A Course in

DC MACHINES AND TRANSFORMERS

Performance, Modelling and Design

K.N. Srinivas

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A Course in

DC Machines and Transformers

Performance, Modelling and Design

(For the diploma and undergraduate engineering students of B.E., B.Tech. and A.M.I.E., of I.E. (I) and Competitive Examinations)

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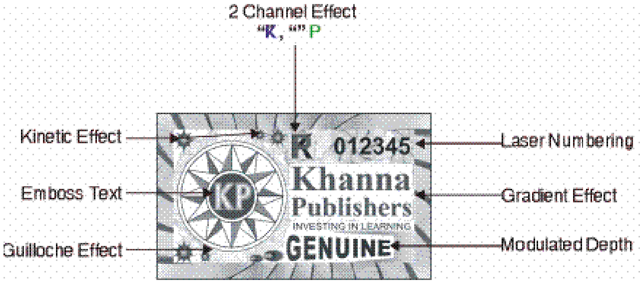

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Preface

I have always wanted to write a comprehensive textbook on electric machines which must be capable of serving diploma students as well as engineering under graduate students and the aspiring engineers preparing for competitive exams. This book is a culmination of the same thought. 'A Course in DC Machines and Transformers: Performance, Modelling and Design' covers the major three aspects of any DC Machines *viz.*, performance, modelling and design.

Chapter 1 deals with the fundamentals of energy transfer that happens in DC Machines. The analytical treatment of the energy equations is readily made comprehensible with the help of numerical examples. MATLAB coding for the examples will enhance the chapter understanding and coding skills.

Chapter 2 is dedicated to discuss the performance of DC Generators. Construction, operating principle, types and the respective voltage and current equations, characteristics of various generators, phenomena called armature reaction and commutation and losses and efficiency are presented to the reader in a lucid approach. Numerical examples are solved and given as exercise.

In the similar way, chapter 3 is allotted to discuss the performance of DC Motors. Application of each DC motor is practically explained. Speed control techniques for DC Motors and electronics control of the same are also available in this chapter.

Testing of DC Machines plays a major role in the operation of them. Chapter 4 provides a comprehensive treatment on different types testing of DC Machines with numerical examples.

Chapter 5 exhaustively, in a step-by-step approach, provides a platform for the beginner to understand the importance of modelling of electric machines. Many mathematical equations involved in the modelling of DC machines have been systematically explained in class-lecture-like fashion, and Simulink based simulations are also provided for an improved perception. Modelling of all types DC Machines is provided. In the under graduate machines course, almost all universities allot one complete unit for modelling of DC Machines, and hence this chapter has been particularly framed.

Chapter 6 is apportioned for DC Machines design. The geometrical design of various parts of a DC machine including its diameter, length, coil turns, electromagnet's dimensions, commutator and brushes design and performance prediction of the machine with the designed parameters (which is a measure to finalize the proper design) are methodically presented.

Chapter 7 deals with a comprehensive treatment on single phase transformers. The principle of operation and performance characteristics are discussed. The tests conducted to pre-determine the efficiency and % voltage regulation of transformers are presented. Numerical examples will make the discussed concepts more clear.

Last chapter 8 has been allotted to present the design of transformers. The design of main dimensions, primary and secondary windings, performance predictions and cooling tubes with classic numeric examples are given.

This is an inclusive book on DC Machines. Instead of moving to three different books to gain knowledge on performance, modelling and design of DC Machines, 'A Course in DC Machines and Transformers: Performance, Modelling and Design' will provide all these three major aspects of a DC Machine in a single go to the reader.

Contents

Principle of Electromechanical Energy Conversion

1 CHAPTER

Course Learning Rationale (CLR):

The purpose of learning this chapter is to :

- Understand the analytics of the electromechanical energy conversion
- Derive the basic equations governing energy conversion in electric machines
- Understand the singly-excited magnetic systems and multi-excited magnetic systems, with numerical examples
- Perform numerical problems highly related with practices

Key Terms

- ☞ Electromagnetic and electromechanical energy conversion
- ☞ Energy balance method
- ☞ Flux-linkages and inductance
- ☞ Energy flow diagram for electrical to mechanical (and vice-versa) energy conversions
- ☞ Singly-excited magnetic systems
- ☞ Doubly-excited magnetic systems
- ☞ Forces and torques in electromagnetic and electromechanical energy conversions

1.1. INTRODUCTION

This chapter is intended to provide an in-depth knowledge about the principle of electromechanical energy conversion and its application to simple devices. The operation of rotating electric machines purely depends upon electromechanical energy conversion and thus a better knowledge on this will enhance the confidence level of understanding electric machines. This chapter initially discusses about principle of induction and principle of interaction that happens in any electromagnetic circuit. The principle of alignment plays a vital role in modelling electric machines, which forms the next discussion. Then the mathematical expressions for the energy stored in magnetic systems and for the forces and torques exhibited in them are treated with numerical examples. Finally, singly excited and multiply excited magnetic systems are discussed with proper derivations and numerical examples.

