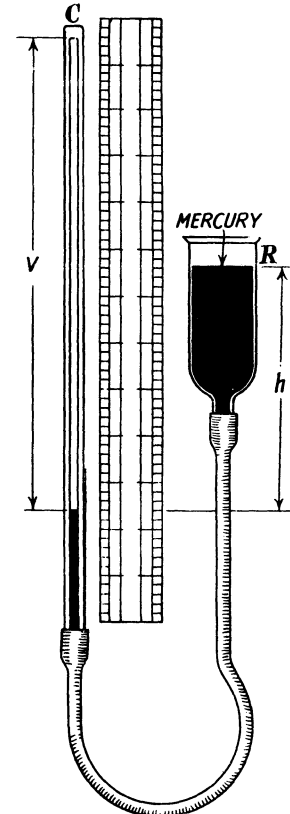


Fig. 1.35

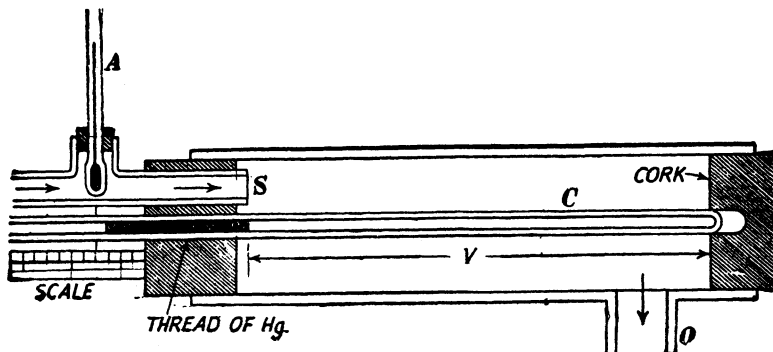


Fig. 1.36

A piece of apparatus that can be used to verify the Charles' Law is shown in Fig. 1.36. A strong capillary tube C , closed at one end, contains air which is sealed by a thread of mercury (Hg). Supports at each end hold C central in a large glass tube. A supply of water whose temperature can be controlled enters at S , the temperature being shown by thermometer at A . An outlet for the water is provided at O . The position of the tube C adjusted so that the thread of the mercury is just showing through the cork as in Fig. 1.36. When cold water at temperature T is flowing through S , for this position of the volume of air represented by V is known. The temperature of water entering at S is raised a few degrees and after the flow of water has been maintained for sometime, the temperature as shown by A and the increased volume as shown by the thread of mercury in C , are recorded. A series of similar readings of increasing temperature and volume are taken. And these when plotted will be found to give a straight line as shown in Fig. 1.37. This line when produced backwards will cut the temperature axis at $-273.16^\circ\text{C} \approx 273^\circ\text{C}$. This temperature is the absolute zero of temperature and since the graph of temperature and volume of air in C is straight line passing through this point, we can say that, at constant pressure the volume is directly proportional to the absolute temperature.

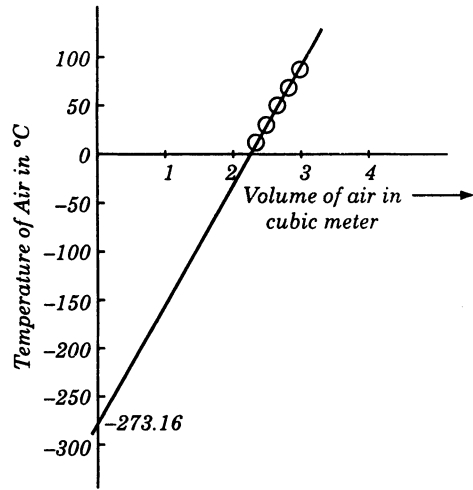


Fig. 1.37

Problem 1.40. Determine the volume occupied by a given mass of air at temperature of 400°C , the same mass of air occupies 2 cubic metre at 15°C . The pressure of the air remains unchanged.

Solution. Since the pressure is constant, Charles' Law applies.

$$\text{We have } \frac{V_1}{T_1} = \text{constant} = \frac{V_2}{T_2}$$

when pressure is constant.

$$\therefore \frac{2}{273 + 15} = \frac{V_2}{400 + 273} \text{ or } V_2 = \frac{2 \times 673}{288} = 4.67 \text{ m}^3.$$

1.37. Characteristic Equation of Gas

In engineering practice, volume, pressure and temperature, all vary simultaneously and, therefore, Boyle's Law or only Charles' Law is not applicable. But from both these laws a general equation for mass of gas undergoing changes in temperature, pressure and volume can be obtained.

$$v \propto \frac{1}{p} \text{ when } T \text{ is constant} \quad (\text{Boyle's Law})$$

$$v \propto T \text{ when } p \text{ is constant} \quad (\text{Charles' Law})$$

$$v \propto \frac{T}{p} \text{ when } T \text{ and } p \text{ both vary.}$$

$$\text{or } v = k \frac{T}{p}$$

where k is constant and depends upon the scale of temperature used and the properties of gas

$$pv = RT \text{ for unit mass}$$

$$\text{and } V = mR \frac{T}{p},$$

where m is the mass of gas and R is the constant depending upon the temperature scale and properties of gas

$$i.e. \quad pV = mRT \quad \dots(1.27)$$

where R is called the characteristic gas constant and equation (1.27) is called characteristic equation of gas.

Equation $pV = RT$ can be used to establish some important relations between thermodynamic parameters of state. Differentiating the equation $pV = RT$, we obtain

$$p dv + v dp = R dt$$

$$at \quad p = \text{constant}; \quad \left(\frac{\partial v}{\partial T} \right)_p = \frac{R}{p}$$

$$at \quad v = \text{constant}; \quad \left(\frac{\partial p}{\partial T} \right)_v = \frac{R}{v}; \text{ or } \left(\frac{\partial T}{\partial p} \right)_v = \frac{v}{R}$$

$$at \quad T = \text{constant}; \quad \left(\frac{\partial v}{\partial p} \right)_T = -\frac{RT}{p^2} \text{ and } \left(\frac{\partial p}{\partial v} \right)_T = -\frac{p}{v} = \frac{RT}{v^2}$$

Product of partial derivatives of an ideal gas turns into -1

$$i.e. \quad \left(\frac{\partial v}{\partial T} \right)_p \left(\frac{\partial p}{\partial v} \right)_T \left(\frac{\partial T}{\partial p} \right)_v = -\frac{R}{p} \cdot \frac{v}{R} \cdot \frac{p}{v} = -1$$

In order to find out the units of the gas constant R .

Let p = Absolute pressure of gas in N/m^2

V = Volume of gas in cubic metric *i.e.* m^3

m = Mass of gas in kg .

T = Absolute temperature of gas in $^{\circ}K$.

$$\text{Therefore,} \quad R = \frac{pV}{mT} = \frac{N \times m^3}{m^2 \times kg \times ^{\circ}K} = Nm/kg \text{ } ^{\circ}K$$

If pressure is expressed in kgf/m^2 , the units of R would be $kgf \cdot m/kg \text{ } ^{\circ}K$

$$R = 29.27 \text{ kgf} \cdot \text{m per kg per } ^{\circ}K \text{ or } 287 \text{ J/kg } ^{\circ}K \text{ for air}$$

$$= 420.6 \text{ kgf} \cdot \text{m per kg per } ^{\circ}K \text{ or } 4124.6 \text{ J/kg } ^{\circ}K \text{ for } H_2$$

$$= 30.26 \text{ kgf} \cdot \text{m per kg per } ^{\circ}K \text{ or } 296.8 \text{ J/kg } ^{\circ}K \text{ for } N_2$$

$$= 26.5 \text{ kgf} \cdot \text{m per kg per } ^{\circ}K \text{ or } 259.9 \text{ J/kg } ^{\circ}K \text{ for } O_2$$