1.2. PRINCIPLE OF INDUCTION

The basic principles related with all electromagnetic devices/machines are: *induction*, *interaction* and *alignment*. These are discussed below.

Principle of Induction:

Whenever an electric conductor is placed in a magnetic field, and if there exists a relative motion between them, then dynamically induced EMF occurs in the electric conductor, when it moves. On the other hand, if the conductor is stationary and the magnetic field is set into rotation, then a statically induced EMF occurs in the electric conductor.

Change of flux linkage in a coil will occur in one of the following three cases:

(*a*) The flux remaining constant and the coil uniformly moving in it. The excitation to the flux producing magnets will be kept constant in this case.

(*b*) The winding (or coil) does not rotate and it remains stationary with respect to the flux in which it is kept; in this case flux varies in magnitude with time. This magnitude variation of flux with respect time is equivalent to it being in motion.

(*c*) Both the magnetic flux and the coil in it are in motion. Physically the coil will be in motion and the flux will be a time-varying one simulating the movement.

Such an induced EMF is given by the Faraday's law:

$$e = -d\lambda/dt \qquad \ldots(1.1)$$

where λ is the flux linkage. Flux linkage can be defined as, as the name itself defines, the total number of flux lines ϕ (in Webers, *Wb*) that links with the total number of electric conductor, *N*. Therefore, $\lambda = N\phi$, *Wb*-turns.

Eqn (1.1) is re-written as, $e = d(N\phi)/dt$ …(1.2)

ϕ depends upon the magnetization and it increases till its saturation limit when the excitation of the magnetic circuit is kept on increasing. So ϕ is a variable, whereas the total number of electric turns remains the same at N which cannot be changed during operation. So,

$$e = -\{N \cdot d(\phi)/dt\} \qquad \ldots(1.3)$$

In eqn. (1.3) the negative (–) sign is prefixed to account for the Lenz's law. This is a law stating that the direction of an induced current is always such as to oppose the change in the circuit or the magnetic field that produces it. This is applicable to any electric circuit of this nature, and hence to account for the opposition, (–) sign is used.

Principle of Interaction:

Consider the illustrations made in Fig. 1.1.

In Fig. 1.1(*a*), there is an undisturbed uniform magnetic field of flux density of vector B, Wb/m^2. Now, into it, a current carrying conductor is introduced. A cross in the conductor indicates that this conductor is assumed to carry current inwards and perpendicular to the plane of the paper.

Cork-screw rule is usually applied to locate the direction of the flux produced by any current carrying conductor. When the rule is applied it can be seen that the flux or field produced by this current carrying conductor is in clockwise CW direction.

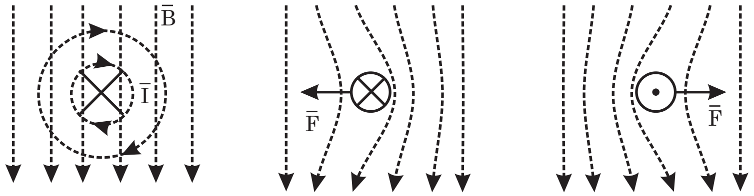
Reflection Section

(*i*) Relate the above definition of Faraday's law with that of its equivalent expression (1.1). What will happen to the induced EMF e, if either of the fields is not moving and why?

(*ii*) Physically verify the model of movement of invisible flux in case (*b*), sec.1.2.

The resultant of these two flux interactions is shown in Fig. 1.1(*b*). As seen by this Fig. 1.1(*b*), the resultant flux density is greater than vector *B* on one side of the current carrying conductor and the same is less than vector B on the other side of the conductor.

Figure. 1.1(*c*) is the other case in which the current direction in the conductor is perpendicular to the plane of the paper but comes out. It is usually denoted by a dot (.). The same explanation on the flux distribution holds good for this case also.