1.38. Universal Gas Constant

It is interesting to note that the product of the characteristic gas constant and the molecular weight of an ideal gas is a constant number. It is equal to 848 in MKS units and 8314 in SI units. Thus $848 \text{ kgf} \cdot \text{m per kg mol per } ^{\circ}K$ or $8314 \text{ J/kg mol } ^{\circ}K$ is called the universal gas constant. It is verified as under :

(M)	Molecular weight M	$\times R$	$= MR$	Standard Gas density kg/m^3
For air	29.00	29.27	848.8	1.295
for H_2	2.016	$\times 420.6$	848	0.090
for N_2	28.016	$\times 30.26$	847.7	1.250
for O_2	32	$\times 26.5$	848	1.429
for NH_3	17.031	$\times 49.78$	847.8	0.771
for CO	28	$\times 30.28$	847.8	1.25
for CO_2	44	$\times 19.36$	851.8	1.977
for H_2O	18.016	$\times 47.07$	848	0.804

Similarly is SI units

(M)	Molecular weight M	$\times R$	= MR Average = 8314	Standard Gas density kg / m ³
For air	29	287	8324	1.295
for H ₂	2.016	4124.4	8315	0.090
for N ₂	28.016	296.8	8315	1.250
for O ₂	32	259.8	8316	1.429
for He	4.003	2077.2	8315	0.178
for CO	28	296.8	8310	1.25
for CO ₂	44	189.9	8334	1.977
for H ₂ O	18.016	461.6	8316	0.804
for NH ₃	17.031	488.2	8315	0.771

1.39. Law III-Avogadro's Law or Avogadro's Hypothesis

Avogadro's Law states that the molecular weights of all the perfect gases occupy the same volume under the same conditions of pressure and temperature

Experiment shows that average volume for one mol of the perfect gases is 22.4136 m³ at standard atmospheric pressure (1 atm = 1.0332 kgf/cm² = 1.01325 bar) and 0°C.

$$\text{Thus } MR = \frac{1.0332 \times 10^4 \times 22.4136}{273.16} = 847.77 \approx 848 \text{ kgf-m/kg mol}^\circ\text{K}$$

$$\text{and in SI units, } MR = \frac{1.01325 \times 10^5 \times 22.4136}{273.16} = 8314.02 \approx 8314 \text{ J/kg mole }^\circ\text{K}.$$

Since MR for all idea gases is constant and is equal to 848 kgf-m per kg. mol per °K, or 8314/kg mol °K it follows that the kg mol volume for all gases at N.T.P. is same. Same will hold good for S.T.P. or any specified conditions of pressure and temperature. MR is also denoted as \bar{R} .

Problem 1.41. Determine the molecular volume of any perfect gas at 600 kPa and 30°C ($kN = \text{kilo Newton}$).

Solution. We have $pV = MRT$

Taking M as the molecular weight, we have

$$MR = 8314 \text{ J/kg mol}^\circ\text{K}; \quad p = 600 \times 1000 \text{ N/m}^2 = 6 \times 10^5 \text{ N/m}^2$$

$$T = (30 + 273) = 303^\circ\text{K}$$

$$\text{Therefore } V = \frac{8314 \times 303}{6 \times 10^5} = 4.198 \text{ m}^3/\text{kg mol}.$$

Problem 1.42. A pressure bottle stores 5 cubic metres of inert gas N₂ at a pressure of 100 bar and temperature of 30°C. Calculate (a) the mass of gas, (b) the mol volume, and (c) the density of the gas.

Molecular weight of N₂ = 28.016.

Solution. For N₂ the characteristic gas content is given by

$$R = \frac{8314}{28.016} = 296.8 \text{ J/kg mol}^\circ\text{K}$$

And we have $pV = mRT$

$$100 \times 10^5 \times 5 = m \times 296.8 \times (273 + 30)$$

$$m = \frac{100 \times 10^5 \times 5}{296.8 \times 303} = 556 \text{ kg}$$

$$\text{Molecular volume} = \frac{8314 \times T}{p} = \frac{8314 \times 303}{100 \times 10^5} = 0.252 \text{ m}^3$$

$$\text{Density of the gas} = \frac{556}{5} = 111.2 \text{ kg/m}^3.$$

Problem 1.43. Pressure vessels of 10 cubic metre are used for storing oxygen, nitrogen and carbon dioxide at a pressure of 15 bar and 40°C. Determine the mass of each gas that can be stored in the tank.