(a) (b) (c)

Fig. 1.1. Illustration for principle of interaction

However, the direction of the mechanical force developed (in both the cases) will be of such a value that attempts to reinstate the magnetic field *B* to its original undistributed and uniform configuration, as evident on Fig.1.1(*b*) and Fig.1.1(*c*).

Torque is fundamentally a product of force and perpendicular distance, in the fields of discussion. Fig. 1.2 indicates a one-turn current carrying coil in a uniform magnetic field. It carries current inwards and outwards as indicated. It experiences mechanical forces as discussed above. Now, the torque exhibited in this one-turn coil by the interaction of coil and magnetic field is shown (Fig. 1.2).

Reflection Section

(*i*) What is reluctance? What is, then, resistance?

(*ii*) Define the terms flux, flux density, magnetic field, self- and mutual-inductances and magnetic vector potential.

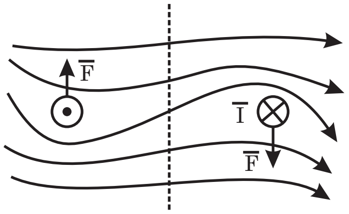


Fig. 1.2. Torque in a 1-turn current carrying coil kept in a uniform magnetic field

The reader shall imagine the case in which the point of discussion is with a multi-turn coil, arranged in the outer slots of a cylindrical structure. All the coil sides carrying current as ‘cross’ will have torque, say as shown downwards, and all the coil sides carrying current as ‘ ’ will have torque, say as shown upwards, thus contributing the production of total torque.

As can be seen, thus as the number of conductors increases in the so called ‘armature’ arrangement, the torque produced also will proportionally increase. This point holds good for the increase in the magnetic field, vector *B*, Wb/m^2.

Principle of Alignment:

There is a magnet, a piece of iron is put in its field. It gets attracted to the magnet instantly. In which path it moves? That is the path of least reluctance. In its path, the movement of iron piece towards the magnet happens through a path in which its movement is least opposed. This is called the principle of alignment.

Let there be highly permeable materials such as iron which can be instantly effected by magnetic field. Let these materials also be surrounded by low permeable medium such as air. Then these magnetic materials experience mechanical forces. In fact, this is the exact magnetic arrangement of electric machines, which will be seen in the forthcoming chapters. These forces exhibit principle of alignment. Such mechanical force make the magnetic material to get aligned with the direction of the magnetic field *B*, such that the path of reluctance is the least.

The principle of alignment is illustrated in Fig. 1.3, presenting the direction of forces. The force is always in such a direction that the net magnetic reluctance is reduced, and, as a consequence, the magnetic flux path is shortened.

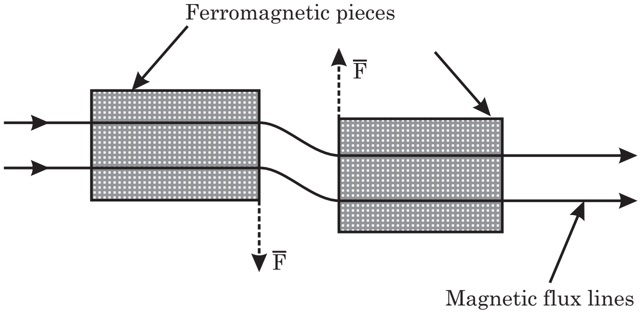


Fig. 1.3. Principle of Alignment

The electromagnetic forces and torques associated with such magnetic field systems can be derived, which forms the next section.

1.3. ENERGY METHOD

Out of various methods available to calculate the net force and torque in electric machines, the 'Energy Method' which is based upon the conservation of energy has become a trustable and hence adopted one. This is a simple and efficient method as discussed below.

This approach has the loss parameters separated from that of energy storage parameters. Thus the electric losses (such as the ohmic losses in the windings) are expressed as a lumped external resistive element. Similarly, the mechanical losses including friction losses and windage losses are included as external elements to the mechanical terminals.

The winding has some resistance associated to it. And, the bearings always have friction associated with them. It is not possible to take away these resistances either from resistors or from bearings. These are represented in the schematic Fig.1.4. Resistance of the winding is shown as 'winding resistance'.