Solution. Assuming that these gases are perfect

$$R \text{ for O}_2 = \frac{8314}{32} = 259.8 \text{ J/kg}^\circ\text{K}; \quad R \text{ for N}_2 = \frac{8314}{28.016} = 296.8 \text{ J/kg}^\circ\text{K}$$

$$R \text{ for CO}_2 = \frac{8314}{44} = 188.95 \text{ J/kg}^\circ\text{K}$$

Therefore, mass of respective gas occupying 10 cubic metre at 15 bar and 40°C, i.e. (40 + 273)°K is given by $pV = mRT$.

$$\text{mass of O}_2 = \frac{15 \times 10^5 \times 10}{259.8 \times 313} = 184.46 \text{ kg}; \quad \text{mass of N}_2 = \frac{15 \times 10^5 \times 10}{296.8 \times 313} = 161.47 \text{ kg}$$

$$\text{mass of CO}_2 = \frac{15 \times 10^5 \times 10}{188.95 \times 313} = 253.6 \text{ kg.}$$

Problem 1.44. An aerostat balloon is filled with hydrogen. It has volume of 1000 m³ at a temperature of 300°K and pressure of 100 kPa. Determine the pay load that can be lifted with the air of the aerostat.

Solution. The mass of air that is displaced by the balloon is

$$m_a = \frac{pV}{RT} = \frac{100 \times 1000 \text{ N/m}^2 \times 1000 \text{ m}^3}{287 \text{ N}\cdot\text{m/kg}^\circ\text{K} \times 300^\circ\text{K}} = 1161.44 \text{ kg}$$

The mass of hydrogen filled in the balloon is

$$m_h = \frac{100 \times 1000 \times 1000}{4124.4 \times 300} = 80.82 \text{ kg}$$

Thus the buoyancy (lift force) of the balloon is

$$\begin{aligned} \text{Pay load} &= (1161.44 - 80.82) \times g = (1161.44 - 80.82) \times 9.81 \text{ N} \\ &= 1080.62 \times 9.81 = 10600.88 \text{ Newtons} \end{aligned}$$

or pay load of 1080.62 kg mass be lifted.

Problem 1.45. In an orbiting space station the station is spinned to induce an artificial gravitational acceleration of 2 m/sec². What would be the weight of 75 kg man inside.

Solution. Force is given by $F = mg = 75 \text{ kg} \times 2 \text{ m/sec}^2 = 150 \text{ N}$

Therefore man weighs 150 N.

On the surface of earth, the man weighs $W = mg = 75 \text{ kg} \times 9.80665 \text{ m/sec}^2 = 735.5 \text{ N}$.

SUPPLEMENTARY READING

1. Joachim E. Lay, Thermodynamics, Charles E. Merrill Books, Inc. Columbus,
2. Callen, H.B. Thermodynamics, John Wiley & Sons, Inc.
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4. Hall, N.A. Ibele, Engineering Thermodynamics, Prentice Hall Inc.
5. Keenan, J.H. Thermodynamics, John Wiley & Sons Inc.
6. Rogers, G.F., Mayhew, Y.R., Engineering Thermodynamics, McGraw- Hill Book Company Inc.

TEST YOUR COMPREHENSION

(Tick Mark Correct Statement)

1. Thermodynamics system may be defined as quantity of matter upon which attention is focussed for study if
 - (a) it is only bounded by real surfaces
 - (b) it is not bounded by imaginary surface
 - (c) the boundary surface are constant in shape and volume.
 - (d) it is bounded by either real surfaces or imaginary surfaces irrespective of shape or volume.