Consider Fig. 1.4. Electric system is the exciting winding set-up. Movement of the plunger causes the mechanical output. This movement, that is the conversion of electrical input to a mechanical output labeled as 'electro-mechanical energy conversion', happens due to the interaction between electric and mechanical terminals, through the medium of the magnetic field energy. Winding resistance will include all losses in this system as I^2R, watts.

So there exist three energy stages viz., input electrical energy, output mechanical energy and the stored magnetic field energy. It is important to know this.

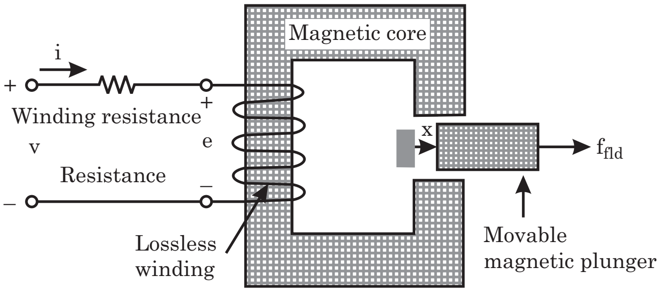


Fig. 1.4. Illustration for Energy balance

If the losses in the whole system are neglected, which is a reasonable approximation to start with, it can be defined that *the time rate of change of the stored magnetic field energy will be the difference between the input and the output*. Thus, referring to the nomenclatures in the Fig. 1.4.

$$\text{Electrical input} = e.i, \text{ watts}$$

Mechanical power output of the energy storage system = mechanical force × mechanical velocity, which is equal to, $f_{fld} \times dx/dt$.

Therefore, the stored magnetic field energy is,

$$dW_{fld}/dt = e.i - (f_{fld} \times dx/dt) \quad \text{...(1.4)}$$

Multiplying by dt,

$$dW_{fld} = e.i\, dt - f_{fld} \times dx$$

By eqn. (1.1), $edt = d\lambda$.

So, $$dW_{fld} = i\, d\lambda - f_{fld} \times dx \quad \text{...(1.5)}$$

Equations (1.4) and (1.5) forms the fundamentals for the energy method. It can be seen from eqn. (1.5) that $W_{fld} = f(\lambda, x)$. This is a potent method to calculate force and torque in electro-mechanical energy conversion systems.

Also, the principle of conservation of energy states that the form of energy is simply changed and it cannot be either created or destroyed.

Applying this to the electro-mechanical energy conversion systems, the system equation formed can be represented as the block diagram in Fig. 1.5.

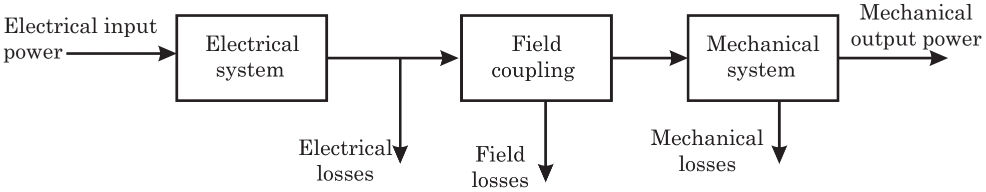


Fig. 1.5. Block diagram for Energy conservation

For a lossless system, $dW_{\text{elec}} = dW_{\text{mech}} + dW_{\text{mag-fld}}$.

Using equations (1.4) and (1.5),

$$i\, d\lambda = (f_{fld} \times dx) + dW_{fld}$$

or $$dW_{\text{elec}} = e\, idt = (dW_{\text{mech}} + dW_{\text{fld}})$$

that is, $$= (f_{fld} \times dx) + dW_{fld} \quad \text{...(1.6)}$$

The use of the energy method expression derived in eqn (1.7) explained for some of the electro-mechanical energy conversion devices in the following sections.

1.4. ENERGY STORED IN A MAGNETIC FIELD

There is a strong relationship between the flux linkage, $\lambda\ (= N\phi)$ and the exciting current of the magnetic flux, i responsible for this, which is shown in Fig. 1.6. This is a non-linear relationship, as the variation of reluctance of the magnetic system is very non-linear.

The x-axis forms the excitation current and y-axis forms the flux linkage λ. For any excitation current i, the energy stored in the field is given by W_f.

Consider this relationship to happen in a simple excitation circuit, with a supply voltage of V_s (volts) supplying a current of i amps through a magnetizing coil of resistance R, (Ohms). (Refer to Fig. 1.7).

$$W_f' = \int_0^l \lambda di$$

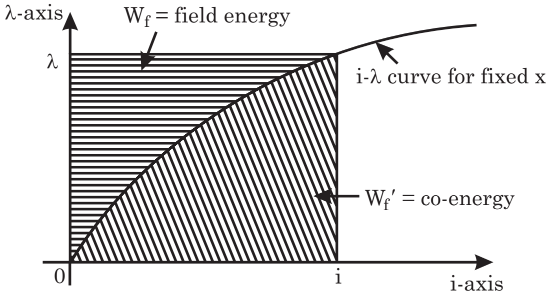


Fig. 1.6. Magnetic field energy and co-energy.

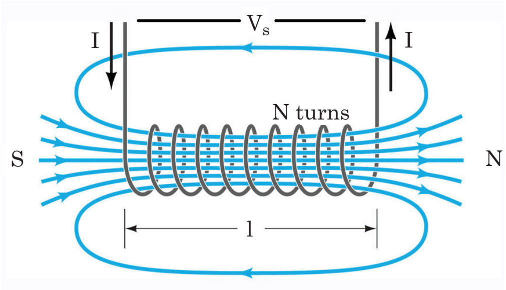


Fig. 1.7. Excitation from V_s to a coil of N turns.

For this simple circuit, *KVL* is $V_s = IR + e$ Multiplying both sides by *i*, it can be seen that,

$$V_s I = I^2R + eI \qquad \text{...(1.7)}$$

This is in Watts units LHS is the total electric input to the circuit. I^2R is the total losses happening in this circuit of resistance *R* Ohms. Consequently *eI* is the power going to the magnetic circuit. Then the magnetic energy or the field energy is given by the time integral of this power.

$$\text{Energy} = \int e.I\, dt \qquad \text{...(1.8)}$$

Reflection Section

(*i*) Why losses occur in electro-mechanical systems?

(*ii*) What are electric losses and how do they dissipate?

(*iii*) What is the physical significance of mechanical losses in a rotating electric machine and how do they dissipate?

Considering the case wherein the excitation current increases from a disconnected value of 0 Amps, to *i*, over a span of time period *t* = 0 to *t*, then the field energy becomes,

$$W_{fld} = \int_0^t ei\, dt = \int_0^t \frac{d\lambda}{dt} idt = \int_0^\lambda id\lambda = \int_0^\lambda id(N\phi)$$

$$= \int_0^\phi N\phi d\phi = \int_0^\phi Fd\phi$$

Using above expression the product of magnetizing force/m(*H*) and length is equal to the magnetic force F, the above expression is rewritten as,

$$= \int_0^\phi Hld(BA) = (Al)\int_0^B HdB = \text{Volume}\int_0^B H\, dB \qquad \text{... (1.9)}$$

Flux density, $B = H(\mu_0\mu_r)$. For air, $\mu_r = 1$. The above equation thus becomes,

$$W_{fld} = \text{Volume}\int_0^B \frac{B}{\mu_0}dB = \frac{1}{2}\frac{B^2}{\mu_0}\times\text{Volume} = \frac{1}{2}\mu_0 H^2\times\text{Volume}$$

$$= \frac{1}{2}BH\times\text{Volume}$$

$$= \frac{1}{2}BH\times Al = \frac{1}{2}F\phi = \frac{1}{2}Ni\phi = \frac{1}{2}\lambda i \qquad \ldots(1.10)$$

Concept of co-energy:

Magnetic field energy, as per the above discussions, is the area between λ-axis and the *i*-λ curve. On the other hand, the area between i-axis and the *i*-λ curve is termed as the **co-energy**.

Coenergy or co-energy – the dual of energy, is a non-physical quantity useful for theoretical analysis of systems storing and transforming energy. Rotating machines fall into this category of transforming energy devices, and thus concept of co-energy is useful for machine's analysis. Co-energy is expressed in the same units as energy and is especially useful for calculation of magnetic forces and torque in electric rotating machines.

Referring Fig. 1.6, total energy in the curve is the whole area of it, that is λi.

Thus, the co-energy is written as, W_{fld}' (or W_f'),

$$W_{fld}' = \lambda i - W_{fld} \qquad \ldots(1.11)$$

Any electric machine has three important parts *viz*., a stationary part (stator), a rotating part (rotor) and an air gap of small length between them.