2. Which statement is correct ?
 (a) Isolated system un-influenced by surroundings is called Universe.
 (b) System and surrounding combined constitute Universe whether these interact with each other or not.
 (c) System which only interacts with surrounding is part of the universe.
 (d) System and surrounding put together form universe only if there is interaction between them.
3. A centrifugal fan forms
 (a) closed system (b) open system (c) isolated system (d) none of the above.
4. Zeroth Law of Thermodynamics defines
 (a) internal energy (b) enthalpy (c) temperature (d) pressure.
5. 1 mm H₂O is equal to
 (a) 100×10^{-6} bar (b) 0.001 kgf/cm² (c) 9.80665 N/m² (d) 0.077 mm Hg.
6. -40°C is equal to
 (a) -40°F (b) 230°K (c) 400°R (d) -72°F.
7. For constant volume process on a gas the value of index n for the path followed on $p-v$ co-ordination is equal to
 (a) 1 (b) zero (c) ∞ (d) 1.4.
8. A gas expands following the law $pV^n = C$ from initial pressure p_1 and volume V_1 to final volume. $5V_1$. The work obtained is maximum when n is
 (a) 1.4 (b) 1 (c) zero (d) ∞ .
9. Air expands from initial pressure p_1 and volume V_1 to final volume $5V_1$ following the law $pV^n = C$
 (a) greater the value of n , greater the work obtained
 (b) smaller the value of n , smaller the work obtained
 (c) for $n = 0$ the work obtained is the greatest (d) for $n = 1.4$ the work obtained is the greatest.
10. Universal Gas Constant of a perfect gas
 (a) increases with temperature (b) decreases with temperature
 (c) increases with increase in molecular weight (d) is always constant.
11. Which of the following can be used as properties of a system
 (a) $\int \left(\frac{dT}{T} - \frac{vdp}{T} \right)$ (b) $\int \left(\frac{dT}{T} - \frac{pdv}{v} \right)$ (c) $\int pdv$ (d) $\int vdp$.
12. Characteristic gas constant of any perfect gas
 (a) increases with increase in temperature (b) increases with increase in pressure
 (c) is a function of pressure and temperature (d) is constant.
13. The ratio of specific heats of gas at constant pressure and at constant volume is
 (a) always constant (b) always varies with pressure
 (c) always varies with temperature (d) none of the above.
14. Characteristic equation of gas is given by ($v =$ specific volume, $m =$ mass of gas)
 (a) $pv = RT$ (b) $pv = mRT$ (c) $pv^n = C$ (d) $pv^\gamma = C$.

ANSWERS

- | | | | | |
|---------|---------|---------|----------|---------|
| 1. (d) | 2. (b) | 3. (b) | 4. (c) | 5. (c) |
| 6. (a) | 7. (c) | 8. (c) | 9. (c) | 10. (d) |
| 11. (a) | 12. (d) | 13. (c) | 14. (a). | |

TEST QUESTIONS

1. Classify each of the following systems whether it is an open or closed system :
 (a) Air Compressor (b) Carburetor (c) Pressure Cooker
 (d) Radiator of an automobile (e) Tea kettle.
2. What do you understand by Intensive and Extensive properties ? Classify the following properties with their units in metric system :
 (a) Specific weight (b) Volume (c) Specific Volume
 (d) Density (e) Molecular weight (f) Pressure
 (g) Temperature (h) Velocity.

3. State the Zeroth Law of Thermodynamics. How is mercury in thermometer able to find the temperature of a body using the Zeroth Law of Thermodynamics.
4. Distinguish between absolute pressure and gauge pressure. How is one related to the other in case of vacuum.
5. Explain Boyle's Law and Charles law as applied to perfect gas. Which of these laws can be used to evaluate Absolute Zero temperature and how ?