Stator can be an electromagnetic field system or an electric winding; similarly the **rotor** can also be. A DC machine has its stator made up of electromagnets called a 'field system.' Its rotor has electric windings. In case of AC generator, rotor has even number of electromagnets whereas its stator has electric windings. As per Faraday's law, either of stator or rotor will be set into motion, thus obtaining the energy conversion.

Now, in the air gap between stator and rotor, considerable amount of magnetic field energy is stored. The stator and rotor have less reluctance whereas the air gap has a larger reluctance. Thus major energy storage happens in the air gap and hence property of the magnetic circuit is determined by the length of air gap.

The above statement can be explained below.

Consider a circular magnetic ring in which a flux density of 1 T is maintained. As the reluctance of the circuit madeup of magnetic material is very low, a small exciting current is sufficient to maintain 1 T, thus requiring a lesser energy.

Reflection Section

(*i*) Find out how the inductance has a relation with the designed dimensions of the device.

(*ii*) Relate the physical significance of inductance (L) in this discussion.

Let a small air gap is cut in the cross-section of the magnetic ring. To maintain the same 1 T in the air gap, where the reluctance is larger, more energy is required. Thus major magnetic energy storage happens in the air gap, making it almost an energy reservoir between stator and rotor, or electric and magnetic system of the device.

The non-linearity of the reluctance is neglected for the analysis purpose. Thus, as the reluctance of the device is taken as a linear quantity, flux and MMF are directly proportional in the complete magnetic circuit. This makes the flux linkage, λ (= $N\phi$) and the exciting current i to vary in direct proportions and linearly related; by an inductance which solely depends on the geometry of the device (and hence on the armature position, x).

$$\lambda = L(x)\, i \qquad \ldots(1.12)$$

1.5. ANALYSIS OF A SINGLY-EXCITED MAGNETIC SYSTEM

Singly-excited magnetic system is one which has only one source of excitation. Examples are electromagnetic relays, moving-iron instruments, reluctance motors, etc.

While, the doubly-excited magnetic systems are those which has two independent source of excitations. Examples are synchronous machines, loudspeakers, DC shunt machines, etc.

Consider a singly-excited system as shown in Fig. 1.8, which is same that of Fig. 1.4. Figure 1.8 has a convenient movable portion which shall be considered as an armature of electric machines. This moves linearly through a distance x, metres whereas the armature revolves angularly with ω rad/sec.

As the armature and mechanical losses can be represented as an external mass, they are presently assumed as massless.

The mechanical energy output of the system can be written as,

$$\begin{aligned} dW_{\text{mech}} &= \text{force} \times \text{displacement} \\ &= f_{fld} \times dx \qquad \ldots(1.13) \end{aligned}$$

The value of W_{fld} is uniquely specified by the values of λ and x. Thus, λ and x are referred to as the state variables as their values uniquely determine the state of the system.

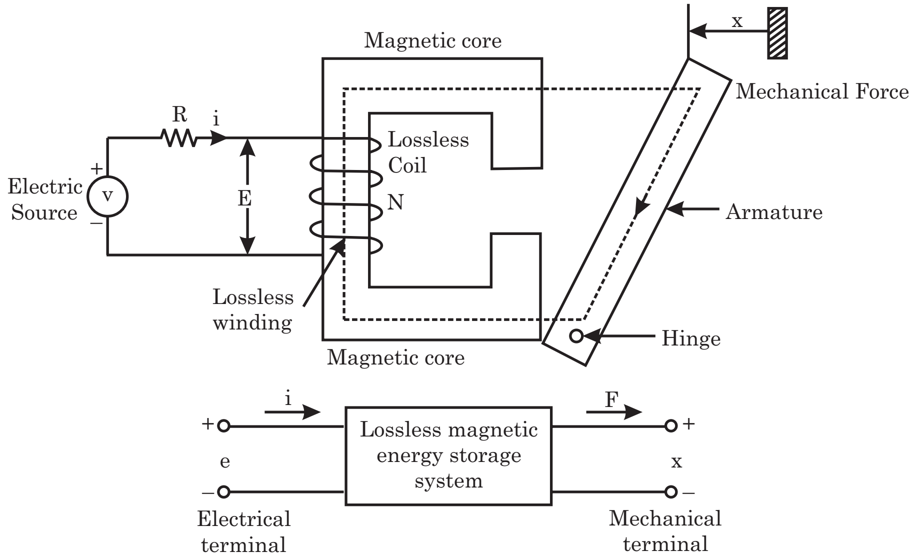


Fig. 1.8. Singly-excited system

R = Resistance of the excitations system.

Mechanical Force = f_{fld} = force produced by the magnetic field x: a displacement.

In Fig. 1.8, for a linear system, in which λ is directly proportional to i, as in eqn. (1.12), eqn. (1.5) gives $W_{fld}(\lambda, x)$ as,

$$\int_0^{\lambda} i(\lambda, x)dx = \int_0^{\lambda} \lambda - L(x)d\lambda$$

$$= 1/2\ [\lambda^2/L(x)] \qquad \text{...(1.14)}$$

The above results were for a linear system. If we had chosen the example of a rotating system, the force would be replace by a torque, and the linear replacement would be replaced by an angular one. The results, otherwise, would be the same.

Determination of magnetic force and torque from energy:

The energy functional (W_{fld}) is determined by the independent state variables λ and x, as W_{fld} depends on these two variables. Recollecting eqn. (1.5), which is $dW_{fld} = i\ d\lambda - f_{fld} \times dx$, the total differential say F with respect to the two state variables λ and x can be written as,

$$dW_{fld}(\lambda, x) = \frac{\partial\, W_{fld}}{\partial \lambda\ (\text{at } x)} d\lambda + \frac{\partial W_{fld}}{\partial x (\text{at} \lambda)} dx \qquad \text{...(1.15)}$$

A and x are two independent variables. So Eqns. (1.5) and (1.15) must be equal for all values of $d\lambda$ and dx. This comparison gives,

$$i = \frac{\partial\, W_{fld}}{\partial \lambda\ (\text{at } x)} d\lambda \qquad \text{...(1.16)}$$

where partial derivative is taken holding x constant,

and

$$f_{fld} = \frac{\partial Wfld}{\partial x (\text{at} \lambda)}\ dx \qquad \text{...(1.17)}$$

where partial derivative is taken holding λ constant.

The force f_{fld} is established explicitly in terms of the electrical state variable λ. The same force can also be expressed in terms of i. This can be done by appropriately expressing λ as a function of i into the expression for f_{fld} using eqn. (1.5).

Recollecting eqn. (1.14), the value of $\left\{-\frac{\partial W_{fld}}{\partial x (\text{at} \lambda)} dx\right\}$ becomes, $\left\{-\frac{\partial}{\partial x}\left(\frac{1}{2}\lambda^2 / L(x)\right)\right\}$.

Thus, f_{fld} is given by,

$$\frac{\lambda^2}{2.L(x)^2} \cdot \frac{dL(x)}{dx} \qquad \text{...(1.18)}$$

The force can also be stated in terms of the current i by substituting eqn. (1.12) into the above equation.

$$f_{fld} = \frac{1}{2} \times i^2 \times dL / dx, \text{ for a linear case.} \qquad \text{...(1.19)}$$

For a device of Fig. 1.8 where the armature provides rotation and undergoes an angular displacement of θ, then the force calculation becomes torque, with the principle of approach remaining the same.

Thus, the modification in the derived equations are:

$$dW_{fld}\ (\lambda, \theta) = i \times \mathrm{d}\ \lambda - T_{fld} \times d\ \theta \qquad \text{...(1.20)}$$

$$T_{fld} = \frac{\partial W_{fld}(\lambda, \theta)}{\partial \theta (at\ \lambda)} \qquad \text{...(1.21)}$$

$$T_{fld} = \frac{1}{2} i^2 \frac{dL(\theta)}{d\theta} \text{, for an angular case} \qquad \text{...(1.22)}$$

Determination of magnetic force and torque from co-energy:

Expression in eqn. (1.5) which states that ($dW_{fld} = i\ d\lambda - f_{fld} \times dx$) can be used to define co-energy. This also can be used to derive force and torque, which is a wide practice amongst engineers.

Reflection Section

(*i*) Prove that induced EMF, $e = Ldi/dt$.