PROBLEMS FOR PRACTICE (NUMERICALS)

1. A piston has area of 5 cm^2 . What mass must the piston have if it exerts a pressure of 50 kPa above atmospheric pressure on the gas enclosed in the cylinder.
2. A pressure gauge reads 2.4 bar and the barometer reads 75 cm of Hg. Calculate the absolute pressure in bar and in the standard atmospheres.
3. A manometer has a liquid of density 800 kg/m^3 , the difference in level of the two legs is 300 mm . Determine the pressure difference read by it in kgf/m^2 ; bar; kPa.
4. A vertical composite liquid column with its upper end exposed to atmosphere of 45 cm of Hg (specific gravity of Hg is 13.5951), 65 cm of water (specific gravity of water is 1) and 80 cm of oil (specific gravity of oil is 0.8). Determine the pressure

- (i) at the bottom of the column (ii) at the inter-surface of oil and water
(iii) at the inter-surface of water and mercury.

5. A manometer containing water and mercury connects two pressure region X and Y as shown in Fig. P-1.1. If $p_y = 400 \text{ Pa}$ gauge, find p_x .
6. The air supply to an internal combustion engine is metered by observing the pressure drop across an orifice in the air line to the engine; the pressure drop is measured by means of a manometer containing paraffin having a specific gravity of 0.81 . Express a difference of level of 25 cm in the manometer in
 - (a) kgf/cm^2
 - (b) metres of air
 - (c) kPa
 - (d) mm H_2O
 - (e) mm Hg
 - (f) bar.

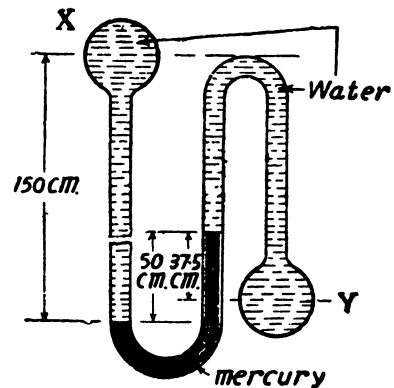


Fig. P-1.1

7. Define a new temperature scale, say $^{\circ}\text{B}$ in which the boiling and the freezing points of water are 500°B and 100°B respectively.
 - (a) Co-relate this scale with centigrade scale.
 - (b) $^{\circ}\text{B}$ reading on the scale is some number on corresponding absolute temperature scale. What is this absolute temperature on $^{\circ}\text{B}$.
8. The length of the mercury column in a mercury in glass thermometer is 5 cm where the thermometer is at the ice point and 25 cm when the thermometer is at the steam point. Consider this length as the property x and a scale t' to be defined by the equation $t' = ax + b$ where $t' = 0^{\circ}$ at the ice point and $t' = 100^{\circ}$ at the steam point. Compare the t' scale with the centigrade scale, where t linearly defined in terms of x (with the same values as t' and ice and steam points). Make the comparison by plotting t' versus t .
9. A certain thermometer, using pressure as the thermometric property gives values to p of 1.86 and 6.81 at the ice point and the steam point respectively. The temperatures of the ice point and the steam point are assigned the number 32 and 212 respectively. Determine the temperature corresponding to $p = 2.5$, if the temperature t is defined in terms of p by the relation

$$t = a \log_e p + b, \text{ where } a \text{ and } b \text{ are constants.}$$
10. The e.m.f. E of a thermocouple having one junction kept at the ice point p and the other junction at a temperature t measured by a centigrade mercury in glass thermometer is

$$E = 0.20 - 5 \times 10^{-12} t^2, \text{ where } E \text{ is in millivolts.}$$
 - (a) Sketch the graph of E versus t and find e.m.f. at $t = 100^{\circ}\text{C}$, 0°C etc.
 - (b) Let the e.m.f. be taken as thermometric property and temperature scale t' be defined by the relation $t' = aE + b$ such that $t' = 0^{\circ}$ at the ice point and $t' = 100^{\circ}$ at the steam point. Sketch the graph of E versus t' by finding the numerical values of a and b .