From the graph of Fig. 1.6, co-energy can be defined as W_{fld}' as a function of exciting current i and the linear displacement x as,

$$W_{fld}' = i\,\lambda - W_{fld}(\lambda, x) \qquad \text{...(1.23)}$$

Now, both i and λ are variables, $d(i, \lambda)$ is written as $i\ d\lambda + \lambda\ di$ ...(1.24)

If eqn. (1.23) is differentiated now,

$$dW_{fld}' = d(i\lambda) - dW_{fld}(\lambda, x) \qquad \text{...(1.25)}$$

$$dW_{fld}' = (i\ d\lambda + \lambda\ di) - (i\ d\lambda - f_{fld} \times dx)$$

i.e.,

$$dW_{fld}' = \lambda\ di + f_{fld} \times dx \qquad \text{...(1.26)}$$

From the eqn. (1.26), co-energy $W_{fld}'(i, x)$ is a state function of the independent variables i and x and thus its diffential can be expressed as,

$$dW_{fld}'(i, x) = \frac{\partial W_{fld}'}{\partial i(\text{at constant } x)} di + \frac{\partial W_{fld}'}{\partial x(\text{at constant } i)} dx \qquad \text{...(1.27)}$$

As eqns (1.26) and (1.27) must be equal for all values of di and dx, it is written that,

$$\lambda = \frac{\partial W_{fld}'}{\partial i(\text{at constant } x)} \qquad \text{...(1.28)}$$

and

$$f_{fld} = \frac{\partial W_{fld}'}{\partial x(\text{at constant } i)} \qquad \text{...(1.29)}$$

The same approach is usable for an angular displacement also, in which the linear motion x is replace with that of the angular motion ω.

Worked Examples in Electro-mechanical Magnetic Devices

Problem 1.1. A magnetic circuit consists of a single-coil cylinder and an oval rotor. Because of the oval structure of the rotor, the air gap (between the cylindrical stator and the oval rotor) becomes non-uniform. The coil inductance thus becomes a function of rotor's angular position and is written as $L(\theta) = L_0 + L_2 \cos(2\theta)$, Henry, where the maximum inductance is 10 mH and the minimum inductance is 2.3 mH. For an exciting current of 3 A, find the torque as a function of θ.

Solution. The torque is given by the expression, $T = \frac{1}{2}\ i^2\ dL/d\theta$. So,

$$T = \frac{1}{2} \cdot 3^2 \cdot d/d\theta\ [L_0 + L_2 \cos(2\theta)$$

$$T = \frac{1}{2} \cdot 9 \cdot [2.3\times10^{-3} \times 2 \times \sin(2\theta)]$$

$$= -0.0207 \sin(2\theta) \text{ Nm}$$

Problem 1.2. Consider the plunger magnet of Fig. 1.9. The uniform air gap of length g is 0.02 cm. 1000 turns coil carries a current of 2.5 A. (*a*) If the plunger is allowed to move slowly,

A Course in
DC MACHINES AND TRANSFORMERS
Performance, Modelling and Design

About the Book

This textbook provides comprehensive treatment on two major Electric Machines, *viz.*, the DC Machines and the Transformers. Initial six chapters are allotted for DC Machines. The performance, design and modelling of DC Machines are thoroughly presented in these chapters in a lecture-type approach with many numerical examples to register the topics profoundly. Then the performance and design of transformers are presented in two separate chapters with number of worked examples of various higher order thinking standards. In almost all the technical institutions offering diploma and undergraduate degree programmes in Electrical Engineering, this textbook will appeal greatly to the students, because this forms the core subject. Engineers inquisitively preparing for competitive exams and higher studies entrance exams such as GATE will benefit in their preparations by studying from this textbook. 'Reflection Section' at different conceptual stages, will help the students to recollect the rudiments in full without further ambiguities. The concept check, multiple choice questions and exercise problems, will assist the students to attain the intended chapter outcomes.

About the Author

Dr. K.N. Srinivas has a rich experience of about 33 years, which includes mainly academics alongwith research and industrial experience on Electric Machines. A PhD holder from Anna University, he has three textbooks to his credit titled 'Basic Electrical Engineering'. He has about 47 ScI/WoS research publications, including the publishers IEEE Transactions, Wiley, Springer, Taylor & Francis and Elsevier. He has travelled abroad including USA, Japan, Thailand, Singapore, Malaysia, London and Dubai to present his research findings at various IEEE international forums. He has successfully executed two funded research projects on Magnetic sensors funded by DRDO and DST. Dr. K.N. Srinivas has produced undergraduate students with research training in Electric Machines who are spread across the globe technically serving the subject. He has produced three PhD scholars successfully and held major academic and administrative positions during his expertise spanning over three decades. He is a recipient of IEEE best research paper award and presently teaching with SRM IST, Ramapuram Campus, Chennai, India.

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