- (c) Sketch the graph of t' versus t by finding the values of t' at $t = 100^\circ\text{C}$, 0°C etc. what kind of scale is t' ?
11. What is meant by fixed points of a thermodynamic scale ? How is a thermometric scale defined in terms of fixed points and physical property of some substance ?
 The volume of a liquid at different temperatures is given by $V = V_0(1 + \alpha t + \beta t^2)$
 where $\alpha = 0.0001$ and $\beta = 0.00003$ and t is the temperature in degrees centigrade measured on the scale of a constant volume air thermometer. A thermometer is constructed with this liquid, the fixed point determined in the same manner, and the thermometer uniformly graduated as centigrade instrument. What temperature this thermometer will read when $t = 50^\circ\text{C}$.
 12. A constant volume gas thermometer containing Helium gives readings of gas pressure p of 100 and 1368 mm of Hg at ice point and steam point respectively.
 - (a) Express the gas thermometer, centigrade temperature t_c in terms of gas pressure p .
 - (b) The thermometer, when left standing in atmospheric, registers 1073 mm. Determine the atmospheric temperature.
 13. Define quasi-static process and derive an expression for the work done. Explain its limitations by giving examples.
 14. 0.084 m^3 of a gas is expanded in a cylinder which causes the pressure to fall from 7 bar to 2.45 bar. If the expansion follows the law $pV^{1.1} = C$, find final volume of the gas.
 15. A non-flow reversible process occurs for which $p = 3V^2 + 1/V$ where p is in bar and V in m^3 . What is the work done when V changes from 0.5 m^3 to 0.1 m^3 ?
 16. Determine the work done by 1 kg fluid system as it expands slowly within the frictionless piston cylinder arrangement from an initial pressure and volume of 5 bar and 0.2 m^3 to a final volume of 0.8 m^3 in accordance with the following defining arrangement :

(a) $p = C$	(b) $pV = C$	(c) $pV^{1.4} = C$	(d) $pV^2 = C$.
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 17. Write short notes on :

(a) System	(b) Properties
(c) Path function and point function	(d) Zeroth law of thermodynamics
(e) Work and heat.	
 18. During a particular process, the specific heat capacity of the system is given by $C = 0.3 + 0.002 T$. Find the heat transferred and mean specific heat of the gas of 0.2 kg from 27°C to 127°C .
 19. A certain system whose mass is 5 kg undergoes a process wherein its temperature changes from 35°C to 75°C . Assume the specific heat of the system to be a function of temperature only, determine the heat absorbed during this process for the following relationship $C = 0.1 e^{0.017 T} \text{ kJ/kg}^\circ\text{C}$.
 20. Air at pressure 1 bar gauge expands to three times its volume, the temperature being constant. Determine the absolute pressure at the end of expansion if the barometer reads 550 mm Hg.
 21. The gaseous products of combustion in the fire box of a steam boiler have a temperature of 1200°C at the beginning in the flue and 25°C at the end. Find the percentage reduction in volume during cooling if the pressure is assumed constant throughout.
 22. Find the volume occupied by 1.5 kg of hydrogen ($R = 4124.4$) at the pressure of 2 bar and temperature of 25°C .
 23. Find the specific weight of nitrogen at 15°C and pressure of 1 bar and also the value of its gas constant R . Molecular weight of nitrogen is 28.02.
 24. Calculate the mass of oxygen ($R = 259.8 \text{ J/kg }^\circ\text{K}$) in a cylinder with a capacity of 100 litres at a temperature of 20°C . The pressure in the cylinder is maintained at 80 bar.
 25. 5 cubic metres of air at 0°C and a pressure 3 bar is heated to 80°C . Determine (i) change of internal energy, (ii) heat supplied, (iii) mechanical work done.
 26. 0.56 cubic metres of gas of a pressure of 2.95 bar and temperature of 30°C is compressed to 20.6 bar. The rise in temperature is 370°C . Determine its final volume. Calculate also the mass of the gas if it is nitrogen with $R = 296.87 \text{ J/kg}^\circ\text{K}$.
 27. A cylinder contains 3.14 kg of oxygen at 4.90 bar and temperature of 27°C . Determine the internal volume of the cylinder and its capacity to contain this mass at one physical atmosphere and 20°C .
 28. Determine the volume of 6 kg air at a pressure of 3.925 bar and temperature of 50°C if C_v for air is 0.71 kJ/kg K and C_p is $1.0 \text{ kJ/kg }^\circ\text{K}$.